

## IMPROVED EXTRACTION OF COUPLING MATRIX AND UNLOADED $Q$ FROM $S$ -PARAMETERS OF LOSSY RESONATOR FILTERS

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**Abstract**—This paper presents a two-stage optimization method for accurately extracting the coupling matrix (CM) and the unloaded quality factor (unloaded  $Q$ ) of each resonator from the measured (or simulated)  $S$ -parameters of lossy cross-coupled resonator bandpass filters. The method can be used in computer-aided tuning (CAT) of microwave filters to accelerate filter design and physical realization. In our method, the Cauchy method is employed for determining characteristic polynomials of the  $S$ -parameters in the normalized low-pass frequency domain, and the CM and unloaded  $Q$  are extracted by two-stage optimization method using genetic algorithm. With respect to the previous methods available in the literatures, the proposed method allows the CM extraction of the filter with source-load coupling. Moreover, the accuracy and robustness of the method can be improved due to the usage of the second stage optimization. The proposed method is applied to the diagnosis of a general coupled resonator filter with/without source-load coupling.

### 1. INTRODUCTION

In order to obtain the frequency response requirements of a coupled resonator filter, the physical realization of the filter would largely depend on a tuning in the stage of design and/or production. The core task in filter tuning is the CM extraction from the filter responses (also called filter diagnosis). By comparing the difference between the extracted CM and the designed CM, what is wrong with the filter can be revealed, and the tuning direction and magnitude can be easily

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decided. The filter diagnosis can speed up the design and realization of a cross-coupled resonator filter.

In recent years, the interest has been growing in methods for extracting the CM of microwave filters from measurements (or simulations) including losses, such as a five-parameter optimization method in [1], Cauchy method in [2] and the analytical method in [3]. A sequential tuning method was also reported in [4]. However, these methods in [1–4] have not taken the source-load coupling into account. The methods in [1,2] require assuming all the resonators with the same unloaded  $Q$ . The method in [3] is attractive, because it overcomes the assumption of uniform unloaded  $Q$  for each resonator. However, complicated degenerate poles of the admittance parameters are required to be handled, and the measurement noise is required to be removed due to the usage of frequency points far below or above the center frequency in [3]. For the sequential tuning, it is not always convenient to segregate each resonator or coupling element in a filter structure such as dielectric resonator filters.

In this paper, a two-stage optimization method is proposed for accurately extracting the CM and the unloaded  $Q$  from the measured (or electromagnetic simulated)  $S$ -parameters of a lossy coupled resonator bandpass filter. The method is based on the presentation in [1] with proper extension to handle the presence of source-load coupling. With respect to the previous method available in [1], the new method has three advantages: 1) the CM extraction of the filter with source-load coupling has been taken into account; 2) unloaded  $Q$  for each resonator may be non-uniform; and 3) the new method has the higher accuracy and robustness. The method can guide the tuning of coupled resonator bandpass filters (as in [5–7]), dual-band resonator bandpass filters (as in [8–10]), and dual-mode bandpass filters (as in [11,12]), which are designed according to the CM, to accelerate filter design and a physical realization.

## 2. BASIC THEORY

In a physical filter model [1, Figure 2], there is always a section of transmission line at I/O ports, which shifts the reference planes. To apply the technique of an ideal circuit model in [13] to extract the CM, the phase-shift effects of the measured  $S$ -parameters should be first removed. The following phase shift caused by the phase loading and the transmission lines should be removed from the raw measured  $S_{11}$  and  $S_{21}$ , respectively [1]

$$\begin{aligned}\Delta\phi S_{11} &= -2(\varphi_{01} + f\theta_{01}/f_0), \\ \Delta\phi S_{21} &= -(\varphi_{01} + \varphi_{02} + f\theta_{02}/f_0 + f\theta_{01}/f_0).\end{aligned}\tag{1}$$

where  $\varphi_{01}$  and  $\varphi_{02}$  are the phase loading at I/O ports, respectively. The phase loading is a frequency-invariant constant, which is revealed for the first time in the field of filter diagnosis in [3].  $\theta_{01}$  and  $\theta_{02}$  are the equivalent electrical length of the transmission line at  $f_0$  in radians at I/O ports, respectively, and  $f_0$  is the center frequency of the filter.

The measured  $S_{21}$  and  $S_{11}$  can be well approximated by three characteristic polynomials  $F$ ,  $P$  and  $E$  as [1, 14]

$$S_{11}(s') = \frac{F(s')}{E(s')} = \frac{\sum_{k=0}^N a_k^{(1)} s'^k}{\sum_{k=0}^N b_k s'^k}, \quad S_{21}(s') = \frac{P(s')}{E(s')} = \frac{\sum_{k=0}^{nz} a_k^{(2)} s'^k}{\sum_{k=0}^N b_k s'^k}. \quad (2)$$

where,  $N$  is the filter order and  $nz$  is the number of finite transmission zeros. A modified frequency transformation used for converting the measured  $S$ -parameters from the bandpass domain  $f$  to the normalized lowpass domain  $s'$  is adopted here as [2]

$$s' = \frac{1}{FBW} \frac{1}{Q_u} + j \frac{1}{FBW} \left( \frac{f}{f_0} - \frac{f_0}{f} \right). \quad (3)$$

where,  $FBW$  is the fractional bandwidth of a filter, and  $Q_u$  is the unloaded  $Q$  of the resonator. The selected values for  $FBW$  and  $f_0$  are obtained from the measured responses or the design values. Knowing the values of the parameters,  $nz$ ,  $N$ ,  $f_0$ ,  $FBW$ ,  $Q_u$  and the  $S$ -parameters samples,  $S_{21}(s'_i)$  and  $S_{11}(s'_i)$  at the finite frequency points  $s'_i$  ( $i = 1, 2, \dots, Ns$ ), the polynomials  $F$ ,  $P$  and  $E$  can be solved in one step by the Cauchy method [14].  $Ns$  is the number of frequency samples. The normalized frequency points  $s'_i$  ( $i = 1, 2, \dots, Ns$ ) can be calculated based on (3) using the selected physical frequency samples  $f_i$  ( $i = 1, 2, \dots, Ns$ ). The emphasis here is that the samples  $f_i$  ( $i = 1, 2, \dots, Ns$ ) should be chosen around the passband in Cauchy method; in fact the frequency samples, as selected too much away from the passband, can reduce the accuracy of the model due to the second order effects of a physical filter, such as the presence of spurious passbands and frequency-dependent couplings. Due to the usage of finite frequency samples around the passband, the measurement noise is not required to be handled in our method. Note that the phase shift of the  $S$ -parameters samples should be first removed from the raw measured  $S$ -parameters.

The parameters,  $r_{21k}$ ,  $r_{22k}$  and  $\lambda_k$  ( $k = 1, 2, \dots, N$ ), used to construct the  $N + 2$  transversal matrix, can be extracted from the polynomials  $F$ ,  $P$  and  $E$  by well-known established technique in [13], and then the  $N + 2$  transversal matrix is determined as shown in Figure 1.

	S	1	2	...	k	...	N	L	
S		$M_{S1}$	$M_{S2}$	...	$M_{Sk}$	...	$M_{SN}$	$M_{SL}$	S — Source
1	$M_{1S}$	$M_{11}$						$M_{1L}$	L — Load
2	$M_{2S}$		$M_{22}$					$M_{2L}$	$M_{SL}$ — Source-Load coupling
⋮	⋮			⋱				⋮	$M_{Lk} = M_{kL}$
k	$M_{kS}$				$M_{kk}$			$M_{kL}$	$M_{Sk} = M_{kS}$ ( $k = 1, 2, \dots, N$ )
⋮	⋮					⋱		⋮	$M_{kk} = -\lambda_k$
N	$M_{NS}$						$M_{NN}$	$M_{NL}$	$M_{Lk} = \sqrt{r_{22k}}$
L	$M_{LS}$	$M_{L1}$	$M_{L2}$	...	$M_{Lk}$	...	$M_{LN}$		$M_{Sk} = r_{21k}/M_{Lk}$

**Figure 1.** The  $N + 2$  transversal matrix, as constructed by the parameters  $r_{21k}$ ,  $r_{22k}$ ,  $\lambda_k$  and  $M_{SL}$ .

The parameter  $r_{22k}$  in Figure 1 will be positive real for a realizable network. The source-load coupling  $M_{SL}$  in Figure 1 is equal to zero, except for the case of  $nz = N$ .  $M_{SL}$  can be evaluated for the case of  $nz = N$  as follows [13]

$$M_{SL} = \frac{1 \pm S_{11\infty}}{S_{21\infty}} \quad (4)$$

Using the polynomials  $F$ ,  $P$  and  $E$ , (4) can be derived as follows:

$$M_{SL} = \frac{(b_N \Omega^N + \dots + b_0) \pm (a_N^{(1)} \Omega^N + \dots + a_0^{(1)})}{a_N^{(2)} \Omega^N + a_{N-1}^{(2)} \Omega^{N-1} + \dots + a_0^{(2)}} \Bigg|_{\Omega=\infty} = \frac{b_N \pm a_N^{(1)}}{a_N^{(2)}} \quad (5)$$

A relatively small value should be chosen for  $M_{SL}$  in practice. Equation (4) was first proposed in [13] and worked well for filter synthesis. However, the accuracy of  $M_{SL}$ , as calculated by (4) or (5), is rather poor for a filter diagnosis due to the second order effects of a physical filter. So, the method in [1] and Cauchy methods (as in [2, 14]) can not diagnose the filters with  $S$ - $L$  coupling.

A normalized  $N + 2$  coupling matrix  $[M']$  including losses is related to the extracted  $S$ -parameters via the following equation:

$$S_{21} = -2j[A^{-1}]_{N+2,1}, \quad S_{11} = 1 + 2j[A^{-1}]_{1,1} \quad (6)$$

where,  $A = [\Omega U - jR + M']$ ,  $\Omega = 1/FBW(f/f_0 - f_0/f)$ ,  $[R]$  is a  $(N + 2) \times (N + 2)$  whose only nonzero entries are  $R_{11} = R_{N+2, N+2} = 1$ , and  $[U]$  is similar to the  $(N + 2) \times (N + 2)$  identity matrix, except that  $U_{11} = U_{N+2, N+2} = 0$ ,  $[M'] = [M_t] - j[G]$ ,  $[M_t]$  is the extracted  $N + 2$  transversal matrix, and  $[G]$  is the diagonal matrix

$[G] = \text{diag}[0, \sigma_1, \dots, \sigma_N, 0]$ , which represents the loss of the filter. The effective unloaded  $Q$  for the  $k$ th resonator  $Q_{uk}$  can be evaluated by using loss factors  $\sigma_k$  via the following equation:

$$Q_{uk} = \frac{1}{FBW \cdot \sigma_k} \quad (k = 1, 2, \dots, N) \quad (7)$$

As  $\sigma_k$  ( $k = 1, 2, \dots, N$ ) approaches zero, (6) is exactly the same as that given in [15], which is suitable for the lossless filter response.

### 3. TWO-STAGE OPTIMIZATION METHOD

Once knowing  $r_{21k}$ ,  $r_{22k}$ ,  $\lambda_k$ ,  $\sigma_k$  ( $k = 1, 2, \dots, N$ ) and  $M_{SL}$ , the  $N + 2$  transversal matrix  $[M_t]$  and loss matrix  $[G]$  can be completed according to the description in Section 2. The aim of the proposed two-stage optimization method is to obtain these parameters from the measured (or simulated)  $S$ -parameters, and then extract the CM and evaluate unloaded  $Q$  for each resonator.

In the first stage of the method, the six parameters,  $\varphi_{01}$ ,  $\theta_{01}$ ,  $\varphi_{02}$ ,  $\theta_{02}$ ,  $Q_u$  and  $M_{SL}$ , are used as the optimized variables. Once they are known, the main steps of determining the desired parameters ( $r_{21k}$ ,  $r_{22k}$ ,  $\lambda_k$ ,  $\sigma_k$  and  $M_{SL}$ ) are: 1) the phase shift of the raw measured  $S$ -parameters can be removed using (1); 2) the characteristic polynomials  $F$ ,  $P$  and  $E$  are solved by (2) using the  $S$ -parameters samples with removing the phase shift; 3) the parameters  $r_{21k}$ ,  $r_{22k}$  and  $\lambda_k$  can be extracted from the polynomials  $F$ ,  $P$  and  $E$  by the technique in [13]; 4) The loss factor  $\sigma_k$  for the  $k$ th resonator can be evaluated by  $\sigma_k = 1/(FBW \cdot Q_u)$ , and then loss matrix  $[G]$  can be formed; and 5) the  $N + 2$  transversal matrix  $[M_t]$  can be constructed according to Figure 1.

The unknown six parameters ( $\varphi_{01}$ ,  $\theta_{01}$ ,  $\varphi_{02}$ ,  $\theta_{02}$ ,  $Q_u$  and  $M_{SL}$ ) are obtained by minimizing the following objective error function using genetic algorithm (GA):

$$F_{obj} = \sum_{i=1}^{Ns} [ |S_{21}^{ext}(f_i)| - |S_{21}^{mea}(f_i)| ]^2 + [ |S_{11}^{ext}(f_i)| - |S_{11}^{mea}(f_i)| ]^2. \quad (8)$$

where  $S_{21}^{ext}(f_i)$  and  $S_{11}^{ext}(f_i)$  are the extracted  $S$ -parameters calculated by (6), and  $S_{21}^{mea}(f_i)$  and  $S_{11}^{mea}(f_i)$  are the measured (or simulated)  $S$ -parameters at the frequency point  $f_i$ .

GA can be used to solve the global minimum value of a multivariate function. In this paper, the GA toolbox for Matlab provided by the University of Sheffield [16] is chosen to minimize the error function in (8).

The first stage of the method has taken the  $S$ - $L$  coupling into account with the assumption of uniform unloaded  $Q$  for each resonator. To handle the case of the resonators with non-uniform unloaded  $Q$  and to further improve the accuracy and robustness of the method, the desired parameters ( $r_{21k}$ ,  $r_{22k}$ ,  $\lambda_k$ ,  $\sigma_k$  and  $M_{SL}$ ) are further used as the optimized variables in the second stage of the method. These desired parameters are obtained by minimizing the same error function in (8) using GA. The first stage of the method provides a good setting range of the optimized variables for the second stage of the method. In numerical process of the second stage, the range of the optimized parameter variation is set to  $\pm 30\%$  of the results obtained in the first stage of the method.

Once  $r_{21k}$ ,  $r_{22k}$ ,  $\lambda_k$ ,  $\sigma_k$  ( $k = 1, 2, \dots, N$ ) and  $M_{SL}$  are determined, the  $N + 2$  transversal matrix  $[M_t]$  can be constructed according to Figure 1. As the result, the transversal matrix  $[M_t]$  is transformed into another CM  $[M_e]$  with the prescribed topology corresponding to the physical structure by the same similarity transformations as the designed one. The CM  $[M_e]$  reflects the actual couplings of a given filter response. Applying (7) to  $\sigma_k$ , the effective unloaded  $Q$  for each resonator can be evaluated.

## 4. FOUR EXAMPLES OF FILTER DIAGNOSIS

### 4.1. Fabricated Fourth-order Bandpass Filter (Filter 1)

The diagnosis technique presented here will be first applied in the measured  $S$ -parameters of a fourth-order filter (filter 1). The filter 1 is fabricated on a Rogers RT/duroid 5880 substrate with a relative dielectric constant  $\epsilon_r = 2.2$ , a thickness  $h = 0.508$  mm, and a loss tangent  $\delta = 0.0009$ .

The novel algorithm is applied with  $N = 4$ ,  $nz = 2$ ,  $N_s = 37$  (frequency interval 2.04–2.22 GHz),  $f_0 = 2.13$  GHz and  $FBW =$

**Table 1.** The desired parameters for constructing the  $N + 2$  transversal matrix of filter 1.

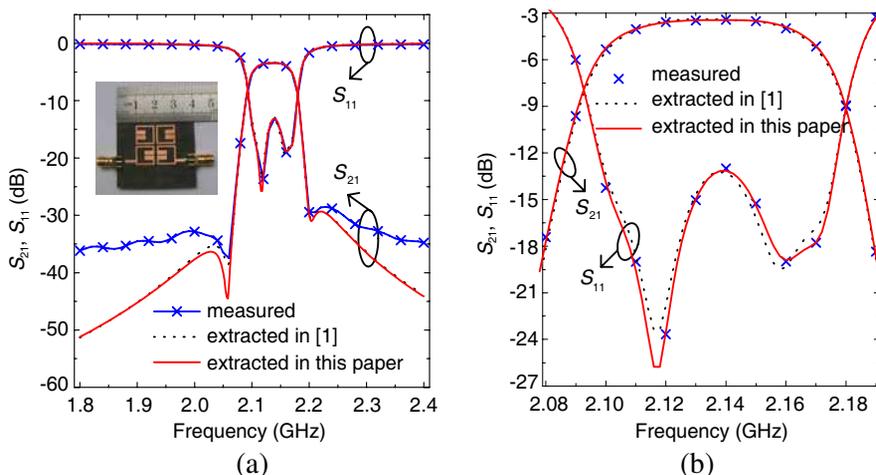
$k$	$r_{21k}$	$r_{22k}$	$\sigma_k$	$\lambda_k$	$M_{SL}$
1	-0.12381	0.10552	0.20879	1.72070	0
2	0.15451	0.14079	0.19563	-1.32944	
3	0.37003	0.37847	0.21100	1.10526	
4	-0.40141	0.32588	0.22026	-0.55967	

2.82%. The desired parameters are obtained, as shown in Table 1. The non-uniform  $\sigma_k$  indicates that the loss contributed by each individual resonator is non-uniform. Applying (7) to  $\sigma_k$  leads to the effective unloaded  $Q$  of 170.02, 181.46, 168.25 and 161.17, for each resonator, respectively.

The  $N + 2$  transversal matrix  $[M_t]$  can be constructed according to Table 1. By applying the same similarity transformation as the designed one to the matrix  $[M_t]$ , the normalized  $N + 2$  CM with the assigned topology corresponding to the physical structure has been extracted as

$$M = \begin{bmatrix} 0 & 1.0822 & 0 & 0 & 0 & 0 \\ 1.0822 & -0.1261 & 1.0370 & 0 & -0.2040 & 0 \\ 0 & 1.0370 & -0.2617 & 0.9011 & 0.1409 & 0 \\ 0 & 0 & 0.9011 & -0.3065 & 1.0315 & 0 \\ 0 & -0.2040 & 0.1409 & 1.0315 & -0.2425 & 0.9750 \\ 0 & 0 & 0 & 0 & 0.9750 & 0 \end{bmatrix} \quad (9)$$

In Figure 2(a), the measured responses of the filter 1 are compared to those calculated by the extracted CM in (9). Very good agreement between the measured and extracted responses can be observed. Figure 2(b) shows the detailed comparison around the passband. The filter 1 has also been diagnosed by the previous method in [1]. To show the improvement offered by the novel approach with respect to



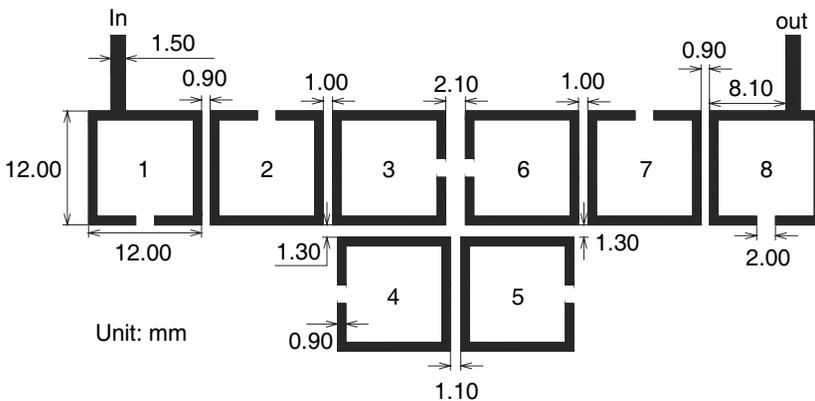
**Figure 2.** Comparison between the measured and extracted  $S$ -parameters of fabricated filter 1 at (a) a wide frequency range, and (b) around the passband.

the previous one, the result obtained by the method presented in [1] is also plotted in Figure 2. As can be seen from the detailed comparison in Figure 2(b), the old method result is less accurate than those obtained by the new technique.

There are somewhat differences between the measured and the extracted  $S$ -parameters at frequencies far away from the passband, which is due to the second order effects of a physical filter, such as spurious passbands, the frequency dispersive behavior, the presence of spurious couplings and frequency-dependent couplings. Moreover, these features of the responses in a practical filter may lead to spurious coupling elements in the extracted CM with respect to the designed CM.

#### 4.2. Simulated Eighth-order Bandpass Filter (Filter 2)

In the second example, the diagnosis technique is applied to the simulated  $S$ -parameters of an eighth-order filter (filter 2), which is designed on a Rogers RT/duroid 5880 substrate ( $\epsilon_r = 2.2$ ,  $h = 0.508$  mm, and  $\delta = 0.0009$ ). The physical structure and dimensions of the filter 2 are shown in Figure 3. The filter has been simulated using a full-wave simulator IE3D. The loss factors (conductor loss and dielectric loss) are included in the simulated responses. The conductivity of the metal is set to  $5.8 \times 10^{-7}$  Siemens in the simulation. The thickness of the metal layer is set to zero in the simulation to improve the simulation speed, which can lead to slightly greater unloaded  $Q$  due to the less loss with respect to the thick metal.



**Figure 3.** The physical structure and dimensions of filter 2.

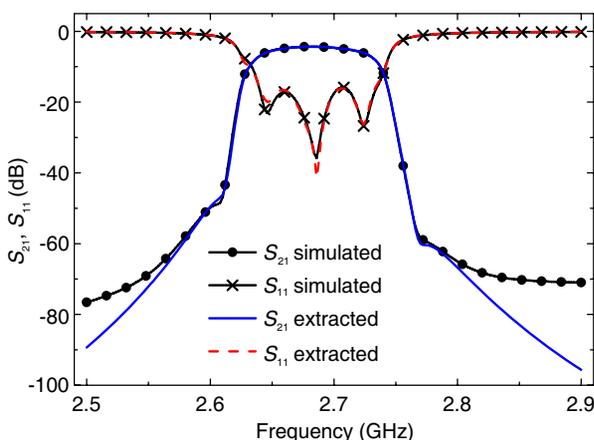
The proposed algorithm is applied with  $N = 8$ ,  $nz = 2$ ,  $N_s = 41$  (frequency interval 2.61–2.77 GHz),  $f_0 = 2.70$  GHz and  $FBW = 3.70\%$ , and then the desired parameters are obtained as follows:

$r_{21k} = (-0.06212, 0.14472, -0.15448, 0.06989, -0.14291, 0.16638, 0.14653, -0.16801)$ ;  $r_{22k} = (0.03574, 0.15640, 0.14343, 0.03115, 0.17254, 0.17178, 0.15267, 0.15761)$ ;  $\lambda_k = (-1.57547, -1.49116, -1.23064, 0.92295, 0.82731, -0.64163, 0.50484, -0.08534)$ ;  $\sigma_k = (0.10072, 0.09677, 0.09766, 0.09765, 0.09955, 0.09789, 0.10072, 0.09903)$ ; and  $M_{SL} = 0$ .

Applying (7) to  $\sigma_k$  leads to the effective unloaded  $Q$  of 268.07, 279.01, 276.47, 276.49, 271.22, 275.82, 268.07 and 272.64, for each resonator, respectively. The extracted normalized  $N + 2$  coupling matrix is listed in Table 2.

**Table 2.** Extracted  $N + 2$  coupling matrix of the filter 2.

0	1.0790	0	0	0	0	0	0	0	0
1.0790	0.3260	0.9323	0	0	0	0	0	0	0
0	0.9323	0.3067	0.6298	0	0	0	0	0	0
0	0	0.6298	0.3093	0.5591	0	-0.1418	0	0	0
0	0	0	0.5591	0.3063	0.8075	-0.0977	0	0	0
0	0	0	0	0.8075	0.5237	0.5393	0	0	0
0	0	0	-0.1418	-0.9077	0.5393	0.3360	0.6226	0	0
0	0	0	0	0	0	0.6226	0.3270	0.8824	0
0	0	0	0	0	0	0	0.8824	0.3340	1.0106
0	0	0	0	0	0	0	0	1.0106	0



**Figure 4.** The simulated and the extracted  $S$ -parameters of filter 2.

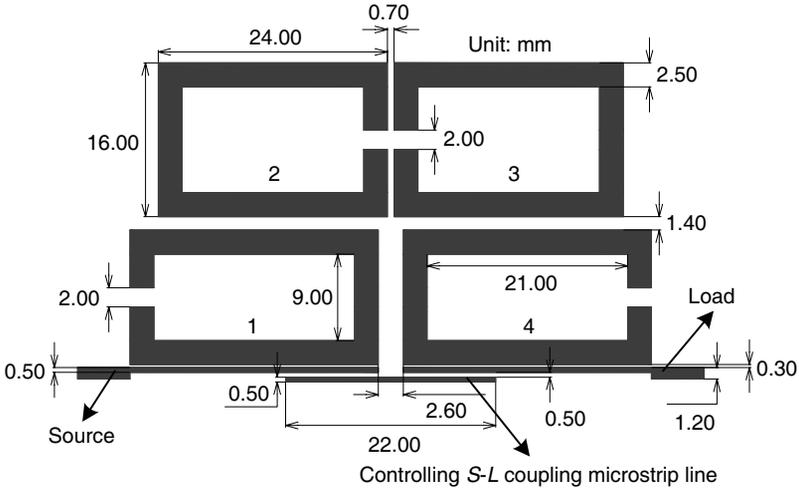
In Figure 4, the original simulated  $S$ -parameters are compared with those calculated by the extracted CM. Very good agreement between the simulated and extracted responses can be observed.

### 4.3. Simulated Fourth-order Bandpass Filter with $S$ - $L$ Coupling (Filter 3)

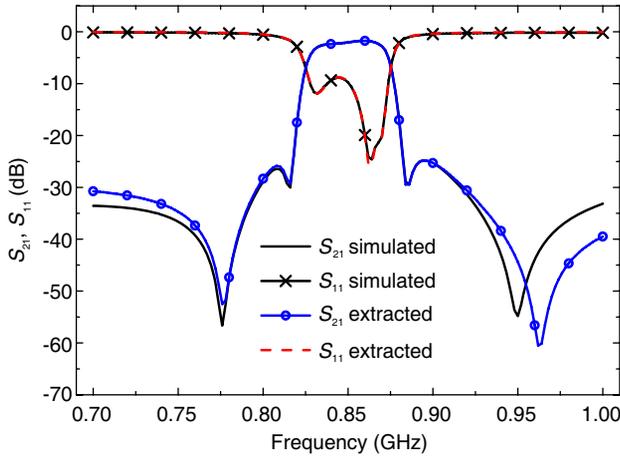
In the third example, the diagnosis technique is applied to the simulated  $S$ -parameters of fourth-order filter with source-load coupling (filter 3), which is designed on a Rogers RO3010 substrate ( $\epsilon_r = 10.2$ ,  $h = 1.27$  mm, and  $\delta = 0.0023$ ). The physical structure and dimensions of the filter 3 are shown in Figure 5. The filter has been simulated using a full-wave simulator IE3D. The loss factors (conductor loss and dielectric loss) are included in the simulated responses. The conductivity of the metal is set to  $5.8 \times 10^{-7}$  Siemens, and the thickness of the metal layer is set to zero in the simulation.

The proposed algorithm is applied with  $N = 4$ ,  $nz = 4$ ,  $N_s = 26$  (frequency interval 0.80–0.90 GHz),  $f_0 = 0.85$  GHz and  $FBW = 4.71\%$ , and then the desired parameters are obtained as follows:

$r_{21k} = (0.14129, -0.10808, -0.36632, 0.36348)$ ;  $r_{22k} = (0.10923, 0.10476, 0.35505, 0.35761)$ ;  $\lambda_k = (-1.40202, 1.29058, -1.07402, 0.71904)$ ;  $\sigma_k = (0.10413, 0.09519, 0.10528, 0.09938)$ ; and  $M_{SL} = 0.01595$ .



**Figure 5.** The physical structure and dimensions of filter 3.



**Figure 6.** The simulated and the extracted  $S$ -parameters of filter 3.

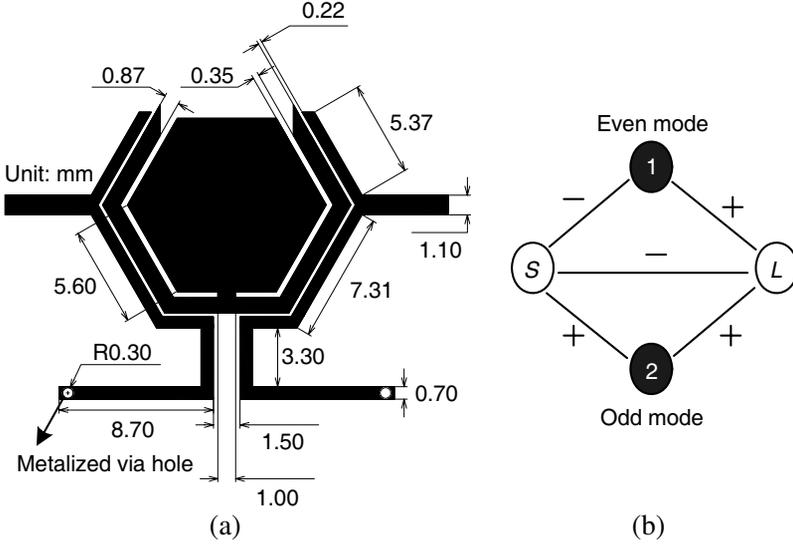
Applying (7) to  $\sigma_k$  leads to the effective unloaded  $Q$  of 204.07, 223.24, 201.84, and 213.83, for each resonator, respectively. The normalized  $N + 2$  coupling matrix is extracted as

$$M = \begin{bmatrix} 0 & 1.0206 & 0 & 0 & 0 & 0.0159 \\ 1.0206 & 0.2425 & 0.9783 & 0 & -0.3305 & 0.0298 \\ 0 & 0.9783 & -0.0037 & 0.7784 & -0.0375 & 0 \\ 0 & 0 & 0.7784 & 0.0539 & 0.9651 & 0 \\ 0 & -0.3305 & -0.0375 & 0.9651 & 0.1737 & 0.9622 \\ 0.0159 & 0.0298 & 0 & 0 & 0.9622 & 0 \end{bmatrix} \quad (10)$$

In Figure 6, the original simulated  $S$ -parameters are compared with those calculated by the extracted CM. Very good agreement between the simulated and extracted responses can be observed.

#### 4.4. Fabricated Dual-mode Filter with $S$ - $L$ Coupling (Filter 4)

In the last example, the diagnosis technique is applied in the measured  $S$ -parameters of a dual-mode transversal bandpass filter with source-load coupling (filter 4). The filter 4 is fabricated on a Rogers RO4003 substrate ( $\epsilon_r = 3.38$ ,  $h = 0.508$  mm, and  $\delta = 0.0027$ ). The configuration and coupling scheme of the filter 4 are shown in Figures 7(a) and (b), respectively. Measurement was performed by Agilent vector network analyzer and  $50 \Omega$  matching load was used in the measurement.



**Figure 7.** (a) Configuration and (b) coupling scheme of the dual mode filter (filter 4).

The proposed algorithm is applied to the measured  $S$ -parameters with  $N = 2$ ,  $n_z = 2$ ,  $N_s = 40$  (frequency interval 2.61–3.39 GHz),  $f_0 = 3.0$  GHz and  $FBW = 4.47\%$ , and then the desired parameters are obtained as follows:

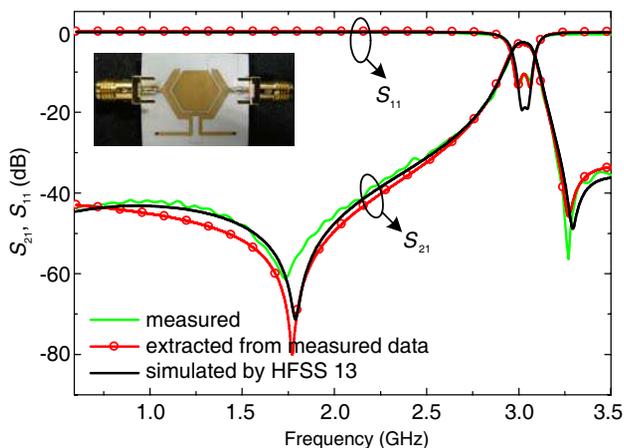
$r_{21k} = (-0.20933, 0.30667)$ ;  $r_{22k} = (0.19219, 0.30470)$ ;  $\lambda_k = (1.14433, -0.33470)$ ;  $\sigma_k = (0.23867, 0.24133)$ ; and  $M_{SL} = -0.00449$ .

Applying (7) to  $\sigma_k$  leads to the effective unloaded  $Q$  of 93.80 and 92.77, for each mode, respectively. Note that the initial  $N + 2$  transversal matrix has corresponded to the topology of the filter structure for a transversal filter. In other words, there is no requirement for a similarity transformation. The normalized  $N + 2$  coupling matrix is extracted from the measured  $S$ -parameters as

$$M = \begin{bmatrix} 0 & -0.4775 & 0.5556 & -0.0045 \\ -0.4775 & -1.1443 & 0 & 0.4384 \\ 0.5556 & 0 & 0.3347 & 0.5520 \\ -0.0045 & 0.4384 & 0.5520 & 0 \end{bmatrix} \quad (11)$$

In Figure 8, the original measured  $S$ -parameters are compared with those calculated by the extracted CM in (11). Very good agreement between the measured and extracted responses can be observed.

The filter 4 has been simulated using a full-wave simulator HFSS



**Figure 8.** The measured, the extracted and simulated  $S$ -parameters of fabricated filter 4.

13.0. The conductivity of the metal is set to  $5.8 \times 10^{-7}$  Siemens, and the metallic thickness is set to zero in the simulation. The simulated  $S$ -parameters are also plotted in Figure 8 to compare the measured with simulated results.

In the diagnosis of the filters 2, 3 and 4, there are also somewhat discrepancies between the simulated (or measured) and the extracted  $S$ -parameters at frequencies far away from the passband, which is due to the second order effects of a physical filter as said in the first example.

## 5. CONCLUSION

A two-stage optimization technique for the accurate extraction of the CM and the unloaded  $Q$  of lossy coupled resonator filters is presented. This method is applied to the diagnosis of any measured (simulated) filter with/without source-load coupling. The filter diagnosis is the core task of CAT of microwave bandpass filter. In this paper, the reason of Cauchy methods unable to diagnose the filters with source-load coupling is revealed. The initial  $N+2$  transversal matrix is obtained by the two-stage optimization method. The similarity transformations are applied to this transversal matrix, and then the CM with the prescribed topology corresponding to the physical structure is extracted. Four filters, including two simulated filters and two fabricated filters, have been diagnosed. The extracted  $S$ -parameters show good agreement with respect to the measured (simulated) responses.

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