Progress In Electromagnetics Research, PIER 9, 31–44, 1994

ENERGY DISSIPATION AND ABSORPTION IN RECIPROCAL BI–ISOTROPIC MEDIA DESCRIBED BY DIFFERENT FORMALISMS

F. Guérin

1. Introduction

Bi-isotropic media are a special subgroup of bianisotropic media where the constitutive parameters are no longer tensors, but scalars. Reciprocal bi-isotropic media, or chiral media, can be described by three complex parameters in the general lossy case. Several formalisms are currently being used to describe the electromagnetic coupling existing in these materials, from which the well–known phenomena of circular birefringence and dichroism originate. One possibility is to express the $\mathbf{D}$ and $\mathbf{B}$ vectors as a function of the $\mathbf{E}$ and $\mathbf{H}$ vectors through

$$\mathbf{D} = \varepsilon_L \mathbf{E} + i\gamma \mathbf{H}$$
$$\mathbf{B} = -i\gamma \mathbf{E} + \mu_L \mathbf{H} \tag{1}$$

This formalism is derived from the one used by Lindell and Sihvola [1]. For a general bi-isotropic medium, characterized by four constitutive parameters, the latter generally write the chirality parameter as κ and the non–reciprocity parameter as χ. Their notations were modified here by writing the chirality coefficient as $\gamma = -\kappa(\varepsilon_o\mu_o)^{\frac{1}{2}}$ and setting χ equal to 0 for reciprocity.

The $\mathbf{D}$ and $\mathbf{H}$ fields may also be expressed in terms of $\mathbf{E}$ and $\mathbf{B}$ as follows

$$\mathbf{D} = \varepsilon_J \mathbf{E} + i\xi \mathbf{B}$$
$$\mathbf{H} = i\xi \mathbf{E} + \frac{1}{\mu_J}\mathbf{B} \tag{2}$$

A similar set of equations was derived by Jaggard et al., based on a heuristic argument [2]. More general tensorial expressions were previously used by Post [3], and by Cheng and Kong, who pointed out the Lorentz covariance of the constitutive relations when written in this fashion [4,5]. A third possibility

$$\mathbf{D} = \varepsilon_D \mathbf{E} + \varepsilon_D \beta \nabla \times \mathbf{E}$$
$$\mathbf{B} = \mu_D \mathbf{H} + \mu_D \beta \nabla \times \mathbf{H} \tag{3}$$

was introduced by Fedorov [6], based on Born's and Drude's work [7,8]. This formalism, in which the non–local character of the constitutive relations is apparent, has been extensively used by Lakhtakia, Varadan and Varadan [9].

In either of the above formalisms, the constitutive parameters are macroscopic properties, resulting from a particular microstructure. Even though the three sets (1), (2), and (3) can be shown to be equivalent for time–harmonic fields [1,10], the meaning of the constitutive parameters is different from one formalism to another. This conducted us to examine how some physical requirements and properties involving the constitutive parameters are affected by the choice of a particular set of constitutive equations. In the following, some known connections between the various formalisms are recalled, and new ones are given. Then, the criteria for energy dissipation in a passive medium are derived (in the non-dispersive case), and the emphasis is put on the differences arising when comparing one system with another. Beyond the theoretical interest, requirements on the values of the constitutive parameters provide a check for experimentally determined quantities, which may lead to improvements in the measurement method or in the computational scheme for the material properties. The case of the normal incidence reflection coefficient of a metal–backed chiral layer is also treated.

2. Connections between the Different Sets of Constitutive Parameters

It is not our purpose to develop this topic here, the interested reader is referred to a paper by Sihvola and Lindell [1], and another paper by Ougier, Chenerie, Sihvola, and Priou, included in this PIER Book issue [10]. However, since our notations are different, we wish to briefly restate the connecting equations and insist on various aspects. A $e^{-i\omega t}$ time dependence is chosen throughout this study. All complex quantities are written in the following fashion: $z = \mathrm{Re}(z) + i\mathrm{Im}(z)$. Considering the Lindell-Sihvola (LS) and the Jaggard-Post-Kong (JPK) formalisms, the following relations are obtained

$$\begin{aligned} \varepsilon_L &= \varepsilon_J + \mu_J \xi^2 \,, \\ \mu_L &= \mu_J \,, \\ \gamma &= \xi \mu_J \,, \\ \varepsilon_J &= \varepsilon_L - \frac{\gamma^2}{\mu_L} \,, \\ \mu_J &= \mu_L \,, \\ \xi &= \frac{\gamma}{\mu_L} \end{aligned} \tag{4}$$

Comparing the LS and the DBF (Drude–Born–Fedorov) systems, one obtains

$$\begin{aligned}
\varepsilon_L &= \frac{\varepsilon_D}{1-\varepsilon_D\mu_D\beta^2\omega^2}\,, \\
\mu_L &= \frac{\mu_D}{1-\varepsilon_D\mu_D\beta^2\omega^2}\,, \\
\gamma &= \frac{\varepsilon_D\mu_D\beta\omega}{1-\varepsilon_D\mu_D\beta^2\omega^2}\,, \\
\varepsilon_D &= \varepsilon_L\left(1-\frac{\gamma^2}{\varepsilon_L\mu_L}\right)\,, \\
\mu_D &= \mu_L\left(1-\frac{\gamma^2}{\varepsilon_L\mu_L}\right) \\
\beta &= \frac{\gamma}{\omega\varepsilon_L\mu_L}\frac{1}{1-\dfrac{\gamma^2}{\varepsilon_L\mu_L}}
\end{aligned} \tag{5}$$

The comparison between the JPK and the DBF systems yields the following results

$$\begin{aligned}
\varepsilon_J &= \varepsilon_D\,, \\
\mu_J &= \frac{\mu_D}{1-\varepsilon_D\mu_D\beta^2\omega^2}\,, \\
\xi &= \omega\varepsilon_D\beta\,, \\
\varepsilon_D &= \varepsilon_J\,, \\
\mu_D &= \frac{\mu_J}{1+\mu_J\dfrac{\xi^2}{\varepsilon_J}}\,, \\
\beta &= \frac{\xi}{\omega\varepsilon_J}
\end{aligned} \tag{6}$$

To be noticed is that for low chirality, i.e., $\varepsilon_D\mu_D\beta^2\omega^2 \ll 1$, $\mu_J\xi^2/\varepsilon_J \ll 1$, or $\gamma^2/\varepsilon_L\mu_L \ll 1$, there is little difference between the permittivities and permeabilities in the various systems. Most of the artificial microwave chiral materials fabricated so far exhibit relatively low chirality, so that the previous conditions are fairly well fulfilled in practice [11], at least for frequencies sufficiently distant from chiral resonances. As far as the chirality parameters are concerned, it is apparent from the examination of (4) to (6) that their meaning depends on the formalism; some of them do not only express the degree of handedness of the medium and its optical activity, but incorporate hybrid information on handedness, permittivity and permeability.

Another interesting point is the low-frequency behavior of the various chirality parameters. Since there is no chirality in electrostatics or magnetostatics, the chirality parameters in the LS and the JPK formalisms satisfy the relation $\mathrm{Lim}_{\omega\to 0}\,\gamma = \mathrm{Lim}_{\omega\to 0}\,\xi = 0$ as can be inferred from constitutive relations (1) and (2). Provided that the permittivity and the permeability go to finite non–zero values as the frequency goes to zero, (5) and (6) both lead to the relation $\mathrm{Lim}_{\omega\to 0}\,\omega\beta = 0$ for the chirality parameter in the DBF formalism (this result can also be found by simply using equation (3) along with Maxwell's equations). Therefore, β is not required to have a finite limit (in particular, it is not required to go to zero) as the frequency goes to zero. The only requirement is that the product $\omega\beta$ goes to zero.

The case of the intrinsic wave impedance can also be treated similarly. Computing the impedance in each formalism separately, one obtains $\eta_L = (\mu_L/\varepsilon_L)^{\frac{1}{2}}$, $\eta_J = \left(\mu_J/\left(\varepsilon_J + \mu_J\xi^2\right)\right)^{\frac{1}{2}}$ and $\eta_D = (\mu_D/\varepsilon_D)^{\frac{1}{2}}$. Using the connecting equations (4), (5) and (6), one then finds that $\eta_L = \eta_J = \eta_D$, i.e. the impedances computed using the three systems of constitutive relations are identical. Since the wavenumbers of the two canonical right– and left–circularly polarized waves propagating in a chiral medium, k_L and k_R, do not depend on the use of specific constitutive relations, one can therefore define a set of three complex scalars which does not depend on the choice of a particular formalism, i.e. k_L, k_R and the intrinsic wave impedance. There are currently several groups involved in chiral media research and using different constitutive equations, which makes it difficult to compare the values they obtain for the constitutive parameters. One way of normalizing the results without imposing the choice of a particular set of constitutive equations would be to deal with the formalism–independent quantities k_L, k_R and the effective wave impedance.

3. Requirements for Passive Reciprocal Bi-isotropic Media

3.1 Requirements Relative to the Energy

The net time–average power flux entering a closed surface S with interior volume V is given for a bi-isotropic reciprocal medium without sources by the usual relation in the non-dispersive case

$$\begin{aligned}\mathcal{P}_{\mathbf{av}} &= -\frac{1}{2}\int_S \mathrm{Re}(\mathbf{E}\times\mathbf{H}^*)\cdot\mathbf{n}_s\,dS \\ &= \frac{\omega}{2}\int_V \mathrm{Im}(\mathbf{H}^*\cdot\mathbf{B}-\mathbf{E}\cdot\mathbf{D}^*)\,dV\end{aligned} \tag{7}$$

where $\mathbf{n_S}$ is the outward normal unit vector, Re is the real part, Im is the imaginary part, and * denotes the complex conjugate. Using the LS formalism, we define a constitutive matrix M by

$$M = \begin{pmatrix} \varepsilon_L & i\gamma \\ -i\gamma & \mu_L \end{pmatrix} \tag{8}$$

which leads to

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = M \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} \tag{9}$$

In a passive medium, $\mathcal{P}_{\mathbf{av}}$ must be positive. By passive medium, we mean here a medium which is passive for all fields. The corresponding requirement for energy dissipation can be written as

$$\forall\ (\mathbf{E},\mathbf{H}),\qquad (\mathbf{E}^*,\mathbf{H}^*)\left(\frac{M-M_t^*}{2i}\right)\begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} \geq 0 \tag{10}$$

(the subscript t denotes matrix transposition) where the strict inequality applies to the strictly lossy case, and the equality denotes the lossless case. After some manipulations, one obtains the following requirements on the constitutive parameters

$$\begin{aligned}&\mathrm{Im}(\varepsilon_L)+\mathrm{Im}(\mu_L)\geq 0 \\ &\mathrm{Im}(\varepsilon_L)\mathrm{Im}(\mu_L)\geq(\mathrm{Im}(\gamma))^2\end{aligned} \tag{11}$$

which, since $(\mathrm{Im}(\gamma))^2 \geq 0$, are equivalent to

$$\begin{aligned} &\mathrm{Im}(\varepsilon_L) \geq 0 \quad , \quad \mathrm{Im}(\mu_L) \geq 0 \\ &\mathrm{Im}(\varepsilon_L)\mathrm{Im}(\mu_L) \geq (\mathrm{Im}(\gamma))^2 \end{aligned} \tag{12}$$

This set of inequalities is in agreement with that found by Lindell [12]. If there is no chirality, i.e. if $\gamma = 0$, then the results applicable to dielectric and magnetic media, $\mathrm{Im}(\varepsilon_L) \geq 0$ and $\mathrm{Im}(\mu_L) \geq 0$, are obtained. Using equations (4) and (12), equivalent conditions for a passive bi-isotropic medium can be obtained in the JPK formalism

$$\begin{aligned} &\mathrm{Im}\left(\varepsilon_J + \mu_J \xi^2\right) \geq 0 \quad , \quad \mathrm{Im}(\mu_J) \geq 0 \\ &\mathrm{Im}\left(\varepsilon_J + \mu_J \xi^2\right) \mathrm{Im}(\mu_J) \geq \left(\mathrm{Im}(\xi \mu_J)\right)^2 \end{aligned} \tag{13}$$

as well as in the DBF system, using equations (5) and (12)

$$\begin{aligned} &\mathrm{Im}\left(\frac{\varepsilon_D}{1-\varepsilon_D\mu_D\beta^2\omega^2}\right) \geq 0 \quad , \quad \mathrm{Im}\left(\frac{\mu_D}{1-\varepsilon_D\mu_D\beta^2\omega^2}\right) \geq 0 \\ &\mathrm{Im}\left(\frac{\varepsilon_D}{1-\varepsilon_D\mu_D\beta^2\omega^2}\right) \mathrm{Im}\left(\frac{\mu_D}{1-\varepsilon_D\mu_D\beta^2\omega^2}\right) \\ &\geq \left[\mathrm{Im}\left(\frac{\varepsilon_D\mu_D\beta\omega}{1-\varepsilon_D\mu_D\beta^2\omega^2}\right)\right]^2 \end{aligned} \tag{14}$$

It should be noticed that only in the LS formalism, have ε_L and μ_L the same meaning in terms of dissipation of energy as have the permittivity and the permeability in ordinary isotropic dielectric and magnetic materials. Also, only in the LS formalism is chirality totally uncoupled and separated from other properties. In the JPK system, only μ_J has the same meaning as the usual magnetic permeability. In the DBF formalism, neither ε_D nor μ_D can be considered as regular permittivity and permeability. Therefore, at least as far as energy dissipation is concerned, the LS formalism seems to be the best suited to the description of bi-isotropic media. When the JPK or the DBF constitutive relations are used, caution is required when dealing with the permittivity or the permeability, especially for media presenting a strong macroscopic chirality.

3.2 Requirements Relative to the Attenuation of the LCP and RCP Fields

We now want to examine the question of energy dissipation when considering the attenuation of the two canonical left- and right-circularly polarized (LCP and RCP) fields existing in a chiral medium, and compare the results to those obtained in 3.1. In the DBF formalism, the two LCP and RCP fields $\mathbf{Q_L}$ and $\mathbf{Q_R}$ are commonly introduced by the so-called Bohren's decomposition as

$$\begin{aligned} \mathbf{E} &= \mathbf{Q_L} + a_R \mathbf{Q_R} \\ \mathbf{H} &= a_L \mathbf{Q_L} + \mathbf{Q_R} \end{aligned} \tag{15}$$

where $a_R = -1/a_L = -i\eta_D$ [13]. $\mathbf{Q_L}$ and $\mathbf{Q_R}$ each satisfy a homogeneous vector Helmholtz equation with respective wavenumbers k_L and k_R given by

$$k_L = \frac{\omega\sqrt{\varepsilon_D\mu_D}}{1-\omega\sqrt{\varepsilon_D\mu_D}\beta} \quad , \quad k_R = \frac{\omega\sqrt{\varepsilon_D\mu_D}}{1+\omega\sqrt{\varepsilon_D\mu_D}\beta} \tag{16}$$

The same LCP and RCP wavenumbers k_L and k_R can be written, using the JPK formalism, as

$$\begin{aligned} k_L &= \omega\left[\mu_J\xi + \left(\varepsilon_J\mu_J + \mu_J^2\xi^2\right)^{\frac{1}{2}}\right] \quad , \\ k_R &= \omega\left[-\mu_J\xi + \left(\varepsilon_J\mu_J + \mu_J^2\xi^2\right)^{\frac{1}{2}}\right] \end{aligned} \tag{17}$$

and, using the LS formalism, as

$$\begin{aligned} k_L &= \omega\left[\gamma + (\varepsilon_L\mu_L)^{\frac{1}{2}}\right] \quad , \\ k_R &= \omega\left[-\gamma + (\varepsilon_L\mu_L)^{\frac{1}{2}}\right] \end{aligned} \tag{18}$$

Requiring that $\mathrm{Im}(k_L) \geq 0$ and $\mathrm{Im}(k_R) \geq 0$ and using (16) to (18), one obtains the following three inequalities

$$\mathrm{Im}\left(\sqrt{\varepsilon_L\mu_L}\right) \geq |\mathrm{Im}(\gamma)| \tag{19}$$

$$\mathrm{Im}\left[\sqrt{\mu_J\left(\varepsilon_J+\mu_J\xi^2\right)}\right]\geq\left|\mathrm{Im}(\mu_J\xi)\right| \tag{20}$$

$$\mathrm{Im}\left(\frac{\sqrt{\varepsilon_D\mu_D}}{1-\varepsilon_D\mu_D\beta^2\omega^2}\right)\geq\left|\mathrm{Im}\left(\frac{\varepsilon_D\mu_D\beta\omega}{1-\varepsilon_D\mu_D\beta^2\omega^2}\right)\right| \tag{21}$$

which can all be written as

$$\mathrm{Im}(k_L+k_R)\geq\left|\mathrm{Im}(k_L-k_R)\right| \tag{22}$$

The conditions previously expressed in equations (12) to (14) can be recast in a generic form writing $\eta_L=\eta_J=\eta_D=\eta$ as

$$\begin{aligned}&\mathrm{Im}\left(\frac{k_L+k_R}{\eta}\right)+\mathrm{Im}\left(\eta(k_L+k_R)\right)\geq 0\\&\mathrm{Im}\left(\frac{k_L+k_R}{\eta}\right)\mathrm{Im}\left(\eta(k_L+k_R)\right)\geq\left[\mathrm{Im}\left((k_L-k_R)\right)\right]^2\end{aligned} \tag{23}$$

From an analytical viewpoint, the condition expressed by (22) is not equivalent to the conditions expressed by (23). However, if we mathematically impose that η be a positive real number, then (23) reduces to (22), and η no longer appears.

The requirements relative to the attenuation of the LCP and RCP fields can thus be viewed as a simplification of the usual requirements on the constitutive parameters for a passive chiral medium. This simplification can be mathematically regarded as a case where the impedance is a purely positive number (i.e. the magnetic loss tangent is equal to the electric loss tangent). A numerical comparison of (22) and (23) leads to the same conclusion. [1]

---

[1] Further work relative to the comparison of requirements on the constitutive parameters of bi-isotropic media derived from different considerations is currently being carried out, so that new results will be available on the particular topic treated in this section at the time of publication of this PIER Book issue. See for instance "On the constitutive parameters of bi-isotropic media" by I. Chenerie, F. Guerin, G. Soum, and S. Bolioli, *Proceedings of Chiral'94,* 111–117, Perigueux, France, May 18–20, 1994.

4. Reflection Coefficient of Reciprocal Bi-isotropic Media and Comments on Their Use for Reducing Microwave Reflection

In the present section, we consider a chiral slab of thickness e backed by a perfectly conducting plate, illuminated by a normally incident linearly polarized plane wave. One motivation is to check if chirality could be useful for applications related to the absorption of microwave energy. Using the generic symbol η for the intrinsic wave impedance as in section 3, one obtains the following reflection coefficient (for a chiral slab with or without metal backing and illuminated at normal incidence, the reflected field is copolarized with the incident field, unlike the transmitted field for the case without metal backing)

$$S_{11}^{sc} = \frac{\frac{\eta - \eta_o}{\eta + \eta_o} - \mathrm{e}^{2ik_{eq}e}}{1 - \frac{\eta - \eta_o}{\eta + \eta_o}\mathrm{e}^{2ik_{eq}e}}, \tag{24}$$

where

$$k_{eq} = \frac{k_L + k_R}{2}$$

The reflection coefficient is written as a function of the impedance η and an equivalent wavenumber k_{eq}, which is the average of the LCP and RCP wavenumbers k_L and k_R (it is the wavenumber of the magneto–dielectric medium that would be equivalent to the present chiral medium as far as reflection is concerned). S_{11}^{sc} is of course formalism–independent; this appears clearly from an analytical viewpoint in (24), where it is written as a function of the two formalism–independent variables η and k_{eq}.

Recalling the expressions obtained in section 2 for the impedance, and writing k_{eq} using the three different formalisms

$$k_{eq} = \omega\sqrt{\varepsilon_L \mu_L} = \omega\sqrt{\mu_J\left(\varepsilon_J + \mu_J \xi^2\right)} = \frac{\omega\sqrt{\varepsilon_D \mu_D}}{1 - \varepsilon_D \mu_D \beta^2 \omega^2} \tag{25}$$

one can discuss the dependence of the reflection coefficient with respect to the various macroscopic chirality parameters. It can be seen that the LS chirality coefficient γ appears neither in the impedance, nor in the equivalent wavenumber. In the JPK formalism, the chirality parameter ξ appears both in the expression of the impedance and that of the equivalent wavenumber. In the DBF formalism, the variable β appears only in the expression of the equivalent wavenumber k_{eq}. Therefore, for two formalisms (JPK and DBF), the reflection coefficient depends on macroscopic chirality, while for the third one (LS), macroscopic chirality is not included at all in its mathematical expression. Furthermore, this LS formalism is the one in which the permittivity and the permeability have the same meaning regarding the dissipation of energy, as they have in ordinary dielectric and magnetic materials. At first sight, such a conclusion does not augur well for the superiority of chiral media as microwave suppressors over less exotic materials. This remark should nevertheless be qualified by saying that only normal incidence on reciprocal bi-isotropic media was considered, and that the case of oblique incidence on a general nonreciprocal bi-isotropic medium is still to be studied in detail, not to mention the case of bianisotropic media.

Another comment should also be made. Only homogeneous media were dealt with so far in this article. However, in a composite containing chiral inclusions in an otherwise nonchiral matrix, the situation is different. In particular, because of the electromagnetic coupling originating from the shape of the inclusions, the macroscopic effective permittivity and the permeability will depend on both the microscopic electric polarizability α_e and microscopic magnetic polarizability α_m, as well as on the microscopic chiral polarizability α_c of the inclusions. Thus, microscopic chirality may play a role at a macroscopic level, and inhomogeneous chiral media may yield combinations of permittivity and permeability which could be difficult to achieve by other means. Besides, if properly understood and controlled, other phenomena such as scattering or multiple scattering may be used to enhance global energy absorption. In any case, it turns out increasingly clear that chiral materials should present some inhomogeneous character if they are to be considered as serious candidates for reflection reduction.

A number of papers treating the potential use of chiral media as microwave suppressors has recently been published in the literature [15–19]. Some of this previously reported work has already been discussed elsewhere [11,14], but some issues had been incompletely addressed then. There has been a great deal of excitement about the possibility of designing and fabricating chiral radar–absorbing materials. Yet, in view of the results of the preceding sections, it seems useful to rediscuss several specific points. It is hoped that these results and the following discussion will be a useful complement to the discussion started in [14].

Whenever simulations are being carried out, it is essential to choose the constitutive parameters of a passive chiral medium according to certain rules. One of these is that the constitutive parameters satisfy energy dissipation requirements (12), (13) and (14) in the frequency regions where dispersion is very small. To be noted is that there is an upper limit on the admissible values of the chirality parameter. Moreover, crossing relations (4), (5) and (6) show that the meaning of the constitutive parameters depends on the initial choice of constitutive relations. In the LS and JPK formalisms, a non-magnetic homogeneous chiral medium can be described by $\mu_L = 1$ or $\mu_J = 1$, but this is no longer true for the DBF formalism because μ_D incorporates more than purely magnetic effects.

The same caution must be taken when dealing with the permittivity or the chirality parameter. For instance, any of the three variables γ, ξ and β can be used to quantify chirality, and they all lead to the same conclusions concerning properties governed by transmission such as circular birefringence and dichroism. Nevertheless, they carry different meanings, as already mentioned in section 2. It turns out that this has an important impact on the analytical expression for the reflection coefficient, and it can lead to opposite conclusions regarding the possibility of using macroscopic chirality as an additional degree of freedom for achieving minimal reflection. It should be emphasized that there is no contradiction when considering that all three macroscopic chirality parameters do not give the same type of information on handedness, and that all three do not incorporate solely chiral effects, but then what is meant by the word "chirality" should be clearly stated. For the common practical case of composites with low chirality, it can be checked a posteriori that even for formalisms where the macroscopic chirality parameter mathematically enters the definition of the reflection coefficient, it only has very little influence on reflection results, unlike the permittivity and the permeability.

5. Conclusion

Several constitutive relations describing homogeneous reciprocal bi-isotropic media were compared for harmonic time dependence, and the relationships between the constitutive parameters expressed using the different formalisms were given. The requirements for dissipation in a passive chiral medium were derived in the non-dispersive case from energy consideration and compared with those obtained by considering the attenuation of propagating fields. The reflection coefficient dependence with respect to macroscopic chirality was found to differ from one formalism to the other. This conducted us to comment on the formalism–dependent meaning of the constitutive parameters of a chiral medium. It appears that the inhomogeneous character of chiral materials may be important for applications involving the absorption of microwave energy. The incorporation of non–reciprocity, indeed anisotropy, in the medium are other possibilities to be examined.

Acknowledgments

The author thanks Dr. Michele Labeyrie from Thomson–CSF Central Research Laboratories for fruitful discussions.

References

1. Sihvola, A. H., and I. V. Lindell, "Bi–isotropic constitutive relations," *Micro. Opt. Tech. Lett.*, Vol. 4, No. 3, 391–396, 1991.
2. Jaggard, D. L., A. R. Mickelson, and C. H. Papas, "On electromagnetic waves in chiral media," *Appl. Phys.*, Vol. 18, 211–216, 1979.
3. Post, E. J., *Formal Structure of Electromagnetics*, North–Holland, Amsterdam, 1962.
4. Cheng, D. K. and J. A. Kong, "Covariant descriptions of bianisotropic media," *Proc. IEEE*, Vol. 56, No. 3, 248–251, 1968.
5. Kong, J. A., "Theorems of bianisotropic media," *Proc. IEEE*, Vol. 60, No. 9, 1036–1046, 1972.
6. Fedorov, F. I., "On the theory of optical activity in crystals-I: The law of conservation of energy and the optical activity tensors," *Optics and Spectroscopy*, Vol. 6, No. 1, 49–53, 1959.
7. Born, M., "Uber die naturliche optische Aktivität von Flussigkeiten und Gasen," *Zeitschrift für Physik*, Vol. 16, 251–258, 1915.

8. Drude, P., *Lehrbuch der Optik,* S. Hirzel, Leipzig, 1900.
9. Lakhtakia, A., V. K. Varadan, and V. V. Varadan, *Time-Harmonic Electromagnetic Fields in Chiral Media,* Springer–Verlag, Berlin, 1989.
10. Ougier, S., I. Chenerie, A. Sihvola, and A. Priou, "Propagation in bi–isotropic media: effect of different formalisms on the propagation analysis," *Progress in Electromagnetics Research,* PIER 9, 19–30, 1994.
11. Guerin, F., M. Labeyrie, and P. Y. Guillon, "Microwave and millimeter–wave free–space measurements of lossy and low–loss chiral material," *PIERS Communication,* see PIERS Proceedings, 729, Pasadena, USA, July 12–16, 1993.
12. Lindell, I. V., *Methods for Electromagnetic Field Analysis,* Oxford University Press, New York, 1992.
13. Bohren, C. F., "Light scattering by an optically active sphere," *Chem. Ph. Lett.,* Vol. 29, 458–462, 1974.
14. Guerin, F., "Microwave chiral materials: a review of experimental studies and some results on composites with ferroelectric ceramic inclusions," *Progress in Electromagnetics Research,* PIER 9, 219–263, 1994.
15. Varadan V. K., V. V. Varadan, and A. Lakhtakia, "On the possibility of designing anti–reflection coatings using chiral composites," *J. Wave-Mat. Int.,* Vol. 2, No. 1, 71–81, 1987.
16. Varadan, V. K., and V. V. Varadan, "Electromagnetic shielding and absorptive materials," U.S. Patent No. 4,948,922, Aug. 14, 1990.
17. Liu, J. C., and D. L. Jaggard, "Chiral layers on planar surfaces," *J. Electro. Waves Applic.,* Vol. 6, No. 5/6, 651–667, 1992.
18. Bhattacharyya, A. K., "Control of radar cross-section and crosspolarization characteristics of an isotropic chiral sphere," *Electron. Lett.,* Vol. 26, No. 14, 1066–1067, 1990.
19. Liao, S. B., and G. J. Yin, "Reflectance of a chiral plate absorber," *Appl. Phys. Lett.,* Vol. 62, No. 20, 2480–2482, 1993.