
PIER 5

Progress in Electromagnetics Research

**Application of Conjugate
Gradient Method to
Electromagnetics and
Signal Analysis**

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Application of Conjugate Gradient Method to Electromagnetics and Signal Analysis

Tapan K. Sarkar

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Editor:

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PREFACE

In the last few years, the conjugate gradient method has been very popular for nonlinear optimization problems and considered as an efficient means for solving matrix equations. The application of the conjugate gradient method in optimization has been well documented by Hestenes [*Conjugate Direction Methods in Optimization*, Springer Verlag, 1980] and Golub and O'Leary [SIAM Review, 1989] who have provided an extensive list of papers including tracing the historical development of the technique. In recent years, the conjugate gradient method has been a popular method in solving a certain class of problems in electromagnetics and signal analysis.

In electromagnetics, the conjugate gradient method has been used in this decade for efficient solution of large matrix problems. Recently, the method has been observed to be quite advantageous for direct application to the solution of operator equations. This has been discussed in Chapter 1 along with its similarity and dissimilarity with other popular numerical methods in electromagnetics. Chapters 2 and 3 describe other iterative techniques and illustrate how improvements can be made on various iterative schemes. Chapter 4 shows that the conjugate gradient method can be quite efficient for the solution of matrix equations arising in electromagnetic scattering from conducting structures. Chapter 5 discusses the rate of convergence of the conjugate gradient method for the electric field, magnetic field, and combined field equations.

It has been observed that when the operator equation to be solved for has a certain particular structure (namely Hankel or Toplitz) when discretized; then, FFT (Fast Fourier Transform) can be introduced to replace matrix multiplications. In this way, matrix equations with tens of thousands of unknowns can be solved in reasonable computer CPU time. Chapters 6-11 describe the class of problems where advantages can be gained by CG and FFT over other conventional techniques. Chapter 12 describes the application of the conjugate gradient to inverse problems. The conjugate gradient method can also be used to analyze time domain problems. In conventional time domain methods, the discretization of space and time are related by the causality condition and sometimes the solution may even diverge for late times.

These problems have been overcome by the conjugate gradient method as described in Chapter 13.

The conjugate gradient method may also be utilized for iteratively finding the smallest/largest eigenvalues of a Hermitian matrix. This method may be computationally quite efficient when the matrix is very large and yields accurate results when the dynamic range of the eigenvalues are very big. Chapter 14 discusses how this technique can be used to efficiently generate the propagation characteristics of various modes in various transmission structures including fin lines. Chapter 15 describes how the various iterative techniques compare for solving the eigenvalue problem in the signal processing literature. The conjugate gradient method has emerged as a very efficient method in adaptive signal processing as compared to conventional LMS and other iterative schemes. It has been used in echo cancellation, adaptive equalizers, and radio direction finding as described in Chapter 16. Recently, it has been implemented in hardware on the AT&T DSP-32 processor performing real time deconvolution for signal processing applications [Fifth ASSP Workshop Spectral Estimation and Modelling, 1990].

In summary, this book provides a survey on how, when, why, and where to apply the conjugate gradient method. Examples have been presented to illustrate for what class of problems the conjugate gradient method is quite advantageous to use. This book can be used for undergraduate research projects and also for a special topic course at the graduate level.

T. K. Sarkar

*Syracuse, New York
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