PERFORMANCE OF GROUND-BASED
HIGH-FREQUENCY RECEIVING ARRAYS WITH
ELECTRICALLY SMALL GROUND PLANES

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1. Introduction

High-frequency (HF) antenna elements for over-the-horizon (OTH) receiving arrays often consist of some form of a vertical monopole element with a ground plane resting on the Earth or in close proximity to it. Its proximity to Earth causes a far-field multipath pattern null on the horizon; multipath ground losses; a leaky quasi-evanescent surface wave which directs energy into the Earth, but not the air medium, causing additional ground losses; and a near-field non-uniform multipath reflection from a nonhomogeneous Earth.
The output of each antenna element (including its matching network, if any) is usually transmitted to its own receiver by means of a transmission line (hereafter referred to as a "feed cable"). Exterior currents are induced on the element feed cables by the tangential component of the monopole element field incident on the cables and by ground-plane currents diffracted by the edge of the ground plane.

Antenna performance degradation by the Earth and by external currents on the feed cables are mitigated by employing ground planes that are as large as is economically feasible. Large ground planes bring the peak of the multipath pattern closer to the horizon, reduce near-field ground losses, provide uniform near-field ground reflection from element-to-element, shield the cables from the element fields, and minimize the ground-plane current incident on the edge of the ground plane.

In the design of very large OTH receiving arrays (comprising hundreds or thousands of elements), electrically large metallic ground planes are prohibitively expensive to construct and maintain. One alternative approach is to employ salt-water ground planes; however, suitable salt-water sites are not readily available and they have unique problems. In this report, we discuss another alternative approach, using elements with electrically small ground planes; and we also estimate the performance degradations that can result from it.

Performance degradations caused by electrically small ground planes include (1) a reduction in element directivity near the horizon (caused by Earth multipath) and distortion of the azimuthal pattern (caused by exterior currents on element feed cables); (2) a decrease in element radiation efficiency with a consequential increase in system internal noise (caused by ground losses); (3) an increase in the array factor root-mean-squared (rms) phase error (caused by non-uniform Fresnel reflection from nonhomogeneous Earth and exterior currents on element feed cables); and (4) an increase in the array factor beam-pointing errors (caused by non-uniform Fresnel reflection from nonhomogeneous Earth).

These performance degradations may be grouped according to their effect on element directivity, the system operating noise factor, and the array factor. The predetection signal-to-noise ratio (SNR) of a receiving system is proportional to the receive antenna directivity (which is proportional to the element directivity and the array factor) and is inversely proportional to the system operating noise factor [1].
For ground-based HF radar or communication systems, the Earth excess propagation loss factor is usually included in the antenna ground-plane system rather than in the propagation path [1]. Ohmic losses of the receive antenna (including its ground-plane system and matching network) are incorporated as part of the system operating noise factor [1] in accordance with International Radio Consultative Committee (CCIR) convention.

The effect of electrically small ground planes on element directivity is discussed in section 2. The system operating noise and array factors are discussed in sections 3 and 4, respectively. Section 5 presents the summary and conclusions.

2. Element Directivity

Monopole elements with perfect ground planes (of infinite extent, conductivity, and density) have a radiation pattern that has its peak on the horizon and is omnidirectional in azimuth. Furthermore, the current on the exterior of the element feed cable is zero. For this case, the interference of the element and the image fields is totally constructive in the direction of the horizon, the feed cable is completely shielded from the element fields, and the current on the bottom surface of the ground plane is zero.

For imperfect ground planes, the direction of peak directivity is at an angle above the horizon. The directivity on the horizon is approximately \(-6\) dBi and \(-\infty\) dBi when the ground plane is in free space [2] and when it is in proximity to flat Earth, respectively. Furthermore, the current on the exterior of the element feed cable is non-zero because the feed cable is not completely shielded from the element fields, and the current on the bottom surface of the ground plane at the feed cable is also non-zero.

This section presents numerical results of the directivity for monopole elements with electrically short circular ground planes resting on flat Earth. In our discussion, we assume that the element and ground planes are fabricated from metal of infinite conductivity. The ground-plane density is infinite for disk ground planes, a function of the number of radial wires and wire radius for radial-wire ground planes, and a function of the mesh spacing and wire radius for screen ground planes. The effects of Earth multipath and exterior current on the element feed cable are discussed in Sections 2.1 and 2.2, respectively.
2.1 Influence of Earth Multipath

Earth multipath introduces a null on the horizon for monopole antennas in proximity to flat Earth unless they have a perfect ground plane. In this section, we discuss the effect of Earth multipath on the element directivity with a condition of zero exterior current on the element feed cable. This section presents a characterization of the antenna parameters, a survey of literature, numerical results for disk ground planes and radial-wire ground planes, and an analytical expression for directivity.

2.1.1 Characterization of antenna parameters

Monopole elements at the center of circular ground planes in proximity to flat Earth are characterized by at least six parameters (see Fig. 2.1): three antenna parameters normalized to the radio-frequency (rf) wavelength $\lambda$ [element length $h/\lambda$, element radius $b/\lambda$, and ground-plane radius $a/\lambda$ or $2\pi a/\lambda$ in radians]; and three Earth parameters [dielectric constant $\epsilon_r$; loss tangent $(\sigma/\omega\epsilon_0\epsilon_r) = (60\lambda\sigma/\epsilon_r)$; and height $z_0/\lambda$ of the Earth's surface relative to the ground plane, where $\sigma =$ Earth conductivity (S/m), $\omega = 2\pi f =$ radian rf frequency (rad/s), $\lambda =$ rf wavelength (m), and $\epsilon_0 =$ free-space permittivity $= \text{8.854} \times 10^{-12}$ (farads/m)]. The Earth constants, loss tangents, and penetration depths for CCIR 527-1 characteristics of Earth in the 3 through 30 MHz band are summarized in table 1. The above six parameters sufficiently characterize the antenna's electrical properties for disk ground planes. For radial-wire ground planes with equally spaced wires, additional parameters are the number $N$ and radius $b_w$ of the wires; for mesh-screen ground planes, the mesh shape, spacing, and wire radius are the additional parameters.

2.1.2 Survey of literature

Numerical results of the magnetic far-field intensity with a ground plane relative to that with no ground plane (for the case of a circular ground plane resting on flat Earth) have been published previously. Wait et al. published results for small radial-wire ground planes [3]; large radial-wire ground planes [4, 5]; small disk ground planes [3]; large disk ground planes [5, 6]; large ground screens [7–10]; nonhomogeneous Earth [11–16]; and assorted ground planes [17]. Rafuse and Ruze [18] published results for large ground screens and radially nonhomoge-
neous Earth. Those results yield the absolute power gain, but not the absolute directivity, because ground losses are included in the relative magnetic far-field intensity. The above references, with the exception of [9, 10, 15, and 18], assume that the current on the finite-extent ground plane is the same as that for a perfect ground plane.

2.1.3 Numerical results for disk ground planes

Weiner [19–21] determined the absolute directivity of quarterwave monopole elements on electrically small disk ground planes resting on flat Earth by using Richmond’s method-of-moments computer programs [22] which compute the actual currents on the element and ground planes and the resulting far fields. The numerical results are shown in Figs. 2.2 through 2.12 for the cases $h/\lambda = 0.25$, $b/\lambda = 10^{-6}$, $2\pi a/\lambda = 0$ to 8 radians, and medium dry ground ($\epsilon_r = 15$, $\sigma = 0.001$ S/m) at a frequency of 15 MHz. Results are compared with those for a perfect ground plane ($\epsilon_r = 1.0$, $\sigma = \infty$) and for free space ($\epsilon_r = 1.0$, $\sigma = 0$).

The peak directivity is approximately independent of disk radius and approximately equal to that of a perfect ground plane (see Figures 2.2 through 2.7). Figures 2.2 through 2.6 are polar plots of numeric directivity on the same linear scale. The angle of incidence of the peak directivity for $0 \leq 2\pi a/\lambda \leq 8$ is approximately independent of ground-plane radius and approximately 30 degrees above the horizon.
<table>
<thead>
<tr>
<th>Cases</th>
<th>LOSS TANGENT</th>
<th>6 (m)</th>
<th>PENETRATION DEPTH</th>
<th>G (Sid)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td>1.5</td>
<td>3.0</td>
</tr>
<tr>
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<td></td>
<td>G (Sid)</td>
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</tr>
<tr>
<td>Sea Water (average</td>
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<td>G (Sid)</td>
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<tr>
<td>Salinity 20°C)</td>
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<td></td>
<td></td>
<td>G (Sid)</td>
</tr>
<tr>
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</tr>
<tr>
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<td></td>
<td>G (Sid)</td>
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<td>Very Dry Ground</td>
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<td></td>
<td>G (Sid)</td>
</tr>
<tr>
<td>Pure Water, 20°C</td>
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<tr>
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<td>G (Sid)</td>
</tr>
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<td>G (Sid)</td>
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<tr>
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<td>G (Sid)</td>
</tr>
<tr>
<td>Free Space</td>
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<td></td>
<td></td>
<td>G (Sid)</td>
</tr>
</tbody>
</table>

Table 2.1: Permittivity, loss tangent, and penetration depth of CCIR 527-1 classification of Earth
Figure 2.2 Numeric directivity polar plot, $2\pi a/\lambda = 0.025$. ($f=15$ MHz, $h/\lambda = 0.25, b/\lambda = 1.0 \times 10^{-5}, Z_0/\lambda = 0$. Case 1: Perfect Ground ($\varepsilon_0 = 1.0, \delta = \infty$), Case 5: Medium Dry Ground ($\varepsilon_0 = 15.0, \delta = 0.001 \text{s/m}$), Case 11: Free Space ($\varepsilon_0 = 1.0, \delta = 0$).)

(see Fig. 2.8). The Earth softens the edge of the ground plane and minimizes changes in directivity resulting from ground-plane edge diffraction. In the absence of Earth, large changes in directivity occur (see case 11 in Figs. 2.7 and 2.8) because ground-plane edge diffraction is more pronounced. The large changes in angle of peak directivity do not represent significant changes in peak directivity because of the broad 3 dB beamwidth of the radiation pattern. The jump in angle of peak directivity between $2\pi a/\lambda = 5.5$ and 5.75 radians (see case 11 of Fig. 2.8) corresponds to a change in directivity of 0.5 dB and is due to a change in beam shape (compare Figs. 2.5 and 2.6). A disk radius of $2\pi a/\lambda > 60$ is required so that the angle of incidence of the peak directivity is within eight degrees of the horizon.
Figure 2.3  Numeric directivity polar plot, $2\pi a/\lambda = 3.0$. ($f=15$ MHz, $h/\lambda = 0.25, b/\lambda = 1.0 \times 10^{-6}, Z_0/\lambda = 0$. Case 1: Perfect Ground ($\varepsilon_0 = 1.0, \delta = \infty$), Case 5: Medium Dry Ground ($\varepsilon_0 = 15.0, \delta = 0.001$ S/m), Case 11: Free Space ($\varepsilon_0 = 1.0, \delta = 0$).

Figure 2.4  Numeric directivity polar plot, $2\pi a/\lambda = 4.0$. ($f=15$ MHz, $h/\lambda = 0.25, b/\lambda = 1.0 \times 10^{-6}, Z_0/\lambda = 0$. Case 1: Perfect Ground ($\varepsilon_0 = 1.0, \delta = \infty$), Case 5: Medium Dry Ground ($\varepsilon_0 = 15.0, \delta = 0.001$ S/m), Case 11: Free Space ($\varepsilon_0 = 1.0, \delta = 0$).
Figure 2.5  Numeric directivity polar plot, $2\pi a/\lambda = 5.0$. ($f=15\text{ MHz}$, \(h/\lambda = 0.25, b/\lambda = 1.0 \times 10^{-6}, Z_0/\lambda = 0\). **Case 1**: Perfect Ground ($\varepsilon_0 = 1.0, \delta = \infty$), **Case 5**: Medium Dry Ground ($\varepsilon_0 = 15.0, \delta = 0.001\text{s/m}$), **Case 11**: Free Space ($\varepsilon_0 = 1.0, \delta = 0$).)

Figure 2.6  Numeric directivity polar plot, $2\pi a/\lambda = 6.5$. ($f=15\text{ MHz}$, \(h/\lambda = 0.25, b/\lambda = 1.0 \times 10^{-6}, Z_0/\lambda = 0\). **Case 1**: Perfect Ground ($\varepsilon_0 = 1.0, \delta = \infty$), **Case 5**: Medium Dry Ground ($\varepsilon_0 = 15.0, \delta = 0.001\text{s/m}$), **Case 11**: Free Space ($\varepsilon_0 = 1.0, \delta = 0$).)
Figure 2.7 Peak directivity

Figure 2.8 Angle of incidence of peak directivity
The directivity at angles of incidence near the horizon \((82^\circ \leq \theta < 90^\circ)\), for \(0 \leq 2\pi a/\lambda \leq 8\), shows no improvement over that with no ground plane at all and, in fact, decreases aperiodically with increasing disk radius by as much as 1 dB (see Figs. 2.9 through 2.13). The directivity at angles of incidence of 82, 84, 86, 88, and 90 degrees is 4, 5, 7, 13, and \(\infty\) dB, respectively, below the peak directivity for these disk radii. Substantial improvement in directivity near the horizon occurs for \(2\pi a/\lambda > 15\) as an approximate lower bound.

### 2.1.4 Numerical results for radial-wire ground planes

Burke, et al. [23–28] obtained the absolute directivity of quarter-wave monopole elements on electrically small radial-wire ground planes in proximity to flat Earth, using a method-of-moments program for wire antennas known as the Numerical Electromagnetics Code (NEC). Numerical results for a test case used in Code version NEC-3 are shown in Fig. 2.14. The antenna parameters are \(h/\lambda = 0.25\), \(b/\lambda = 1.67 \times 10^{-4}\), \(2\pi a/\lambda = 1.26\), \(\varepsilon_r = 10.0\), \(\sigma = 0.01\) S/m, \(N = 6\), and \(b_w/\lambda = 1.67 \times 10^{-4}\), at a frequency of 5 MHz. The directivity is approximately the same as that for a disk ground plane of the same radius.
Figure 2.10 Directivity at six degrees above the horizon

Figure 2.11 Directivity at four degrees above the horizon
Figure 2.12 Directivity at two degrees above the horizon

Figure 2.13 Directivity on the horizon
2.1.5 Analytical expression for directivity

The numeric directivity determined by numerical methods or by measurements, of electrically short monopole elements on ground planes resting on lossy Earth, may be approximated analytically by a best-fit expression of the form

\[
d_r(\theta) = \begin{cases} 
  A \cos^m \theta \sin^n \theta; & 0 \leq \theta \leq \pi/2 \text{ rad} , m > 0, n > 1 \\
  0, & -\pi/2 \leq \theta < 0 \text{ rad}
\end{cases}
\]

where the exponents \( m \) and \( n \) may have non-integer values. The directivity given by equation (2-1) is zero for \( \theta = 0 \) and \( \pi/2 \) radians. The exponents \( m \) and \( n \) are chosen to yield a specific directivity in a specific direction that provide a best-fit to the actual theoretical or measured directivity. The coefficient \( A \) is chosen to satisfy the condition

\[
(1/4\pi) \int_0^{2\pi} \int_0^{\pi/2} d_r(\theta) \sin \theta d\theta d\phi = 1
\]

Accordingly,

\[
A = 2 \left( \int_0^{\pi/2} \cos^m \sin^{n+1} \theta d\theta \right)^{-1}
\]  

An analytical expression that approximates the directivity obtained by numerical methods for medium dry ground and \( 2\pi a/\lambda = 3 \) (see figure 2.15) is given by

\[
d_r(\theta) = \begin{cases} 
  10 \cos \theta \sin^3 \theta; & 0 \leq \theta \leq \pi/2 \text{ rad} \\
  0, & -\pi/2 \leq \theta < 0 \text{ rad}
\end{cases}
\]  

Equation (2-3) has a peak directivity \( d_{r\text{peak}} = 3.25 = 5.12 \) dBi in the direction \( \theta = \pi/3 \) radians. The directivity of electrically short monopole elements on ground planes resting on lossy Earth does not vary appreciably with different types of lossy Earth or for normalized ground-plane radii \( 0 \leq 2\pi a/\lambda \leq 8 \) radians. Therefore, equation (2-3) is also an approximation of the directivity obtained by numerical methods for cases (3) through (10) of table 2.1 and normalized ground-plane radii \( 0 \leq 2\pi a/\lambda \leq 8 \) radians. A more accurate analytical expression
of the directivity for a specific case is obtained by optimizing the exponents \( m \) and \( n \) in equation (2–1), and modifying the coefficient \( A \) in accordance with equation (2–2).

2.2 Degradation by Feed-Cable Exterior Current

The condition of zero exterior current on the feed cable is never realized for monopole elements with imperfect ground planes. However, this condition is approximated in practice with lossy ferrite toroidal cases around the feed cable [2]. The ferrite toroids must extend along the cable to a distance so that the element field impinging upon the cable is sufficiently weak and the current (edge-diffracted underneath the ground plane to the feed cable) is adequately attenuated.
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Figure 2.15 Numeric directivity of a quarter-wave element on a disk ground plane resting on medium dry ground, $2\pi a/\lambda = 3.0$. (a) Richmond's method-of-moments; (b) $10 \cos \theta \sin^3 \theta$.

The exterior current on the feed cable may also be minimized by burying the cable below the surface of the Earth at a sufficient depth. The depth can be much less than the field penetration depth. For example, the penetration depth for a plane-wave incident normally on medium dry ground at 15 MHz is 21m (see Table 2.1). If the feed cable is buried at a depth of 21cm (1/100 of the penetration depth), most of the exterior current that is generated on the feed cable will not be reradiated into the air medium; instead, it will be leaked off into the Earth.

A more detailed discussion of performance degradation by feed-cable exterior current is given in reference [29].
3. System Operating Noise Factor

The predetection SNR of a receiving system is inversely proportional to the system operating noise factor $f$. If the ambient temperatures of the receiving antenna, matching network, and transmission line are equal to the reference temperature, then [1]

$$ f = (f_a - 1 + l_cl_ml_nf_r)f_p $$

(3-1)

where

$f_a$ = receive antenna external noise factor integrated over the antenna pattern function (numeric)
$c, l_m, l_n$ = available loss factors of the receive antenna, matching network, and transmission line, respectively (numeric ≥ 1)
$f_r$ = receiver noise factor (numeric)
$f_p$ = signal-to-noise processing factor (numeric)

For an external noise-limited system, the system operating noise factor given by equation (3-1) reduces to

$$ f \approx f_af_p, \quad f_a \gg l_cl_ml_nf_r - 1 $$

(3-2)

where $l_c, l_m, l_n$, and $f_r$ are internal noise parameters generated by the receiving system. When the receiving system is externally noise-limited, which is one of the design objectives of the HF receiving system, the system operating noise factor is minimized and the predetection SNR is maximized. This condition is expressed as

$$ l_cl_ml_nf_r - 1 \ll f_a, \quad \text{externally noise-limited receiving system.} $$

The external noise factor $f_a$ is a function of the directivity of the receive antenna [1]. The available loss factors $l_c, l_m, l_n$ and the receiver noise factor $f_r$ are evaluated in references [1, 30] and [31] as functions of the circuit impedance parameters and source impedances of the respective circuits. The available loss factors are equal to the ratio of the input to output available powers of the respective circuits, which is unity when the ohmic loss of the circuit is zero. The ohmic loss
of the receive antenna earth-ground system is conventionally included in the available loss factor \( \ell_c \) of the receive antenna (rather than in the excess propagation loss factor) in HF receive systems [1]. The receiver noise factor \( f_r \) is a function of the receiver source admittance and four empirical noise parameters of the receiver [30].

The parameters \( f_a, \ell_c, \ell_m, \ell_n, \) and \( f_r \) are all functions of the extent, conductivity, and density of the receive antenna ground plane. The parameters \( \ell_m, \ell_n, \) and \( f_r, \) which are indirect functions of these ground-plane parameters, become stabilized (as a function of ground-plane radius and Earth permittivity) for small ground planes (see Section 3.2). The antenna available loss factor \( \ell_c, \) a direct monotonic function of these ground-plane parameters, requires a much larger ground plane before becoming stabilized (see Section 3.1).

3.1 Influence of Ground Losses on the Antenna Available Loss Factor

The antenna available loss factor \( \ell_c \) is defined as the ratio of the available power at the input of the antenna to the radiated far-field power. For an antenna in free space or in proximity to dielectric Earth \((\sigma = 0)\), the far-field power is radiated into the hemispheres above and below the antenna. However, for an antenna in proximity to lossy Earth \((\sigma > 0)\), the radiated far-field power is confined to the hemisphere above the Earth because any power radiated downward into lossy Earth is attenuated at large distances at a rate greater than \(1/\tau^2\). The antenna available loss factor \( \ell_c \) is given in reference [30] as:

\[
\ell_c = 1/\eta = R_{in}/R_{rad} = 1 + (R_c/R_{rad}) \tag{3-3}
\]

where

\[\eta = \text{antenna radiation efficiency} = \text{fraction of the available input power that is radiated to the far field (numeric \leq 1)}\]

\[R_{in} = \text{antenna input resistance} = R_c + R_{rad} \text{ (ohms)}\]

\[R_{rad} = \text{antenna radiation resistance} \text{ (ohms)}\]

\[R_c = \text{series resistance due to ohmic losses of the antenna circuit including ground losses, but excluding losses in the matching network (ohms)}\]
The series resistance $R_c$ is only from ohmic losses in the Earth because the element and ground plane are assumed to be fabricated from metal of infinite conductivity.

Hansen [32] has determined the radiation efficiency $\eta$ of vertical elements in proximity to lossy Earth ($\sigma > 0$) for Hertzian and half-wave dipoles above lossy Earth. Weiner [19–21] has determined the radiation efficiency for quarter-wave monopole elements on disk ground planes resting on Earth, and Weiner, Zamoscianyk, and Burke [33, 34] have determined it for quarter-wave elements on radial-wire ground planes in proximity to Earth. For an element above dielectric Earth ($\sigma = 0$) or in free space the radiation efficiency $\eta = 1$ since the element and ground plane have been assumed to be fabricated from metal of infinite conductivity.

In addition to the radiation efficiency $\eta$, we define a modified radiation efficiency $\eta_d$ that is equal to the ratio of the far-field power radiated in the hemisphere above the Earth to the available input power. Their relationship, for the cases of an element above lossy Earth ($\sigma > 0$) or dielectric Earth ($\sigma = 0$), is given by:

$$\eta_d = \begin{cases} 
\eta, & \text{element above lossy Earth (\(\sigma > 0\))} \\
\text{or a perfect ground plane (\(\sigma = \infty\))}
\end{cases}$$

$$P_{\text{air}}/(P_{\text{air}} + P_{\text{earth}}), \quad \text{element above dielectric Earth (\(\sigma = 0\))}$$

(3–4)

where

$$P_{\text{air}} = \text{far-field power radiated into upper hemisphere (air)}$$

$$P_{\text{earth}} = \text{far-field power radiated into lower hemisphere (earth)}$$

King [36] and Burke [37] have determined the modified radiation efficiency $\eta_d$ for a Hertzian dipole above dielectric Earth.

The following subsections present numerical results of Earth-ground losses for these cases: a vertically polarized dipole above lossy Earth, a vertically polarized Hertzian dipole above dielectric Earth, a quarter-wave monopole element on a disk ground plane resting on Earth, and a quarter-wave monopole element on a radial-wire ground-screen in proximity to Earth. A discussion of the role of the surface wave in directing energy into the Earth is included.
3.1.1 Vertically polarized dipole above lossy earth

The radiation efficiency of a vertical dipole above lossy Earth increases non-monotonically with increasing height above Earth; non-monotonically with increasing length of the dipole; and non-monotonically with increasing conductivity of the Earth [32]. Minimum radiation efficiency occurs for a Hertzian dipole at zero height above the Earth. Its numerical value is not clear in reference [32]. For lossy earth ($\sigma > 0$), computer runs of the NEC-3 program using the Sommerfeld option yield a minimum radiation efficiency $\eta = 0$ in the limit of the dipole length $h$ approaching zero and the dipole height $|z_0|$ approaching zero (see Table 3.1). A Fresnel reflection coefficient model is grossly inaccurate in computing absolute power gain, ground losses, radiation efficiency, or input impedance for an element above lossy Earth (see Table 3.1 for electrically short dipoles whose base rests on Earth), although such a model accurately computes the absolute directivity [38]. A Fresnel reflection coefficient model yields results for directivity similar to those obtained by Richmond’s method-of-moments for disk ground planes when the disk radius approaches zero, but it yields different results from Richmond’s method-of-moments for input impedance, absolute power gain, and radiation efficiency. The Fresnel reflection coefficient model accurately calculates the relative energy reflected in various directions into the upper hemisphere, but it neglects the leaky quasi-evanescent surface wave that is generated by the incident spherical wavefront in proximity to the air-earth interface (see Section 3.1.5).

3.1.2 Vertically polarized Hertzian dipole above dielectric earth

For a vertically polarized Hertzian dipole at zero height ($|z_0| \rightarrow 0$) above dielectric Earth, the modified radiation efficiency $\eta_d$ is greater than zero for $\epsilon_r < \infty$ and is equal to $0.09 - 0.10$ for $9 \leq \epsilon_r \leq 81$ (see Figs. 3.1 and 3.2 showing data generated by King [36] and Burke [37]). King’s results, based on a Sommerfeld integral model, are approximate because his analytical formulation is subject to the condition $\epsilon_r \gg 1$ (or equivalently, $k_1/k_2 \geq 3$). However, Burke’s results, which are based on the NEC-3 method-of-moments program, are considered to be the best available.

The absolute directivity pattern of a vertically polarized Hertzian dipole at zero height above dielectric Earth is shown in Fig. 3.3 for
<table>
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<th>CCIR 527-1 Earth Classification (Except (10))</th>
<th>Radiation Efficiency $\eta$ (numeric)</th>
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</thead>
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<tr>
<td>$\lambda = 0.01$</td>
<td>$\lambda = 0.02$</td>
</tr>
<tr>
<td>---------------------------------------------</td>
<td>--------------------------------------</td>
</tr>
<tr>
<td>(1) Perfect Ground</td>
<td></td>
</tr>
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</tr>
<tr>
<td>Fresnel</td>
<td>1.000</td>
</tr>
<tr>
<td>(2) Sea Water</td>
<td></td>
</tr>
<tr>
<td>Sommerfeld*</td>
<td>2.33 (10^{-2})</td>
</tr>
<tr>
<td>Fresnel</td>
<td>8.67 (10^{-3})</td>
</tr>
<tr>
<td>(3) Fresh Water</td>
<td></td>
</tr>
<tr>
<td>Sommerfeld*</td>
<td>7.34 (10^{-3})</td>
</tr>
<tr>
<td>Fresnel</td>
<td>2.82 (10^{-3})</td>
</tr>
<tr>
<td>(4) Wet Ground</td>
<td></td>
</tr>
<tr>
<td>Sommerfeld*</td>
<td>4.42 (10^{-4})</td>
</tr>
<tr>
<td>Fresnel</td>
<td>9.46 (10^{-5})</td>
</tr>
<tr>
<td>(5) Medium Dry Ground</td>
<td></td>
</tr>
<tr>
<td>Sommerfeld*</td>
<td>1.30 (10^{-3})</td>
</tr>
<tr>
<td>Fresnel</td>
<td>1.73 (10^{-3})</td>
</tr>
<tr>
<td>(6) Very Dry Ground</td>
<td></td>
</tr>
<tr>
<td>Sommerfeld*</td>
<td>1.05 (10^{-3})</td>
</tr>
<tr>
<td>Fresnel</td>
<td>6.30 (10^{-4})</td>
</tr>
<tr>
<td>(7) Pure Water, 20°C</td>
<td></td>
</tr>
<tr>
<td>Sommerfeld*</td>
<td>3.63 (10^{-2})</td>
</tr>
<tr>
<td>Fresnel</td>
<td>6.65 (10^{-2})</td>
</tr>
<tr>
<td>(8) Ice, -1°C</td>
<td></td>
</tr>
<tr>
<td>Sommerfeld*</td>
<td>1.16 (10^{-3})</td>
</tr>
<tr>
<td>Fresnel</td>
<td>6.99 (10^{-3})</td>
</tr>
<tr>
<td>(9) Ice, -10°C</td>
<td></td>
</tr>
<tr>
<td>Sommerfeld*</td>
<td>3.84 (10^{-3})</td>
</tr>
<tr>
<td>Fresnel</td>
<td>2.30 (10^{-4})</td>
</tr>
<tr>
<td>(10) Average Land</td>
<td></td>
</tr>
<tr>
<td>Sommerfeld*</td>
<td>1.16 (10^{-4})</td>
</tr>
<tr>
<td>Fresnel</td>
<td>1.97 (10^{-4})</td>
</tr>
<tr>
<td>(11) Free Space</td>
<td></td>
</tr>
<tr>
<td>Sommerfeld*</td>
<td>1.000</td>
</tr>
<tr>
<td>Fresnel</td>
<td>1.000</td>
</tr>
</tbody>
</table>

*More accurate result:

 Sommerfeld = NEC-3 program in Sommerfeld option, $N = 9$ segments, $b/\lambda = 10^{-5}$, voltage excitation at 5th segment.
 Fresnel = NEC-3 program in Fresnel reflection coefficient option, $N = 9$ segments, $b/\lambda = 10^{-5}$, voltage excitation at 5th segment.

Table 3.1 Radiation efficiency of a vertically polarized thin dipole of length $h$ whose base has zero current and zero height above Earth, $f = 15$ MHz.

the case $\epsilon_r = 9$. It was approximated by using the Sommerfeld option of the NEC-3 program with a voltage excitation source between dielectric Earth and the base of an electrically short ($h/\lambda = 2 \times 10^{-4}$), electrically thin ($b/\lambda = 2 \times 10^{-4}$) monopole element. In the upper hemisphere (air), the radiation pattern is similar to that for the case of lossy Earth, except that the peak directivity is reduced by approximately 11 dB (compare with Figs. 2.2 and 2.7). In the lower hemisphere (dielectric Earth), however, the radiation pattern is remarkable because most of the energy is radiated at the critical angle $\theta_c$. The peak direc-
tivity is 15.6 dBi at an angle of transmission $\theta_t = 19.5$ degrees which corresponds to

$$\theta_t = \theta_c = \arcsin \left( (\epsilon_r)^{-1/2} \right),$$

Hertzian dipole on dielectric Earth

\( (3-5) \)

where

$\theta_c =$ critical angle = angle of reflection for a plane-wave incident

from a less dense medium at an angle of incidence approaching $\pi/2$ radians.

The modified radiation efficiency is $\eta_d = 0.103$ (see Fig. 3.3). The fraction of the available input power that is transmitted into the dielectric Earth is $1 - \eta_d = 0.897$ (compare with 1.0 for lossy Earth), which is transmitted to the far field (compare with zero for lossy Earth) at the critical angle $\theta_c$. The role of the surface wave in directing energy into the earth is discussed in Section 3.1.5.

### 3.1.3 Quarter-wave monopole element on a disk ground plane resting on earth

Weiner [19-21] has used Richmond's method-of-moments [22] to obtain the radiation efficiency of quarter-wave monopole elements on disk ground planes resting on Earth for the CCIR-527-1 classifications given in Table 2.1. The radiation efficiency $\eta$ increases monotonically with increasing disk radius $a$ (see Fig. 3.4). The radiation efficiency incorrectly differs from unity for Case 11 (free space) because of a computer program artifact for that case.

### 3.1.4 Quarter-wave monopole element on a radial-wire ground-screen in proximity to earth

Numerical results for the radiation efficiency of quarter-wave monopole elements with radial-wire groundscreens in proximity to Earth have been obtained by Weiner, Zamoscianyk and Burke [33-35] using Burke's method-of-moments code NEC-GS. These results are plotted in Fig. 3.5 for the groundscreen at a height $z_0/\lambda = -10^{-4}$ above Earth of relative complex permittivity $\epsilon^*/\epsilon_0 = 15 - j1.5$. This permittivity corresponds approximately to medium dry ground at a
tivity is 15.6 dBi at an angle of transmission $\theta_t = 19.5$ degrees which corresponds to

$$\theta_t = \theta_c = \arcsin \left[ (\epsilon_r)^{-1/2} \right],$$

Hertzian dipole on dielectric Earth

(3–5)

where

$\theta_c =$ critical angle $= \text{angle of reflection for a plane-wave incident from a less dense medium at an angle of incidence approaching } \pi/2 \text{ radians.}$

The modified radiation efficiency is $\eta_d = 0.103$ (see Fig. 3.3). The fraction of the available input power that is transmitted into the dielectric Earth is $1 - \eta_d = 0.897$ (compare with 1.0 for lossy Earth), which is transmitted to the far field (compare with zero for lossy Earth) at the critical angle $\theta_c$. The role of the surface wave in directing energy into the earth is discussed in Section 3.1.5.

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Figure 3.1 Radiation efficiency of vertically polarized Hertzian dipole above dielectric Earth, $|z_0|/\lambda = 0$ and $|z_0|/\lambda = \infty$.

frequency of 15 MHz (see case 5 of Table 2.1). The radiation efficiency increases monotonically with increasing number of radial wires. The results for $N = 128$ radial wires as a function of ground plane radius, are in close agreement with Richmond’s method of moments for disk ground planes.

3.1.5 *Role of surface wave in directing energy into the earth*

Why is the radiation efficiency (and modified radiation efficiency) of an electrically short element in close proximity to Earth so small? If the source wavefront were a plane-wave incident at small grazing angles onto a semi-unbounded denser medium, most of the energy would be reflected. Instead, most of it is transmitted into the denser medium because a surface wave is generated.
Hertzian dipole at height $|z_0|$ in air ($k_2$) over a dielectric half-space ($k_1 = k_2 \sqrt{\varepsilon_r}$)

$$\eta_d = \frac{P_{\text{air}}}{P_{\text{air}} + P_{\text{earth}}}$$

<table>
<thead>
<tr>
<th>$k_1/k_2$</th>
<th>King</th>
<th>Burke</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>$0.087$</td>
<td>$0.089$</td>
</tr>
<tr>
<td>6</td>
<td>$0.094$</td>
<td>$0.094$</td>
</tr>
<tr>
<td>3</td>
<td>$0.088$</td>
<td>$0.103$</td>
</tr>
</tbody>
</table>

Figure 3.2 Radiation efficiency of vertically polarized Hertzian dipole above dielectric Earth, $0 \leq |z_0|/\lambda \leq 2$.

In addition to energy being transmitted and absorbed into Earth by reflection of the direct and indirect waves, it is also directed into the Earth by a leaky quasi-evanescent surface wave that is generated in the air medium in proximity to the air-earth interface. The surface wave is a quasi-evanescent field in the air-medium only, but leaks energy into the Earth medium, not into the air medium [36]. The surface wave is evanescent in the air medium in the sense that it does not radiate into the far field but instead is confined to the space near the Earth’s surface. However, the surface wave is not truly evanescent in that medium penetration into the air medium is a function of range rather than being a simple exponential function of distance from the Earth’s surface. The dipole radiation efficiency is small when the dipole is in close proximity to Earth because energy leaks into the Earth in the immediate vicinity of the dipole where the fields are strongest.

The explanation of the surface wave directing most of the available input energy into the Earth is consistent with the radiation pattern of
Fig. 3.3. Most of the available input power is transmitted into the dielectric Earth at an angle of transmission equal to the critical angle. Goos and Hanchen [39] demonstrated the role of the surface wave in displacing the location of the reflected wave using a collimated narrow beam of light incident onto a less dense medium at angles of incidence greater than or equal to the critical angle.

The generation and analysis of surface waves are described in the literature extensively on the basis of the Sommerfeld integral (see list of references within reference [40]). Booker and Clemmow [41, 42] determine the equivalent aperture distribution in the vertical plane that gives the surface and space-wave fields in the air medium above the Earth. The aperture distribution consists of the dipole, its image, and an infinite line source extending downward from the image. The line source, which generates the surface wave, disappears when the Earth is perfectly conducting. The aperture distribution yields an angular spectrum of reflected plane waves whose complex amplitude is determined by the Fresnel reflection coefficient for each angle of the spectrum. The
amplitude spectrum of the Fresnel coefficients is the Fourier transform of the aperture distribution. Brekhovskikh [43] combines the method of steepest descents (saddle points) with the decomposition of a spherical wavefront into an angular spectrum of plane waves to give a complete description of the direct wave, reflected wave, and surface wave at all points in space for a wide variety of cases, including the case of Goos-Hänchen. Brekhovskikh's analysis is applicable to spherical-wave refraction, whereas Fresnel's reflection and transmission coefficients are valid for plane-wave refraction by a semi-unbounded medium.

3.2 Ground-Plane Stabilization of the Available Loss Factors and Receiver Noise Factor

The available loss factors of the receive antenna, matching network, and transmission line ($\ell_c$, $\ell_m$, $\ell_n$, respectively), and the receiver noise factor $f_r$ are functions of the source impedance of each of the respective circuits. Consequently, these parameters are a function of the receive antenna input impedance which, in turn, is a function of the receive antenna ground-plane system.

Figure 3.4 Radiation efficiency of a quarter-wave element on a disk ground plane resting on medium dry ground.
3.2.1 Functional dependence on antenna input impedance

The impedance equivalent circuit of the receiving system is given in reference [30]. Sequentially, the receiving system consists of the antenna, matching network, transmission line, and receiver. The receiving system available loss factors $\ell_c$, $\ell_m$, and $\ell_n$ of the antenna, matching network, and transmission line, respectively, are given by references [30] and [45] as:

$$\ell_c = \frac{R_{in}}{R_{rad}} \quad (3-6)$$

$$\ell_m = 1 + \frac{[R_m + R_s]}{R_{in}} \quad (3-7)$$

$$\ell_n = \frac{\exp(2ad)}{1 - |\Gamma|^2 - 2[\text{Im}\ Z_0/\text{Re}\ Z_0]\text{Im}\ \Gamma \cdot \left\{1 - |\Gamma|^2 \exp(-4ad) - 2[\text{Im}\ Z_0/\text{Re}\ Z_0]\text{Im}\ [\Gamma \exp(-2\gamma d)]\right\}}$$

(3-8)
where

\[ R_{\text{in}} = \text{antenna input resistance (ohms)} = R_{\text{rad}} + R_c \]
\[ R_{\text{rad}} = \text{antenna radiation resistance (ohms)} \]
\[ R_c = \text{series ohmic resistance of antenna circuit (ohms)} \]
\[ R_m, R_s = \text{series resistances of matching network and its} \]
\[ \text{switching circuit, respectively (ohms)} \]
\[ \Gamma = \text{voltage reflection coefficient looking back at the matching} \]
\[ \text{network} = (Z - Z_0)/(Z + Z_0) \]
\[ Z = \text{output impedance of matching network (ohms)} = \]
\[ \begin{cases} 
Z_0, & \text{perfect matching network} \\
Z_{\text{in}}, & \text{no matching network} 
\end{cases} \]
\[ Z_0 = \text{characteristic impedance of transmission line (ohms)} \]
\[ Z_{\text{in}} = \text{antenna input impedance} = R_{\text{in}} + jX_{\text{in}} \text{ (ohms)} \]
\[ \alpha = \text{attenuation coefficient of transmission line (m}^{-1}) \]
\[ \beta = \text{phase constant of transmission line (m}^{-1}) \]
\[ d = \text{length of transmission line (m)} \]
\[ \gamma = \alpha + j\beta = \text{propagation constant of transmission line (m}^{-1}) \]

Equations (3–6), (3–7), and (3–8) are functions of the antenna input impedance \( Z_{\text{in}} \). In the absence of a matching network, \( \ell_m = 1 \). With a perfect matching network (\( R_m = R_s = 0 \), antenna is conjugate-matched to transmission line), \( \ell_m = 1 \) and \( \ell_n = \exp(2\alpha d) \).

The receiver noise factor \( f_r \) is a function of the source admittance \( Y_{\text{source}} \) seen by the receiver looking back at the transmission line, given by reference [30] as

\[ Y_{\text{source}} = \frac{1}{Z_0} \left[ \frac{1 - \Gamma \exp(-2\gamma d)}{1 + \Gamma \exp(-2\gamma d)} \right] \quad (3-9) \]

For a long lossy transmission line or a perfect matching network, \( Y_{\text{source}} = 1/Z_0 \). The receiver noise factor \( f_r \) is given by reference [29] as:

\[ f_r = f_0 + \left( r_n/\text{Re} Y_{\text{source}} \right)|Y_{\text{source}} - y_{\text{no}}|^2 \quad (3-10) \]
\[ f_0 = \text{minimum noise factor of receiver for any possible source admittance } Y_{\text{source}} \]

\[ r_n = \text{empirical receiver noise parameter with the dimensions of resistance that account for the sensitivity to source impedance of the receiver noise factor} \]

\[ y_{no} = g_{no} + j b_{no} = \text{complex empirical receiver noise parameter with the dimensions of admittance (siemens)} \]

Equations (3–9) and (3–10) are generally functions of the antenna input impedance \( Z_{\text{in}} \).

3.2.2 Input impedance of a quarter-wave monopole element on a disk ground plane resting on earth

The antenna input resistance \( Z_{\text{in}} \) is given by

\[ Z_{\text{in}} = R_{\text{in}} + j X_{\text{in}} = (R_{\text{rad}} + R_{\text{c}}) + j X_{\text{in}} \quad (3-11) \]

where

\[ R_{\text{rad}} = \text{antenna radiation resistance (ohms)} = R_{\text{in}} \eta \]

\[ \eta = \text{antenna radiation efficiency (numeric)} \]

\[ R_{\text{c}} = \text{series ohms resistance of antenna (ohms)} \]

\[ = R_{\text{in}} - R_{\text{rad}} = R_{\text{in}} (1 - \eta) \]

Weiner \[19,20\] used Richmond's method-of-moments \[22\] to obtain the radiation resistance \( R_{\text{rad}} \), input resistance \( R_{\text{in}} \), and input reactance \( X_{\text{in}} \) of a quarter-wave monopole element on a disk ground plane resting on Earth for the CCIR-527-1 classifications of Earth given in Table 2.1. Numerical results for medium dry ground at \( f = 15 \text{ MHz} \) are presented in Figs. 3.6, 3.7, and 3.8. The radiation resistance \( R_{\text{rad}} \) increases periodically as the disk radius increases (see Fig. 3.6). The
Figure 3.6 Radiation resistance of a quarter-wave monopole element on a disk ground plane resting on medium dry ground

input resistance decreases aperiodically to an asymptotic value of 36 ohms as the disk radius increases (see Fig. 3.7). The input reactance increases aperiodically to an asymptotic value of 22 ohms for a very thin monopole element (see Fig. 3.8). The input reactance is a stronger function of the element radius than the radiation resistance [2].

3.2.3 Input impedance of a quarter-wave monopole element on a radial-wire groundscreen in proximity to earth

Weiner, Zamoscianyk, and Burke [33,34] have used the NEC-GS program to obtain numerical results of the input resistance $R_{in}$ and input reactance $X_{in}$ of a quarter-wave monopole element on a radial-wire groundscreen in proximity to Earth. These results are plotted in Figs. 3.9 and 3.10 for the groundscreen buried at a depth $z_0/\lambda = 10^{-4}$ in medium dry ground at $f = 15$ MHz.

The input resistance and reactance asymptotically approach the result for a disk ground plane as the groundscreen density approaches infinity (that is, as the number of radial wires $N \to \infty$). The reso-
nances that occur in input impedance for a sparse number of radial wires (see Figs. 3.9 and 3.10) provided that the wires are not in proximity to earth of high conductivity [32,33], are a unique characteristic of radial-wire ground planes. They occur apparently because the currents on the sparse radial wires are not closely coupled, unlike those of the case for a high density of radial wires [33,34].

3.2.4 Antenna input impedance stabilization with respect to ground-plane radius and earth permittivity

Antenna impedance mismatch is not as critical for receiving systems as it is for transmitting systems because the rf power levels in receiving systems are orders of magnitude less. Nevertheless, antenna impedance mismatch can cause a significant increase in the internal noise factor [30]. This problem is particularly acute in HF receiving systems because they usually employ electrically short elements that must operate over a wide range of frequencies. Therefore, there is interest in stabilizing the antenna receive mode input impedance so that
Figure 3.8 Input reactance of a quarter-wave monopole element on a disk ground plane resting on medium dry ground

the system remains externally noise-limited.

The antenna input impedance is a function of the normalized element length $h/\lambda$ (the most influential parameter); the ground-plane system; and the normalized element radius $b/\lambda$ which influences input reactance more than input resistance. The normalized ground-plane radius $2\pi a/\lambda$; groundscreen density (or number $N$ of radial wires); normalized ground-plane height $z_0/\lambda$ of the earth’s surface relative to the ground plane; and Earth relative permittivity $\varepsilon^*/\varepsilon_0$ also affect input impedance.

Normalized ground-plane radii of at least $2\pi a/\lambda = 2$ radians are required to stabilize the input impedance of quarter-wave elements with high-density ground planes to within 20 percent of that for a perfect ground plane (see Figs. 3.7 through 3.9 for stabilization as a function of ground plane radius and reference 20 for stabilization as a function of Earth relative permittivity). The minimum number of radial wires required to achieve this degree of stabilization is approximately $N = 32$ (see Figs. 3.8 and 3.9). The input impedance of
monopole antennas with low-density ground planes is stabilized with ground-plane radii smaller than those required for high-density ground planes.

3.3 Comparison of External Noise Factor with CCIR Predictions

The external noise factor $f_a$ in (3–1), (3–2), and (3–3) is a function of the directivity (and, consequently, ground-plane characteristics) of the receive antenna unless the angular distribution $f(\theta, \phi)$ of the external noise is uniform in the hemisphere above lossy Earth. Within this section, we determine the external noise factor as a function of the ground-plane radius and compare the results with CCIR predicted values [45–47]. For normalized ground-plane radii less than at least eight radians, the external noise factor is approximately independent of the ground-plane radius and approximately equal to the CCIR predicted values in the HF band for the same angular distribution of CCIR external noise.
Figure 3.10 Input reactance of a quarter-wave monopole element on a radial-wire groundscreen; $z_0/\lambda = 10.4$, $\varepsilon^*/\varepsilon_0 = 15 - j1.5$

The external noise factor of a receiving system is given in reference 1 as

$$f_a = \frac{1}{4\pi} \int_{0}^{2\pi} \int_{-\pi/2}^{\pi/2} f(\theta, \phi) d_r(\theta, \phi) \sin \theta d\theta d\phi$$

(3–12)

where

$$f(\theta, \phi) = \text{external noise angular distribution (numeric with the same dimensionality as } f_a \text{ and normalized to the same reference noise as } f_a)$$

$$d_r(\theta, \phi) = \text{numeric directivity of the receive antenna}$$

For an antenna in proximity to lossy earth, $f(\theta, \phi) = 0$ and $d_r(\theta, \phi) = 0$ and in the hemisphere below the ground. Therefore, (3–12) reduces to
\[ f_a = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{2\pi} f(\theta, \phi) \, d_r(\theta, \phi) \sin \theta \, d\theta \, d\phi \]  

(3-13)

antenna in proximity to lossy earth

For angular distributions of external noise that are either uniform 
\([f(\theta, \phi) = f_a0]\) or a point source 
\([f(\theta, \phi) = (4\pi / \sin \theta_0)f_a0 \delta(\theta - \theta_0) \delta(\phi - \phi_0)]\), Eq. (3-13) reduces to

\[
f_a = \begin{cases} 
  f_a, & \text{uniform angular distribution} \\
  f_a0d_r(\theta_0, \phi_0), & \text{point source angular distribution for a source at } \theta = \theta_0, \phi = \phi_0 
\end{cases} \tag{3-14}
\]

The external noise factor is independent of directivity for uniform external noise, but it is proportional to directivity for a point source of external noise.

Section 2 demonstrates that the directivity of a monopole element with a circular ground plane resting on lossy Earth is approximately independent of the ground-plane radius for normalized ground-plane radii \(0 \leq 2\pi a/\lambda \leq 8.0\) radians (see Figs. 2.2 through 2.13). Therefore, the external noise factor \(f_a\) is approximately independent of ground-plane radius for normalized ground-plane radii at least as small as eight radians.

The CCIR (French) published statistical values of \(f_a\) for atmospheric noise in the frequency range 0.01 through 20 MHz based on measurements as a function of location, time of day, and season [44,46]; and man-made noise in the frequency range 0.25 through 250 MHz based on measurements as a function of type of location [47]. These values, denoted by \(f_{aCCIR}\), are claimed to be normalized to correspond to those values that would be measured with an electrically short vertically polarized monopole element mounted on a perfect ground plane [45–47]. It should be noted, however, that the antenna used for the CCIR measurements is a monopole element 21.75 feet long and a ground plane consisting of 90 radial wires (each 100 feet long and situated eight feet above the ground) [48]. The HF CCIR values most likely correspond to those values that would be measured with an electrically short vertically polarized monopole element mounted
on an electrically small ground plane in proximity to lossy Earth, as discussed in the remainder of this subsection.

The CCIR external noise factor $f_{CCIR}$ is given by

$$
f_{aCCIR} = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{2\pi} f_{CCIR}(\theta, \phi) d_{rCCIR}(\theta) \sin \theta d\theta d\phi \quad (3-15)$$

where

$$f_{CCIR}(\theta, \phi) = \text{angular distribution of external noise for CCIR database}$$

$$d_{rCCIR}(\theta) = \text{numeric directivity of antenna for CCIR database}$$

A receiving system's external noise factor $f_a$ expressed in terms of CCIR external noise factor $f_{aCCIR}$ is found from Eqs. (3-13) and (3-15) to be

$$\frac{f_a}{f_{aCCIR}} = \frac{\int_0^{2\pi} \int_0^{2\pi} f(\theta, \phi) d_r(\theta, \phi) \sin \theta d\theta d\phi}{\int_0^{2\pi} \int_0^{2\pi} f_{CCIR}(\theta, \phi) d_{rCCIR}(\theta, \phi) \sin \theta d\theta d\phi} \quad (3-16)$$

Consider a receive antenna consisting of an electrically short monopole element with an electrically small ground plane resting on flat lossy Earth. Its directivity $d_r(\theta, \phi) = d_r(\theta)$ is analytically approximated by Eq. (2-3) for medium dry Earth and $2\pi a/\lambda = 3$ and is approximately valid for cases (3) through (11) of Table 2.1 and normalized ground-plane radii $0 \leq 2\pi a/\lambda \leq 8$ radians. What is the ratio $f_a/f_{aCCIR}$ for the condition $f(\theta, \phi) = f_{CCIR}(\theta, \phi)$?

Since $f_{CCIR}$ is unknown (the electrically short monopole element used to obtain the CCIR database of $f_{CCIR}(\theta, \phi)$ is omnidirectional in the azimuthal direction and has poor angular resolution in the vertical plane), consider these arbitrary cases:
Case 1: \( \frac{f_{CCIR}(\theta, \phi)}{f_{aCCIR}} = 1 \)

Case 2: \( \frac{f_{CCIR}(\theta, \phi)}{f_{aCCIR}} = b \sin^8 \theta \)

where

\[
b = 2 \left[ \int_{0}^{\pi/2} \sin^9 \theta \, d_{rCCIR}(\theta) \, d\theta \right]^{-1}
\]

Case 3: \( \frac{f_{CCIR}(\theta, \phi)}{f_{aCCIR}} = b \cos \theta \sin^3 \theta \)

where

\[
b = 2 \left[ \int_{0}^{\pi/2} \cos \theta \sin^4 \theta \, d_{rCCIR}(\theta) \, d\theta \right]^{-1}
\]

Case 1 corresponds to a uniform angular distribution. Case 2 corresponds to an angular distribution with a maximum gain on the horizon and a 50 percent value at \( \theta = 66 \) degrees. Case 3 corresponds to an angular distribution with a maximum gain at \( \theta = 60 \) degrees and a shape identical to that of \( d_r(\theta) \) given by Eq. (2–3) for medium dry Earth and \( 2\pi a/\lambda = 3.0 \).

If the values of \( f_{aCCIR} \) have been normalized to correspond to those values that would be measured with an electrically short monopole element on a perfect ground plane, then

\[
d_{rCCIR}(\theta) = \begin{cases} 
3 \sin^2 \theta, & 0 \leq \theta \leq \pi/2 \\
0, & -\pi/2 \leq \theta < 0 
\end{cases} \quad (3–17)
\]

perfect ground plane

However, we contend that the values of \( f_{aCCIR} \) in the HF band correspond more closely to those values that would be measured with an electrically small ground plane. The ground plane radial wires used to obtain the CCIR database are of length \( a = 100 \) feet, corresponding to \( 2\pi a/\lambda = 1.9 \) and 19 radians at 3 and 30 MHz, respectively. In Section 2, we showed that the directivity pattern of monopole elements with ground planes resting on lossy Earth is not appreciably
different for normalized ground-plane radii $0 \leq 2\pi a/\lambda \leq 8$ radians. We suspect that the directivity pattern for $2\pi a/\lambda \leq 19$ radians more closely resembles that for an electrically small ground plane than for an electrically large ground plane. Except for normalization to correct for ohmic losses of the antenna system, it is doubtful that the CCIR values of $f_{aCCIR}$ are normalized to correspond to those for a perfect ground plane. That kind of normalization would require a knowledge of the elevation angular distribution $f_{CCIR}(\theta)$, which was experimentally not known; and the directivity of monopole elements with electrically small ground planes resting on lossy Earth, which was not known theoretically for disk ground planes until 1990 [19,20] and for radial-wire ground planes until 1989 [28]. Although the antenna input impedance for the radial-wire ground plane used to obtain the CCIR database is not appreciably different from that for a perfect ground plane, the directivity pattern in the HF band at angles near the horizon is significantly different. Therefore, we contend that the numeric directivity $d_{rCCIR}(\theta)$ of the antenna used to obtain the CCIR database could not have been normalized to that for a perfect ground plane. We further contend that the CCIR database [45–47] in the HF band is better approximated by that for an electrically small ground plane for normalized ground-plane radii $0 \leq 2\pi a/\lambda \leq 8.0$ radians and possibly larger radii. Accordingly, $d_{rCCIR}(\theta)$ is better approximated by Eq. (2–3) than by Eq. (3–17); namely,

$$
d_{rCCIR}(\theta) = \begin{cases} 
10 \cos \theta \sin^3 \theta, & 0 \leq \theta \leq \pi/2, \\
0, & -\pi/2 \leq \theta < 0 
\end{cases} \quad (3-18)
$$

electrically small ground planes on lossy earth, $0 \leq 2\pi a/\lambda \leq 8.0$ radians.

The external noise factor of electrically short monopole elements with electrically small ground planes resting on lossy Earth is equal to the CCIR measured values [45–47] for the same angular distribution of external noise, assuming that the values are normalized to those for a similar antenna (see Table 3.2). However, if the CCIR measured values were normalized to those for an electrically short monopole element with a perfect ground plane (which we dispute), then the external noise factor would be unchanged for uniform external noise; reduced
Receive Antenna: Electrically short monopole element with an electrically small ground plane resting on lossy earth

\[ d_1(\theta) = 10 \cos \theta \sin^2 \theta; \quad 0 \leq \theta \leq \pi/2 \text{ rad}, \text{ zero otherwise} \]

CCIR Antenna (1): Electrically short monopole element with a perfect ground plane

\[ d_{\text{CCIR}}(\theta) = 3 \sin^2 \theta; \quad 0 \leq \theta \leq \pi/2 \text{ rad}, \text{ zero otherwise} \]

CCIR Antenna (2): Electrically short monopole element with an electrically small ground plane resting on lossy earth

\[ d_{\text{CCIR}}(\theta) = 10 \cos \theta \sin^2 \theta; \quad 0 \leq \theta \leq \pi/2 \text{ rad}, \text{ zero otherwise} \]

<table>
<thead>
<tr>
<th>Case</th>
<th>CCIR No.</th>
<th>Angular Distribution of CCIR External Noise</th>
<th>External Noise Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>Noise Distribution</td>
<td>( f_{\text{CCIR}}(\theta)/f_{a\text{CCIR}} )</td>
<td>( f_{a}/f_{a\text{CCIR}} )</td>
</tr>
<tr>
<td>1</td>
<td>Uniform</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>Peak on Horizon</td>
<td>1.805 ( \sin^4 \theta )</td>
<td>2.6 ( \sin^4 \theta )</td>
</tr>
<tr>
<td>3</td>
<td>Peak at ( \theta = 60 ) deg</td>
<td>4.667 ( \cos \theta \sin^3 \theta )</td>
<td>3.9 ( \cos \theta \sin^3 \theta )</td>
</tr>
<tr>
<td>4</td>
<td>Point Source on Horizon</td>
<td>0.667 ( \delta [\theta - (\pi/2)] )</td>
<td>A ( \delta [\theta - (\pi/2)], A = \infty )</td>
</tr>
</tbody>
</table>

Table 3.2. Comparison of external noise factor with CCIR measured values, \( f(\theta) = f_{\text{CCIR}}(\theta) \).

by a factor \( f_a/f_{a\text{CCIR}} = 0.69 = -1.6 \) dB for external noise with a peak on the horizon; and increased by a factor \( f_a/f_{a\text{CCIR}} = 1.19 = 0.8 \) dB for external noise with a peak at 30 degrees above the horizon (see Table 3.2).

These results suggest that the CCIR measured values of the external noise factor \( f_{a\text{CCIR}} \) are not appreciably affected by the extent of the ground plane that was assumed in normalizing the CCIR values, provided that the external noise was angularly distributed over a large solid angle. If the external noise factor is from a point source near the horizon, then the external noise factor is significantly affected by whether the ground plane is electrically small or perfect. If it is electrically small, the numeric directivity is approximately zero near the horizon; however, if it is perfect, then the directivity is a maximum on the horizon [see case 4 of Table 3.2 and Eq. (3–17)].
Similarly, the peak intensity of the angular distribution of external noise is not significantly affected by the extent of the ground plane for a given external noise factor and shape of the angular distribution, provided that the external noise is angularly distributed over a large solid angle. For example, the peak intensity of the angular distribution for electrically small ground planes relative to that for a perfect ground plane will be unchanged for uniform external noise; increased by a factor of \((2.6/1.805) = 1.44 = 1.6\) dB for external noise with a peak at 30 degrees above the horizon (see Table 3.2). However, if the external noise is from a point source on the horizon, the amplitude of the delta function distribution will increase by a factor of infinity (see Table 3.2).

4. Array Factor Degradation by Nonhomogeneous Earth

The electric field, at each element of a ground-based HF receiving array, is the sum of a direct field and an indirect (multipath) field. For elements with sufficiently small ground planes, the indirect field is not reflected from the ground plane; instead, it is reflected from the Earth in proximity to that element. The indirect field, relative to the direct field, is the product of the earth Fresnel reflection coefficient and a pathlength phase delay that is proportional to the height of the element above the Earth. The Fresnel reflection coefficient model is valid for computing directivity, but not the absolute power gain (see Section 3.1.1). If the Earth beneath the array is nonhomogeneous, then the argument of the total electric field at each element (after allowance for the true phase advance of the direct field at each element) is not uniform from element-to-element. The non-uniform argument causes an array rms phase error and beam-pointing errors when the nonhomogeneous earth is systematically distributed.

The earth Fresnel reflection coefficients and the arguments of the total field for a vertically polarized Hertzian dipole at height \(h\) above the earth are tabulated in reference [49] for CCIR 527-1 classifications of Earth, and \(h/\lambda = 0, 0.54, \) and 0.270. The normalized heights \(h/\lambda = 0.54\) and 0.270 correspond to the midpoints at 6 MHz and 30 MHz, respectively, of a 5.4 m-length vertical monopole. The rms phase errors and beam-pointing errors are modeled in reference [52] for arbitrary distributions of nonhomogeneous Earth.
Table 4.1. Summary of rms Phase Errors, Randomly Distributed Two-Level Inhomogeneous Earth.

Numerical results are summarized in Tables 4.1, 4.2, and 4.3 for randomly distributed and systematically distributed Earth inhomogeneities for cases where one-half of the array elements are located in proximity to one type of Earth, while the remaining half are located in proximity to a second type. Very dry ground/medium dry ground, medium dry ground/wet ground, and very dry ground/wet ground combinations of Earth types are considered. The beam-pointing errors for randomly-distributed Earth inhomogeneities are not shown in tables 4.1, 4.2, and 4.3 because the beam-pointing errors are zero for this case.

The rms phase error is an increasing monotonic function of \( h/\lambda \) (for modest values of \( h/\lambda \)), but generally a non-monotonic function of the angle of incidence \( \theta \).
Table 4.2. Summary of rms phase errors, two-level inhomogeneous Earth, systematically distributed with each half of array over different Earth.

The maximum expected values of the rms phase errors at the best diffraction focus of the array for the cases examined are 18 degrees and nine degrees for randomly distributed and systematically distributed inhomogeneities, respectively. The rms phase error is less for systematically distributed inhomogeneities because the linear phase error caused by beam-pointing errors has been subtracted. The maximum expected values of the beam-pointing error (in elevation and in azimuth) are zero and 0.3 beamwidths for randomly distributed and systematically distributed inhomogeneities, respectively. The maximum rms phase errors and beam-pointing errors occur for very dry ground/wet ground, \( \theta = 85 \) degrees, \( h/\lambda = 0.270 \).
Table 4.3. Summary of elevation beam-pointing errors, two-level inhomogeneous Earth, systematically distributed with each half of array over different Earth, $\phi = 0$ degrees.

The above numerical results suggest that the influence of inhomogeneous earth is appreciable, but not significant, on the performance degradation of an HF receiving array with electrically small ground planes.
5. Summary and Conclusions

Ground-based HF receiving arrays often employ some form of a vertical monopole element with a ground plane that is made as large as is economically feasible to mitigate the effects of the Earth and feed-cable exterior current on system performance. In the design of very large ground-based HF receiving arrays comprising hundreds or thousands of elements, electrically large metallic ground planes are prohibitively expensive to construct and maintain. Economically, electrically small ground planes are a relatively low-cost solution to this problem, provided the system performance is not too adversely affected. The performance of HF receiving arrays with electrically small ground planes is investigated herein.

Electrically small ground planes do degrade the performance of ground-based HF receiving arrays by reducing element directivity near the horizon, distorting the element azimuthal pattern, increasing the system internal noise factor, and increasing the array factor rms phase error and beam-pointing errors. These performance degradations, however, are not significant for most applications.

Earth multipath reduces the directivity near the horizon. The peak directivity is approximately the same as that for a perfect ground plane except that the peak directivity is approximately 30 degrees above the horizon instead of near the horizon. The directivity pattern is unaffected by the size of the ground plane for normalized ground-plane radii at least as large as $0 \leq 2\pi a/\lambda \leq 8$ radians. The numeric directivity, of electrically short monopole elements with electrically small ground planes resting on lossy Earth, is given approximately by $d_r(\theta) = 10\cos \theta \sin^3 \theta$; $0 \leq \theta \leq 90$ degrees zero otherwise. At angles near the horizon and for normalized ground-plane radii at least as large as eight radians, the directivity is reduced from the peak directivity by approximately 4 dB at $\theta = 82$ degrees, 5 dB at 84 degrees, 7 dB at 86 degrees, 12 dB at 88 degrees, and $\infty$ dB at 90 degrees.

The exterior current on the element feed cable distorts the element azimuthal pattern and modifies the phase center of the element.

The system internal noise factor is increased by increased ground losses and antenna impedance mismatch. Whenever the antenna proximity to earth is not mitigated (by an electrically large ground plane), the element ground losses are increased because of the generation of a leaky quasi-evanescent surface wave that directs energy into the Earth,
but not into the air medium. The radiation efficiency of a vertically polarized Hertzian dipole resting on Earth is zero if the Earth is lossy, and is approximately ten percent if the Earth is a perfect dielectric. The radiation efficiency of monopole antennas increases monotonically with increasing radius and density of the ground planes, and increasing length of the monopole element; it increases quasi-monotonically with increasing height of the ground plane above the Earth.

Antenna impedance mismatch can cause a significant increase in the system internal noise factor, particularly in HF receiving systems, because such systems usually employ electrically short elements that must operate over a wide range of frequencies. Normalized ground-plane radii of at least $2\pi a/\lambda = 2$ radians are required to stabilize the input impedance of quarter-wave elements with a high-density ground plane to within 20 percent of that for a perfect ground plane. The input impedance for a low-density ground plane is stabilized by ground-plane radii smaller than those for a high-density ground plane. Impedance-matching networks are generally necessary to achieve impedance matching over a wide range of frequencies. The ohmic loss of the matching network should be minimized so that the reduction in internal noise factor achieved by reduced mismatch loss is not offset by an increase in internal noise factor caused by those ohmic losses.

The external noise factor of HF receiving systems with electrically small ground planes is expected to be comparable to CCIR predicted values. Although CCIR claims that their predicted values have been normalized to those for an electrically short monopole element on a perfect ground plane, the ground-plane radial wires used to obtain the database are of length $a = 100$ ft, corresponding to $2\pi a/\lambda = 1.9$ and 19 radians at 3 MHz and 30 MHz, respectively. The ground-plane radii are too small to yield a directivity pattern at HF wavelengths that is appreciably different from that of a vertically polarized Hertzian dipole resting on lossy Earth. It is doubtful that the CCIR values of the external noise factor are normalized to correspond to those for a perfect ground plane. This kind of normalization would require a knowledge of the elevation angular distribution of external noise, which was not experimentally determinable; it would also require knowledge of the directivity of monopole elements with electrically small ground planes resting on lossy Earth, but that was not known theoretically until recently.
The electric field at each element of a ground-based HF receiving array is the sum of a direct field and an indirect (multipath) field. For elements with electrically small ground planes in proximity to inhomogeneous Earth, the indirect field causes an array rms phase error and beam-pointing errors when the Earth inhomogeneities are systematically distributed. The maximum expected values of rms phase error, at the best diffraction focus of the array for very dry/wet inhomogeneous Earth, are 18 degrees and nine degrees for randomly distributed and systematically distributed inhomogeneities, respectively. The maximum expected value of the beam-pointing error is zero and 0.3 beamwidths for randomly distributed and systematically distributed non-homogeneities, respectively.

The most significant performance degradation is the reduced directivity at angles near the horizon. For applications that require better directivity at those angles, the ground plane must be made electrically large.

Acknowledgements

The computer plots of Figs. 3.4, 3.8, and 3.9 were developed by S. Zamoscianyk based on numerical results supplied by G. J. Burke of Lawrence Livermore National Laboratory. G. J. Burke also supplied the data for Table 3.1. S. Lamoureux typed the manuscript.

References


[37] Private communication from G. J. Burke, Lawrence Livermore National Laboratory, June 19, 1990.


