IMPROVING THREE-DIMENSIONAL ELECTRICAL CAPACITANCE TOMOGRAPHY IMAGING USING APPROXIMATION ERROR MODEL THEORY

R. Banasiak\textsuperscript{1}, Z. Ye\textsuperscript{2}, and M. Soleimani\textsuperscript{2, *}

\textsuperscript{1}Department of Computer Engineering, Technical University of Lodz, Lodz, Poland
\textsuperscript{2}Department of Electronic & Electrical Engineering, University of Bath, Bath, UK

Abstract—Electrical capacitance tomography (ECT) is a non-invasive technique that aims to reconstruct images of internal permittivity values of a volume of interest, based on measurements taken on the external boundary. Since most reconstruction algorithms rely on model-based approximations, it is important to ensure numerical accuracy for the model being used. Finite element methods (FEM) are the most commonly used modelling technique for the ECT forward problem. Due to the characteristic of the ECT technique, the size of the electrodes needs to be built as big as possible so as to cover the entire image area. To accurately solve the forward model a large number of elements are required in and around electrodes. Notably complete sensor model (CSM) will provide the best solution of the forward model. It is also known that denser mesh provides more accurate solution; however, it also requires longer computation time. This paper demonstrates the advantages of the incorporating discretisation and modelling error in 3D ECT problem. The approximation error model (AEM) has been used to account for discretisation error, and this allows using a coarse mesh for the forward model and achieving the reconstruction accuracy close to that of denser mesh.

1. INTRODUCTION

Electrical Capacitance Tomography (ECT) is a technique which uses external capacitance measurements to obtain the dielectric

\textsuperscript{Received 20 December 2011.}

\textsuperscript{* Corresponding author: Manuchehr Soleimani (m.soleimani@bath.ac.uk).}
permittivity distribution within an object. The imaging method can be used for non-invasive visualization for industrial applications in both 2D \([7, 12]\), 3D mode \([4, 6, 8]\) and for 4D dynamic mode \([13]\). The capacitance sensor for 2D mode and 3D mode consists of a non-conducting pipe, on which an array of electrodes is mounted externally, with an earthed screen around it. 3D ECT sensor layout is discussed in \([5]\). This earthed screen protects the data readings being affected by external electrical field changes. The earthed screen may have radial earthed screens in an attempt to improve the quality of the image by preventing flux density between adjacent electrodes. The design and effect of these radial screens are discussed in \([1]\). If a metal vessel is used, then the electrodes have to be mounted internally, and the vessel acts as the earthed electrical screen itself \([3]\). The capacitance measurements data will be obtained by applying potential in turn to one electrode at a time, and by measuring the inter-electrode capacitance between this electrode and the remaining electrodes. For image reconstruction an ill-posed inverse problem associated with electrical capacitance tomography has to be solved. A considerable amount of research has taken place into creating both forward and inverse problems \([4–6]\).

The ill-posed inverse problem in ECT makes it sensitive to measurement noises and modelling errors \([11]\). 3D ECT is considered in this study. A complete sensor model (CSM) in 3D ECT is the most accurate forward model. The CSM requires meshing the exterior shielding, inter-electrode shielding and the air behind the electrodes and external shields \([5]\). This will make the 3D ECT computationally intensive. This paper proposes to investigate the error between CSM and a simple sensor model with coarse mesh for the forward problem. The approximation error model (AEM) has been developed \([10]\) to provide a statistical way to compensate for these errors \([2]\). AEM has been successfully applied to optical tomography and electrical impedance tomography \([9]\). The proposed AEM algorithm can be extended to similar inverse problem applications \([14–19]\). A major difference in 3D ECT is the setup of the capacitive sensors, which makes it very mesh intensive in area around electrodes. This paper aims to use AEM to 3D ECT image reconstruction.

2. ECT IMAGE RECONSTRUCTION

The forward problem is the simulation of measurement data for give value of excitation and material (permittivity) distribution and the inverse problem is the imaging result for a given set of measurement data. Before solving the inverse problem the forward problem needs to
Figure 1. (a) Cross-sectional view of 3D sensor. (b) Three-dimensional view of 3D sensor.

be solved. Figure 1 shows the ECT sensor set up including electrodes and outer screen.

Low frequency approximation to the Maxwell’s equations has been considered. Let’s take \( E = -\nabla u \) and assume no internal charges. Then the following equation holds.

\[
\nabla \cdot (\varepsilon \nabla u) = 0 \quad \text{in } \Omega \tag{1}
\]

where \( u \) is the electric potential, \( \varepsilon \) is complex conductivity and \( \Omega \) is the region containing the field. The potential on each electrode is known as

\[
u = V_k \quad \text{on electrode } e_k \tag{2} \]

where \( e_k \) is the \( k \)-th electrode held at the potential \( V_k \). Using finite element discretization of the Equation (1) using first order with the boundary condition (2) linear system of equations is obtained as

\[
K(\varepsilon)U = B \tag{3}
\]

where the matrix \( K \) is the discrete representation of the operator \( \nabla \cdot \varepsilon \nabla \) and the vector \( B \) is the boundary condition term and \( U \) is the vector of electric potential solution. The total charge on the \( k \)-th electrode is given by

\[
Q_k = \int_{E_k} \varepsilon \frac{\partial u}{\partial \hat{a}_n} \, dx^2 \tag{4}
\]

where \( \hat{a}_n \) is the inward normal on the \( k \)-th electrode, the surface integral in Equation (4) is done over the surface of electrode \( (E_k) \)
and the capacitance is calculated by $C = Q/V$. The inverse problem is: given the electrical response $u$, find the permittivity distribution $\varepsilon$, if

$$J(\varepsilon_b)x = y$$

where $\varepsilon_b$ is the permittivity estimate used in the forward problem; $J$ is the sensitivity matrix which is the Jacobian of the capacitance with respect to pixels evaluated at $\varepsilon_b$; $x = \varepsilon - \varepsilon_b$ is the difference between the permittivity distribution solution and the previous estimate of the permittivity distribution; and $y = f(\varepsilon) - f(\varepsilon_b)$, where $f(\varepsilon)$ is the measured capacitance $C$ with zero system noise error $e$ and $f(\varepsilon_b)$ is the capacitance calculated using the $u$ from solving the forward problem and will be discussed in more detail later.

Investigation has shown that by making improvements to the forward model and nonlinear inversion technique, the reconstruction of complicated geometrical shapes is possible using ECT [5]. The measured capacitance data $C$ contains an error $e$ which is caused by noise. This level of error is specified by the systems engineer. The other source of error is due to the discretisation of the problem using Finite Element Method (FEM). Finite element theory says that, when the mesh is getting denser, and thus the element size $h$ tends to zero, the accuracy in the model will increase, i.e., $f_h(\varepsilon_h)$ tends to $C$ as $h$ tends to 0. This results in the fact that for some inaccurate models large discretization error ($\psi(\varepsilon)$) can be created that can be even higher than the measurement error [2]. These errors can greatly reduce the quality of an image for coarser mesh based FEM models. This paper demonstrates how the incorporation of this error into the model enhances reconstruction quality achieved with using computationally efficient coarser meshes.

### 3. APPROXIMATION ERROR MODEL

The aim is to improve the quality of the reconstructed images achieved by using coarser meshes and thus reduce the computational time required to obtain a sufficient level of accuracy which is very important to 3D ECT fast imaging. The measurement error model is given by the approximate equation

$$C = f_h(\varepsilon_h) + e$$

where $e$ is the noise error in the measurement data, $h$ is the mesh parameter controlling the level of discretisation, and $f_h(\varepsilon_h) \to f(\varepsilon)$ as $h \to 0^+$ according to the theory of finite element method, that is that the equation becomes exact within the measurement accuracy. The
enhanced error model, which incorporates the discretisation error, is given by

\[ C = f_h(\varepsilon_h) + [f(\varepsilon) - f_h(\varepsilon_h)] + e = f_h(\varepsilon_h) + \psi(\varepsilon) + e \] (7)

where \( \psi(\varepsilon) \) is the discretisation error.

By determining the probability distribution for the modelling error \( \psi(\varepsilon) \), one can treat it as measurement noise, and thus use a coarser mesh to obtain better reconstruction quality by correcting for the discretisation error. The first step in achieving this goal is to assume that the continuous model \( \varepsilon \rightarrow f(\varepsilon) \) can be approximated to an appropriate level of accuracy by a densely discretized finite-dimensional model \( \varepsilon_\delta \rightarrow f_\delta(\varepsilon_\delta) \) where \( \delta > 0 \) is small and \( h > \delta \).

Suppose \( f_\delta(\varepsilon_\delta) = f(\varepsilon) \) within this level of accuracy and so the enhanced error model can be written as

\[ C = f_h(\varepsilon_h) + [f_\delta(\varepsilon_\delta) - f_h(\varepsilon_h)] + e = f_h(\varepsilon_h) + \psi(\varepsilon_\delta) + e \] (8)

A set of random samples can be generated of the permittivity solutions of size \( N \), using both the denser mesh with permittivity solutions \( \varepsilon_\delta \) and the coarser mesh with permittivity solutions \( \varepsilon_h \). This can be achieved by randomising the location, size and permittivity distribution spherical 3D objects in material under examination, and then by using Tikhonov regularization approach to solve the inverse problem, for both the permittivity distribution \( \varepsilon_\delta \) in the denser mesh and the permittivity distribution \( \varepsilon_h \) in the coarser mesh, for each random case. Assume that the solutions \( \varepsilon_\delta \) from the denser mesh, are accurate and the discretization error in the capacitance are calculated in each case using

\[ \Psi_h = f(\varepsilon^{(i)}_\delta) - f(\varepsilon^{(i)}_h) \] (9)

The distribution of the discretisation approximation error from the samples will be calculated by assuming Gaussian error term, \( \psi_h \sim N(\psi^*_h, \Gamma_{\varepsilon_h}) \), using the approximations [2].

\[ \psi^*_h = \frac{1}{N} \sum_{i=1}^{N} \psi^{(i)}_h \] (10)

And

\[ \Gamma_{\varepsilon_h} = \frac{1}{N-1} \sum_{i=1}^{N} \left[ \psi^{(i)}_h \psi^{(i)T}_h - \psi^*_h \psi^*_T \right] \] (11)

Assuming the noise error is mutually independent and Gaussian-type, and considering \( e \sim (e^*, \Gamma_e) \), where \( e^* \) is the mean and \( \Gamma_e \) is the symmetric and positive definite covariance matrix. The average discretisation error plus the average noise \( e^* \) provides us with the mean
$E(n)$ of the noise vector, $n = \psi + e$. The covariance of the noise vector can be calculated using

$$
\Gamma_n = \Gamma_\psi + \Gamma_e
$$

(12)

By using the Cholesky factorizations

$$
L_{\epsilon h}^T L_{\epsilon h} = \Gamma_{\epsilon h}^{-1}, \quad L_{\psi + e}^T L_{\psi + e} = \Gamma_n^{-1}
$$

(13)

These can then be incorporated into the 3D ECT model, in order to correct for the discretization error, along with the noise error, so that the inverse problem becomes:

$$
\text{Minimise} \| L_{e + \psi}(C - f_h(\epsilon_h) - \psi^* - e^*) \|^2 + \| L_{\epsilon_h}(\epsilon_h - \epsilon_h^*) \|^2
$$

(14)

which is equivalent to

$$
\| L_{e + \psi}(J(\epsilon_h)(\epsilon - \epsilon_h^*) - \psi^*) \|^2 + \| L_{\epsilon_h}(\epsilon_h - \epsilon_h^*) \|^2
$$

(15)

Given that the actual $\epsilon$ is approximated by the simulated $\epsilon_\delta$, there will be measurement error $e$ in both sets of solutions for $\epsilon$. This means that the distribution for the measurement error can be taken as $e \sim N(0, \Gamma_e)$ where $\Gamma_e = \text{diag}(\sigma_{e,j}^2, \ldots, \sigma_{e,m}^2)$, where $\sigma_{e,j} = r el_e|C_j^*|/100$ and $rel_e$ is the relative noise of the system as a percentage.

### 3.1. AEM Results

For three-dimensional ECT mode a 32-electrodes based complete sensor FEM model was used according to a solution presented in [5]. Two 3D tetrahedral meshes were developed for AEM examination purpose: the dense one for complete model composed of 109 k elements and the coarse one composed of 20 k elements. The sensor model was composed of 4 layers with 8 electrodes at each layer. The diameter of the sensor was 150 mm with 304 mm of height. 3D ECT FEM models used in this study are visualized in Figure 2.

For purpose of AEM, the forward model was trained by using 3000 randomly located spherical-shaped probes with random value of radius between 10 mm and 40 mm. Relative permittivity of each sphere was random value between 1.0 and 4.0. All spheres positions $(x, y, z)$ were generated with constraints for its centre coordinates $(x \in [-60; 60]; \ y \in [-60; 60]; \ z \in [50; 250])$ according to FEM sensor model dimensions given above, for keeping them in active zone of 3D ECT sensor located between the 1st and the 4th layer [14]. To obtain the AEM model trained 6000 FEM solutions were carried out. Due to large amount of computational complexity of this process the GP GPU CUDA technology was applied for speed-up the process. Normally the AEM training can take few days using Intel Core i7 — conventional CPU unit. Our Matlab AccelerEyes
Improving ECT imaging using AEM theory

Figure 2. 3D ECT meshes. (a) Coarse mesh (20 k) for simple sensor model. (b) Dense mesh (109 k) for complete sensor model.

Jacket implementation provided about 9 times speed-up using Nvidia GT555M graphic processing unit. Thus the AEM training process can be completed in a reasonable time (8–9 hours). For the inverse solution single-step Tikhonov Regularization algorithm was applied for absolute value based capacitance data with regularization parameter chosen empirically by trial and error (in practice it was selected between 0.001 and 0.1). For the simulation two different type of objects were examined. For the enhanced model examination purpose three spatially distributed small spheres as can shown in Figure 3(a) at the top is considered. As a more challenging object the L-shaped permittivity distribution was simulated, as presented in Figure 3(a) at the bottom, to further verify the AEM implementation. The Figures 3(b), (c) presents 3D ECT images obtained with and without AEM correction.

The spherical objects, as can be seen in Figures 3(b), (c), were resolved properly including central zone of the 3D ECT sensor which is less sensitive to permittivity changes. The L-shaped object, which is typically a difficult task for 3D ECT imaging, was reconstructed with an acceptable quality — there are some artefacts visible according to the linear-based inversion algorithms used. The enhanced model correction added to the coarse mesh based inverse solution with absolute value based capacitance data provided significant improvement in image quality in comparison to uncorrected 3D ECT coarse model. This is due to the fact that approximation error
Figure 3. (a) “True” 3D image. (b) 3D ECT image with use of simple 3D ECT model. (c) 3D ECT image corrected with AEM model.

Figure 4. Comparison of the measurement data distribution between the simple model and the AEM model.
estimated from the training process is relatively high as presented in Figure 4. It can be clearly seen for the neighbouring measurements (see arrows presented in Figure 4). It obviously comes from the fact that complete error due its close-to-real numerical implementation is able to reflect the electric field behaviour more accurately especially when objects are located close to the sensor electrodes as was proved and discussed in [5].

4. CONCLUSIONS

The use of FEM has been shown as a promising and effective method of modelling electrical capacitance problems for 3D ECT mode. However, discretisation and model error causes major problems in ECT reconstruction. An accurate model of the forward problem in 3D ECT requires a substantial amount of computational time and memory. This paper shows that significant reduction in computational time of the image reconstruction can be achieved by modelling the discretisation error using proposed AEM algorithm. High quality images can be achieved by implementing the AEM algorithm and use of a simple coarse mesh for forward model. Computational time for CSM is very high and even with the high performance computing techniques such as GPU method the real time imaging is very challenging. This is a main problem for potential industrial use of the 3D ECT, making real time 3D ECT hard to achieve. The observation of this paper shows that the AEM approach has shown visibly better image quality for three spheres than the “L-shaped” structure. This could be because the training samples were spherical shaped. We recommend a wide range of shapes and multiple objects should be included in training stage. Nonetheless, the proposed method provides a promising tool to reduce the computational time in 3D ECT and potentially pave the way for real time 3D imaging using trained Tikhonov method. The proposed AEM algorithm will help a step closer to real time imaging in 3D. Experimental verification of proposed method will be subject of a future study.

ACKNOWLEDGMENT

This work is partially co-funded by The Rector of Technical University of Lodz (TUL) in Poland and The Dean of The Faculty of Electrical, Electronic, Computer and Control Engineering at the TUL, in the frame of young researcher’s supporting programme. This work is partially supported by the Polish Committee of Science and Research (Project number 4664/B/T02/2010/38).
REFERENCES


