Progress In Electromagnetics Research, Vol. 121, 381–389, 2011

ULTRATHIN CYLINDRICAL CLOAK

J. Zhang* and N. A. Mortensen

DTU Fotonik, Department of Photonics Engineering, Technical University of Denmark, DK-2800 Kongens, Lyngby, Denmark

Abstract—We propose a cylindrical invisibility cloak achieved utilizing two dimensional split-ring resonator structured metamaterials at microwave frequencies. The cloak has spatially uniform parameters in the axial direction, and can work very well even when the cloak shell is very thin compared with the concealed object and the working wavelength. Numerical simulation is performed to verify the functionality of the cloak, where the cloak layer is only around 1/4 of the operating wavelength. Our work provides a feasible solution to the experimental realization of cloaks with ideal parameters.

1. INTRODUCTION

In recent years, a number of approaches have been proposed to realize the invisibility cloaks for electromagnetic (EM) waves [1–25]. The one that wins most attention is based on transformation optics, a mathematic tool which takes advantage of the form invariance of Maxwell's equations under certain coordinate transformations. A spherical cloak was proposed which can perfectly hide arbitrary objects inside from EM illumination [1], and a cylindrical cloak which is uniform and infinite in its axial direction was proposed afterwards as a two-dimensional simplification of the spherical cloak [6]. Since the constitutive material parameters (μ_ρ, μ_ϕ, ε_z for TE case, or ε_ρ, ε_ϕ, μ_z for TM case) are space dependent in three directions and difficult to realize, Schurig et al. simplified the material parameters so that only μ_ρ (or ε_ρ) is spatially varying, leading to the first experimental realization of such a cloak at microwave frequency [7]. Other kinds of simplifications have also been considered, including those based on high order transformations to minimize the inherent reflection at the outer

Received 12 September 2011, Accepted 24 October 2011, Scheduled 1 November 2011

* Corresponding author: Jingjing Zhang (jinz@fotonik.dtu.dk).

surface of the simplified cloak [8, 9], or the use of simplified parameters for non-magnetic optical cloaks [10]. Further analytical scattering models of imperfect cloaks [11–18] and theoretical studies on other intrinsic problems associated with cloaks [19–28] and transformation optics [29–33] have also been presented [18–25].

The simplified approaches for cloak designs reduce the difficulty of the experimental realizations. However, theoretical work shows that the scattering of such kind of cloaks can not be totally removed [9, 10, 13, 15], which greatly limits the applications of cloaks. In particular, the performance of the cloak may be jeopardized as the thickness of the cloak shell is reduced. As a solution, a perfect cylindrical cloak with only two spatially variant parameters is considered [18]. For TE wave illumination, a scale transformation in cylindrical coordinate (TM case can be similarly derived) is applied: $\rho' = f(\rho)$, $\varphi' = \varphi$, $z' = z$, where f is an arbitrary smooth function. Under the above mapping, the parameters of the transformation media can be expressed as

$$\mu_\rho = \frac{f(\rho)}{\rho f'(\rho)}, \quad \mu_\varphi = \frac{\rho f'(\rho)}{f(\rho)}, \quad \varepsilon_z = \frac{f'(\rho) f(\rho)}{\rho}. \tag{1}$$

Letting $\varepsilon_z = \frac{f'(\rho) f(\rho)}{\rho} = C$, where C is an undetermined coefficient, and considering the conditions $f(R_1) = 0$ and $f(R_2) = R_2$, we obtain the solution for $f(\rho)$ and the corresponding material parameters of the cloak

$$\mu_\rho = \frac{\rho^2 - R_1^2}{\rho^2}, \quad \mu_\varphi = \frac{\rho^2}{\rho^2 - R_1^2}, \quad \varepsilon_z = \frac{R_2^2}{R_2^2 - R_1^2}. \tag{2}$$

where R_1 and R_2 represent inner and outer boundaries of the cloak. We note that ε_z is a constant larger than 1. At the same time, since only μ_ρ and μ_φ are spatially variant, this cloak can be realized with 2D metamaterials [34–36].

We first consider an ideal invisibility cloak with extremely thin shell, and then evaluate the performance of the cloak when four-layer stepwise approximation is adopted. While an extremely thin cloaks is commonly considered impractical, we offer a scheme to realize this cloak with cross-embedded split-ring resonators (SRR) [35], which allows for the independent control of the permeability along two orthogonal directions. The cloak shell is thus composed of four layers of SRR structures, the total thickness of which is approximately 1/4 of the operating wavelength. Numerical simulations show that although small forward scattering can still be noticed, the cloak effectively reduces the backward scattering of the object whose radius is four times that the thickness of the cloak shell. This work provides a possible solution to many practical cases wherein a thin concealing device is required.

2. PARAMETERS

Figure 1 shows the parameter profile of the cloak (solid lines) for a TE polarized incidence. In principle, as long as the material parameters take the exact forms addressed in Equation (2), a cloak with perfect invisibility can be realized. The COMSOL Multiphysics finite element-based electromagnetics solver is used to simulate the z-directed electric field distribution of a perfect electric conducting (PEC) cylinder (the radius is 0.16 m) illuminated by a TE plane wave along x direction. In this case, remarkable scattering is observed, as shown in Figure 2(a). In comparison, a cloak with the parameters characterized by Equation (1) whose inner and outer radiuses of the cloak are 0.16 m and 0.2 m, respectively can perfectly guide the incoming wave and reduce the scattering outside (see Figure 2(b)).

Addressing real fabrication process, we of course have to consider a stratified version of the cloak with discretized parameters to approximate the ideal one. Quite naturally, it can be estimated that the more layers we use, the better performance we will get. In order to model the physical realization of the ideal continuous cloaking material, we simulate a 4-layer cloak with the discretization of the material parameters. In Figure 1, the dashed lines indicate the material parameters for the configuration of a 4-layer cloak. The corresponding electric field distribution is also simulated, as shown in Figure 2(c). Compared with the ideal case, we find that though small forward scattering is introduced by this layered cloak, most of the incident wave is guided around the inner cylinder.

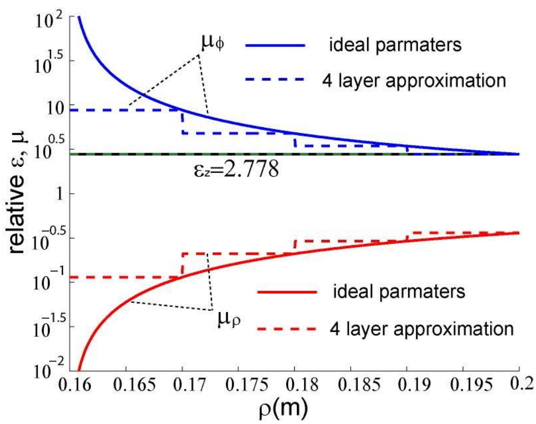

Figure 1. The permittivity and permeability components of the cloak.

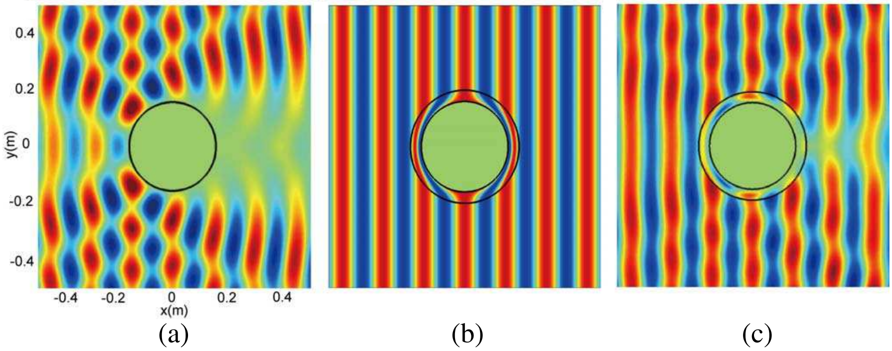

Figure 2. E_z field distribution of (a) a bare PEC cylinder, (b) a perfect cloak with the axially invariant parameter, (c) a four-layer stepwise approximation of the perfect cloak. In the calculation, we use the parameters retrieved from the 2D metamaterial shown in Figure 3. The imaginary part of permeability is also included, which accounts for the forward scattering.

3. RESULTS

We use 2D SRR structures to realize this cloak for TE polarized wave. Consistently with the above discussion, the inner and outer boundaries of the cloak are also 0.16 m and 0.2 m, respectively. The elementary cell of the metamaterial is shown in Figure 3. We propose to use two kinds of printed circuit boards with the S-rings to assemble the 2D cell: one small square board (side length 10 mm) with the SRR mirrored printed on both sides of the 2 mm thick substrate and the other with the SRR printed only on one side of the 1 mm thick substrate (side length 13.6 mm). Two pieces of the one-side printed boards are inserted into the slots of the two-side printed board to form a 2D unit cell of the metamaterial, as indicated in Figure 3. The cloak is composed of four layers, and each layer is implemented by repeating the unit cell along the circle. The 2D structure for each layer is separately designed to meet the parameter requirement specified in Figure 1. For the interior layer (first layer), the dimensions of the sample are $h1 = 8.96$ mm, $g1 = 1.6$ mm, $w1 = w2 = 1.02$ mm, $h2 = 9.4$ mm, and $g2 = 1.74$ mm, The relative permittivity of the substrate is 4.0. The dimensions for other layers are as follows: the second layer, $h1 = 8.8$ mm, $g1 = 1.57$ mm, $w1 = 1.002$ mm, $w2 = 1.024$ mm, $h2 = 9.44$ mm, and $g2 = 1.75$ mm; the third layer, $h1 = 8.68$ mm, $g1 = 1.55$ mm, $w1 = 0.988$ mm, $w2 = 1.03$ mm, $h2 = 9.49$ mm, and $g2 = 1.76$ mm; the outer layer, $h1 = 8.64$ mm, $g1 = 1.54$ mm, $w1 = 0.983$ mm, $w2 = 1.04$ mm, $h2 = 9.78$ mm, and $g2 = 1.77$ mm.

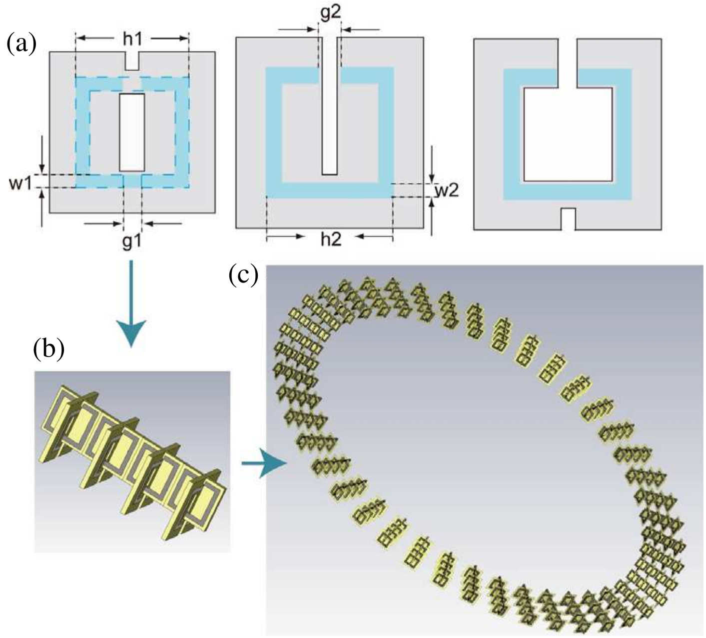

Figure 3. Schematic of the fabrication of the cloak, (a) the SRR structures composing the 2D unit cell, (b) unit cells in the four layers, (c) a cylindrical cloak constructed with SRRs.

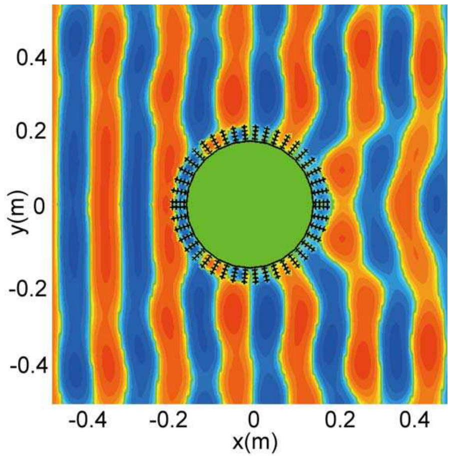

Figure 4. E_z field distribution of a PEC cylinder covered by the ultrathin cloak under a plane wave incidence at 1.88 GHz.

The performance of the full cloaking structure, resolving all spatial details down to the unit-cell scale, is evaluated with the commercial software CST STUDIO SUITE. In the simulation, the electromagnetic wave is incident from the left side of the cloak, propagating in the x direction, and the boundary condition in z axis is set as perfect electric boundary. The z component of the electric field at 1.88 GHz is shown in Figure 4, where we can see that only in the region behind the cloaked object some perturbations of the wave front can be noticed. The scattering is mainly caused by the discretization of the ideal parameters, and partly due to the loss introduced by the metamaterials.

4. CONCLUSIONS

In conclusion, we have proposed an ultrathin cylindrical cloak for TE wave made of 2D SRR metamaterial structures. Since the design of this cloak is based on the ideal parameters without compromising the performance of the cloak, it can overcome the drawbacks of those cloaks relying on simplified parameters. In principle, the only issues that would introduce scattering are the discretization of the parameters and the loss of the composing materials. With the advent of more and more 2-D metamaterial structures, the approaches we proposed here can be implemented with some manual efforts, but without need for advanced fabrication approaches.

ACKNOWLEDGMENT

The work is supported by a Hans Christian Ørsted postdoctoral fellowship as well as by the Danish Research Council for Technology and Production Sciences (FTP grant #274-07-0080 and 274-07-0379). Jingjing Zhang wants to acknowledge Yu Luo and Hongsheng Chen for insightful discussions.

REFERENCES

1. Pendry, J. B., D. Schurig, and D. R. Smith, "Controlling electromagnetic fields," *Science*, Vol. 312, 1780, 2006.
2. Leonhardt, U., "Optical conformal mapping," *Science*, Vol. 312, 1777, 2006.
3. Greenleaf, A., M. Lassas, and G. Uhlmann, "Anisotropic conductivities that cannot be detected by EIT," *Physiol. Meas.*, Vol. 24, 413, 2003.

4. Alù, A. and N. Engheta, "Achieving transparency with plasmonic and metamaterial coatings," *Phys. Rev. E*, Vol. 72, 016623, 2005.
5. Milton, G. W., M. Briane, and J. R. Willis, "On cloaking for elasticity and physical equations with a transformation invariant form," *New J. Phys.*, Vol. 8, 248, 2006.
6. Cummer, S. A., B.-I. Popa, D. Schurig, D. R. Smith, and J. B. Pendry, "Full-wave simulations of electromagnetic cloaking structures," *Phys. Rev. E*, Vol. 74, 036621, 2006.
7. Schurig, D., J. J. Mock, B. J. Justice, S. A. Cummer, J. B. Pendry, A. F. Starr, and D. R. Smith, "Metamaterial electromagnetic cloak at microwave frequencies," *Science*, Vol. 314, 977–980, 2006.
8. Cai, W., U. K. Chettiar, A. V. Kildishev, and V. M. Shalaev, "Optical cloaking with metamaterials," *Nature Photon.*, Vol. 1, 224, 2007.
9. Weder, R., "A rigorous analysis of high-order electromagnetic invisibility cloaks," *J. Phys. A: Math. Theor.*, Vol. 41, 065207, 2008.
10. Zhang, J., Y. Luo, and N. A. Mortensen, "Minimizing the scattering of a nonmagnetic cloak," *Appl. Phys. Lett.*, Vol. 96, 113511, 2010.
11. Chen, H., B.-I. Wu, B. Zhang, and J. A. Kong, "Electromagnetic wave interactions with a metamaterial cloak," *Phys. Rev. Letts.*, Vol. 99, 063903, 2007.
12. Ruan, Z., M. Yan, C. W. Neff, and M. Qiu, "Ideal cylindrical cloak: Perfect but sensitive to tiny perturbations," *Phys. Rev. Letts.*, Vol. 99, 113903, 2007.
13. Yan, M., Z. Ruan, and M. Qiu, "Cylindrical invisibility cloak with simplified material parameters is inherently visible," *Phys. Rev. Letts.*, Vol. 99, 233901, 2007.
14. Cummer, S. A., B.-I. Popa, D. Schurig, D. R. Smith, J. B. Pendry, M. Rahm, and A. Starr, "Scattering theory derivation of a 3D acoustic cloaking shell," *Phys. Rev. Letts.*, Vol. 100, 024301, 2008.
15. Cai, W., U. K. Chettiar, A. V. Kildishev, V. M. Shalaev, and G. W. Milton, "Nonmagnetic cloak with minimized scattering," *Appl. Phys. Letts.*, Vol. 91, 111105, 2007.
16. Luo, Y., H. Chen, J. Zhang, L. Ran, and J. A. Kong, "Design and analytical full-wave validation of the invisibility cloaks, concentrators, and field rotators created with a general class of transformations," *Phys. Rev. B*, Vol. 77, 125127, 2008.
17. Peng, L., L. Ran, and N. A. Mortensen, "The scattering of a cylindrical invisibility cloak: Reduced parameters and

optimization," *J. Phys. D: Appl. Phys.*, Vol. 44, 135101, 2011.

18. Luo, Y., J. Zhang, H. Chen, S. Xi, and B.-I. Wu, "Cylindrical cloak with axial permittivity/permeability spatially invariant," *Appl. Phys. Lett.*, Vol. 93, 033504, 2008.
19. Chen, H., X. Jiang, and C. T. Chan, "Extending the bandwidth of electromagnetic cloaks," *Phys. Rev. B*, Vol. 76, 241104, 2007.
20. Luo, Y., J. Zhang, B.-I. Wu, and H. Chen, "Interaction of an electromagnetic wave with a cone-shaped invisibility cloak and polarization rotator," *Phys. Rev. B*, Vol. 78, 125108, 2008.
21. Yao, P., Z. Liang, and X. Jiang, "Limitation of the electromagnetic cloak with dispersive material," *Appl. Phys. Letts.*, Vol. 92, 031111, 2008.
22. Chen, H., J. Ng, C. W. J. Lee, Y. Lai, and C. T. Chan, "General transformation for the reduced invisibility cloak," *Phys. Rev. B*, Vol. 80, 085112, 2009.
23. Jiang, W., T. Cui, X. Yang, Q. Cheng, R. Liu, and D. R. Smith, "Invisibility cloak without singularity," *Appl. Phys. Lett.*, Vol. 93, 194102, 2008.
24. Zhang, P., Y. Jin, and S. He, "Obtaining a nonsingular two-dimensional cloak of complex shape from a perfect three-dimensional cloak," *Appl. Phys. Lett.*, Vol. 93, 243502, 2008.
25. Jiang, W., J. Y. Chin, Z. Li, Q. Cheng, R. Liu, and T. Cui, "Analytical design of conformally invisible cloaks for arbitrarily shaped objects," *Phys. Rev. E*, Vol. 77, 066607, 2008.
26. Han, T., X. Tang, and F. Xiao, "The petal-shaped cloak," *Journal of Electromagnetic Waves and Applications*, Vol. 23, No. 14–15, 2055–2062, 2009.
27. Han, T., C.-W. Qiu, and X. Tang, "Creating rigorous open cloaks," *Journal of Electromagnetic Waves and Applications*, Vol. 24, No. 13, 1839–1847, 2010.
28. Zhang, J. J., Y. Luo, H. Chen, and B.-I. Wu, "Sensitivity of transformation cloak in engineering," *Progress In Electromagnetics Research*, Vol. 84, 93–104, 2008.
29. Cojocaru, E., "Illusion devices with internal or external circular objects designed by the coordinate transformation method," *Journal of Electromagnetic Waves and Applications*, Vol. 24, No. 16, 2309–2317, 2010.
30. Mei, Z.-L., J. Bai, and T.-J. Cui, "Illusion devices with quasi-conformal mapping," *Journal of Electromagnetic Waves and Applications*, Vol. 24, No. 17–18, 2561–2573, 2010.

31. Luo, Y., J. Zhang, H. Chen, B.-I. Wu, and L.-X. Ran, "Wave and ray analysis of a type of cloak exhibiting magnified and shifted scattering effect," *Progress In Electromagnetics Research*, Vol. 95, 167–178, 2009.
32. Luo, Y., J. B. Pendry, and A. Aubry, "Surface plasmons and singularities," *Nano Lett.*, Vol. 10, 4186, 2010.
33. Luo, Y., A. Aubry, and J. B. Pendry, "Electromagnetic contribution to surface-enhanced Raman scattering from rough metal surfaces: A transformation optics approach," *Phys. Rev. B*, Vol. 83, 155422, 2011.
34. Shelby, R. A., D. R. Smith, S. C. Nemat-Nasser, and S. Schultz, "Microwave transmission through a two-dimensional, isotropic, left-handed metamaterial," *Appl. Phys. Letts.*, Vol. 78, 489, 2001.
35. Zhang, J., H. Chen, L. Ran, Y. Luo, B.-I. Wu, and J. A. Kong, "Experimental characterization and cell interactions of a two-dimensional isotropic left-handed metamaterial," *Appl. Phys. Letts.*, Vol. 92, 084108, 2008.
36. Cheng, Q., H.-F. Ma, and T.-J. Cui, "A complementary lens based on broadband metamaterials," *Journal of Electromagnetic Waves and Applications*, Vol. 24, No. 1, 93–101, 2010.