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TRANSIENT WAVE PROPAGATION IN A GENERAL DISPERSIVE MEDIA USING THE LAGUERRE FUNCTIONS IN A MARCHING-ON-IN-DEGREE (MOD) METHODOLOGY

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Abstract—The objective of this paper is to illustrate how the marching-on-in-degree (MOD) method can be used for efficient and accurate solution of transient problems in a general dispersive media using the finite difference time-domain (FDTD) technique. Traditional FDTD methods when solving transient problems in a general dispersive media have disadvantages because they need to approximate the time domain derivatives by finite differences and the time domain convolutions by using finite summations. Here we provide an alternate procedure for transient wave propagation in a general dispersive medium where the two issues related to finite difference approximation in time and the time consuming convolution operations are handled analytically using the properties of the associate Laguerre functions. The basic idea here is that we fit the transient nature of the fields, the permittivity and permeability with a series of orthogonal associate Laguerre basis functions in the time domain. In this way, the time variable can not only be decoupled analytically from the temporal variations but that the final computational form of the equations is transformed from FDTD to a FD formulation in the differential equations after a Galerkin testing. Numerical results are presented for transient wave propagation in general dispersive materials which use for example, a Debye, Drude, or Lorentz models.

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1. INTRODUCTION

The FDTD method has been widely used in the numerical computation of electromagnetic fields [1–4]. However, any practical medium in nature, such as snow, ice, plasma, and soil, has frequency-dependent properties. The permittivity and permeability of the dispersive medium are a function of frequency. When a FDTD method is applied to the transient wave propagation in a dispersive media, the constitutive relations have convolutional forms in the time domain. This convolution is very time consuming to calculate for arbitrary permittivity or permeability variation in the time domain. To simplify this calculation, researchers have classified materials in different categories and there are three popular material dispersion models that are amenable to FDTD modeling [4]: the Debye relaxation, the Drude model, and the Lorentzian resonance. For simplicity, we consider only the electric permittivity $\varepsilon(t)$. The extension to magnetic permeability $\mu(t)$ is straightforward. In the time domain, we have

$$\varepsilon(t) = \varepsilon_0 \left[\varepsilon_\infty \delta(t) + \chi(t)\right] \tag{1}$$

where ε_0 is the permittivity of free space. ε_∞ is the relative permittivity at the infinite frequency, and $\chi(t)$ is the electric susceptibility. The electric susceptibility function for the three different generic material dispersions can be expressed as:

1) Debye medium

$$\chi(t) = \sum_{p=1}^{P} \frac{\varepsilon_{s,p} - \varepsilon_\infty}{\tau_p} e^{-t/\tau_p} u(t) \tag{2}$$

where $\varepsilon_{s,p}$ refers to the relative permittivity at static state or zero frequency, τ_p represent electric pole relaxation time, P is the number of poles, and $u(t)$ is the unit step function.

2) Drude medium (plasma)

$$\chi(t) = \sum_{p=1}^{P} \frac{\omega_p^2}{\nu_{c,p}} \left(1 - e^{-\nu_{c,p} t}\right) u(t) \tag{3}$$

where ω_p is the Drude pole frequency (plasma frequency) and $\nu_{c,p}$ is the inverse of the pole relaxation time (collision frequency).

3) Lorentz medium

$$\chi(t) = \sum_{p=1}^{P} \gamma_p e^{-\alpha_p t} \sin(\beta_p t) u(t) \tag{4}$$

$$\alpha_p = \delta_p, \quad \beta_p = \sqrt{\omega_p^2 - \delta_p^2}, \quad \gamma_p = \frac{G_p(\varepsilon_s - \varepsilon_\infty)\,\omega_p^2}{\beta_p}, \quad \sum_{p=1}^{P} G_p = 1,$$

where ω_p is the frequency of the conjugate pole pair, and δ_p is the damping coefficient.

Several frequency-dependent FDTD methods have been developed, such as recursive convolution (RC), piecewise linear recursive convolution (PLRC), auxiliary differential equation (ADE), Z-transform, trapezoidal recursive convolution (TRC), JE convolution techniques, and so on [3–9]. Basically there are two popular algorithms to apply for analysis of wave propagation in a dispersive media. One is the RC and the more accurate PLRC and the other algorithm is the ADE method. Both the traditional FDTD methods for media dispersion have three disadvantages:

1) The three 'generic' media dispersions are mathematical models of permittivity and permeability. The analytic expressions may not accurately approximate the general media dispersion in reality.
2) Both RC/PLRC and ADE are designed for one of the three specific media dispersions. For an arbitrary media dispersion, we have different formulations.
3) Sampling along time is required for both RC/PLRC and ADE. As a matter of fact, discretization in the time domain is the requirement for all the traditional FDTD methods. It will introduce numerical errors and could make FDTD potentially unstable since the time and the spatial samples have to satisfy the Courant-Friedrich-Levy sampling criteria.

As a summary, traditional FDTD methods for handling general dispersive media have disadvantages because they approximate the time derivatives by finite differences and the time domain convolutions by finite discrete summations.

In this paper, we propose an efficient and accurate method to apply FDTD for transient wave propagation in a general dispersive media using the MOD scheme with the associate Laguerre functions. This MOD methodology has been successfully implemented in a FDTD formulation [10–14] and in integral equations dealing with skin effects in conductors, and propagation in non-dispersive dielectric and in a dispersive media [15–17]. The basic idea here is that we fit the fields, the permittivity and permeability with a series of orthogonal associate Laguerre basis functions in the time domain. The time domain variable t can be analytically eliminated from the differential equations after a Galerkin testing. Due to the analytic properties of the derivatives and the convolutions using the associate Laguerre functions, the calculations using FDTD in an arbitrary dispersive media can be fast and accurately. This is illustrated next. Finally, the validity of the proposed MOD methodology is provided using three typical forms of the dispersive model, i.e., Debye, Drude, and Lorentz medium.

2. FORMULATION

In the time domain, the general FDTD formulation in a dispersive media can be written as [4]

$$\frac{\partial}{\partial t}\left[\varepsilon(\mathbf{r},t)\otimes\mathbf{E}(\mathbf{r},t)\right] = \nabla\times\mathbf{H}(\mathbf{r},t)-\mathbf{J}(\mathbf{r},t) \tag{5}$$

$$\frac{\partial}{\partial t}\left[\mu(\mathbf{r},t)\otimes\mathbf{H}(\mathbf{r},t)\right] = -\nabla\times\mathbf{E}(\mathbf{r},t)-\mathbf{M}(\mathbf{r},t) \tag{6}$$

where ε is the permittivity and μ is the permeability and $\otimes$ denotes a convolution. $\mathbf{E}$ and $\mathbf{H}$ are the electric field and magnetic field, and $\mathbf{J}$ and $\mathbf{M}$ are the volume electric and magnetic current densities, respectively. Note that all the variables in (5) and (6) are all functions of both space and time. Since the media is dispersive, ε and μ may vary for different frequencies. In the time domain they are now arbitrary functions of time for a general dispersive medium. Their multiplications with the appropriate fields in the frequency domains are transformed to convolution in the time domain which can be defined as:

$$f(t)\otimes g(t)=\int_0^t f(t-\tau)g(\tau)d\tau. \tag{7}$$

For simplicity, we consider a one-dimensional problem with the field component E_y and H_z which are propagating along the x-direction. Then Equations (5) and (6) become

$$\frac{\partial}{\partial t}\left[\varepsilon(x,t)\otimes E_y(x,t)\right] = -\frac{\partial H_z(x,t)}{\partial x}-J_y(x,t) \tag{8}$$

$$\frac{\partial}{\partial t}\left[\mu(x,t)\otimes H_z(x,t)\right] = -\frac{\partial E_y(x,t)}{\partial x}-M_z(x,t). \tag{9}$$

To eliminate the tedious integration of convolution and to carry out the derivatives analytically, we expand all the temporal quantities (both known and unknown) in terms of the associate Laguerre functions given by [18]

$$\phi_p(st)=e^{-st/2}L_p(st) \tag{10}$$

where s is a time scale parameter which takes care of the units along the time axis, and L_p is the Laguerre polynomial with degree p [19, 20].

The scaling factor s can be chosen a priori [21]. This temporal basis functions are orthonormal as

$$\int_0^\infty \phi_p(st)\phi_q(st)d(st) = \delta_{pq} \tag{11}$$

where δ_{pq} is Kronecker delta with value 1 when $p = q$ and 0 otherwise.

A continuous function $F(x,t)$ defined for any value of time $t \geq 0$ can be expanded by the associate Laguerre basis functions as

$$F(x,t) = \sum_{p=0}^{\infty} F_p(x)\phi_p(st) \tag{12}$$

where F_p is the coefficient which can be obtained from

$$F_p(x) = \int_0^\infty F(x,t)\phi_p(st)d(st). \tag{13}$$

First we expand the time domain functions E_y, H_z, J_y, and M_z with (12):

$$E_y(x,t) = \sum_{p=0}^{\infty} E_y^p(x)\phi_p(st) \tag{14}$$

$$H_z(x,t) = \sum_{p=0}^{\infty} H_z^p(x)\phi_p(st) \tag{15}$$

$$J_y(x,t) = \sum_{p=0}^{\infty} J_y^p(x)\phi_p(st) \tag{16}$$

$$M_z(x,t) = \sum_{p=0}^{\infty} M_z^p(x)\phi_p(st) \tag{17}$$

where E_y^p, H_z^p, J_y^p, and M_z^p are the coefficients of the associate Laguerre basis functions for E_y, H_z, J_y, and M_z, respectively. We expand the time domain parameters ε and μ by the associate Laguerre series as:

$$\varepsilon(x,t) = \sum_{n=0}^{\infty} \varepsilon^n(x)\phi_n(st) \tag{18}$$

$$\mu(x,t) = \sum_{n=0}^{\infty} \mu^n(x)\phi_n(st) \tag{19}$$

where

$$\varepsilon^n(x) = \int_0^\infty \varepsilon(x,t)\phi_n(st)d(st) \tag{20}$$

$$\mu^n(x) = \int_0^\infty \mu(x,t)\phi_n(st)d(st). \tag{21}$$

Here it is assumed that the general permittivity and permeability profiles for the dispersive media are known and so that we know the coefficients ε^n and μ^n. However, the field quantities E_y^p, and H_z^p are unknown and our goal is to estimate these coefficients from the given excitations J_y and M_z.

We focus on the solution of (8) first. As long as the formulation using the associate Laguerre basis functions of (14)–(19) can be derived, the formulation for the other Equation (9) can be similarly obtained. By expanding ε and E_y, putting them in the left hand side of (8), and by changing the sequence of summation and integration we have

$$\varepsilon(x,t) \otimes E_y(x,t) = \sum_{n=0}^{\infty} \varepsilon^n(x) \sum_{p=0}^{\infty} E_y^p(x) \int_0^t \phi_n(st - s\tau)\phi_p(s\tau)d\tau. \quad (22)$$

The convolution between ε and E_y can be transformed to the convolution between the associate Laguerre basis functions with different degrees. Due to the property of convolution between the associate Laguerre basis functions [18] we have

$$\int_0^t \phi_m(st - s\tau)\phi_n(s\tau)d\tau = \frac{1}{s}\left[\phi_{m+n}(st) - \phi_{m+n+1}(st)\right]. \quad (23)$$

Using the result in (23), the integration for the convolution in (22) can be eliminated analytically and we have

$$\varepsilon(x,t) \otimes E_y(x,t) = \frac{1}{s} \sum_{n=0}^{\infty} \varepsilon^n(x) \sum_{p=0}^{\infty} E_y^p(x)\left[\phi_{p+n}(st) - \phi_{p+n+1}(st)\right]. \quad (24)$$

In addition to the property of the convolution, the derivative of the associate Laguerre basis functions can be written as [18]

$$\frac{d}{dt}\phi_n(st) = -s\left[\frac{1}{2}\phi_n(st) + \sum_{k=0}^{n-1}\phi_k(st)\right]. \quad (25)$$

Putting (25) in (24), we have

$$\frac{\partial}{\partial t}\left[\varepsilon(x,t) \otimes E_y(x,t)\right] = \frac{1}{2} \sum_{n=0}^{\infty} \varepsilon^n(x) \sum_{p=0}^{\infty} E_y^p(x)\left[\phi_{p+n+1}(st) + \phi_{p+n}(st)\right]. \quad (26)$$

By expanding H_z and J_y in the right hand side of (8) and using (26), we obtain the simplified equation without any explicit convolutions and temporal derivatives as:

$$\frac{1}{2}\sum_{p=0}^{\infty} E_y^p(x) \sum_{n=0}^{\infty} \varepsilon^n(x) \left[\phi_{p+n+1}(st) + \phi_{p+n}(st)\right]$$
$$= -\frac{\partial}{\partial x}\sum_{p=0}^{\infty} H_z^p(x)\phi_p(st) - \sum_{p=0}^{\infty} J_y^p(x)\phi_p(st). \tag{27}$$

To eliminate the variable t and the infinite summation in (27), we test this equation in a Galerkin's methodology with $\phi_q(st)$. Due to the orthogonal property in (11) we have

$$\frac{1}{2}\left[\varepsilon^0(x)E_y^q(x) + \sum_{p=0}^{q-1}\left\{\varepsilon^{q-p}(x) + \varepsilon^{q-p-1}(x)\right\}E_y^p(x)\right]$$
$$= -\frac{d}{dx}H_z^q(x) - J_y^q(x). \tag{28}$$

Note that in (28), we have successfully eliminated the time variable t from the final computational form of the differential equation, and have eliminated the explicit convolution and the temporal derivative analytically. No numerical approximation or discrete sampling has been applied so far and the Equation (28) is theoretically equivalent to (8). Similarly, we have for Equation (9) by using a similar procedure as

$$\frac{1}{2}\left[\mu^0(x)H_z^q(x) + \sum_{p=0}^{q-1}\left\{\mu^{q-p}(x) + \mu^{q-p-1}(x)\right\}H_z^p(x)\right]$$
$$= -\frac{d}{dx}E_y^q(x) - M_z^q(x). \tag{29}$$

Using the finite difference in space to approximate the spatial derivatives is similar to the traditional FDTD methods. By using the spatial difference in (28) at $x = i\Delta x$, we have

$$\varepsilon_i^0 \left.E_y\right|_i^q = -\frac{2}{\Delta x}\left[\left.H_z\right|_{i+1/2}^q - \left.H_z\right|_{i-1/2}^q\right] - 2\left.J_y\right|_i^q - \left.P_y\right|_i^{q-1} \tag{30}$$

where

$$\left.P_y\right|_i^{q-1} = \sum_{p=0}^{q-1}\left[\varepsilon_i^{q-p} + \varepsilon_i^{q-p-1}\right]\left.E_y\right|_i^p. \tag{31}$$

Similarly, from (29) at $x = (i + 1/2)\Delta x$, we get

$$\mu_{i+1/2}^0 \left.H_z\right|_{i+1/2}^q = -\frac{2}{\Delta x}\left[\left.E_y\right|_{i+1}^q - \left.E_y\right|_i^q\right] - 2\left.M_z\right|_{i+1/2}^q - \left.Q_z\right|_{i+1/2}^{q-1} \tag{32}$$

where

$$Q_z\Big|_{i+1/2}^{q-1} = \sum_{p=0}^{q-1} \left[\mu_{i+1/2}^{q-p} + \mu_{i+1/2}^{q-p-1}\right] H_z\Big|_{i+1/2}^{p}. \tag{33}$$

The left hand sides in (30) and (32) have the coefficients for the qth degree. The right hand sides of these equations contain the previous computed coefficients with degree less than q. By inserting (32) into (30) for $H_z|_{i\pm1/2}^{q}$, we have

$$\alpha_{i(i-1)} E_y\Big|_{i-1}^{q} + \alpha_{ii} E_y\Big|_{i}^{q} + \alpha_{i(i+1)} E_y\Big|_{i+1}^{q} = \beta_i^q \tag{34}$$

where

$$\alpha_{i(i-1)} = \frac{2}{\Delta x \mu_{i-1/2}^0} \tag{35}$$

$$\alpha_{ii} = -\left(\frac{\Delta x \varepsilon_i^0}{2} + \frac{2}{\Delta x \mu_{i-1/2}^0} + \frac{2}{\Delta x \mu_{i+1/2}^0}\right) \tag{36}$$

$$\alpha_{i(i+1)} = \frac{2}{\Delta x \mu_{i+1/2}^0} \tag{37}$$

$$\begin{aligned}\beta_i^q = {} & \Delta x J_y\Big|_i^q - 2\left(\frac{M_z\Big|_{i+1/2}^{q}}{\mu_{i+1/2}^0} - \frac{M_z\Big|_{i-1/2}^{q}}{\mu_{i-1/2}^0}\right) \\ & + \frac{\Delta x}{2} P_y\Big|_i^{q-1} - \left(\frac{Q_z\Big|_{i+1/2}^{q-1}}{\mu_{i+1/2}^0} - \frac{Q_z\Big|_{i-1/2}^{q-1}}{\mu_{i-1/2}^0}\right).\end{aligned} \tag{38}$$

We can build a matrix equation form from (34)–(38) with a proper boundary condition. Here we use the dispersion boundary condition derived using the associate Laguerre basis functions in [10]. By solving this matrix equation recursively using the MOD, the electric field is obtained with M temporal basis functions.

3. NUMERICAL RESULTS

To validate the proposed algorithm, examples involving three typical kinds of dispersive model, i.e., Debye, Drude, and Lorentz are taken into consideration. With susceptibility functions given in (2)–(4), Equation (20) can be computed analytically as

$$\varepsilon^q = \varepsilon_0\left(s\varepsilon_\infty + \chi^q\right) \tag{39}$$

where

$$\chi^q = \sum_{p=1}^{P} \frac{\varepsilon_{s,p} - \varepsilon_\infty}{\tau_p} \left(\frac{1}{s\tau_p} - \frac{1}{2}\right)^q \left(\frac{1}{s\tau_p} + \frac{1}{2}\right)^{-(q+1)} \quad \text{for Debye model} \tag{40}$$

$$\chi^q = \sum_{p=1}^{P} \frac{\omega_p^2}{\nu_{c,p}} \left[2(-1)^q - \left(\frac{\nu_{c,p}}{s} - \frac{1}{2}\right)^q \left(\frac{\nu_{c,p}}{s} + \frac{1}{2}\right)^{-(q+1)}\right] \quad \text{for Drude model} \tag{41}$$

$$\chi^q = \mathrm{Re}\left[\sum_{p=1}^{P} j\gamma_p \left(\frac{\alpha_p + j\beta_p}{s} - \frac{1}{2}\right)^q \left(\frac{\alpha_p + j\beta_p}{s} + \frac{1}{2}\right)^{-(q+1)}\right] \quad \text{for Lorentz model} \tag{42}$$

and Re[·] denotes extracting the real part of a complex number. In these analytic integrations to get (40)–(42), we used the transform of the Laguerre functions [18] as

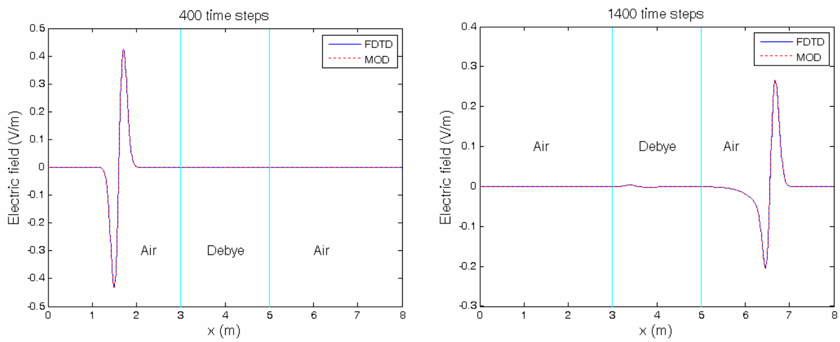
$$\int_0^\infty e^{-bx} \phi_n(x)\, dx = \left(b - \frac{1}{2}\right)^n \left(b + \frac{1}{2}\right)^{-(n+1)}. \tag{43}$$

We assume the permeability is that of free space, $\mu(t) = \mu_0 \delta(t)$ and Equation (21) becomes $\mu^q = s\mu_0$.

The geometry to be analyzed here is a one-dimensional dispersive dielectric slab. The incident field used in this work is the Gaussian pulse plane wave as defined in [22]

$$E^{\mathrm{inc}}(x,t) = E_0 \frac{4}{T\sqrt{\pi}} e^{-\gamma^2}, \quad \gamma = \frac{4}{T}\left(ct - ct_0 - x + x_s\right) \tag{44}$$

where T is the pulse width in lm (lightmeter). c is the velocity of propagation in free space. t_0 is a time delay which represents the time at which the pulse peak at the origin, and x_s is the source position of the plane wave incidence. We set $E_0 = T\sqrt{\pi}/4$. When the derivative of the Gaussian pulse is incident on the slab, $E_0 = T^2\sqrt{\pi}/(32c)$. The time delay is set as $t_0 = 1.5(T/c)$ in the numerical computation. When a plane wave with y-polarization is incident from the x-direction, the current densities in (38) are given in [11]. We compare the electric field computed by the proposed MOD scheme with the solution obtained using the PLRC-FDTD [9] and the analytic solution. In the computation of the PLRC-FDTD, we set the time step size $\Delta t = \Delta x/2c$. The analytic solution is obtained by using the inverse discrete Fourier transform of the frequency domain solution as described in [23].



(a) (b)

Figure 1. Electric field along the x-position for a plane wave with the derivative of a Gaussian pulse incident on the Debye material slab. (a) $t = 400\Delta t$, (b) $t = 1,400\Delta t$.

The first geometry to be analyzed here is a one-dimensional 0.25-dB loaded foam absorber which was considered in [8]. The parameters for this material are $\varepsilon_{s,1} = 1.16$, $\varepsilon_\infty = 1.01$, and $\tau_1 = 6.497 \times 10^{-10}$. The problem space consists of 800 cells with $\Delta x = 10\,\text{mm}$ and 301–500 cells for the slab. The incident field is a plane wave with the derivative of a Gaussian pulse. We set $T/c = 2\,\text{ns}$ and $x_s/\Delta x = 50$. The number of Laguerre basis functions is $M = 300$ and the time scale parameter is set to $s = 8.85\times10^9$. Figure 1 shows the electric field along the x-position at 400 and 1,400 time steps of the FDTD computation. The time step is $\Delta t = 16.67\,\text{ps}$. The agreement between the proposed MOD and PLRC-FDTD solution is excellent. Figure 2 compares the exact solution and numerical results at $x = 400\Delta x$, calculated using the PLRC-FDTD method and the proposed MOD scheme. All three solutions show excellent agreement.

For the next example, calculations are made for a plasma slab [6]. The one-dimensional problem space consists of 800 spatial cells each 75 µm thick, with the plasma slab occupying cells 301 through 500. The plasma considered has $\varepsilon_\infty = 1$, $\omega_1/2\pi = 28.7\,\text{GHz}$, and $\nu_{c,1} = 2 \times 10^{10}$. In order to eliminate the incident energy at zero frequency, calculations were made for a normally incident plane wave with a time behavior given by the derivative of a Gaussian pulse. We set $T/c = 25\,\text{ps}$ and $x_s/\Delta x = 50$. The number of Laguerre basis functions is $M = 250$ and the time scale parameter is set to $s = 7 \times 10^{11}$. Figure 3 shows the electric field along the x-position at 1,100 and 1,650 time steps of the PLRC-FDTD computation. The time step is 0.125 ps. The agreement between the MOD and PLRC-FDTD solution is excellent. Figure 4 compares the exact solution and the numerical results at $x = 200\Delta x$, calculated using the PLRC-FDTD method and the proposed MOD scheme. All the three solutions agree well, as is evident from the figure.

Finally, the transient fields for a pulsed plane wave incident on a dispersive medium with two second-order Lorentz poles are calculated [7]. The parameters for this material are $\varepsilon_s = 3$, $\varepsilon_\infty = 1.5$, $\omega_1/2\pi = 20\,\text{GHz}$, $\delta_1 = 0.1\omega_1$, $G_1 = 0.4$, $\omega_2/2\pi = 50\,\text{GHz}$, $\delta_2 = 0.1\omega_2$, and $G_2 = 0.6$. The problem space consists of 1,200 cells with $\Delta x = 37.5\,\mu\text{m}$ and 401–800 cells for the slab. The excitation is a Gaussian

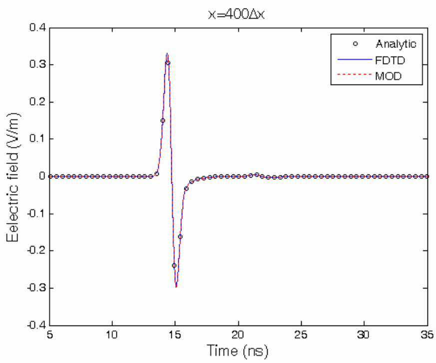


Figure 2. Transient electric field at $x = 400\Delta x$ versus time for a plane wave incident with the derivative of a Gaussian pulse on the Debye material slab.

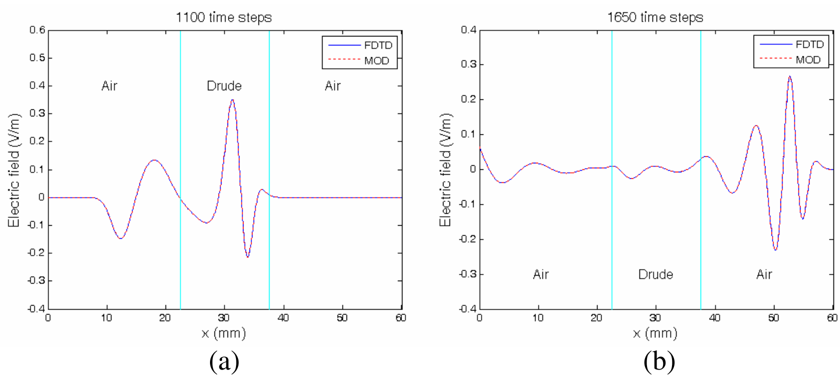


Figure 3. Electric field along x-position for a plane wave with the derivative of a Gaussian pulse incident on the plasma slab. (a) $t = 1,100\Delta t$, (b) $t = 1,650\Delta t$.

pulse plane wave. We set $T/c = 25$ ps and $x_s/\Delta x = 50$. The number of Laguerre basis functions is $M = 150$ and the time scale parameter is set to $s = 6.9 \times 10^{11}$. Figure 5 shows the spatial plots of the electric field versus position of the x-coordinates at temporal locations of 900 and 1,800 time steps of the FDTD computation. The time step is 0.0625 ps. Agreement between the solutions obtained using the proposed MOD method and the conventional PLRC-FDTD is excellent. Figure 6 shows the transient electric field at $x = 300\Delta x$ computed by the MOD and PLRC-FDTD method, and the inverse Fourier transform of the analytic solution. All three solutions show a good agreement.

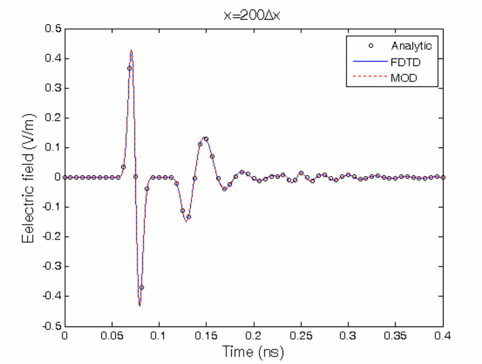


Figure 4. Transient electric field at $x = 200\Delta x$ versus time for a plane wave incident with the derivative of a Gaussian pulse on the plasma slab.

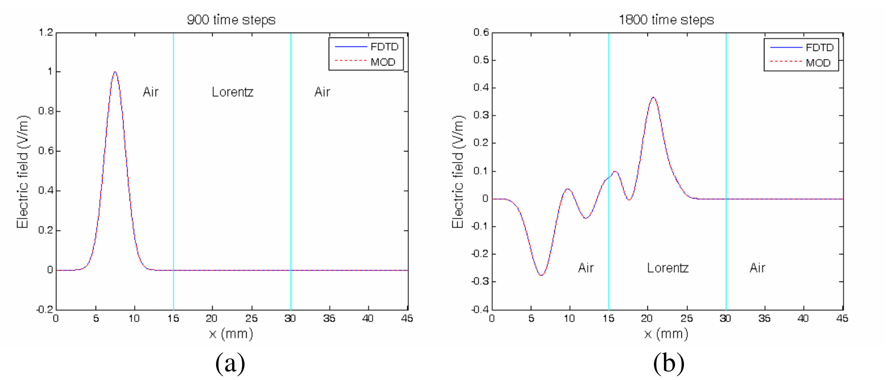


Figure 5. Electric field along the x-position for a plane wave incident with a Gaussian pulse on a Lorentz slab. (a) $t = 900\Delta t$, (b) $t = 1,800\Delta t$.

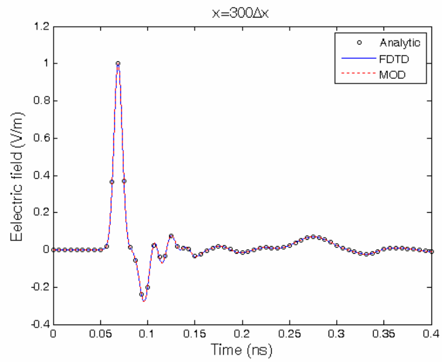


Figure 6. Transient electric field at $x = 300\Delta x$ versus time for a plane wave incident with a Gaussian pulse on a Lorentz slab.

4. CONCLUSIONS

We have proposed an efficient and accurate marching-on-in-degree method when using the FDTD technique for the transient solution of wave propagation in a general dispersive media. The time domain known and unknown coefficients for the material parameters and fields are approximated by a set of orthogonal Laguerre functions. Through the use of associate Laguerre functions, the time variable can be eliminated analytically from the solution procedure so that the computation methodology reduces to a simple finite difference procedure. In addition, we have successfully eliminated the explicit convolutions and the temporal derivatives analytically. Numerical results are presented for various dispersive materials, i.e., Debye, Drude, Lorentz models. The agreement between the solutions obtained using the proposed method and the traditional FDTD method, and the inverse Fourier transform of the frequency domain analytic solution is excellent as a function of both the spatial and the temporal variables.

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