ACTIVE LEARNING METHOD FOR THE DETERMINATION OF COUPLING FACTOR AND EXTERNAL $Q$ IN MICROSTRIP FILTER DESIGN

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Abstract—In the final step of any filter design process, the desired center frequency, coupling factor and external quality factor ($Q_{\text{ext}}$) are used to determine the physical parameters of the filter. Although in the most cases the physical dimensions of a single resonator for a given center frequency are determined using exact analytical or simple approximate equations, usually such simple equations cannot be found to easily relate the required coupling factor and $Q_{\text{ext}}$ to the physical parameters of the filter. Analytical calculation of coupling factor and $Q_{\text{ext}}$ versus dimensions are usually complicated due to the geometrical complexities or in some cases such as microstrip resonators due to the lack of exact solution for the field distribution. Therefore coupling factor and $Q_{\text{ext}}$ of various kinds of resonators, especially microstrip resonators, are related to the physical parameters of the structure by the use of time consuming full wave simulations. In this paper a surprisingly fast and completely general approach based on a soft computing pattern-based processing technique, called active learning method (ALM) is proposed to overcome the time consuming process of coupling factor and $Q_{\text{ext}}$ determination. At first the ALM technique and the steps of modeling are generally described, then as an example and in order to show the ability of the method this modeling approach is implemented to model the coupling factor and $Q_{\text{ext}}$ surfaces of microstrip open-loop resonators versus physical parameters of the structure. Using the ALM-based extracted surfaces for coupling factor and $Q_{\text{ext}}$, two four pole Chebychev bandpass filters are designed and fabricated. Good agreement between the measured and simulated results validates the accuracy of the proposed approach.

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1. INTRODUCTION

Modern communication systems need a complex arrangement of frequency allocation therefore a lot of different structures for the resonators and filters have been proposed to satisfy these requirements. In 1995 square open-loop resonators (SOLRs) were proposed by Hong [1]. They have been widely used in microwave filters due to their small size, planar structure and narrow realizable bandwidth. Although these resonators or other modified versions of them such as hexagonal [2–4] and spiral resonators [5,6] have been used to realize canonical [7–13], dual [14–21], triple [22,23] and quad-passband filters [24] with complicated transmission characteristics, all of these filters suffer from the time consuming full-wave-based process of determining the physical parameters of the filter from the desired coupling factor and $Q_{ext}$. The ability of soft computing techniques in modeling complicated problems in a vanishingly short time instead of using numerical or analytical approach [25–32] may provide a fast and accurate solution to this problem. Among the soft computing techniques the ability of fuzzy inference method in solving complicated electromagnetic problems such as microwave filter tuning [33,34], EMC problems [35], resonant frequency computation [36,37], determination of the transmission lines characteristic parameters [38], determination of the relative magnetic permeability [39] and also antenna modeling [40–42] has been proved in several publications. Artificial neural network (ANN) which is also a well-known soft computing technique has been used recently in microwave filter design [43]. Although the modeling steps of these methods seem to be very similar to the human logical thinking the amount and the complexity of the mathematics which is used, even in common fuzzy-based modeling techniques [44,45], is usually forgotten. The time consuming process of ANN training and difficult interpretation of the knowledge embedded in the trained ANNs in a comprehensive form decreases the interest of using these modeling techniques if there is any alternative. ALM which is a relatively new soft computing technique does not suffer from the mathematical complexity of fuzzy algorithms and the difficult interpretability of ANN-based techniques [46,47]. It has an intelligent information-handling process such as human brain and can be interpreted as a recursive fuzzy method which can express any multi-input single-output (MISO) system as the combination of some single-input single-output (SISO) one. Behavior of each SISO system which is a curve and a spread related to it will be extracted by the ink drop spread (IDS) method versus the corresponding independent variables. Then
using a proper combination rule of inference, the general behavior of the system is extracted.

Considering the simplicity, interpretability, fastness and accuracy of the ALM method, in this paper a completely general ALM-based approach for coupling factor and $Q_{ext}$ computation is proposed. The paper is organized as follows. In the second section of the paper the concept of coupling factor and $Q_{ext}$ is discussed briefly. The third and the fourth section of the paper are devoted to the basic definitions of ALM, clarifying the modeling steps using an illustrative example and its general formulation. In the fifth section the ALM technique is implemented to model the coupling factor and $Q_{ext}$ of SOLRs versus three important physical parameters. Finally using the extracted coupling factor and $Q_{ext}$ surfaces two four pole Chebychev bandpass filters are designed and fabricated. It should be mentioned that the modeling approach is completely general. It can be implemented to model coupling factor and $Q_{ext}$ for each required physical parameter, other resonators or other feeding configurations. Considering more than three modeling parameters by the proposed approach is simply possible by adding its initial data to the former ones.

2. COUPLING STRUCTURES AND $Q_{ext}$ DEFINITION

Shown in Figure 1 are three possible structures for the coupled SOLRs. The coupled structures result from different orientations of a pair of identical SOLRs which are separated by the spacing $S$. According to the nature of coupling which is through the fringe fields, three different types of coupling structures can be produced which are referred to as electric, magnetic and mixed coupling (Figure 1).

![Coupled SOLRs](image)

**Figure 1.** Coupled SOLRs in the case of, (a) electric coupling, (b) magnetic coupling, and (c) mixed coupling.

To calculate the coupling coefficient between two coupled resonators the following equation is used.

$$K = \frac{f_e^2 - f_m^2}{f_e^2 + f_m^2}$$
where $f_e$ and $f_m$ are the even and odd resonant frequencies of the two coupled resonators which occur in the frequencies that $S_{11}$ has it’s minimum value [7].

Another important step in the filter design process is the determination of $Q_{ext}$ which is the coupling factor of a resonator to an external circuit. For every resonant circuit $Q_{ext}$ can be computed by the aid of unloaded quality factor ($Q_U$) and loaded quality factor ($Q_L$) as it is shown in (2) [48].

$$\frac{1}{Q_{ext}} = \frac{1}{Q_L} - \frac{1}{Q_U} \quad (2)$$

Figure 2 shows a typical tapped-line feeding structure for a SOLR.

Using ALM method which will be described in the following sections the electric and magnetic coupling factor of SOLRs for three variables, i.e., the substrate permittivity ($\varepsilon_r$), the resonator length ($L$ as shown in Figure 2) and the spacing between resonators ($S$) normalized to $L$ ($S_N$ which is equal to $S/L$) will be modeled in the range of $[3 \ 12]$, $[6 \text{mm} \ 12 \text{mm}]$ and $[0.08 \ 0.35]$, respectively. Then $Q_{ext}$ will be modeled versus the length ($L_f$), the width ($W_f$) and the position ($D_f$) of the feed line for two different tapped-line feeding structures which have been shown in Figures 2(a) and 2(b). The variation range of $L_f$, $W_f$ and $D_f$ are $[5 \text{mm} \ 20 \text{mm}]$, $[0.2 \text{mm} \ 1 \text{mm}]$, and $[0 \ 5 \text{mm}]$, respectively for the structure of Figure 2(a) and $[5 \text{mm} \ 13 \text{mm}]$, $[0.2 \text{mm} \ 1 \text{mm}]$, and $[0 \ 2 \text{mm}]$, respectively for the structure of Figure 2(b). Considering the modeled $Q_{ext}$ surfaces provides a more efficient way in modeling this quantity. As it will be shown in the modeled surfaces, the variation of $D_f$ itself can provide a wide variation range of $Q_{ext}$ even when the $L_f$ and $W_f$ have been set to the fixed values. Considering this fact we have modeled $Q_{ext}$ versus the resonator length, the substrate permittivity, and the normalized feed line position ($D_N = D_f^2/L$) in the variation range of $[6 \text{mm} \ 12 \text{mm}]$, $[3 \ 12]$, and $[0 \ 0.6]$, respectively. Finally using these modeled $Q_{ext}$ and coupling factor surfaces, one can design a SOLR filter.
within a relatively wide variation range of center frequency, fractional bandwidth and other required characteristics.

3. ALM DEFINITION

This section is devoted to the introduction of the ALM modeling technique, the major steps toward this modeling and its mathematical formulation which is accompanied by an illustrative example.

3.1. Major Steps of ALM Modeling

ALM uses the following steps to model an unknown system or function versus the total number of independent variables that affect its behavior or output value.

(i) At first, sample data is gathered by any numerical method or measurement technique.

(ii) In the second step which is usually called projection step, all the gathered data is projected on each $x_i$-$y$ plane where $x_i$ is the $i$th input variable and $y$ stands for the output. In this step we are trying to imagine that the system is composed of some SISO one. If the system was really a SISO one the projected data would provide a narrow unique path but because of the effect of other inputs a spread is detectable around each narrow path. This spread shows the effect of other input variables on the corresponding $x_i$-$y$ plane. It can be easily deduced that, the narrower this spread, the effects of other input parameters on the output computation are less.

(iii) In the third step the narrow path and its related spread for each $x_i$-$y$ plane is extracted by a method called ink drop spread (IDS) algorithm. This algorithm will be thoroughly discussed in the following sub-section.

(iv) Then appropriate fuzzy rules of inference are implemented to combine all the extracted narrow paths and spreads.

(v) Finally the calculated output is compared with the original output by a predefined threshold error.

(vi) If the model is not accurate enough, the input domains will be divided in more sections and if it is necessary intermediate data points will be added to the previous ones. The active learning method is run again to reduce the error of the model.

In the next sub-section the mathematical formulation of the method will be described. The modeling steps will also be clarified by an example.
3.2. Illustrative Example and Mathematical Formulation

In order to explain the modeling process of ALM, we use a two-input function that has an input-output relationship as shown in Figure 3(a). For the simplicity of explanation each input domain is divided into two parts as shown in Figure 3(b), therefore four SISO systems are generated, i.e., A, B, C and D. In this figure $A_{ij}$ is the $j$th membership function for the $i$th input variable. Projected data of parts B and C on the corresponding $x_i$-$y$ plane are shown in Figure 4.

In order to extract the behavior of the system in the shaded area of Figure 3(b) which is in fact the intersection of SISO systems, B and C, it is only necessary to consider $x_1$ in the interval $[-1 0]$ of the part B and $x_2$ in the interval $[0 0.5]$ of the part C. As it can be seen in Figure 4, for the mentioned interval of $x_2$ in the part C a wide spread
is detectable, therefore no information about the output can be elicited from this projected data. On the other hand, in the mentioned interval for the $x_1$ in the part B there is a thin spread which means a high correlation between $x_1$ and the output. Therefore part B effectively elicits the system feature of the shaded area. In order to extract the behavior of the system in the total variation range of the inputs all data of parts A to D should be utilized. For each of these parts the related narrow path which explains the behavior of the corresponding SISO system and the related spread which explains the effect of other inputs on the extracted narrow path are obtained using IDS algorithm. Therefore following paragraphs are devoted to the explanation of the IDS method.

In the IDS method we assume each data point in each $x_i$-$y$ plane as a light source which has a cone shape beamwidth. When the vertical distance from this point increases its pattern spreads and interferes with the pattern of other data points which are now other sources of light. Indeed these cones are three dimensional membership functions which show the degree of confidence to other points near data points. If the mixed patterns are plotted on the corresponding input-output plane in grayscale as shown in Figure 5, a narrow path can be extracted using (3). The process of grayscale representation of the data in each $x_i$-$y$ plane is called IDS.

\[ \psi(x_p) = \left\{ (k\Delta y + y_{\min}) \left| \sum_{q=0}^{k} d(x_p, y_q) \approx \sum_{q=k+1}^{N} d(x_p, y_q) \right\} \right. \]

\[ x_p = x_{\min} + p\Delta x; \quad y_q = y_{\min} + q\Delta y; \quad x = [x_{\min}, x_{\min} + M\Delta x] \]

\[ y = [y_{\min}, y_{\min} + N\Delta y]; \quad p = 0, 1, \ldots, M; \quad q = 0, 1, \ldots, N; \quad 0 < k < N \]

where $\psi(x)$ is the extracted narrow path for the input parameter $x$. $d(a, b)$ is the value of darkness in point $(a, b)$. This value is 1 for the darkest points which are the most confident points in the plane and is 0 for the lightest ones. To compute the spread for each value of input

**Figure 5.** A typical IDS pattern.
the width of the dark area in $y$ direction is calculated using (4).

$$\sigma(x) = \max\{y|d(x,y) > 0\} - \min\{y|d(x,y) > 0\}$$

(4)

In Figure 6 the IDS pattern and the narrow path for each of the SISO systems in Figure 4 has been extracted using the mentioned approach. Spreads can be extracted using (4). All the extracted narrow paths and spreads are used in the inference process. In the above example each input domain was divided into two sections therefore we have only four inference rules as follows:

$$R_{11}: \text{If } x_2 \text{ is } A_{21}, \text{ then } y \text{ is } \psi_{11}$$

$$R_{12}: \text{If } x_2 \text{ is } A_{22}, \text{ then } y \text{ is } \psi_{12}$$

$$R_{21}: \text{If } x_1 \text{ is } A_{11}, \text{ then } y \text{ is } \psi_{21}$$

$$R_{22}: \text{If } x_1 \text{ is } A_{12}, \text{ then } y \text{ is } \psi_{22}$$

(5)

where $\psi_{ij}$ is the $j$th narrow path for the $i$th input variable and $A_{ij}$ denotes the $j$th membership function for the $i$th input variable. Assuming $x$ as variable, the general form of the membership functions can be expressed as follows:

$$\begin{cases}
0.5 \times (1 + \cos \pi((x - a)/(b - a))) & \text{for } x:a \to b \\
0.5 \times (1 - \cos \pi((x - a)/(b - a))) & \text{for } x:a \to b
\end{cases}$$

(6)

Finally the model output is obtained by aggregating the narrow paths as follows:

$$y \text{ is } \beta_{11}\psi_{11} \text{ or } \beta_{12}\psi_{12} \text{ or } \beta_{21}\psi_{21} \text{ or } \beta_{22}\psi_{22}$$

(7)

where $y$ is the output of the model, or is the union operator and $\beta_{ij}$ denotes the weight of $j$th narrow path for the $i$th input variable.
value of $\beta$ is determined from the spread and the degree of truth of the antecedent part in (5). It can be calculated as follows:

$$\beta_{1k_1} = \frac{(\omega_{1k_1} \times A_{2k_1})/(\omega_{11} \times A_{21} + \omega_{12} \times A_{22})}{\omega_{21} \times A_{11} + \omega_{22} \times A_{12}}$$

$$\beta_{2k_2} = \frac{(\omega_{2k_2} \times A_{1k_2})/(\omega_{11} \times A_{21} + \omega_{12} \times A_{22})}{\omega_{21} \times A_{11} + \omega_{22} \times A_{12}}$$

$$\omega_{ij} = \frac{1}{\sigma_{ij}}; \quad i = 1, 2; \quad j = 1, 2; \quad k_1 = 1, 2; \quad k_2 = 1, 2$$

(8)

4. ALM FOR THREE-INPUT SYSTEMS

In this section the ALM formulation for modeling of an unknown function with three independent variables which are referred to as $x_1$, $x_2$ and $x_3$ is described. After gathering input-output data in the range of independent variables each input domain is divided into $m_1$, $m_2$ and $m_3$ partitions, respectively. In the next step the gathered data are projected on the corresponding $x_i$-$y$ plane for IDS processing which means the extraction of narrow path and spread for each SISO system. The number of IDS units is calculated as follows:

$$L = \sum_{i=1}^{3} l_i \quad l_i = \prod_{j=1, j \neq i}^{3} m_j$$

(9)

where $L$ is the total number of all IDS units and $l_i$ denotes the number of IDS units for the input variable $x_i$.

All the extracted narrow paths and spreads are utilized in the inference process. The number of inference rules is equal to $L$. Some of these inference rules are shown in (10).

$$R_{11}: \text{If } x_2 \text{ is } A_{21} \text{ and } x_3 \text{ is } A_{31} \text{ then } y \text{ is } \psi_{11}$$

$$\vdots$$

$$R_{1l_1}: \text{If } x_2 \text{ is } A_{2m_2} \text{ and } x_3 \text{ is } A_{3m_3} \text{ then } y \text{ is } \psi_{1l_1}$$

$$R_{21}: \text{If } x_1 \text{ is } A_{11} \text{ and } x_3 \text{ is } A_{31} \text{ then } y \text{ is } \psi_{21}$$

$$\vdots$$

$$R_{3l_3}: \text{If } x_1 \text{ is } A_{1m_1} \text{ and } x_2 \text{ is } A_{2m_2} \text{ then } y \text{ is } \psi_{3l_3}$$

(10)

Finally the output of the system is calculated by aggregating the narrow paths using (11).

$$y \text{ is } \beta_{11} \psi_{11} \text{ or } \ldots \text{ or } \beta_{ik} \psi_{ik} \text{ or } \ldots \text{ or } \beta_{3l_3} \psi_{3l_3}$$

$$k = 1, 2, \ldots, l_i; \quad i = 1, 2, 3$$

(11)
where $\beta_{ik}$ is calculated as follows:

$$
\beta_{ik} = \frac{\omega_{ik} \times \Gamma_{ik}}{\sum_{i=1}^{3} \sum_{k=1}^{l_i} \omega_{ik} \times \Gamma_{ik}}
$$

(12)

In this equation $\Gamma_{ik}$ is calculated using (13).

\[
\Gamma_{11} = A_{21} \land A_{31}, \ldots, \Gamma_{1l_1} = A_{2m_2} \land A_{3m_3} \\
\Gamma_{21} = A_{11} \land A_{31}, \ldots, \Gamma_{2l_2} = A_{1m_1} \land A_{3m_3} \\
\Gamma_{31} = A_{11} \land A_{21}, \ldots, \Gamma_{3l_3} = A_{1m_1} \land A_{2m_2}
\]

(13)

where $\land$ is the intersection operator of the fuzzy sets.

5. COUPLING AND $Q_{\text{ext}}$ MODELING

In this section the effect of substrate permittivity, resonator length and spacing between resonators on the electric and magnetic coupling factor will be modeled using the discussed modeling technique. Then for the structures shown in the Figure 2 the effect of feed line length, feed line position and feed line width on the $Q_{\text{ext}}$ will be modeled. For all of these modeling the height of the substrate ($h$), the width of the resonator ($W$) and the gap size ($g$) are selected to be equal to 1.27 mm, 1 mm, and 1 mm, respectively. In the $Q_{\text{ext}}$ modeling, for the structure of Figures 2(a) and 2(b), $L$ is selected equal to 10.5 mm, and 6.5 mm, respectively. Parameters variation range have been mentioned in the last paragraph of the Section 2.

The following steps have been carried out to construct the ALM-based coupling factor and $Q_{\text{ext}}$ surfaces:

(i) Electric and magnetic coupling factors have been calculated using Ansoft HFSS-13 for 176 points within the variation range of the modeling parameters, i.e., $L$, $\varepsilon_r$ and $S_N$. These are the input data of our model for coupling factor computation. In the same way, $Q_{\text{ext}}$ of Figures 2(a) and 2(b) has been calculated using Ansoft HFSS-13 for 140 and 120 data points, respectively within the variation range of $W_f$, $L_f$ and $D_f$. These calculations provide the input data of our model for $Q_{\text{ext}}$ computation.

(ii) Concerning the electric and magnetic coupling factor modeling, the variation range of $L$, $\varepsilon_r$ and $S_N$ has been divided into 3, 3, and 10 sections, respectively. Concerning the $Q_{\text{ext}}$ modeling, the variation range of $W_f$, $L_f$ and $D_f$ has been divided into 4, 3, and 6 sections, respectively for the structure of Figure 2(a) and 5, 3, and 4 sections, respectively for the structure of Figure 2(b).

(iii) The narrow path and spread of each of these divisions has been computed by (3) and (4), respectively.
(iv) Using the inference rules (10), the computed narrow paths, and spreads the output has been calculated by (11).

Each of the extracted ALM-based surfaces, i.e., electric and magnetic coupling factor surfaces and the $Q_{ext}$ surfaces, has three variables. Therefore it is not possible to show each of them in one figure. Figures 7 and 8 show the extracted surfaces versus two of the modeling parameters. In each of these figures one of the independent variables has been set to three different values and the other ones sweep their own range of variations.

Considering Figures 7(a)–7(c) it can be deduced that the electric coupling factor is mainly influenced by the normalized spacing, mildly influenced by the resonator length, and weakly influenced by the substrate permittivity. In the same way, considering Figures 7(d)–7(f) it can be seen that the magnetic coupling is also mainly influenced by the normalized spacing, mildly influenced by the resonator length, but very weakly influenced by the substrate permittivity.

Figures 8(a)–8(c) illustrate the fact that the $Q_{ext}$ for the structure which has been shown in Figure 2(a) is equally influenced by the length, the width, and the position of the feed line. $Q_{ext}$ of the
Figure 8. Extracted $Q_{ext}$ for the structure of Figure 2(a); (a) $W_f = 0.2$ mm (top), $W_f = 0.35$ mm (middle), $W_f = 1$ mm (bottom), (b) $L_f = 20$ mm (top), $L_f = 12.5$ mm (middle), $L_f = 5$ mm (bottom), and (c) $D_f = 0$ (top), $D_f = 2.5$ mm (middle), $D_f = 5$ mm (bottom). Extracted $Q_{ext}$ for the structure of Figure 2(b); (d) $W_f = 0.2$ mm (top), $W_f = 1$ mm (middle), $W_f = 0.6$ mm (bottom), (e) $L_f = 11$ mm (top), $L_f = 13$ mm (middle), $L_f = 5$ mm (bottom), and (f) $D_f = 2$ mm (top), $D_f = 1.5$ mm (middle), $D_f = 0$ (bottom).

Figure 2(b) is shown in Figures 8(d)–8(f). These figures show that the $Q_{ext}$ drastically influenced by the feed line position while the influence of the feed line width and length is much weaker. Figures 8(a)–8(f) also illustrate that the feeding structure of Figure 2(a) can not be used to design very narrow band filters due to the fact that this structure can not provide high values of $Q_{ext}$.

Considering the $Q_{ext}$ surfaces depicted in Figures 8(a)–8(f) provides a more efficient way in modeling this quantity. As it can be concluded from these figures variation of the feed line position ($D_f$) itself can provide a wide variation range of $Q_{ext}$ even when the feed line length and feed line width have been set to the appropriate fixed values. Considering this fact and the capability of the structure shown in Figure 2(b) in providing different values of $Q_{ext}$, we have modeled the $Q_{ext}$ of Figure 2(b) versus the resonator length, the substrate permittivity, and the normalized feed line position ($D_N$). These modeled $Q_{ext}$ and coupling factors provide the required tool for designing SOLR filters in a wide variation range of electrical
Figure 9. Extracted $Q_{\text{ext}}$ for the structure of Figure 2(b) for the new modeling parameters; (a) $\varepsilon_r = 3$ (top), $\varepsilon_r = 7.5$ (middle), $\varepsilon_r = 12$ (bottom), (b) $L = 6\text{ mm}$ (top), $L = 9\text{ mm}$ (middle), $L = 12\text{ mm}$ (bottom), and (c) $D_N = 0.6$ (top), $D_N = 0.3$ (middle), $D_N = 0$ (bottom).

characteristics such as center frequency and fractional bandwidth. In this modeling process, $L_f$, $W_f$, $W$, $g$, and $h$ have been selected equal to 10 mm, 0.8 mm, 1 mm, 1 mm, and 1.27 mm, respectively. Modeling process is similar to the procedure which has been mentioned before. Figure 9 shows the sample results of the extracted data.

These $Q_{\text{ext}}$ and coupling factor surfaces not only provide a powerful design tool for SOLR filters but also provide the possibility of estimating the sensitivity of the electrical characteristics of the filter to each one of the modeling parameters. In other words the effect of fabrication or the substrate permittivity tolerances can be easily taken into account using these surfaces.

It should be mentioned that the coupling factor and $Q_{\text{ext}}$ surfaces have been extracted within the total variation range of the three modeling parameters. Resolution of the extracted data is equal to 0.01 mm for $L$, $W_f$, $L_f$, and $D_f$, 0.01 for $\varepsilon_r$, and is equal to 0.001 for $S_N$ and $D_N$. Considering the above mentioned resolution each of the magnetic or electric coupling surfaces consists of $601 \times 901 \times 271$ points. ALM needs less than half an hour for modeling each of these electric or magnetic coupling surfaces while Ansoft HFSS-13 needs 3 minutes for each point of these coupling surfaces. In other words, Ansoft HFSS-13 needs about 837 years for computing each of these surfaces.

In order to verify the model accuracy, the error between the target function (full-wave-based extracted data) and the constructed model (ALM-based extracted data) was measured using two different criteria, i.e., the fraction of variance unexplained (FVU) and correlation.
coefficient (CC) which are defined as follows.

\[
FVU = \frac{\sum_{l=1}^{N} (y_l - \hat{y}_l)}{\left(\sum_{l=1}^{N} (y_l - \bar{A})^2\right)^{\frac{1}{2}}}
\]

(14)

\[
CC = \frac{\sum_{l=1}^{N} (y_l - \bar{A}) (\hat{y}_l - \bar{B})}{\sqrt{\left(\sum_{l=1}^{N} (y_l - \bar{A})^2\right) \left(\sum_{l=1}^{N} (\hat{y}_l - \bar{B})^2\right)}}
\]

(15)

where \(y_l\) and \(\hat{y}_l\) denotes the \(l\)th data point of the output vector and the constructed model, respectively. \(\bar{A} = \frac{1}{N} \sum_{l=1}^{N} y_l\), \(\bar{B} = \frac{1}{N} \sum_{l=1}^{N} \hat{y}_l\), and \(N\) is the total number of output vector.

FVU is proportional to the mean square error. As the model accuracy increases, the FVU approaches zero and CC approaches one. Shown in Figure 10 are three coupling curves result from setting the substrate permittivity equal to 9.8 and the resonator length equal to the three different fixed values (6.5, 8.5, and 10.5 mm) in the extracted three-input ALM-based electric and magnetic coupling factors. Full-wave-based simulation results which have been calculated using Ansoft HFSS-13 are also included in these figures for the comparison purpose. As it is seen they are in good agreement. FVU and CC calculation results validate again the accuracy of the extracted surfaces quantitatively. FVU is equal to 0.0002, 0.0007, and 0.0003 for the Figures 10(a), 10(b), and 10(c), respectively, and CC is equal to 0.9999 for all of these figures.

The above mentioned results clearly show the ability of ALM technique in providing a completely fast, accurate and general modeling tool for coupling factor and \(Q_{ext}\) computation within a

![Figure 10](image_url)

**Figure 10.** Coupling factor curves result from setting substrate permittivity equal to 9.8 and the resonator length \((L)\) equal to the three different values in the extracted three-input ALM-based electric and magnetic coupling factors accompany with the full wave simulation results, (a) \(L = 6.5\) mm (electric coupling), (b) \(L = 8.5\) mm (magnetic coupling), and (c) \(L = 10.5\) mm (magnetic coupling).
wide variation range of the physical parameters. Providing such a design tool using full wave simulators is almost impossible due to the tremendous amount of simulation time.

6. FILTER DESIGN USING EXTRACTED SURFACES

To show the accuracy of the extracted coupling factor and $Q_{ext}$ surfaces, two four pole Chebychev bandpass filters were designed with the fractional bandwidth of 4% and the center frequencies of 1.64 GHz and 2.95 GHz which are referred to as filter 1 and filter 2, respectively. Figures 11(a) and 11(b) show the physical structure of the fabricated filters.

For the above mentioned fractional bandwidth $Q_{ext}$ is equal to 24 and the coupling matrix is given by:

$$
\begin{bmatrix}
0 & 0.0359 & 0 & 0 \\
0.0359 & 0 & 0.0277 & 0 \\
0 & 0.0277 & 0 & 0.0359 \\
0 & 0 & 0.0359 & 0 \\
\end{bmatrix}
$$

Both of the filters were fabricated on a Rogers substrate (TMM10i) with a relative permittivity of 9.8 and thickness of 1.27 mm. As it was mentioned before the resonator gap and width both were selected to be 1 mm. To satisfy required center frequencies, $L$ (Figure 2) was selected equal to 10.5 mm and 6.5 mm for the filters 1 and 2, respectively. Considering the physical structure of the filters 1 and 2 (Figure 11) it can be easily deduced that for both of the filters the first and the last two resonators are electrically coupled while the second and the third resonators are magnetically coupled. Therefore the spacing between the electrically coupled resonators, i.e., $S_{12}$ and $S_{34}$ (Figure 11) can be determined easily using extracted electric coupling factor surfaces. Considering the ($L = 10.5 \text{ mm, } \varepsilon_r = 9.8$) cut of these surfaces for the first filter and ($L = 6.5 \text{ mm, } \varepsilon_r = 9.8$) cut of them for the second filter will result in two curves which determines the electrical coupling values.
for both of the filters versus spacing between resonators. These curves result in $S_{12} = S_{34} = 1.3\text{ mm}$ for the filter 1 and $S_{12} = S_{34} = 1.2\text{ mm}$ for the filter 2. In the same way, the spacing between magnetically coupled resonators, i.e., $S_{23}$ can be determined easily using extracted magnetic coupling factor surfaces and the resulted magnetic coupling curves for each of the filters. These curves result in $S_{23} = 2.7\text{ mm}$ for the filter 1 and $S_{23} = 2.4\text{ mm}$ for the filter 2. Using ALM-based extracted $Q_{\text{ext}}$ surfaces, for $Q_{\text{ext}} = 24$ various triplets of $(L_f, W_f, D_f)$ were calculated. Among these different choices, $L_f$, $W_f$ and $D_f$ were selected $19\text{ mm}$, $0.4\text{ mm}$, and $0.2\text{ mm}$, respectively for the filter 1 and $11.25\text{ mm}$, $0.8\text{ mm}$, and $0$, respectively for the filter 2.

The simulated, the measured, and the ideal responses of the filters are shown in Figure 12. As it is seen they are in good agreement which shows the accuracy of the ALM-based extracted coupling factor and $Q_{\text{ext}}$ surfaces.

7. CONCLUSIONS

A novel approach based on ALM has been proposed to model coupling factor and $Q_{\text{ext}}$ of SOLRs. This ALM-based modeling approach has been implemented to model the coupling factor of electrically and magnetically coupled SOLRs versus three important effective parameters, i.e., spacing between resonators, resonator length and substrate permittivity. Using the same approach $Q_{\text{ext}}$ of a SOLR has been modeled versus important parameters which are feed line length, feed line width and feed line position. Afterwards $Q_{\text{ext}}$ was modeled versus a new set of modeling parameters, i.e., the position of the feed line, the length of the resonator and the permittivity of the

Figure 12. Responses of the filters, (a) filter 1, and (b) filter 2.
substrate. Based on the extracted coupling factor and $Q_{\text{ext}}$ surfaces two filters were designed and fabricated. Good agreement between the measured responses of the filters and the simulated ones validates the accuracy of the extracted surfaces. Although the model is very simple in its structure it is surprisingly fast and as accurate as the other time consuming and complicated numerical methods. The modeling approach is general and can be used for other kinds of coupling, other resonators or other feeding structures with the same degree of simplicity. It is also possible to consider more than three parameters in the proposed modeling approach easily if we add initial required data of the new parameters to the previous ones. This surprisingly fast method provides the possibility of modeling unknown functions within a wide variation range of modeling parameters with a high resolution of the extracted data. This resolution is not accessible through the use of full wave simulators due to the tremendous amount of simulation time.

REFERENCES


