EVOLUTION TIME OF THE ELECTROMAGNETIC WAVE INSTABILITY IN METALS UNDER THE ACTION OF A CURRENT PULSE

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1. Introduction

Nonlinear electromagnetic phenomena may occur in metals and semimetals at low temperatures. Under the action of strong waves the surface impedance undergoes a series of bifurcation with increasing amplitude and becomes a chaotic function of time [1]. Certain nonlinear phenomena are caused by a high electric current flowing in the conductor. If the carriers drift speed is higher than the velocity of the elastic waves, phonons are generated. Owning to this, breaks appear in the voltage-current characteristic. This effect tells upon the spectrum of electromagnetic (EM) waves in the resonator and results in the formation of the second harmonic [2]. Flowing of a sufficiently high electric current in the conductor is accompanied by the setting-up of autooscillations and turbulence [3,4].

Earlier we observed a nonlinear interaction between an EM wave on a section of the meander line and a sequence of strong current pulses in its central conductor. The interaction is realized at cryogenic temperatures in a strong magnetic field. The current and the wave interact
near the surface on the scale of the order of the skin-depth of the metal. The interaction takes place under conditions where the current-carrier drift speed is much smaller than the speed of electromagnetic waves in the line or that of elastic waves in metal. A nonlinear interaction between the current and the wave shows up as generation of the wave harmonics \[5\] and a nonmonotonic change and instability of the EM wave amplitude. These nonstationary phenomena occur as long as a current pulse is applied and persist during a time interval of up to several milliseconds after the current pulse is removed \[6,7\].

The present paper deals with the instability evolution and persistence and also considers the influence of certain experimental conditions on these time characteristics. This study seems to be appropriate since the physical nature of instability is still vague.

2. Methods

The experimental scheme is as follows: A section of the transmission line is placed in a strong magnetic field at the cryostat temperature of 4.2 K. As distinct from the method \[6,7\], the central conductor is a plate and therefore we deal with a section of a strip line. The plate is made of a single crystal of pure metal. The induction vector of the external magnetic field \(\vec{B}_o\), that of the current in the plate \(\vec{j}\), and wavevector \(\vec{q}\) are parallel.

An installation whose block diagram is given in Fig. 1 was used for measurements. The pulse generator PG1 serves as a synchronizer. It generates pulses for modulation of the radio-frequency generator RFG and activates the other pulse generator PG2. Pulses are applied from the PG2 output to the power amplifier PA. The last amplifier produces current pulses at the low-pass filters LPF and the metal sample, which serves as the load.
The test unit also receives a continuous or pulse radio-frequency (RF) signal. T-section high-pass filters protect the radio-frequency circuits from strong current pulses. The signal passes through the transmission line and the sample and arrives to the sampling oscilloscope SO and the receiver REC. The oscilloscope SO is synchronized from the auxiliary output of the generator RFG. Thus it becomes possible to observe the shape of the radio-frequency oscillations. The receiver REC amplifies and demodulates the signal. The envelope signal corresponding to the temporal change of the EM wave amplitude is seen on the oscilloscope OS. The same oscilloscope shows the signal proportional to the strength of the pulse current. The spectrum of the RF signal envelope is seen on the spectrum analyzer SA.

The range of the EM-wave frequency $f$ is from 0.3 up to 2.3 GHz, the generator output is $10^{-7} - 10^{-2}$ W. The receiver bandwidth is 4.5 MHz. The typical current pulse length $\tau_j$ is from several tens to several hundreds of microseconds long. The pulse repetition period $T_r$ equals several tens of milliseconds. The amplitude variation was measured with an accuracy of 0.3 dB. The test unit is placed in a low-temperature cryostat.
cryostat with a superconducting solenoid. The solenoid can produce a magnetic field with an induction of up to 8.3 T. Since $\vec{B}_z//\vec{j}$, the sample is not effected by a ponderomotive force, which tends to change position of the sample relative to the base of the test unit – the screen of the strip line. But the said force arises in the conductors supplying the current pulse. Therefore measures were taken to preclude displacement of the test sample. The contacts were of a rigid construction. Besides, all the components of the strip line were glued together. Indium was used for the high-current contact with the sample.

Pure-aluminum and tungsten samples were cut from single crystals by such a way that all the planes of the samples corresponded to crystallographic planes of the (001) type. The resistance ratio $\rho_{300}/\rho_{4.2}$ of the tungsten sample was over 50,000. Dimensions of the sample were $24 \times 6 \times 0.4$ mm and the same of the aluminum sample were measured $24 \times 12 \times 0.5$ mm.

3. Results

Unsteady-state phenomena in the amplitude of an EM wave that passed the meander line were observed in Ref. [6] and received a comprehensive study in Refs. [7,8]. As the current strength exceeds a certain threshold $j_c$ (the critical current), there arise nonstationary variations in the amplitude of the EM wave that passed the line. If the current is increased further, instability regions appear, where the wave amplitude changes from one pulse to another. As a rule, the onset of the instability region and the beginning of the current pulse are spaced by a time interval $\tau_{in}$. During this period, instability develops without showing up in the wave amplitude. A number of experiments were carried out with a superconducting colenoid in the frozen current regime, then observed instability does not relate to the magnetic flux unsteadiness.

The threshold value of the current strength depending on the magnetic induction and wave frequency was revealed. As the threshold values are exceeded, the wave amplitude changes randomly. These phenomena also arise in a strip line, where the central conductor – the sample – has the shape of a plate. The measurements were fulfilled with the samples made of both tungsten and aluminum crystals. It turns out that the results are qualitatively similar in both cases.
Because of this, the results obtained only on the tungsten sample will be presented, excluding Fig. 6, which was received on aluminum.

The schematic diagram exhibiting the envelope of current and wave pulses, and characteristic temporal scales is presented at Fig. 2. The period of the RF signal is too small to be indicated there. By this means, the following system of inequalities took place: $f^{-1} < \tau_j < \tau_{in} \leq \tau_e$ where $\tau_e$ is the lifetime of instability. Besides, this article discusses the situation $\tau_e < T_r$, thus the instability caused by one current pulse, remains only as fluctuations by the time of the next pulse.

![Figure 2. Schematic diagram of the wave envelope EM and current pulses $j$ supported by temporal scales.](image)

It was found experimentally (Fig. 3) that the amplitude variations are accompanied by an unstable phase shift of the EM signal. The shape of the RF signal, which is shown with a dashed line in Fig. 3, differs from sinusoidal (solid line in Fig. 3). Within the time intervals, where the signal amplitude is stable even at high currents, the phase is stable too. Such time intervals correspond to the solid line, Fig. 3. The distortion of the RF signal is equivalent to the generation of the wave harmonics under instability conditions. This effect was actually observed [5].
Figure 3. Unstable phase shift of RF signal. $B_0 = 7$ T, $f = 794$ MHz, $\tau_j = 160\mu s$; $J_M = 20$ A (a), 30 A (b), 60 A (c).

Variations of the RF signal envelope under the action of the current pulse with magnitude $J_M = 16$ A are given in Fig. 4a at the frequencies $f = 1975$ MHz and $2067$ MHz. When the current is turned off, the envelope exhibits no changes and the spectrum contains only the receiver noise of -55 to -60 dB on the lower oscillogram, Fig. 4b. If changes appear on the RF envelope, which are reproduced from one pulse to another, the envelope spectrum consists of a series of peaks spaced by the repetition frequency. The previous level of the noise spectrum remains unchanged.

Variations of the amplitude at $f = 1975$ MHz that take place within the time intervals marked with double-head arrows change from pulse to pulse. An aperiodic character of these changes shows up in the
appearance of a continuous component of the spectrum at the level of -45 to -50 dB. Appearance of the continuous spectrum agrees with the suggestion [6] that a dynamic chaos regime is realized.

The EM wave instability regions, which are shadowed in Fig. 2, are spaced by the interval \( \tau_{in} \) from the beginning of the current pulse before the first region. As it is seen from Fig. 5, the instability evolution time decreases dramatically as the current strength is increased.
Figure 5. Instability development time vs. current strength. $B_0 = 4.5$ T, $f = 1750$ MHz, $\tau_j = 100\mu$s.

Figure 6. Instability development $\tau_{in}$ and end $\tau_e$ time. $B_0 = 4$ T, (a) $\tau_j = 110\mu$s, 1 - 1880 MHz, 2 - 1898 MHz, (b) $\tau_j = 160\mu$s, (c) $J_M = 60$ A.
Changes in the wave amplitude are observed during the time $\tau_e$ several milliseconds long. It might be conceived that $\tau_e$ should correlate with the time during which the energy imparted to the sample’s electrons by the current pulse dissipates. But Fig. 6a shows that $\tau_e$ and its dependence on $J_M$ are different for different-frequency waves. Note only that $\tau_e$ increases with $J_M$ close to the threshold values of the current strength. Quantities $\tau_e$ and $\tau_{im}$ measured for the wave with $f = 2338$ MHz are presented in Fig. 6b. The data of Fig. 6c show that duration of the current pulse has no regular effect on the instability time $\tau_e$. Our measurements show that $\tau_{im}$ decreases as $B_0$ is lowered, but this takes place when the threshold value has been exceeded. Near the threshold value the instability time $\tau_e$ increases with $B_0$.

![Image](image-url)

*Figure 7. Effect of rectangular (1) and trapezoidal (2) pulses. $B_0 = 6.5$ T, $J_M = 9$ A, $f = 1211$ MHz; (a) current pulse, (b) RF envelope, (c) envelope of coil signal*

To elucidate the physical nature of the effect a strong current and the mechanism of its influence on the EM wave, we perform a series of experiments. It turns out that no instability in wave parameters depending on the magnetic field intensity exists when dc current regime is used. Figure 7 shows the effects observed when a current pulse of rectangular (1) or trapezoidal (2) shape is applied to sample. Changes of the amplitude are lower in the case of the trapezoidal pulse. When
\( \tau_m \gg \tau_j \), it is reasonable to assume that instability arises from an external effect similar to an impact. An irregularly varying signal whose duration is comparable with \( \tau_e \) (Fig. 7c) was registered using a coil installed on the sample surface. The coil had 25 turns, was 3 mm in diameter, and its axis was normal to the plate plane. The signal whose envelope is presented in Fig. 7c was measured at frequency of 11 kHz using a receiver with a 9-kHz bandwidth. The magnitude of the signal equals units of microvolts at \( B_o = 6.5 \) T and decreases as magnetic induction is lowered.

To ascertain the range parameter values where the wave amplitude exhibits instability, the measurements were performed at the threshold values of current strength at different values of magnetic induction. For the results refer to Fig. 8a. Three sections can be noted on the curve. The first where \( J' = \frac{dJ}{dB_o} < 0 \) at lower values of \( B_o \), the second where \( J' > 0 \) and the third where \( J' < 0 \) at higher values of \( B_o \). Different forms of the RF envelope, which is shaped as a rectangular pulse at the input, correspond to these sections (see Fig. 8b).

Figure 8. Critical current vs. magnetic induction (a) and the RF signal envelope at different \( B_o \). (b) \( f = 1750 \) MHz, \( \tau_j = 100 \mu s \).
4. Discussion

Let us turn first to the reasons responsible for the effect of a strong current pulse on a metal. Any explanations should take into account a sufficiently long (compared to the electron relaxation time and the time constant of a strip line section) interval of instability existence. Therefore three options deserve attention: heating with a current pulse; excitation and relaxation of elastic oscillations or waves; durable persistence of currents in the sample after the current pulse in the external circuit has been removed.

Thermal conditions largely influence the phenomena in question [8]. If the temperature is raised considerably, current has no longer influence on the wave parameters. Experiments [8] show however that duration of unsteady-state phenomena is longer than the time during which temperature gradients exist in the sample. Besides, a temperature gradient of about 1 K/cm, which is purposefully produced in the sample, does not lower the threshold value of current strength. Then the observed phenomena cannot be explained by excitation and relaxation of thermo-electromagnetic waves.

Excitation of elastic waves is possible despite the fact that the carrier drift speed is lower than the sound velocity [9]. In this case the threshold values of current strength and magnetic induction should be related by the condition \( j' = \frac{d\phi}{dB} > 0 \). This inequality sign is actually observed, Fig. 8, but only for a certain range of currents and magnetic fields. Excitation of elastic oscillations in a metal plate under the action of a current pulse is possible but cannot be regarded as the main cause of anomalies in the EM wave. Therefore it is expedient to consider also the third option of the effect of current on a metal.

According to this option, a sufficiently strong pulse current is passed through the sample so that the intrinsic magnetic field of the current influences dynamics of conduction electrons (similarly to consideration [10]). Since a complex oscillation process is realized in the above-described experiments, then, first, conditions are provided for appearance and evolution of instability and, second, a mechanism exists for its nonlinear limitation. The reason can be connected with a magnetodynamic mechanism. Electrons move in the sample in a strong homogeneous magnetic field \( B_0 = B_z \) parallel to the current and in an inhomogeneous field \( B_y \), which acquires at the opposite faces of the plate the values \( \pm (\mu_0 / 4\pi)^{1/2} \frac{2j'_{\pi}}{D} \), with \( D \) being the plate width. In our
experiments inhomogeneity of this component for the field is high owing to the skin-effect for the current pulse.

In the field $B_o$ conduction electrons move in cyclotron orbits. Simultaneously, centers of the orbits drift in crossing electric and magnetic fields of the current (Fig. 9). Bulk (1), flying (2) and trapped (3) electron trajectories in metal are distinguished. The last group is especially important for the formation of instability since this group is responsible for the descending section of the voltage-current characteristics [10]. For this group of electrons the $x$-component of the Lorentz force becomes zero: $[\vec{v}, B_y(x)\vec{j} + B_o\vec{k}]_x = 0$. The velocity components $\vec{v}$ of these electrons are estimated as $v_y \sim v_F B_y B_o$ and $v_x \approx \left(\frac{d}{R_s}\right)^{1/2} v_F$, with $v_F$ being the Fermi velocity and $R_s$ the cyclotron orbit radius in the field $B_y$. When $B_y \ll B_o$ and $R_s \gg d$, trapped electrons occupy small areas on the Fermi surface with low $v_y$ and $v_x$, and the voltage-current characteristic exhibits a descending section $V \sim J^{-1/2}$ ($V$ is the voltage drop across the sample). If a pulse current passes through the plate, this group of electrons is spatially localized within the effective skin depth $\delta_s$ for the characteristic frequencies of the pulse spectrum. The inequalities indicating relations between spatial scales of the problem may be written as $\delta \ll \delta_s \ll d \leq 1 < D$, where $\delta$ is the skin-depth for the wave and 1 is the electron mean free path.

Figure 9. Movement of electrons in the metal plate. 1 - bulk electrons, 2 - flying electrons, 3 - trapped electrons.
It was shown [10] that homogeneous distribution of a strong current along the plate (Oy axis) is unstable relative to perturbations with the wavenumbers $k < k_o = \frac{2\pi}{d}$. This leads to the appearance of the magnetic induction component $B_x$, which is inhomogeneous along the x and y coordinates and which has inhomogeneity scale $\Delta y > d$. In the experiment we deal with a thick plate where the free mean path is $l < d$. Bulk electrons contribute to conductivity of this sample, distinguishing an anomalous contribution from trapped electrons to the voltage-current characteristic. However, stratification of current is possible in this case too, since a latent negative differential resistance $r_d$ exists for the group of trapped electrons ($r_{tr}^d < 0$) [11].

Let us discuss the mechanism through which a current pulse transfers energy. We shall follow the line of reasoning adopted in [12] and make changes to suit our experimental conditions. Assume that the external permanent magnetic field $\vec{H}_o$ is parallel to the current $\vec{j}$. The amplitude of the RF wave field $\mathcal{H}$ at the metal boundary is much less than the amplitude of the field $\mathcal{H}_s$ at characteristic frequencies of the current pulse spectrum $\omega_s$, $\mathcal{H} / \mathcal{H}_s / O_y$. The anomalous skin-effect conditions is fulfilled and, besides, the inequality $\omega_s \ll \omega \ll \nu$ ($\nu$ being the electron scattering rate) holds at both frequencies.

If the field of a current pulse induces a magnetic field similar to the field of the current state [13], the latter field is inhomogeneous in the sample owing to the instability and varies with time at a characteristic frequency $\tilde{\omega} \ll \omega_s$, amplitude $\mathcal{H}$ and damping scale $\tilde{\delta} \gg \delta_s$. Under anomalous skin-effect conditions metal conductivity is known to be governed by electron dynamics. Dynamics that are rather sensitive to the strong intrinsic magnetic field of the current pulse. As a result, responsibility for the observed instability effects can be on the magnetodynamical nonlinearity mechanism. These arguments are confirmed by researches [10-15], where instability effects caused by the magnetodynamical nonlinearity were studied both theoretically and experimentally. So, the proposed approach makes it possible to clarify to some extent many features of the phenomena observed: the influence of current on the RF wave amplitude, a threshold character of the influence by the current amplitude, the influence of magnetic field strength.

Pay attention to an analogy between the instability of pulse current in question and the current-convective instability of plasma. It is well known that a species of the current-convective instability – screw
instability – is realized [16] in the plasma of semiconductors. When \( \vec{B}_o \parallel \vec{j} \), the influence of the intrinsic magnetic field leads to inhomogeneity of the magnetic field with the curvature radius \( R_s = \frac{p_f}{eH_s} \), where \( p_f \) is the Fermi momentum. Electrons drift along the magnetic lines of force and move in cyclotron orbits. They are influenced by the force \( \vec{F}_c = n\frac{m_c}{R_s}(v_\|^2 + v_\perp^2) \) acting in the direction \( \vec{n} \) from the curvature center to the force point, \( |\vec{n}| = 1 \); \( m_c \) is the cyclotron mass; \( v_\| \) and \( v_\perp \) are the longitudinal and transverse components of the electron velocity respectively (see Fig. 8). This force is equivalent to the action of a “gravity field” \( \vec{g} = \vec{n}\frac{1}{R_s}(v_\|^2 + v_\perp^2) \). For trapped electrons the mean value of the \( x \)-component of the force \( \vec{F}_c \) turns to zero.

If the carrier density gradient exists, a convective instability similar to instability of an inhomogeneous fluid in a gravity field may arise. In our case this gradient takes place in the neighborhood of the metal boundary. Evolution of this type instability is connected with convection. For this reason, auto-oscillations of the \( x \)-component of the low frequency magnetic field are observed in our experiment (see Fig. 7). A pulse character of strong current stimulates appearance and evolution of instability. The experimental dependence of the latent instability-evolution time \( \tau_m \) on current strength at \( j_M > j_C \), Fig. 5, shows a qualitative agreement with the dependence of the increment of the current-convective instability \( (\text{Im} \omega)^{-1} \sim v^{-1/2} \sim j_M^{-1/2} \). After the current pulse is removed, the instability relaxation time should not be less than the low frequency oscillation period \( \frac{2\pi}{\omega} \). Long duration of this time compared to the time of electron momentum relaxation is due to the large imaginary nondiagonal components of the metal conductivity in a strong magnetic field. As it was noted above, a low temperature character of the instability is related to the requirement of the skin-effect anomaly.

The threshold values of the electric field intensity \( E_c \) under the surface current-convective instability are related by the inequality \( E_cB_o\mu^2 \geq \text{const} \), where \( \mu \) is the carrier mobility. Considering that in our case \( j = \sigma E \), \( j \sim \frac{\sqrt{\sigma}}{\delta_s p_\perp} \), \( \delta_s \) is the skin depth for a current pulse, \( \sigma \) the conductivity, and \( p_\perp \) the cross-sectional perimeter of the sample, we obtain the following condition for the threshold values:

\[
\frac{j_cB_o\mu^2}{\delta_sp_\perp\sigma} \geq \text{const}
\] (1)
This condition roughly agrees with the experimental results.

A current pulse imparts a drift velocity to conduction electrons. The drift velocity is extremely nonuniform along the cross-section owing to the fact that $\delta_s$ is much lower than the plate thickness $d$. The electron subsystem passes to nonequilibrium state. Evolution of instability results in redistribution of the energy to collective modes, which correspond to an oscillatory movement of the carriers. This leads to appearance of a metastable turbulence. The energy of oscillatory modes dissipates with time. Two alternatives are possible. Dissipation ends before the next current pulse comes and then the process is nearly reproduced from pulse to pulse. If oscillations persist to the beginning of the next pulse, the regime of intermittent laminar and turbulent intervals is realized. We study the system at a small excess over the threshold value. Available experimental results agree with the assumption on appearance of a low-dimensional strange attractor.

Current pulses change the wave amplitude up to 15–20 dB. Any reasonable values of the sample's conductivity cannot account for such changes in the transmission strip line by Joule losses. The reason for such a strong effect may be distortion of lines of surface high-frequency currents in the sample owing to nonuniformity of the surface impedance on the plate surface.

If the pulse current contains a turbulent component, electrons are transferred from the skin layer to the bulk of the metal. At low temperatures and under an anomalous skin effect conditions the mean free path of the electrons exceeds the skin depth. Electrons, which come to the skin layer from the bulk of the metal, initially do not participate in formation of high frequency currents. This is equivalent to an extraneous field intensity $\vec{E}_{ex}(z)$ induced on the sample surface. The extraneous field intensity depends on the $z$ coordinate along which current flows and the wave propagates. The electric field intensity is expressed by expansion of the field with respect to the normalized modes $\vec{e}_n$ and $\vec{h}_n$ of the strip line:

$$E(z) = -\frac{1}{2} \sum_n \frac{W_n}{|W_n|} \vec{e}_n \int_S [\vec{E}_{ex}(z), \vec{h}_n^*] d\vec{s},$$  \hspace{1cm} (2)$$

where $W_n$ is the wave resistance of the line for the $n^{th}$ mode and the integration is taken over the perturbation surface $S$ of RF currents within the portion of the line passed by the wave to the reference point. For this reason the $z$-coordinate dependence of the amplitude $|E(z)|$
is nonexponential [7].

Formation and evolution of instability give rise to a complicated spatially inhomogeneous process. However, in most experiments we were interested only in the signal strength amplitude $A(t)$ at the end point of the strip line. The time $\tau_{in}$ after the onset of the current pulse predictably of the wave amplitude is lost. Data of Fig. 3 indicate to an unstable oscillation phase shift. One may try to use the Kolmogrov entropy $h$ [17] for description of instability. Let us denote the amplitude fluctuations by $\delta A_o$ at the moment of time $t = 0$ corresponding to onset of the current pulse and assume the fluctuation development process to be exponential: $|\delta A(t)| = |\delta A_o| \exp(ht)$. Estimate the dependence of the entropy $h$ on the pulse current strength from experimental data. At the moment of time $t = \tau_{in}$ the change in the amplitude $|A(t)|$ becomes of the order $A$ and

$$h = \frac{1}{\tau_{in}} \ln \left| \frac{\delta A(t)}{\delta A_o} \right|$$

(3)

It was found experimentally that the logarithm in (3) is a quantity weakly depending on $J_M$ and equal approximately to 2. It may be assumed that $h \sim \frac{2}{\tau_{in}}$. The Kolmogrov entropy $h$ vs $J_M$ is shown in Fig. 10 as measured at a frequency $f = 1.75$ GHz.

![Figure 10. Kolmogorov entropy vs. current strength. $B_o = 4.5T$, $f = 1750$ MHz, $\tau_j = 100\mu$s.](image)
5. Conclusion

Summing up, we outline the basic results of the study. It was found that an unstable phase shift of the electromagnetic wave takes place under the action of a current pulse in the strip line with a pure-metal conductor. Time boundaries of the wave amplitude instability and influence of the current strength on the latent time of instability evolution are analyzed. Some considerations on the character of instability induced by a current pulse are given and a physical reason by which the instability affects the amplitude of a radio-frequency wave is proposed. It is mentioned that the regime of a linear response is realized for a radio-frequency wave.

The turbulent stage of the wave amplitude variation is connected with formation of a low-dimensional chaotic attractor in the metal plate by which is carried to a nonequilibrium state by a current pulse. This concept is supported by the existence of threshold value of the current, and also by laminar-turbulent intermittence in the amplitude of electromagnetic waves.

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References