

## **HYBRID PIFA-PATCH ANTENNA OPTIMIZED BY EVOLUTIONARY PROGRAMMING**

**R. Sánchez-Montero, S. Salcedo-Sanz  
and J. A. Portilla-Figueras**

Department of Signal Theory and Communications  
Escuela Politecnica Superior  
Universidad de Alcala  
Campus Universitario, Ctra. Madrid-Barcelona, Km. 33.600  
Alcala de Henares, Madrid 28805, Spain

**R. J. Langley**

Department of Electronic and Electrical Engineering  
University of Sheffield  
Mappin Street, Sheffield, S1 3JD, UK

**Abstract**—In this paper we study the optimization process of a novel hybrid antenna, formed by a Planar Inverted-F Antenna (PIFA) and a coplanar patch in the same structure, and intended to be used in mobile communications and WIFI applications simultaneously. This hybrid device has been recently proposed and characterized in the literature, and it has been shown that it allows a bandwidth of 850 MHz (49%) in the lower band and 630 MHz (11.25%) in the upper band. In spite of these good performance results, the fine tuning of the joint PIFA-patch parameters in the hybrid antenna is a hard task, not easy to automatize. In this paper we propose the use of an Evolutionary Programming (EP) approach, an algorithm of the Evolutionary Computation family, which has been shown to be very effective in continuous optimization problems. We use a real encoding of the antenna's parameters and the CST Microwave Studio simulator to obtain the performance of the antenna. The simulator is therefore incorporated to the EP algorithm as a part of the antenna's evaluation process. We will show that the EP is able to obtain very good sets of parameters in terms of the designer necessities, usually a larger bandwidth at the design frequencies. In this case, the bandwidth of

the EP optimized antenna results in 980 MHz (55%) for the lower band and 870 MHz (17%) for the upper band.

## 1. INTRODUCTION

The booming of wireless communications technologies in the last few years has allowed an unstoppable movement to portable devices (mobile handsets, tiny laptops, netbooks, etc.), as small and light as possible, without compromising functionality. Nowadays, the trend is to integrate a large number of wireless services into these small devices, so the necessity for small antennas, operating over a wide bandwidth or multiple bands and manufactured on low cost materials, is greater every day.

One possibility to obtain a reduced size antenna with large bandwidth is the Planar Inverted-F Antenna (PIFA) [1, 2]. Another possibility for obtaining small size devices is based on a coplanar stripline feeding into a monopole [3, 4]. This construction allows a wider bandwidth and more band spacing flexibility than that obtained from a classical microstrip patch antenna. Both antennas have been widely analyzed in the literature [5–8] and they can be optimized using specific design equations when they work separately [9].

Recently, a hybrid antenna based on the joint disposition of a PIFA and a coplanar patch in the same structure has been proposed [9]. Other existing technologies for compact and multiband antennas are stacked patch [10, 11] and two feeding ports [12]. The proposed design allows a higher bandwidth, reduced size and a more symmetrical radiation pattern, which is a significant improvement over each antenna working on its own. The main problem with this hybrid antenna is to obtain an accurate optimization of its parameters. The joint structure of the device makes really difficult to obtain an exact expression based on design equations, so optimization algorithms working on the antenna performance must be used. Usually, the main problem of the optimization of antennas is that these devices must be simulated for each set of parameters, so the computation is usually time-consuming, and very few objective function evaluations are available to complete the optimization process in a reasonable time. In these conditions, soft-computing approaches work quite well, and specifically, there are several works in the literature applying novel meta-heuristics approaches, such as evolutionary computation algorithms to antennas optimization [13–15]. The good performance of these previous approaches even with the constrain of very few function evaluations, made us to consider a class of evolutionary computation algorithm to carry out the parameters' optimization

of the hybrid PIFA-patch antenna. There are many algorithms based on evolutionary computation which have been applied to continuous optimization problems such as the optimization of radiant devices. Among others, the Evolutionary Programming (EP) [17], the Particle Swarm Optimization (PSO) [19], or Differential Evolution approach (DE) [18], and hybrid approaches mixing these and other local search heuristics could be good methods to solve the problem. In this paper we have focused on the performance of Evolutionary Programming approach [17] to the optimization of the hybrid PIFA-Patch antenna, and we left for future work the comparison with alternative evolutionary-based algorithms. In this paper we describe the antenna's structure and the EP algorithm proposed, including the encoding, algorithm's structure and objective function, given by the CST simulator. We will show that this approach is quite effective, and obtains antenna's design of high quality in terms of a design objective function proposed by the antenna designer.

The structure of the rest of the paper is the following: In Section 2 we present the hybrid prototype to be studied in the paper, built with two antennas (PIFA and Circular patch antenna). Section 3 describes the Evolutionary Programming approach proposed as optimization method. A detailed discussion of the results obtained with the EP algorithm are given in Section 4. Finally, conclusions are summarized in Section 5.

## 2. ANTENNA DESIGN

The proposed antenna structure, also proposed in [9], is shown in Figure 1 whose dimensions are:  $L_g = 57$  mm,  $W_g = 24$  mm,  $L_p = 22$  mm,  $W_p = 15$  mm,  $W_s = 3.5$  mm,  $h = 6$  mm,  $A = 5$  mm,  $B = 10$  mm,  $r_{in} = 7.7$  mm,  $r_{out} = 11.2$  mm,  $L = 30$  mm,  $S = 5$  mm, and  $d = 0.5$  mm. The antenna was constructed from two single antennas: a circular patch antenna and a PIFA. The prototype was achieved by using two tier processes in order to increase the bandwidth and reducing the antenna dimensions.

First, the coplanar circular antenna and the coplanar waveguide feeding structure were etched onto the same side of a single sided substrate. The PIFA antenna was shorted to the ground plane using a  $W_s$  width strip located at  $(x = 5$  mm,  $y = 7.5$  mm,  $z = 6$  mm) on the top plate from origin. The resonant frequency of each basic antenna was adjusted independently. On one hand, the patch antenna was designed to operate near 5 GHz for WLAN applications. On the other hand, the PIFA antenna was fixed to work near 1.8 GHz for mobile applications. The geometry of the PIFA has been taken from [20].

Finally, both antennas were inserted into the same structure. In order to achieve a simplified structure, the PIFA was feeded by electromagnetic coupling, so the device maintains the enough impedance bandwidth as it is required at the two operating bands. Moreover, when the antennas are jointly used, it is necessary to reduce the sizes of both devices to achieve the correct resonant frequencies. Another consequence of this union is an enhancement of the bandwidth for the hybrid antenna.

The individual behaviour of these structures has been widely studied [5, 7, 21]. However, when merging both antennas, the behaviour is slightly different. In this case, it is necessary to make a parametric study.

The proposed hybrid antenna has been made on a low cost material, FR4 substrate, with relative permittivity 4.5, loss tangent 0.025 and 1.54 mm thickness.

### 3. AN EVOLUTIONARY PROGRAMMING ALGORITHM FOR ANTENNA PARAMETERS OPTIMIZATION

Evolutionary algorithms (EAs) [17, 22, 23] are robust problems' solving techniques based on natural evolution processes. They are population-based techniques which codify a set of possible solutions to the problem, and evolve it through the application of the so called *evolutionary operators* [22]. Among EAs, Evolutionary Programming (EP) approaches are usually applied to continuous optimization problems. This algorithm is characterized by only using mutation and selection operators (no crossover is applied). Several versions of the algorithm have been proposed in the literature: The Classical Evolutionary Programming algorithm (CEP) was first described in the work by Bäck and Schwefel in [23], and analyzed later by Yao et al. in [17] and Lee and Yao in [24]. It is used to optimize a given function  $f(\mathbf{x})$  ( $\psi$  or  $\varphi$  in our case), i.e., obtaining  $\mathbf{x}_o$  such that  $f(\mathbf{x}_o) < f(\mathbf{x})$ , with  $\mathbf{x} \in [\lim_{inf}, \lim_{sup}]$ . The CEP algorithm performs as follows:

- (i) Generate an initial population of  $\mu$  individuals (solutions). Let  $t$  be a counter for the number of generations, set it to  $t = 1$ . Each individual is taken as a pair of real-valued vectors  $(\mathbf{x}_i, \sigma_i)$ ,  $\forall i \in \{1, \dots, \mu\}$ , where  $\mathbf{x}_i$ 's are objective variables, and  $\sigma_i$ 's are standard deviations for Gaussian mutations.
- (ii) Evaluate the fitness value for each individual  $(\mathbf{x}_i, \sigma_i)$  (using the problem's objective function).

- (iii) Each parent  $(\mathbf{x}_i, \sigma_i)$ ,  $\{i = 1, \dots, \mu\}$  then creates a single offspring  $(\mathbf{x}_i, \sigma_i)$  as follows:

$$\mathbf{x}'_i = \mathbf{x}_i + \sigma_i \cdot \mathbf{N}_1(\mathbf{0}, \mathbf{1}) \quad (1)$$

$$\sigma'_i = \sigma_i \cdot \exp(\tau' \cdot N(0, 1) + \tau \cdot \mathbf{N}(\mathbf{0}, \mathbf{1})) \quad (2)$$

where  $N(0, 1)$  denotes a normally distributed one-dimensional random number with mean zero and standard deviation one, and  $\mathbf{N}(\mathbf{0}, \mathbf{1})$  and  $\mathbf{N}_1(\mathbf{0}, \mathbf{1})$  are vectors containing random numbers of mean zero and standard deviation one, generated anew for each value of  $i$ . The parameters  $\tau$  and  $\tau'$  are commonly set to  $(\sqrt{2\sqrt{n}})^{-1}$  and  $(\sqrt{2n})^{-1}$ , respectively [18], where  $n$  is the length of the individuals.

- (iv) If  $x_i(j) > \lim\_sup$  then  $x_i(j) = \lim\_sup$  and if  $x_i(j) < \lim\_inf$  then  $x_i(j) = \lim\_inf$ .
- (v) Calculate the fitness values associated with each offspring  $(\mathbf{x}'_i, \sigma'_i)$ ,  $\forall i \in \{1, \dots, \mu\}$ .
- (vi) Conduct pairwise comparison over the union of parents and offspring: for each individual,  $p$  opponents are chosen uniformly at random from all the parents and offspring. For each comparison, if the individual's fitness is better than the opponent's, it receives a "win".
- (vii) Select the  $\mu$  individuals out of the union of parents and offspring that have the most "wins" to be parents of the next generation.
- (viii) Stop if the halting criterion is satisfied, and if not, set  $k = k + 1$  and go to Step 3.

A second version of the algorithm is the so called Fast Evolutionary Programming (FEP). The FEP was described and compared with the CEP in [17]. The FEP is similar to the CEP algorithm, but it performs a mutation following a Cauchy probability density function, instead of a Gaussian based mutation. The one-dimensional Cauchy density function centered at the origin is defined by

$$f_k(x) = \frac{1}{\pi} \frac{k}{k^2 + x^2} \quad (3)$$

where  $k > 0$  is a scale parameter. See [17] for further information about this topic. Using this probability density function, we obtain the FEP algorithm by substituting step 3 of the CEP, by the following equation:

$$\mathbf{x}'_i = \mathbf{x}_i + \sigma_i \cdot \delta \quad (4)$$

where  $\delta$  is a Cauchy random variable vector with the scale parameter set to  $k = 1$ .

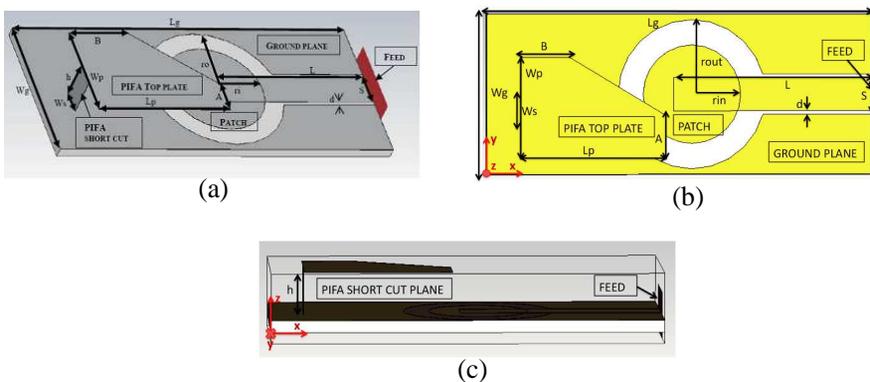
Finally, in [17] the *Improved FEP* (IFEP) is also proposed, where the best result obtained between the Gaussian mutation and the Cauchy mutation is selected to complete the process.

The EP algorithm shown above can be directly applied to estimate the optimal parameters set of our antenna model: each individual  $\mathbf{x}$  represents a different set of antenna parameters and the corresponding variances for the mutation operators, in the following way:  $(\mathbf{x}, \sigma) = \{r_{in}, r_{out}, L_g, W_g, A, B, \sigma_{rin}, \sigma_{rout}, \sigma_{Lg}, \sigma_{Wg}, \sigma_A, \sigma_B\}$ . The objective function of the problem is given by Equation (5), and it is related to the antenna bandwidth. Note that this objective function value is calculated by using the CST Microwave Studio [25].

#### 4. NUMERICAL SIMULATION RESULTS AND COMPARISON

The resonant frequencies of the antennas can be adjusted independently following the equations given in [5–8]. Nevertheless, the bandwidth of each band depends on the complete hybrid device and it is not connected with only one parameter of the prototype. For this reason, the optimization process must take into account the variation of several parameters simultaneously [13–15].

The EP algorithm presented in the previous section is applied to the case of PIFA antenna mounted on a coplanar circular patch antenna which is described in Section 2. The circular patch antenna is etched onto the dielectric, and the ground plane is mounted on the substrate of the antenna. We first seek to determine the main parameters of the



**Figure 1.** Antenna configuration. (a) Complete antenna; (b) Top view; (c) Front view.

antenna related to the bandwidth. As a result of this study, we focus on the following antenna parameters:  $A$ ,  $B$ ,  $L_g$ ,  $W_g$ ,  $r_{in}$  and  $r_{out}$ . All these variables are defined in Figure 1.

Next step consists of defining the fitness function according to the following rules:

- (i) The antenna analysis should be focused on the  $S_{11}$  values from 1 GHz to 7 GHz with 1001 samples. The reflection coefficient must be fewer than  $-10$  dB.
- (ii) The working bands are defined from 1.4 GHz to 2.6 GHz for the lower band and from 4.9 GHz to 6.1 GHz for the higher band. So, the antenna can operate in GSM, UMTS and WLAN applications [5, 16].
- (iii) The samples, where  $S_{11}$  does not achieve the criterion mentioned above, are counted for each band. Their values are dened in the variables called  $N_1$  (for the lower band) and  $N_2$  (for the upper band).

According to these remarks the fitness function is calculated by using the following Equation:

$$\text{Fitness} = f(x) = k_1 N_1 + k_2 N_2 \quad (5)$$

where  $k_1$  and  $k_2$  are the coefficients which allow different weights for the lower band or the higher band respectively. In our case  $k_1 = k_2 = 1$ . Obviously, the purpose of the optimization process by the EP algorithm is to minimize the objective function given by Equation (5).

During the simulations, we adopted the following parameter set for the EP:

- Population size: pop = 20.
- Maximum number of generations: maxgen = 50.
- IFEP running mode.

The EP algorithm has been programmed in MATLAB in connection with the CST simulator for the calculation of the fitness value of each individual. The process for communicating CST and MATLAB is as follows: First, using EP as it is described in Section 3, the values for the parameters to be optimized are generated using MATLAB. These values are saved in a text file. A Visual Basic for Applications macro (VBA) is the more straightforward way to run the CST simulation from MATLAB. Second, MATLAB calls CST and a VBA macro loads the data from the text file and stores the values in CST variables. Then, a new macro is called for creating the structure of the proposed device according to the values of the variables previously loaded and starts the simulation. When the simulation finishes, the

results for  $S_{11}$  are saved in a new text file and CST is closed. Finally, MATLAB reads the results of  $S_{11}$  from the file and the fitness function is calculated. This process is repeated for each individual until all the population is evaluated.

Using this process, we have analyzed the influence in the  $S_{11}$  response when we applied the EP for the optimization of two parameters in the antenna: First  $r_{in}$  and  $r_{out}$  are modified, later  $A$  and  $B$  are used in the algorithm and finally  $L_g$  and  $W_g$  are varied. Figure 2 shows the comparison between the original and the results of the optimization responses. The values are given in Table 1, where LBW means Lower Bandwidth, UBW means Upper Bandwidth and TBW means Total Bandwidth which is obtained by adding the LBW and the UBW. LBW and UBW are centered at 1.8 GHz and 5.2 GHz respectively.

Then, the prototype has been optimized by the EP with four parameters in two different ways. Initially, we have modified two parameters,  $r_{in}$  and  $r_{out}$  ( $2 + 0$  schema). After that, when the EP is finished, we start again the process with another two parameters (variables  $W_g$  and  $L_g$ ) ( $2 + 2$  schema). Then, we carried out the optimization process with all four parameters together ( $r_{in}$ ,  $r_{out}$ ,  $W_g$ , and  $L_g$ ) ( $4 + 0$  schema). Table 1 shows that the results

**Table 1.** Bandwidth values obtained by the EP.

Parameters	Fitness	LBW (MHz)	UBW (MHz)	TBW (MHz)
Original	-	850 (47.2%)	630 (12.11%)	1480 (59.31%)
$r_{in}$ and $r_{out}$	111	1000 (55.5%)	738 (14.19%)	1738 (69.69%)
$A$ and $B$	129	858 (47.6%)	774 (14.8%)	1632 (62.4%)
$L_g$ and $W_g$	153	762 (42.3%)	777 (14.92%)	1539 (57.22%)
$r_{in}$ and $r_{out}$ + $L_g$ and $W_g$	154	1000 (55.5%)	534 (10.2%)	1534 (65.7%)
$r_{in}, r_{out}$ , $L_g$ and $W_g$	99	978 (54.3%)	838 (16.1%)	1816 (70.4%)
$r_{in}, r_{out}$ , $W_g$ , $L_g, A$ and $B$	93	980 (54.4%)	870 (16.7%)	1850 (71.1%)

in each case are slightly different. Figure 2 shows the comparison between the  $S_{11}$  response of the original and the optimized antennas. Finally, we have carried out the optimization of the antenna with six parameters simultaneously (6 + 0 schema). The results are shown in Figure 2 and the values are indicated in Table 1. The results show that there is a bandwidth enhancement, specially when four or six parameters are optimized simultaneously.

Regarding the computation time, note that the simulation process of each antenna for a given set of parameters (done by the CST) takes time. All the simulations have been carried out in an Intel Core 2 Duo computer with 2.53 GHz and 4 GB RAM, and the computation time depends much on the number of parameters to be optimized. This can be seen in Table 2, where the computation time of the complete EP process is shown. Note that the CST simulator takes much more time when the number of antenna parameters is increased, which affect the final computation time of the algorithm. Note however,

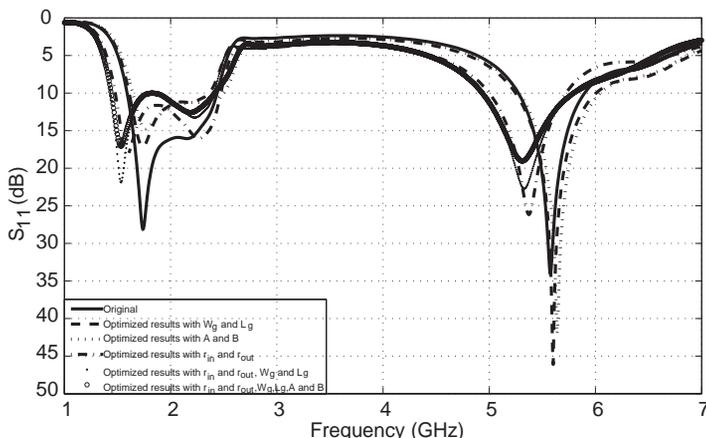


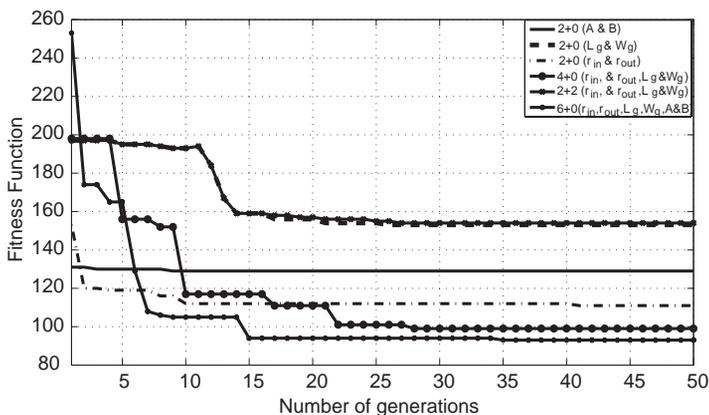
Figure 2.  $S_{11}$  original and optimized results.

Table 2. Computational time variation.

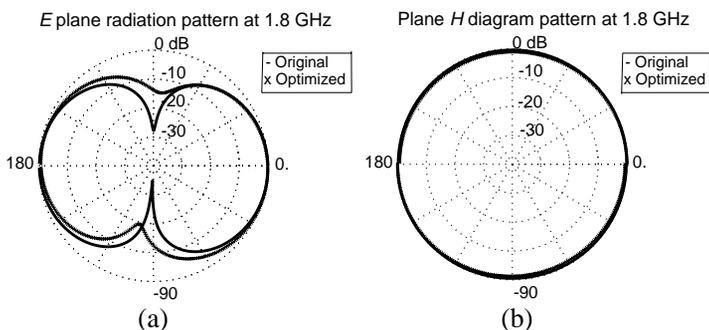
Number of parameters	Computation time (minutes)
2 + 0	2900
4 + 0	4320
2 + 2	5040
6 + 0	6480

that the antenna design process must be carried out just once and no real time is required at all, so the computation time required by the system is completely assumable by a designer as long as the antenna characteristics are improved.

Figure 3 represents the fitness function evolutions with the number



**Figure 3.** Fitness function evolution in the EP process.



**Figure 4.** Radiation pattern at 1.8 GHz. (a) *E* plane; (b) *H* plane.

**Table 3.** Values of the original and optimized parameters.

Antenna	$W_g$ (mm)	$L_g$ (mm)	$r_{in}$ (mm)	$r_{out}$ (mm)	$A$ (mm)	$B$ (mm)
Original	24	57	7.7	11.2	5	10
EP Optimized	24.43	63.6	7.4	12.2	0.1	7.6

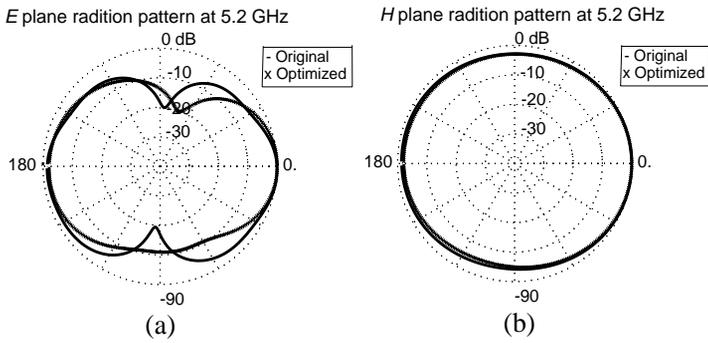


Figure 5. Radiation pattern at 5.2 GHz. (a) *E* plane; (b) *H* plane.

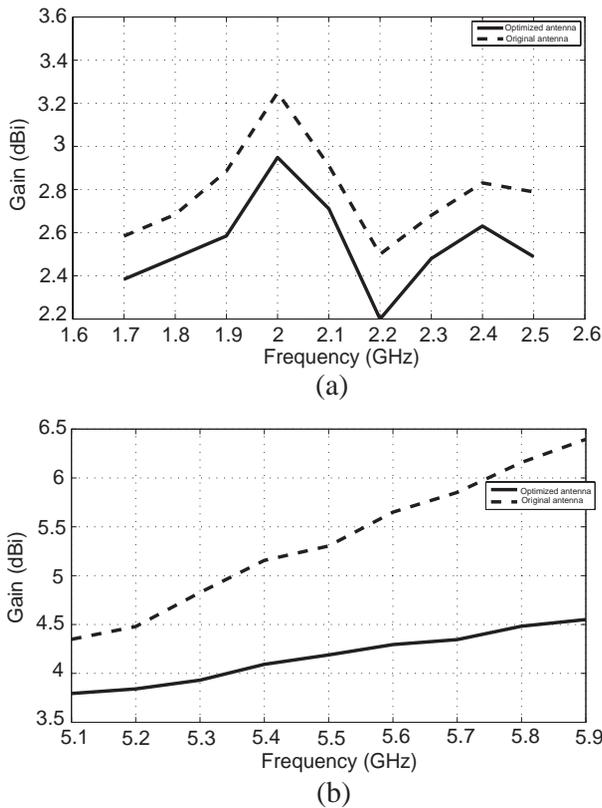


Figure 6. Gain Values; (a) Gain in the lower band; (b) Gain in the upper band.

of generations to verify the convergence of the EP algorithm. The total bandwidth is inverse to the value of the fitness function. As the best value is obtained a few generations before that the EP algorithm finishes, the set of parameters is proven to be optimal. In this work, the best solution is obtained in the last case (6 + 0 schema), when all parameters are modified at the same time. Finally, for the best performance antenna, Table 3 shows the comparison between the parameters values of the original and the EP optimized antenna. The rest of the parameters have the same value as the original antenna, shown in Section 2. Radiation pattern and gain for original and best performance antenna are also shown in Figures 4, 5, and 6. As can be seen there are not noticeable differences between them. The purposed optimization process of the input impedance improves slightly the gain and does not affect to the radiation pattern.

## 5. CONCLUSION

In this paper we have presented an optimization process based on an evolutionary programming (EP) algorithm, to improve the characteristics of a hybrid PIFA-patch antenna. The EP algorithm has been implemented in MATLAB and the prototype simulations have been carried out using the CST Microwave Studio simulator. We have shown that the proposed automatic optimization process is able to obtain a high quality PIFA-patch antenna, with good properties of bandwidth, as required by the antenna designer.

## REFERENCES

1. Dimousios, T. D., C. D. Nikolopoulos, S. A. Mitilineos, and C. N. Capsalis, "A new low-profile and cost SPA-PIFA for mobile 2.4 GHz ISM applications," *Journal of Electromagnetic Waves and Applications*, Vol. 24, No. 7, 881–891, 2010.
2. Xiao, H.-L. and Z.-P. Nie, "Design of compact planar inverted-L diversity antenna for handheld terminals," *Progress In Electromagnetic Research C*, Vol. 1, 185–189, 2008.
3. Chang, T. N., G. Y. Shen, and J. M. Lin, "CPW-fed antenna covering Wimax 2.5/3.5/5.7 GHz bands," *Journal of Electromagnetic Waves and Applications*, Vol. 24, No. 2–3, 189–197, 2010.
4. Lin, D.-B., I.-T. Tang, and Y.-Y. Chang, "Flower-like CPW-FED monopole antenna for quad-band operation of mobile handsets,"

- Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 17–18, 2271–2278, 2009.
5. Balanis, C. A., *Antenna Theory: Analysis and Design*, 3rd edition, John Wiley and Sons, 2005.
  6. Mahattanajatuphat, C., P. Akkaraekthalin, S. Saleekaw, and M. Krairiksh, “A bidirectional multiband antenna with modified fractal slot fed by CPW,” *Progress In Electromagnetics Research*, Vol. 95, 59–72, 2009.
  7. Kashiwa, T., N. Yoshida, and I. Fukai, “Analysis of radiation characteristics of planar inverted-F antenna on conductive body of hand-held transceiver by spatial network method,” *Electronic Letter*, Vol. 25, No. 16, 1044–1045, 1989.
  8. Saidatul, N. A., A. A. H. Azremi, R. B. Ahmad, P. J. Soh, and F. Malek, “Multiband fractal planar inverted F antenna (F-PIFA) for mobile phone application,” *Progress In Electromagnetics Research B*, Vol. 14, 127–148, 2009.
  9. Deng, S. M., C. L. Tsai, C. K. Yeh, and S. S. Bor, “CPW-fed PIFAS with a capacitively coupling slot for dual wide-band operations,” *Journal of Electromagnetic Waves and Applications*, Vol. 24, No. 5–6, 721–733, 2010.
  10. Vazquez Antuna, C., G. Hotopan, S. Ver Hoeye, M. Fernandez Garcia, L. F. Herran Ontanon, and F. Las Heras, “Microstrip antenna design based on stacked patches for reconfigurable to dimensional planar array topologies,” *Progress In Electromagnetics Research*, Vol. 97, 95–104, 2009.
  11. Dahele, J. S., K. F. Lee, and D. P. Wond, “Dual frequency stacked annular ring microstrip antenna,” *IEEE Trans. Antennas Propag.*, Vol. 49, 1094–1100, 2001.
  12. Secmen, M. and A. Hizal, “A dual-polarized wide-band patch antenna for indoor mobile communication applications,” *Progress In Electromagnetics Research*, Vol. 100, 189–200, 2010.
  13. Kouveliotis, N. K., S. C. Panagiotou, P. K. Varlamos, T. D. Dimousios, and C. N. Capsalis, “Optimizing a PIFA using a genetic algorithms approach,” *Journal of Electromagnetic Waves and Applications*, Vol. 22, No. 2–3, 453–461, 2008.
  14. Zhou, S. G., B. H. Sun, J. L. Guo, Q. Z. Liu, and Y. Hua, “A new fitness function for optimizing the matching network of broadband antennas by genetic algorithm,” *Journal of Electromagnetic Waves and Applications*, Vol. 22, No. 5–6, 759–765, 2008.
  15. Luo, Z., X. Chen, and K. Huang, “A novel electrically-small microstrip genetic antenna,” *Journal of Electromagnetic Waves*

- and Applications*, Vol. 24, No. 4, 513–520, 2010.
16. IEEE 802 LAN/MAN Standards Committee, <http://ieee802.org>, Last visit September 3, 2010.
  17. Yao, X., Y. Liu, and G. Lin, “Evolutionary programming made faster,” *IEEE Trans. Evol. Comput.*, Vol. 3, No. 2, 82–102, 1999.
  18. Price, K., R. Storn, and J. Lampinen, *Differential Evolution—A Practical Approach to Global Optimization*, Springer-Verlag, 1998.
  19. Kennedy, J. and R. C. Eberhart, *Swarm Intelligence*, Morgan Kaufmann, 2001.
  20. Kim, B. C., et al., “Tapered type PIFA design for mobile phones at 1800 MHz,” *The 57th IEEE Semiannual Vehicular Technology Conference*, 1012–1014, April 2003.
  21. Fujimoto, K., A. Henderson, K. Hirasawa, and J. R. James, *Small Antennas*, Research Studies Press Ltd, 1987.
  22. Goldberg, D. E., *Genetic Algorithms in Search, Optimization and Machine Learning, Reading*, Addison-Wesley, MA, 1989.
  23. Bäck, T. and H. P. Schwefel, “An overview of evolutionary algorithms for parameter optimization,” *Evolutionary Computation*, Vol. 1, No. 1, 1–23, 1993.
  24. Lee, C. Y. and X. Yao, “Evolutionary programming using mutations based on the Levy probability distribution,” *IEEE Transaction on Evolutionary Computation*, Vol. 8, No. 1, 1–13, 2004.
  25. CST-Microwave Studio, *User’s Manual*, 2009.