

RADIATION PATTERN SYNTHESIS FOR MAXIMUM MEAN EFFECTIVE GAIN WITH SPHERICAL WAVE EXPANSIONS AND PARTICLE SWARM TECHNIQUES

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Abstract—A new Mean Effective Gain (MEG) expression using Spherical Wave Expansions (SWE) is presented in order to evaluate the impact of mobile environments on radiating structures. The proposed approach takes into account the pattern polarization and transforms a continuous functional optimization problem into an approximate discrete formulation. It allows to synthesize efficient antenna radiation patterns in terms of the Mean Effective Gain when it is combined with modern heuristic optimization techniques. In addition, antenna performance limits are evaluated by means of certain bounds. These depend on the modal number which is required to describe accurately far fields and depend ultimately on the antenna size. The method estimates the optimum patterns for two different wireless scenarios that are characterized by the statistical probability density functions of incoming waves and particularized in the case of Gaussian statistics. The numerical evaluation has been performed by means of the Particle Swarm Optimization (PSO) technique, which is slightly modified to include a specific constrain and whose parameters have been computed previously by solving a canonical problem. Finally, representative results in outdoor and mixed wireless scenarios are discussed, pointing out some useful consequences in antenna design.

1. INTRODUCTION

There has been an increasing interest recently in researching the wireless environment impact on radiating structures. An important challenge in particular, motivated by the necessity of quality of service in telecommunication systems, focuses on modeling the effect of different propagation scenarios on antenna parameters. Engineers must consider those possible effects and adopt several design policies to minimize the degradation. As a result of the antenna interaction with complex media (e.g., mobile environments), electromagnetic field and equivalent circuit performance measures such as antenna gain, directivity, or impedance, cannot be treated as deterministic values. Instead, these should be look at as random variables in different degrees. Although some statistical approaches have been developed to study some features [1, 2], modelling the scenario influence on antenna parameters is a difficult task. One of these best known magnitudes is the Mean Effective Gain (MEG), which takes into account the statistics of the incoming waves and their polarization when the antenna is studied from the reception point of view.

There are still open questions concerning the MEG that require to be answered. The overall communication system performance (with regard to the capacity) is determined, among other elements, by the antenna behavior. It is important to achieve the maximum Mean Effective Gain in order to maximize signal parameters [3], but few authors have analyzed the problem. Glazunov et al. [4] have proposed a method to evaluate the channel influence based on spherical wave modes, calculating some bounds as well as a possible optimal solution. However, their approach may not be feasible because it does not consider the copolar and crosspolar component relationship of the radiation pattern.

In this work, the Spherical Wave Expansion (SWE) technique is applied to obtain a mathematical expression of the MEG that includes the polarization relationship between the directivity components through the SWE coefficients. We use the integral approach provided by [5] to carry out the evaluation. This expression is defined as a cost function where the variables are the SWE coefficients. It comprises theoretically an infinite number of complex variables, but can be solved if the number of modes is truncated. The approximation is justified because only a few modes are necessary in practice to describe accurately radiation patterns. Besides, the existence of a maximum is assured since, according to Harrinton's work [6], there is always a maximum achievable directivity which relies upon the number of modes and it is easy to prove that the directivity is a maximum bound of the

MEG.

Once the SWE-MEG is found, the problem states as: “Compute the coefficients and the far field pattern which maximize the MEG, given some incoming wave probability density functions (pdfs)”. Therefore, some prior knowledge about the statistical channel is required. Different studies deal with this issue and may be found in the literature [7, 8]. They suggest Gaussian and uniform pdfs, hence we compute the optimum patterns for those densities. Due to the large number of variables, which increases with the number of modes, heuristic techniques such as Genetic Algorithms (GA) or Particle Swarm Optimization (PSO) seem suitable candidates. This paper exploits the classical PSO algorithm to compute the optimum far fields, although modified PSO versions [9, 10] can be applied to improve the performance.

The article is organized as follows. First, Section 2 reviews theoretically the Mean Effective Gain. Then, the Spherical Wave Expansion of directivity in terms of its polarization components as well as a new MEG cost function are proposed in Section 3. Section 4 presents the classical PSO scheme used for reaching the global optimum far field patterns, computed for outdoor and mixed (indoor-outdoor) wireless scenarios. Finally, some conclusions are outlined in Section 5.

2. MEAN EFFECTIVE GAIN

The Mean Effective Gain was originally proposed by Taga as a measure to take into account the stochastic behavior of the incoming field caused by the environment complexity. It quantifies the degradation of gain (or directivity) arising from random vertical and horizontal polarization when electromagnetic waves propagate through the wireless channel and evaluates the crosspolar effect on the receiving antenna.

Let P_V and P_H be the averaged incident power of vertical and horizontal-polarized waves when the antenna is moving in a random path and $P_V + P_H$ the total mean power averaged on this path. The mobile antenna Mean Effective Gain (MEG) is the ratio between the total received power (P_{rec}) by the structure in a random path and the total incident power,

$$MEG = \frac{P_{rec}}{P_V + P_H} \quad (1)$$

The relation between the mean incident power corresponding to the vertical and horizontal polarization is the so-called crosspolariza-

tion ratio Γ ,

$$\Gamma = \frac{P_V}{P_H}, \tag{2}$$

which is a channel-dependent magnitude. A few hypothesis are assumed to expand (1):

- (i) The directivity components in $\hat{\theta}$ and $\hat{\phi}$ are

$$D(\theta, \phi) = D_\theta(\theta, \phi) + D_\phi(\theta, \phi) = \frac{\wp_\theta(\theta, \phi)}{\frac{P_r}{4\pi r^2}} + \frac{\wp_\phi(\theta, \phi)}{\frac{P_r}{4\pi r^2}} \tag{3}$$

- (ii) The mobile antenna is moving in the XY-plane and has an efficiency $\eta = 1$.
- (iii) The spatial channel is modeled through the probability density functions P_θ and P_ϕ , which must fulfil

$$\int_0^{2\pi} \int_0^\pi P_\theta(\theta, \phi) \sin \theta d\theta d\phi = 1; \int_0^{2\pi} \int_0^\pi P_\phi(\theta, \phi) \sin \theta d\theta d\phi = 1. \tag{4}$$

Under these conditions, the closed-form expression corresponding to the Mean Effective Gain is [5]:

$$MEG = \oint \left\{ \frac{\Gamma}{1 + \Gamma} P_\theta(\Omega) D_\theta(\Omega) + \frac{1}{1 + \Gamma} P_\phi(\Omega) D_\phi(\Omega) \right\} d\Omega \tag{5}$$

where $\Omega \equiv (\theta, \phi)$ denotes an arbitrary direction (in a spherical coordinate system) and $d\Omega$ is the differential of solid angle.

It is important to notice that, as $D(\Omega)$ is a directivity function, the constrain

$$\iint_{4\pi} D(\Omega) d\Omega = \iint_{4\pi} (D_\theta(\Omega) + D_\phi(\Omega)) d\Omega = 4\pi \tag{6}$$

implies a relationship between its components.

Some authors [5, 7, 8] have identified experimentally some parametric models for the incoming waves, whose moments are summarized in Table 1, where μ and σ are the expected elevation angle and the standard deviation, respectively, for densities like

$$\begin{aligned} P_\phi(\theta, \phi) &= A_\phi e^{-(\theta - [\pi/2 - m_H])^2 / 2\sigma_H^2}, & 0 \leq \theta \leq \pi, -\pi \leq \phi \leq \pi \\ P_\theta(\theta, \phi) &= A_\theta e^{-(\theta - [\pi/2 - m_V])^2 / 2\sigma_V^2}, & 0 \leq \theta \leq \pi, -\pi \leq \phi \leq \pi \end{aligned} \tag{7}$$

and A_θ and A_ϕ are constants calculated with Eq. (4).

Table 1. Statistical moments of incoming waves.

Environment	Azimuth	El. Pol. θ	El. Pol. ϕ	XPR
Outdoor	Uniform	$\mu_\theta = 10^\circ$	$\mu_\phi = 19^\circ$	5 dB
	Uniform	$\sigma_\theta = 15^\circ$	$\sigma_\phi = 50^\circ$	
Outdoor-Indoor	Uniform	$\mu_\theta = 1^\circ$	$\mu_\phi = 2^\circ$	9 dB
	Uniform	$\sigma_\theta = 8^\circ$	$\sigma_\phi = 15^\circ$	
Indoor	Laplacian	$\mu_\theta = 4^\circ$	$\mu_\phi = 2^\circ$	7 dB
	$\sigma = 24$ Laplacian	$\sigma_\theta = 9^\circ$	$\sigma_\phi = 11^\circ$	

3. SPHERICAL WAVE EXPANSION AND MEAN EFFECTIVE GAIN OPTIMIZATION

According to Eq. (5), the channel modifies the expected antenna gain. However, if we desire to maximize the mean effective gain, it is only allowed to modify the directivity pattern. Any admissible mathematical function able to describe this should be obtained from antenna far fields, i.e., from possible solutions to the Maxwell Equations. Our main idea consists of expressing the directivity and the MEG using a SWE in a vector basis.

Appropriate vector basis functions have been studied and calculated by some authors with the aid of Maxwell Equations. They take certain limits to their solutions to obtain far field basis [11, 12] by means of the Generalized Legendre Polynomials, $\bar{P}_n^m(\cos \theta)$,

$$\vec{K}_{1mn}(\theta, \phi) = \sqrt{\frac{2}{n(n+1)}} \left(-\frac{m}{|m|}\right)^m e^{im\phi} (-i)^{n+1} \left\{ \frac{im\bar{P}_n^m(\cos \theta)}{\sin \theta} \hat{\theta} - \frac{d\bar{P}_n^m(\cos \theta)}{d\theta} \hat{\phi} \right\} \tag{8}$$

$$\vec{K}_{2mn}(\theta, \phi) = \sqrt{\frac{2}{n(n+1)}} \left(-\frac{m}{|m|}\right)^m e^{im\phi} (-i)^n \left\{ \frac{d\bar{P}_n^m(\cos \theta)}{d\theta} \hat{\theta} + \frac{im\bar{P}_n^m(\cos \theta)}{\sin \theta} \hat{\phi} \right\} \tag{9}$$

where $n = 1, \dots, \infty$ and $m = -n, \dots, n$ are the spherical modal numbers. It is possible to compute the radiated fields from a source with the so-called Far Field Pattern Function $\vec{K}(\theta, \phi)$,

$$\vec{E}(\theta, \phi) = \frac{k\sqrt{\eta}}{\sqrt{4\pi}} \frac{e^{-ikr}}{kr} \vec{K}(\theta, \phi) \quad \vec{H}(\theta, \phi) = \frac{k}{\sqrt{\eta}} \frac{1}{\sqrt{4\pi}} \frac{e^{-ikr}}{kr} \hat{r} \times \vec{K}(\theta, \phi) \tag{10}$$

being η the medium specific impedance and k the wave number. This Far Field Pattern Function is a linear combination of the basis

functions from (8) and (9) and therefore

$$\vec{K}(\theta, \phi) = \sum_s \sum_m \sum_n T_{smn} \vec{K}_{smn}(\theta, \phi) \quad (11)$$

being T_{smn} complex coefficients. In addition, the relation between the directivity and the far field pattern function is

$$D(\theta, \phi) = \frac{\left| \sum_{smn} T_{smn} \vec{K}_{smn}(\theta, \phi) \right|^2}{\sum_{smn} |T_{smn}|^2} = \frac{\left| \vec{K}(\theta, \phi) \right|^2}{\sum_{smn} |T_{smn}|^2} \quad (12)$$

If we assume no losses and perfect matching, then

$$\sum_s \sum_m \sum_n |T_{smn}|^2 = 1 \Rightarrow D(\theta, \phi) = \left| \vec{K}(\theta, \phi) \right|^2 \quad (13)$$

However, Eq. (13) is not useful to expand the MEG equation, because two different orthogonal-polarized functions are required. We propose to perform the decomposition by means of the same technique that separates copolar and crosspolar components. Instead of

$$D(\theta, \phi) = D_{co}(\theta, \phi) + D_{cross}(\theta, \phi) = |\vec{K}(\theta, \phi) \cdot \hat{i}_{co}^*|^2 + |\vec{K}(\theta, \phi) \cdot \hat{i}_{cross}^*|^2 \quad (14)$$

being \hat{i}_{co} and \hat{i}_{cross} unitary and orthogonal vectors, we use

$$D(\theta, \phi) = D_{\theta}(\theta, \phi) + D_{\phi}(\theta, \phi) = |\vec{K}(\theta, \phi) \cdot \hat{\theta}|^2 + |\vec{K}(\theta, \phi) \cdot \hat{\phi}|^2 \quad (15)$$

Let α and β be defined as

$$\begin{aligned} \alpha_{mn} &= \sqrt{\frac{2}{n(n+1)}} \left(-\frac{m}{|m|} \right)^m e^{im\phi} (-i)^{n+1} \\ \beta_{mn} &= \sqrt{\frac{2}{n(n+1)}} \left(-\frac{m}{|m|} \right)^m e^{im\phi} (-i)^n, \end{aligned} \quad (16)$$

and

$$\vec{K}(\theta, \phi) = \sum_{smn} T_{smn} \vec{K}_{smn}(\theta, \phi) = \sum_m \sum_n T_{1mn} \vec{K}_{1mn}(\theta, \phi) + T_{2mn} \vec{K}_{2mn}(\theta, \phi) \quad (17)$$

The θ -component is calculated projecting $\vec{K}(\theta, \phi)$ onto $\hat{\theta}$

$$\begin{aligned} \vec{K}(\theta, \phi) \cdot \hat{\theta} &= \left[\sum_{mn} \left(T_{1mn} \vec{K}_{1mn}(\theta, \phi) + T_{2mn} \vec{K}_{2mn}(\theta, \phi) \right) \right] \cdot \hat{\theta} \\ &= \sum_{mn} \left(T_{1mn} \vec{K}_{1mn}(\theta, \phi) \cdot \hat{\theta} + T_{2mn} \vec{K}_{2mn}(\theta, \phi) \cdot \hat{\theta} \right) \\ &= \sum_{mn} \left(T_{1mn} \alpha_{mn} \frac{im \bar{P}_n^m(\cos \theta)}{\sin \theta} + T_{2mn} \beta_{mn} \frac{d \bar{P}_n^m(\cos \theta)}{d \theta} \right) \end{aligned} \quad (18)$$

Projecting the Far Field pattern onto $\hat{\phi}$ leads to the ϕ -component

$$\begin{aligned} \vec{K}(\theta, \phi) \cdot \hat{\phi} &= \sum_{mn} \left(T_{1mn} \vec{K}_{1mn}(\theta, \phi) \cdot \hat{\phi} + T_{2mn} \vec{K}_{2mn}(\theta, \phi) \cdot \hat{\phi} \right) \\ &= \sum_{mn} \left(-T_{1mn} \alpha_{mn} \frac{d\bar{P}_n^m(\cos \theta)}{d\theta} + T_{2mn} \beta_{mn} \frac{im\bar{P}_n^m(\cos \theta)}{\sin \theta} \right) \end{aligned} \quad (19)$$

Finally, combining Eqs. (5), (18) and (19) the MEG can be rearranged as a function of coefficients instead of functionals,

$$\begin{aligned} \text{MEG}(T_{smn}) &= \Gamma \iint_{4\pi} \frac{P_\theta(\theta, \phi) \cdot |\vec{K}(\theta, \phi) \cdot \hat{\theta}|^2}{1 + \Gamma} d\Omega \iint_{4\pi} \frac{P_\phi(\theta, \phi) \cdot |\vec{K}(\theta, \phi) \cdot \hat{\phi}|^2}{1 + \Gamma} d\Omega \\ &= \frac{\Gamma}{1 + \Gamma} \iint_{4\pi} P_\theta(\theta, \phi) \cdot \left| \sum_{mn} \left(T_{1mn} \alpha_{mn} \frac{im\bar{P}_n^m(\cos \theta)}{\sin \theta} + T_{2mn} \beta_{mn} \frac{d\bar{P}_n^m(\cos \theta)}{d\theta} \right) \right|^2 d\Omega \\ &+ \frac{1}{1 + \Gamma} \iint_{4\pi} P_\phi(\theta, \phi) \cdot \left| \sum_{mn} \left(-T_{1mn} \alpha_{mn} \frac{d\bar{P}_n^m(\cos \theta)}{d\theta} + T_{2mn} \beta_{mn} \frac{im\bar{P}_n^m(\cos \theta)}{\sin \theta} \right) \right|^2 d\Omega \end{aligned} \quad (20)$$

The original optimization problem which seeks (D_θ, D_ϕ) functions such that, given the pdf field (P_θ, P_ϕ) , maximize expression (5) constrained to Eq. (6) is transformed to find the complex coefficients T_{smn} such that

$$\begin{aligned} &\max \text{MEG}(T_{smn}) \\ &\sum_s \sum_m \sum_n |T_{smn}|^2 = 1 \end{aligned} \quad (21)$$

Two advantages come out from the proposed method. On the one hand, both partial directivity functions are connected by the coefficients, as Eq. (20) establishes. Therefore, each integral cannot be maximized independently and this process might be not straightforward even if other techniques (like a variation principle or the Schwarz inequality) can be applied to solve the original problem. On the other hand, some computer techniques are suitable for optimizing the proposed cost function whereas the main problem seems mathematically intractable. However, there is one disadvantage: the modes need to be truncated to apply some numerical technique and setting $\max n = N$ leads to obtain an approximation rather than an exact solution. Nevertheless, the truncated modal number which represents accurately radiated fields is related to the antenna size [6, 11], and 5 or 6 modes are usually enough to achieve low error [13]. Since the pdfs may be probably well approximated by a spherical harmonics basis with a small number of elements, the computed patterns will be close to the actual optimal values.

4. PARTICLE SWARM OPTIMIZATION RESULTS

Particle Swarm optimization (PSO) was first proposed by Eberhart and Kennedy [14] in 1995 as a global optimum finding technique. Since then, the algorithm has been studied, improved, and applied successfully to many electromagnetic synthesis problems. It is based on the random movement of a group or particles exploring a multidimensional search space. Every point in this space is a possible solution of the synthesis problem, and thus, each particle position is a potential optimum. The group coordination is the key of the algorithm convergence and it allows to find a global optimum.

We propose to use the PSO method to solve (21), although other techniques such as Genetic Algorithms, Differential Evolution, etc. may be applied as well. In addition, we focus on the solution rather than in the analysis technique. Therefore, we use the PSO as a mathematical tool and improved versions which could accelerate the convergence rate are not the subject of our work.

4.1. Particle Swarm Scheme

The PSO begins with a random population of individuals (a particle swarm). Let T_{1nm} and $T_{2nm} \in \mathbb{C}$ be the coefficients involved in equations (20) and (21). These coefficients can be rearranged in a matrix \mathbb{T} whose dimension determines the optimization problem size. The steps to find the optimal solution are the following:

- (i) Initialize a random swarm of M particles, compute the MEG for every particle using (20) and evaluate $pbest$ and $gbest$, where $pbest$ and $gbest$ are the best solution in each iteration and thorough the whole iteration process, respectively.
- (ii) Update the velocity, \mathbb{V} , and particle position, \mathbb{T} , according to the following equations:

$$\begin{cases} \mathbb{T}_{k+1} = \mathbb{T}_k + \mathbb{V}_k \\ \mathbb{V}_{k+1} = \mu \mathbb{V}_k + c_1 r_1 \cdot (p_{best} - \mathbb{T}_k) + c_2 r_2 \cdot (g_{best} - \mathbb{T}_k) \end{cases} \quad (22)$$

where μ is the inertial weight, c_1 and c_2 are the cognitive and social rates. The random variables r_1 and r_2 are distributed uniformly between 0 and 1.

- (iii) Evaluate the fitness function and update $pbest$ and $gbest$.
- (iv) Repeat previous steps (ii) and (iii) until we achieve the desired fitness value.

Unless there is a prior knowledge about the parameter space, the initial particles are typically distributed uniformly on a presumed

initial space to facilitate the global search. Thus, the number of iterations will depend on the required fitness value, the dimension and the search space size.

In the classical scheme, the search space is a multidimensional compact domain where every particle position and velocity are within the intervals $[t_d^{\min}, t_d^{\max}]$ and $[v_d^{\min}, v_d^{\max}]$. These limits are selected according to the problem to deal with, and play an important role in the final PSO performance. In the case of the MEG optimum problem, the restriction

$$\sum_s \sum_m \sum_n |T_{smn}|^2 = 1 \tag{23}$$

can be used for decreasing the computational cost, since it constrains the particle movement to a spherical manifold. Therefore, there is no need to explore the whole domain but only the surface manifold. We propose then a modification of the classical PSO: first, the random particle position matrix \mathbb{T}^i is generated in the whole multidimensional space. Afterwards, the transformation

$$\tilde{\mathbb{T}}^i = \frac{\mathbb{T}^i}{\sum_s \sum_m \sum_n |T_{smn}|^2} \tag{24}$$

leads to the new particle position constrained in a unity sphere. These are the new coordinates to evaluate the cost function.

In order to estimate the problem dimension as a function of the number which controls total spherical harmonics “ N ”, notice $s = 1, \dots, 2$, $m = -N, \dots, N$ and $n = 0, \dots, N$ and that every \mathbb{T} matrix (with positive and negative m) is complex. Then

$$\mathbb{T}_{smn} = \begin{pmatrix} t_{i00} & 0 & \dots & 0 \\ t_{i01} & t_{i11} & \dots & 0 \\ t_{i02} & t_{i12} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ t_{i0N} & t_{i1N} & \dots & t_{iNN} \end{pmatrix} \tag{25}$$

Since four almost triangular complex matrices like (25) define completely the set of possible solutions, the total number of coefficients (or equivalently, the PSO dimension) is

$$Dim_{PSO} = 4 \cdot 2 \cdot \left(N \frac{N+1}{2} + N \right) - 4 \tag{26}$$

The complexity is high even for a small harmonic number, hence the PSO technique is a suitable choice for this problem.

The particle as well as the iteration number are estimated from the study of a canonical problem (the synthesis of maximum directivity patterns) and it is a testing method to evaluate the algorithm performance.

4.2. Parameter Selection

In order to find out the convergence rate and the suitable parameters for the synthesis problem, the proposed algorithm is carried out on an easier canonical problem. Once the spherical harmonic number N is truncated, the maximum directivity is

$$D_{max} = N^2 + 2N \quad (27)$$

This bound is achieved when the coefficients are [15]

$$T_{smn} = \left(\vec{K}(\theta, \phi)_{smn} \cdot \hat{i}_{co}^* \right)^* \quad (28)$$

and consequently, with some particular pattern. Using a similar approach, the directivity function may be computed from the coefficients T_{smn} .

The directivity optimization was carried out with the MATLAB environment and truncated orders from $N = 2$ up to $N = 6$. Different sets of particles were evaluated (with 20, 30 and 40 elements), showing that 40 particles achieved a good compromise between convergence rate and complexity. The iteration number was fixed to a maximum (300) with the higher dimension ($N = 6$). Increasing the harmonic number decelerates the convergence rate (Fig. 1), namely because of the complexity growth in terms of the PSO dimension (Eq. (26)).

Figure 1 shows a good agreement between the analytical values and those found by means of the PSO. In addition, the total synthesized radiation pattern (Fig. 2) resembles the analytical results provided by Harrington.

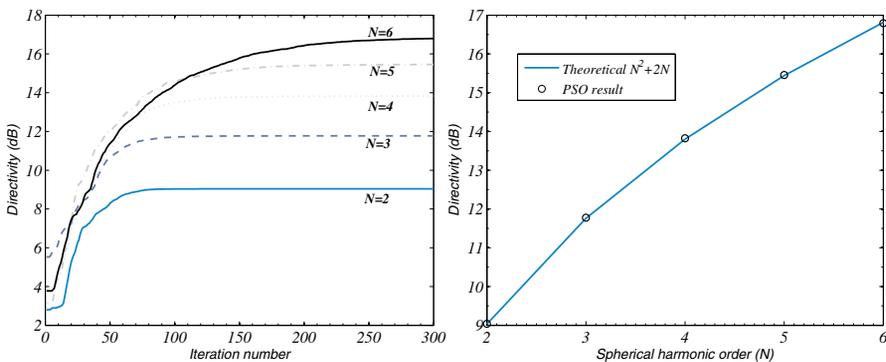


Figure 1. PSO performance with final parameters in the case of the canonical problem and comparison with the theoretical solution from Eq. (27).

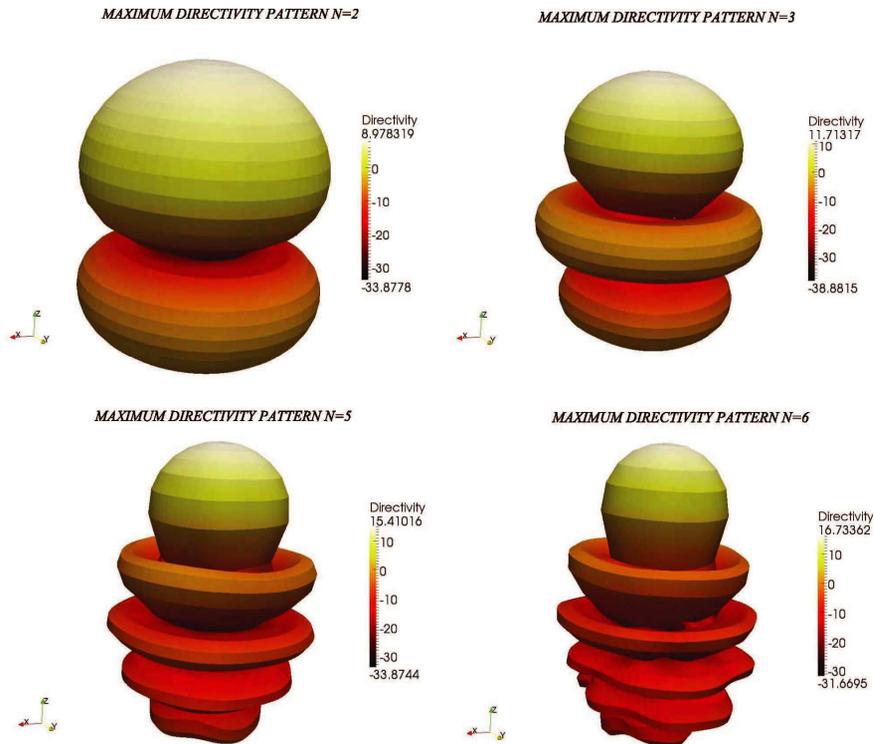


Figure 2. Maximum available directivity radiation patterns.

5. MEG BOUNDS AND OPTIMUM PATTERN RESULTS

The Particle Swarm method has been applied to the synthesis problem, mathematically summarized in Eq. (22), using the proposed parameter selection. We have focused on outdoor and mixed environments, as they have similar statistics with different moments, what is useful to compare between numerical solutions. Fig. 3 shows the convergence when the mode number increases in the case of the outdoor environment. As expected, the convergence rate is very similar to that accomplished by the application of the PSO to the canonical problem. There is a remarkable difference on the numerical values, because the MEG cost function does not grow as fast as the directivity when the mode number increases. In fact, the Mean Effective Gain values are clearly lower than the maximum directivities. This difference is due to the channel effects and cannot be mitigated using another antenna. The higher the directivity is, the less efficient the antenna is, thinking

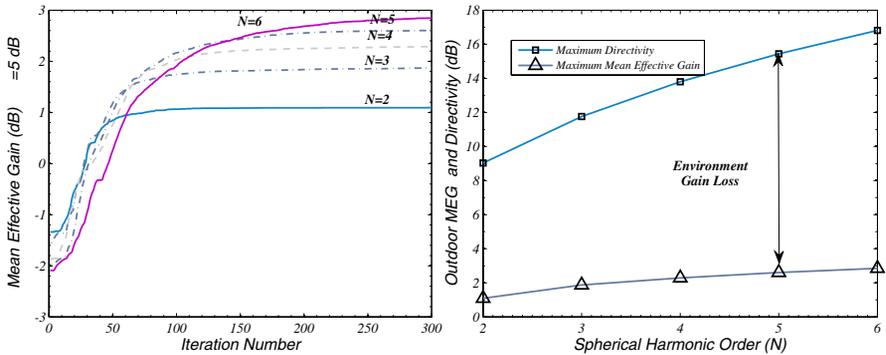


Figure 3. PSO convergence rate in the MEG optimization process (outdoor environment) and comparison with the maximum possible value.

in terms of the MEG. Thus, there must be a trade-off between efficiency and directivity.

The radiation pattern series shows the effect on increasing the modal number (Fig. 4). The maximum achievable effective gain improves, but at a slower rate than that of the actual directivity. The former is lower compared to the maximum directivity.

Obviously, optimum radiation patterns depend on the channel properties. This dependence is based on two facts: the results change for every modal number solution, and their directivities and main pattern parameters do not match. The maximum directivity angular direction (Fig. 5) depends, as expected, on the statistical moments of the incoming waves. Furthermore, Fig. 6 summarizes the computed results for every scenario and harmonic numbers 4, 5 and 6. These figures show the available directivity, the optimum MEG corresponding to the subspace order and the environment, and the optimum pattern directivity. As far as the environment is concerned, any antenna behaves better in a mixed scenario. This is a consequence of the crosspolar ratio value, which is 4 dB larger in indoor-outdoor than in outdoor environments. In addition, the actual optimum directivity is below the maximum available directivity. This means that increasing directivity does not always improve the performance of an antenna in a mobile environment. In fact, if an antenna were designed with a higher directivity than the actual optimum value, it would have on average worse performance because the radiation pattern would result in a minor MEG.

Finally, a deep analysis of these curves reveals in which conditions the optimum MEG achieves the maximum available directivity. Taking

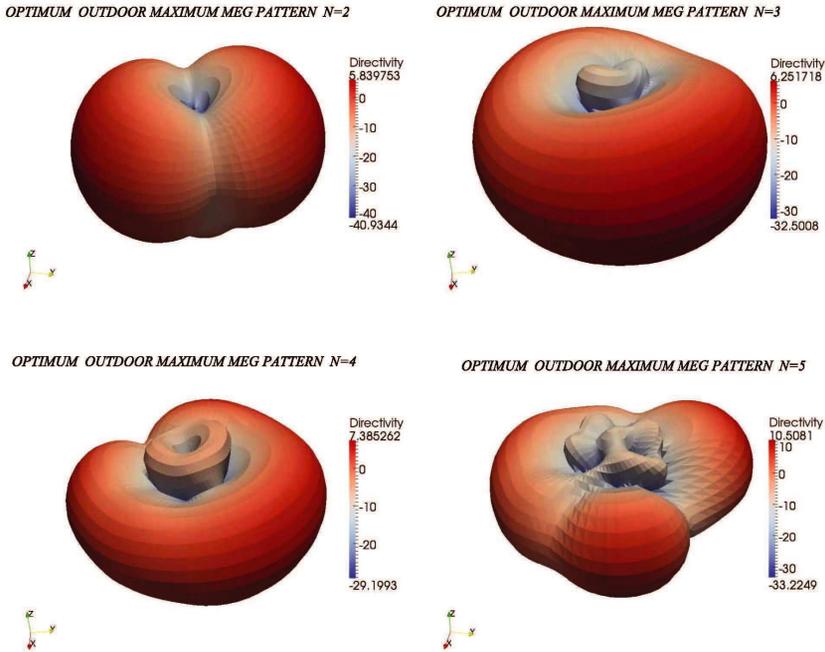


Figure 4. Maximum MEG radiation patterns in outdoor wireless scenarios with different harmonic numbers.

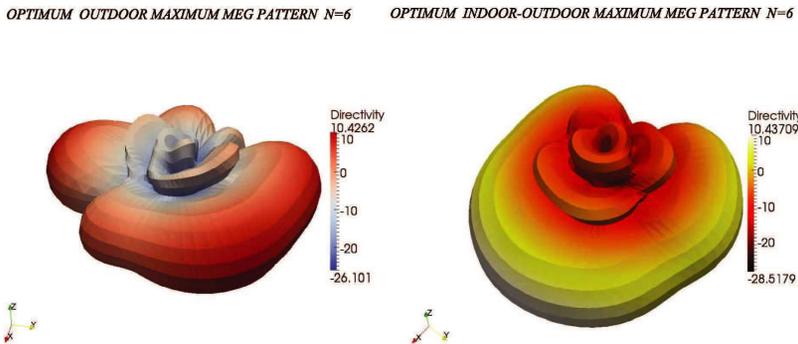


Figure 5. Maximum MEG radiation pattern comparison between outdoor and mixed wireless scenarios.

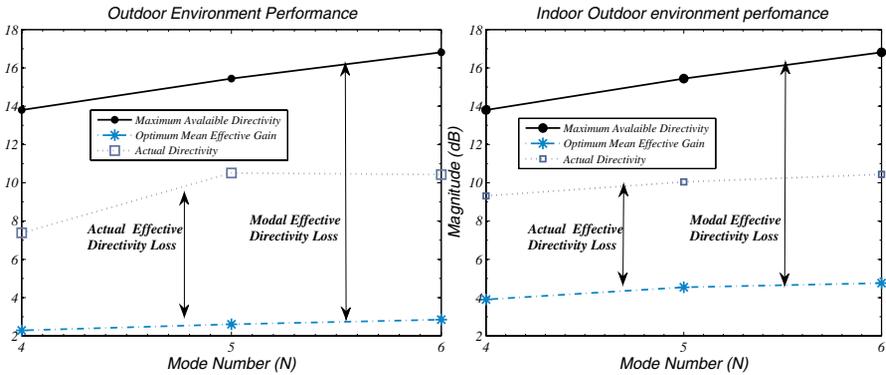


Figure 6. Available, achievable and actual optimum MEG values in wireless environments.

into account that the channel influence is described by the crosspolar ratio, the statistical distribution and its moments, if the crosspolar ratio is increased, the optimum MEG approximates to the actual directivity. If the probability density function tended to a Dirac distribution with suitable statistical moments, the actual directivity would reach to the maximum available directivity. Both conditions must hold to attain the limit, provided the modal number is truncated.

6. CONCLUSIONS

In this paper, we have proposed a new method based on Spherical Wave Expansions and Particle Swarm techniques in order to evaluate and optimize the Mean Effective Gain of antennas in wireless scenarios. The SWE applied to the integral form of the MEG led to a discrete formulation that offers a new approach to synthesize optimum radiation patterns to maximize the MEG, given the statistical description of the incoming waves. The discrete problem is highly dimensional and therefore heuristic techniques such as PSO have demonstrated to be very suitable. In addition, two case studies have confirmed the validity of this method, whose parameters have been previously selected by solving a canonical problem (the synthesis of maximum directivity patterns). Although the procedure is computationally expensive and slow, yields to very useful conclusions, such as it is not always possible to improve the MEG by only increasing the directivity or by turning around the antenna properly. Besides, we provide some bounds which cannot be improved and

the optimum radiation patterns to achieve those limits, stressing the importance of the channel on antenna performance and allowing a better understanding of the antenna-channel interaction.

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