An Advanced Numerical Technique for a Low-Frequency Electromagnetic Field Simulation based on the Finite-Difference Time-Domain Method

Minhyuk Kim\textsuperscript{1}, SangWook Park\textsuperscript{2}, and Hyun-Kyo Jung\textsuperscript{1}

\textsuperscript{1} Department of Electrical and Computer Engineering, Seoul National University, Seoul 151-742, Korea.
\textsuperscript{2} ICT Convergence Research Team, EMI/EMC R&D Center, Corporation Support & Reliability Division, Korea Automotive Technology Institute, Cheonan, 330-9112, Korea.

Abstract- In this paper, an advanced numerical technique is proposed for a low-frequency electromagnetic field simulation. The finite difference time domain (FDTD) method is one of the best methods for analyzing electromagnetic field problems, which require high levels of precision, such as a heterogeneous whole-body voxel human model. However, directly using the standard FDTD method in the quasi-static frequency band requires an extra-large number of iterations. We overcome this drawback using the hybrid of surface equivalence theorem and quasi-static FDTD (QS-FDTD) method. The surface equivalence theorem is used to consider arbitrary shape of source and the QS-FDTD is applied to obtain electromagnetic response quickly in the low-frequency region. The results of the proposed method are in good agreement with those from a commercial electromagnetic simulator.

1. INTRODUCTION

Owing to the recent development of electric/electronic/information technology, many of these types of devices have become available to use with wireless communication and wireless power transfer. The interest in electromagnetic field (EMF) hazards as well is growing due to the undesirable EMF generated from the devices. The term EMF dosimetry refers to “evaluation of electromagnetic radiation dose.” The EMF dosimetry throughout the experiment can be applied only for the homogeneous medium and have a limited frequency range from 150 MHz to 9 GHz. There are theoretical solutions to simple geometries such as a sphere, or an ellipse. However, it is difficult to analytically solve most real problems as certain phenomena take place, such as scattering, attenuation, and phase delays. Therefore, numerical technique is usually applied to calculate EMF dosimetry for any frequencies with inhomogeneous whole-body voxel model.

It is important to use a suitable numerical method, as the numerical technique depends on the analysis target. Many numerical techniques are currently being studied such as the finite element method (FEM), the finite difference time domain (FDTD) method, the method of moments (MOM), and the hybrid algorithm.

At present, the FDTD method is a popularly used method for interpreting electromagnetic problems. Its strengths include its computing time and memory using, as it is an explicit method. Thus, no matrix calculation process is required, unlike with other popular methods, such as FEM and MOM. Therefore, this method is very well suited to problems that require high resolutions with dielectrics and lossy materials, such as a human model unlike MOM whose algorithm is not applicable to analyze the dielectric material. Hence, it is the most widely used method for bio-electromagnetic applications, which exceed a few MHz [1].

However, the analysis time becomes excessively long as the number of iterative computations increases to converge to a steady state in the low-frequency band. To solve this problem, implicit-FDTD approaches [2-5] and several FDTD variations have been devised. The implicit FDTD methods are theoretically unconditionally stable. However, this method still presents problems in conditions with only a few MHz because the acceleration factor
reduces not only the simulation time but also the accuracy when the factor exceeds the limit.

There have been several studies of FDTD variations for application to the low-frequency band. Examples include an impedance network method [6, 7], current vector potential method [8], and a scalar potential finite-difference method [9]. These methods make it easy to simulate the incident magnetic field problem as it can be calculated by Faraday’s law. However, it is not easy to implement for the analysis of an incident electric field problem as the charge on the surface of the object must be known to analyze the electrical coupling. There is a study that induced electric field is not always negligible when calculating the dosimetry of the human body model under low frequencies [10]. Therefore, there is a need to develop a method to calculate both electric and magnetic fields easily.

The frequency-scaling FDTD method was first introduced [11]. The method analyzes the problem at a higher frequency than the frequency of interest. The results are then converted using the ratio of the previous frequency relative to that of the target. This approach can significantly reduce the simulation time of a low-frequency analysis; however, the following problems can nonetheless arise: i) the FDTD method still requires a considerable amount of simulation time at frequencies of few MHz, and ii) errors occur because the skin depth is not negligible at the frequency used in the original paper. The quasi-static FDTD (QS-FDTD) method [12] resolved this problem using a ramp source. However, it mainly focused on the plane wave excitation. When directly applying this method to conductor or/and dielectric models, i.e., antennas, which radiate electromagnetic fields, calculation of the radiation characteristics of the antenna cannot be done.

It was difficult to calculate a response of electromagnetic field by arbitrary source under the low frequency by the above-mentioned problems. Therefore, we propose a new approach to address these problems. We refer to the proposed method as the surface-equivalence-theorem-based quasi-static FDTD (SQ-FDTD). In the proposed method, surface equivalent theorem is used to calculate arbitrary source excitation, and the quasi-static approximation is applied to converge quickly under low frequencies. The method, taking into account both electric and magnetic coupling, simulate inhomogeneous complex-shaped problem in the low frequency extremely quickly by arbitrary source excitation. In this study, the radiation characteristics of the antenna are calculated by numerical method in advance. Then, the entire field is calculated by applying the results of the previous step to the SQ-FDTD method. To verify our approach, the electromagnetic field responses of certain models are calculated with the proposed method, and the results are compared with those of a commercial software package (FEKO [13]).

2. THE PROPOSED METHOD

The basic concept of the proposed method is using the QS-FDTD method with surface equivalent theorem to solve the low-frequency problem quickly by exciting arbitrary source. The proposed method is composed of two separate parts, whereas the standard FDTD solves the problem all at once. The first part involves the following steps: i) selecting arbitrary surface surrounding sources and/or scatterers, ii) calculating the currents on these surfaces by any theoretical or numerical techniques. In the next part, the whole electromagnetic field responses are obtained using the QS-FDTD method by applying the results of the previous part as the source. The following are detailed explanations of each part.

2.1 Surface equivalence theorem

The surface equivalence theorem is a concept that is used when determining the electromagnetic scattering
of complex scatterers [14].

\[ E_1 \times \hat{n} = M_s \]
\[ \hat{n} \times (H_2 - H_1) = J_s \]  

Here, \( E_1 \) and \( H_1 \) are the field outside an equivalent surface, and \( E_2 \) and \( H_2 \) are the field inside an equivalent surface.

Any type of numerical technique can be used to determine the currents on an equivalent surface. These currents are used in the proposed method as sources on the equivalent surface. Simple Fourier transforms are applied if the calculated currents are in the frequency domain as expressed in (2).

\[ J_s = J_{s0} \sin(\omega t + \phi) \]  

2.2 QS-FDTD

The FDTD method is a numerical analysis technique, which discretized the time-dependent Maxwell's equations using central-difference approximations to the space and time partial derivatives. The standard FDTD method requires a simulation time of a few periods of the frequency of interest to reach a steady state. In the method, the time step is fixed by the cell size. Since simulation for a few periods is required, as the frequency goes down, the period gets longer and the number of time steps increases. Thus, it is impossible to apply this method under a few MHz, as it is extremely time-consuming given its need for a large number of iterative simulation steps to reach convergence. The quasi-static approximation (QSA) is applied to the previously calculated currents to avoid this problem in our approach [12]. In the QSA, the propagation effects could be negligible when the sizes of the involved objects are small in comparison with the free space wavelength. The QSA approach is often assumed up to a few tens of megahertz [15]. Thus, quick convergence can be achieved considering the fact that the phase is known in the steady-state quasi-static case [15 ,16]. That is to say, the fields
exterior to the object have the same phase as the incident field, and the interior fields are first-order fields that are proportional to the time derivative of the incident field [12]. Thus, it is possible to quickly calculate the electromagnetic field to be analyzed inside and outside the target as it is sufficient to record two successive field values after the transient response has finished [12, 17, 18].

The function used in this method was altered to start smoothly such that it suppresses high-frequency contamination. For example, the electric current (2) is expressed as follows [12]:

$$J_s = \begin{cases} 0 & -\infty < t < t_0 \\ (t - (\tau / \pi) \sin(\pi t / \tau)) / 2 & t_0 < t < \tau \\ t - \tau / 2 & t > \tau \end{cases} \quad (3)$$

The magnetic current is obtained in the same manner. Throughout the above process, the quasi-static approximated currents are obtained on the equivalent surface induced from the electromagnetic waves generated by the source. The approximated currents are used in the time-domain Maxwell equations as expressed in (4).

$$\nabla \times \tilde{E} = \frac{\partial \tilde{D}}{\partial t} + \tilde{J}_s$$

$$\nabla \times \tilde{H} = -\frac{\partial \tilde{B}}{\partial t} - \tilde{M}_s \quad (4)$$

The whole electromagnetic fields generated by the traveling waves on the equivalent surface are calculated by the standard FDTD scheme. As the steady state of the quasi-static system converges in much less than one full cycle, all the electromagnetic fields are obtained exceptionally faster than they are with conventional methods. The flow chart of the proposed method is illustrated in Fig. 2.

---

Fig. 2. Flowchart of the proposed method.
To obtain steady-state solution, generally, hybrid numerical techniques need iteration process for considering the interaction between the methods. However, the interaction between dielectric material and source is not included in the proposed method as the effect could be skipped to rapid computation in case that the effect of the interaction is negligible [10].

3. METHOD VERIFICATION

In this research, the currents on the equivalent surface of the rectangular box including the antennas are calculated by MOM in the first step. The following step converts frequency-domain currents into the quasi-static time domain. The electromagnetic fields are then calculated by solving the standard FDTD scheme with quasi-static approximated currents as a source on the equivalent surface.

![Simple system for calculating the interior electromagnetic field of a sphere from antenna radiation.](image)

The simulation system, which is composed of an equivalent surface, including an antenna and the target model, is illustrated in Fig. 3. In this paper, tests were conducted to verify the proposed method using a single-layered dielectric sphere with a dipole antenna for considering and a loop antenna, as shown in Figs. 4(a) and (b), respectively. The operating frequency is 1 MHz in this paper.

![Tested antennas: (a) dipole, (b) loop.](image)

The first test is conducted with a dipole antenna whose length $L$ is 100 mm. A dielectric sphere with a diameter $r$ of 400 mm and an equivalent surface with $12 \times 12 \times 32$ are placed in the center of the $y$-axis. The distance $R$ between the center of the equivalent surface and the sphere is 940 mm. The domain is constructed by $70 \times 70 \times 70$ cubical cells $\Delta x = \Delta y = \Delta z = 20$ mm. The sphere has a conductivity of $\sigma = 0.3$. The sinusoidal function is converted by (3) with $\tau = 100 \delta t$ and $\delta t = 33.3$ ps.
The proposed method converged after 600 time steps. The results comparison is summarized in Table 1 between two methods, which is made based on maximum and average relative difference as expressed in (5) and (6), respectively.

\[
\text{Maximum difference} = \text{Max} \left( \frac{\text{FEKO-Proposed method}}{\text{Proposed method}} \times 100 \right) \quad (5)
\]

\[
\text{Average difference} = \frac{\sum_{i=1}^{n} \left( \frac{\text{FEKO-Proposed method}}{\text{Proposed method}} \right) \times 100}{n} \quad (6)
\]

Table 1. Electric field and magnetic field comparisons for the dipole antenna.

<table>
<thead>
<tr>
<th>Plane x=0</th>
<th>Plane y=0</th>
<th>Plane z=0</th>
<th>Plane x=0</th>
<th>Plane y=0</th>
<th>Plane z=0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum difference (%)</td>
<td>29.42</td>
<td>24.85</td>
<td>20.21</td>
<td>31.16</td>
<td>25.32</td>
</tr>
<tr>
<td>Average difference (%)</td>
<td>10.42</td>
<td>7.08</td>
<td>7.66</td>
<td>11.02</td>
<td>7.15</td>
</tr>
</tbody>
</table>

The internal electric and magnetic field results obtained by the proposed method and FEKO are shown in Fig. 5 and Fig. 6, respectively.

![Fig. 5. Absolute internal electric field cross-sections from the dipole antenna in the plane z=0: (a) FEKO, (b) the proposed method.](image)
Fig. 6. Absolute internal magnetic field cross-sections from the dipole antenna in the plane $z=0$: (a) FEKO, (b) the proposed method.

The next simulation is with a circular loop antenna with a diameter $D_L$ of 60 mm. A dielectric sphere with material properties identical to those used in the previous test with a diameter $r$ of 400 mm, and an equivalent surface of $16 \times 16 \times 16$ are placed in the center of the $y$-axis. The central distance $R$ is 800 mm. The domain is constructed with $80 \times 80 \times 80$ cubical cells with the same mesh size used in the dipole antenna simulation. The identical source properties are used with the dipole antenna induction. Quick convergence is also achieved, as in the previous test, and the results of the proposed method are well matched to those of the FEKO. The electromagnetic field computed by the two methods is compared with maximum and average relative difference and summarized in Table 2.

Table 2. Electric field and magnetic field comparisons for the loop antenna.

<table>
<thead>
<tr>
<th></th>
<th>Electric field</th>
<th>Magnetic field</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Plane $x=0$</td>
<td>Plane $y=0$</td>
</tr>
<tr>
<td>Maximum difference (%)</td>
<td>21.85</td>
<td>22.65</td>
</tr>
<tr>
<td>Average difference (%)</td>
<td>8.65</td>
<td>8.87</td>
</tr>
</tbody>
</table>

The internal electric and magnetic field results obtained by the proposed method and by FEKO are shown in Fig. 7 and Fig. 8, respectively.
Fig. 7. Absolute internal electric field cross-sections from the loop antenna in the plane z=0: (a) FEKO, (b) the proposed method.

Fig. 8. Absolute internal magnetic field cross-sections from the loop antenna in the plane z=0: (a) FEKO, (b) the proposed method.

From the above results, the proposed method is able to calculate low-frequency electromagnetic field problem regardless of the type of incidence field, in contrast to the conventional methods mentioned in the introduction. Furthermore, the result is computed much more rapidly than that by the standard FDTD method. The number of required iterations for one period in the standard FDTD method is determined as follows:

\[
N = \frac{\sqrt{n}c_0}{\text{frequency} \cdot \Delta x}
\]  

Here, \( n \) is the dimension of the simulation, \( c_0 \) is the speed of light, and \( \Delta x \) is the mesh size [19].

Thus, the simulation time is reduced by approximately 200 times when simulating the above problems with the proposed method when compared with the standard FDTD. Moreover, the required iteration number \( N \) is inversely proportional to the decrease in frequency. For example, the speed gain of the proposed method is
nearly two million iteration steps when the target frequency is 100 Hz when compared with the standard FDTD method, as the proposed method reaches convergence rapidly regardless of a change in frequency. Thus, the method shows much stronger performance with low-frequency problems.

4. CONCLUSION

In this paper, the SQ-FDTD method is proposed. Using this method, it is possible to analyze low-frequency problems in a short time regardless of the shape of the source as well as plane wave excitation using quasi-static approximation, unlike the conventional methods. It is especially well suited for a heterogeneous dielectric and conductive body with arbitrary shaped scatterers. The internal electric and magnetic fields of the dielectric sphere are calculated from the excitation of two antennas to validate the SQ-FDTD method. This paper successfully demonstrates the possibility of analyzing near-field problems at low frequencies with the proposed approach.

ACKNOWLEDGEMENT

This research was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (NRF-2012R1A1A2006794).

REFERENCES

10. S. W. Park, K. Wake, and S.Watanabe, “Calculation errors of the electric field induced in a human body


Author
1. Minhyuk Kim // ejnp@snu.ac.kr
2. SangWook Park // parksw@katech.re.kr
3. Hyun-Kyo Jung // hjkung@snu.ac.kr
Dear Editors and Reviewers,

Many thanks for your valuable feedback. We have made extensive corrections and expansions in the manuscript after going over the reviewers’ comment. Detailed explanation of the proposed method and analyses of the results have been added as suggested. All major changes in the revised version of the manuscript are highlighted in yellow.

Reviewer 1's Comments:
The English is awkward throughout the paper.
Thanks for your comment. The paper has been taken editing service from IEEE partner to resubmit this paper.

2. Proposed Method

The first paragraph explains the method very badly. Mention the equivalence theorem in conjunction with the closed surface. Explain that a frequency-domain method such as MOM will be used to find the amplitude and phase of the electric field and the magnetic field over the closed surface. Explain that these will then be converted to cosine functions in the time domain, and the FDTD method in the time domain will be used to find the near field in the object.

Thanks for your careful examination.
We have extensively changed the description of the proposed method. The major pieces have been explained in the first paragraph. The detailed description has been described separately in each section.

Basically, the proposed method is composed of two separated steps. The aim of the first step is obtaining currents on the equivalent surfaces. We can use any theoretical or numerical techniques to obtain those currents. We used MOM just for antenna used in this paper as the method is suited to the wired structure. The point has been explained in the first paragraph of the proposed method. (page 2, lines 35-36)

Equation 2 makes no sense because vector Js appears on the left and the same vector Js appears on the right multiplied by sin ( omega t + phi ). This cannot be an equality.
The awkward equation has been corrected as the reviewer suggested.
The paragraph following equation 2 is difficult to understand. This para is central to understanding the proposed method yet is hard to follow.
We agree with the reviewer about that. We have added and changed some sentences as the reviewer suggested. The detailed explanation of the corrected sentences is following.

In FDTD the time step is fixed by the cell size. Since simulation for “a few periods” is required, as the frequency goes down, the period gets longer (T=1/f) and the number of time steps increased as 1/f. It would be insightful to explain in this manner.
We agree with the reviewer about that. The point has been explained in the first paragraph of the QS-FDTD section. (“Since simulation for a few periods is required, as the frequency goes down, the period gets longer and the number of time steps increases.”)

The language about quasi-static behavior is confusing.
We agree with the reviewer about that. Our approach can be applied under the quasi-static condition. In the quasi-static condition, the fields exterior to the object have the same phase as the incident field, and the interior fields are first order fields that are proportional to the time derivative of the incident field. We used the quasi-static behavior to express this. However, we have changed all “quasi-static behavior” to “in the quasi-static condition” to not make reader confuse as suggested.

Why is the phase known? The currents over the surface enclosing the antenna have phase which is a function of position over the surface. Is the position-varying phase being replaced by a constant phase?

Thanks for your question. In the QSA, the propagation effects could be negligible when the sizes of the involved objects are small compared to the free space wavelength. We have added the sentence in the first paragraph of the QS-FDTD section. (“In the QSA, the propagation effects could be negligible when the sizes of the involved objects are small in comparison with the free space wavelength. The QSA approach is often assumed up to a few tens of megahertz.”) Two papers demonstrating the possibility of using QSA have been added to support this.


Equation (3) gives a ramp function with a gradual startup and the words imply that the start of the “target frequency sinusoidal wave” is approximated with a ramp. For t>tau in (3) we have a ramp but when do we switch to a sinusoidal wave?
Thanks for your question. We don’t need to switch the source. It is possible to quickly calculate the electromagnetic field to be analyzed inside and outside of the target as it is sufficient to record two successive field value after the transient response has finished as we know the phase. Two papers demonstrating the possibility of using ramp source to achieve quick convergence have been added to support this.


3. Results

In the example what is the frequency of operation of the dipole? Of the loop? This basic fact is not stated.

The frequency of operation was written in the submitted paper. (second paragraph of page 5, second paragraph of page 6) However, we have moved the sentence in the first paragraph of the page 5.

The method proposed in the paper has the weakness that the antenna fields are calculated by MOM (or some other frequency-domain method) inside the closed surface, in the absence of the scattering object (the dielectric sphere in the example). The antenna may interact strongly with the dielectric object especially if it is close to the object. Then the currents on the antenna in the presence of the dielectric object might be quite different than the currents of the antenna in isolation.

We agree with the reviewer about that. To obtain a steady-state solution, generally, hybrid numerical techniques need to iteration process for considering the interaction between the methods. However, the interaction between dielectric material and source is not included in the proposed method as the effect could be skipped to rapid computation in case that the effect of the interaction is negligible. We have added the sentence about the interaction effect in the last paragraph of QS-FDTD section on page 4. A paper demonstrating the possibility of ignoring the effect has been added to support this.


The paper would be greatly strengthened by a realistic example. Give us a context: are we talking about 60 Hz fields due to the power-distribution system? The paper talks about applying the method to a whole-body human model, so choose a sphere the size of a head and use realistic tissue parameters for the sphere. What is a realistic source for such a field? I am assuming that the dipole source is used
because it primarily generates a vertical electric field in the sphere, and the loop because it makes an approximately vertical magnetic field.

Thanks for your suggestion. Actually, the size of dielectric sphere is similar to the human head and the material properties are set to be 2/3 of that of muscle tissue, which represents the average dielectric properties of the human body [10]. And the frequency is one of the target frequency of a wireless power transfer system. However, this paper is for proposing a new numerical technique. Thus, we have focused on the verification of the proposed method with the simplified human phantom model such as a sphere. The numerical dosimetry using the heterogeneous structure of the human voxel model will be one of our future works.

The comparisons with FEKO for both the electric field and the magnetic field due to the dipole look significantly different. The RMS comparison of (5) emphasizes the large field values and so will show 90% agreement if the large fields at the left are nearly the same. The small field values elsewhere in the field are compared poorly by the RMS comparison.

We agree with the reviewer about that. The significantly different is occurring at the boundary. We have changed RMS comparison to the maximum and the average relative difference in table 1 and 2 to take into account the differences in all of the regions.

The loop’s field in Fig. 7 does not penetrate the sphere very much and the 98% RMS agreement is misleading.

We are afraid if the reviewer might misunderstand our proposed approach. The magnetic field can penetrate the sphere easily. However, the electric field does not pass easily as reviewer mentioned. Thus, external electric field induces charge on the surface of the sphere. And the charge generates the internal electric field. The values of the internal electric field are lower than those of external electric field. Even if the values are small, it is induced from external field. We have compared the internal electric field results by FEKO and propose method. We have added the maximum and average results for loop antenna simulation in Table 2.