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EXPERIMENTAL AND THEORETICAL STUDIES OF A BROADBAND SUPERLUMINALITY IN FABRY-PEROT INTERFEROMETER

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Abstract—This study experimentally demonstrates a broadband (20%) superluminality in a Fabry-Pérot-like interferometer implemented on a waveguide system. A narrow wave packet propagating with an effective group velocity of $5.29^{+4.28}_{-1.70}c$ without distortion was observed. The underlying mechanism is attributed to the multiple-reflection interference and the modal effect, which provide an approach for controlling the wave characteristics through manipulating the geometry of the system. Besides, the criteria of the renowned generalized Hartman effect are explicitly clarified.

1. INTRODUCTION

Signal propagating with a group velocity that exceeds the speed of light in vacuum (c) is intriguing for physicists, as it seems to violate the causality and Einstein's special relativity. Such phenomenon called superluminality is generally attributed to steep anomalous dispersion [1–8] and tunneling effect [9–17], which have been experimentally demonstrated in the electromagnetically-induced-absorption [1–5]/ring-resonator [6–8] optical systems and the photonic/waveguide [9–17] systems, respectively. However, the systems employing the anomalous dispersion usually operate near the steep absorption line, which leads to a narrow superluminal frequency range as well as low transmission (for passive systems), while the systems associated with the tunneling mechanism also suffer from extremely low transmission [16–26]. Therefore, finding a new mechanism that supports both relatively high transmission and wide superluminal bandwidth might facilitate various applications.

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On the basis of the superluminal studies in waveguide system [11–14], the propagation of an electromagnetic wave in a waveguide is considered to be an ideal resemblance to a time-independent quantum system [11] due to the similarity between cutoff frequency and quantum potential. A potential barrier/well can be modeled by either inserting a dielectric material into a uniform waveguide [14–17, 20, 21] or using an undersized/oversized waveguide changed in width [11–13]. Notably, the discontinuities of the undersized/oversized waveguide will excite high-order evanescent modes in addition to the fundamental mode, which is called the modal effect. The importance of the modal effect had long been noted [11–13, 27–31], but its effect and extended application on the superluminality has never been studied.

This study proposes a scheme to induce a broadband faster-than-light phenomenon with relatively high transmission by using a waveguide system with geometric discontinuities changed in height, but not width like done so previously. Since the cutoff frequency of operating mode is constant throughout, this new configuration is not analogous to potential-barrier system and the modal effect becomes critical. An electromagnetic wave packet was generated and its transmitted characteristics were measured. A broadband (20%) superluminality with a transmission of 3.2% was observed, in which the wave packet can propagate with an effective group velocity of $5.29^{+4.28}_{-1.70}c$ without significant distortion.

The group delay and the bandwidth of the superluminality can be simply controlled by varying the height (through modal effect) and the length (through interference) of the waveguide, respectively. Moreover, the group delay in the proposed system is linearly proportional to the selected lengths, which indicates that the understanding of the generalized Hartman effect should be further clarified [11, 20–22]. A complete equation set derived based on multiple-reflection interference is displayed to elucidate the observed phenomena and to clarify that the generalized Hartmann effect is valid only for the cases of extremely low transmission. This study may shed the light on controlling group delay for optical wave packet switching/modulating [32] or high-speed signal network processing in the present communication industry.

2. THEROETICAL MODEL FOR SUPERLUMINALITY IN A FABRY-PEROT-LIKE INTERFEROMETER

Consider a wave that passes through two separated discontinuities as shown in Figs. 1(a) and 1(b). The widths (a) of the waveguides are identical, indicating the constant cutoff frequency for the fundamental operating TE_{10} mode. The non-zero reflection R and R' shown in

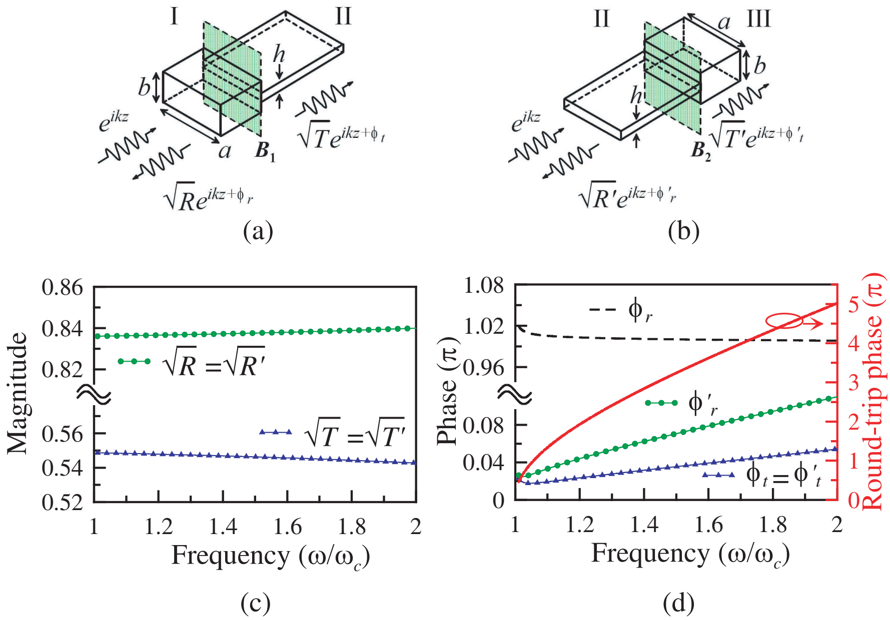

Figure 1. Discontinuities and the transmitted/reflected properties due to modal effect. (a) Schematic diagram for the single discontinuity. The wave is incident from region I to region II. (b) Schematic diagram for another discontinuity, while the wave is incident from region II to region III. Magnitude part and phase part of the transmission and reflection coefficients are displayed in (c) and (d), respectively. Notably, $\sqrt{T}(\sqrt{R})$ equals to $\sqrt{T'}(\sqrt{R'})$, obeying the reciprocity. These values are calculated using the modal analysis [27–31]. The round-trip phase change ($2kL + 2\phi'_r$ with $L = 10\,\text{cm}$) is illustrated in (d).

Fig. 1(c) is strictly due to the excitation of the higher-order evanescent modes. The conversion between high-order modes and the TE_{10} mode modifies the magnitudes of transmission ($\sqrt{T}$ and $\sqrt{T'}$) and reflection ($\sqrt{R}$ and $\sqrt{R'}$) coefficients of the fundamental propagating wave (TE_{10}), as well as induces the extra phase changes (ϕ_t, ϕ'_t, ϕ_r, and ϕ'_r) during the transmission and reflection processes. Such phenomenon is called the modal effect [27–31]. In this study, the regions I and III are WR-284 waveguides with a width of 2.84 inches ($a = 72.14\,\text{mm}$) and a height of 1.34 inches ($b = 34.04\,\text{mm}$). The region II is an undersized waveguide which has the same width as that of WR-284 waveguide but the height (h) is reduced to 3.00 mm. The frequency responses of the magnitudes and phases are shown in Figs. 1(c) and 1(d), respectively.

An integrated waveguide system with an undersized middle section of length L is shown in Fig. 2(a), in which the regions I and III are input and output sections, respectively. It can be treated as a combination of the two discontinuities shown in Figs. 1(a) and 1(b). Two geometrically-discontinuous interfaces (B_1 and B_2) between regions I/II and II/III form the boundaries of a Fabry-Pérot cavity, and thus the wave will bounce back and forth in the middle region, resulting in multiple reflections. After analyzing the multiple-reflection trace of a propagating wave, the final transmitted signal (F_T defined in Fig. 2(a)) can be expressed as

$$F_T = \sqrt{T}\sqrt{T'} \exp\left[i\left(\phi_t + \phi_t' + kL\right)\right] \sum_{n=0}^{\infty} \left\{R' \exp\left[i\left(2\phi_r' + 2kL\right)\right]\right\}^n, \quad (1)$$

where k is the propagation constant of the fundamental mode; $\sqrt{R'}$ and ϕ_r' are the magnitude and phase of the reflection coefficient from region II to I or III; ϕ_t and ϕ_t' represent the phase differences associated with signal passing through the first (from I to II) and the second (from II to III) boundaries, respectively. These parameters have been defined in Figs. 1(a) and 1(b), while their frequency responses for the case of $L = 10\,\text{cm}$ are given in Figs. 1(c) and 1(d).

The transmitted wave shown in (1) can be further simplified to

$$F_T = \frac{\sqrt{T} \times \sqrt{T'}}{\sqrt{1 - 2R' \cos\left(2\phi_r' + 2kL\right) + R'^2}} e^{i\varphi_T}, \quad (2a)$$

$$\varphi_T = kL + \tan^{-1}\left[\frac{R' \sin\left(2kL + 2\phi_r'\right)}{1 - R' \cos\left(2kL + 2\phi_r'\right)}\right] + (\phi_t + \phi_t'), \quad (2b)$$

where φ_T represents the total phase difference between the final transmitted wave and the incident wave. Because of the reciprocity,

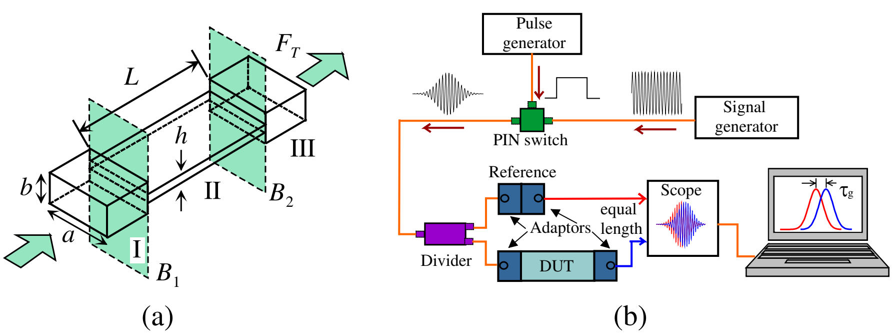

Figure 2. Experimental setup for group delay measurement. (a) Device under test (DUT). (b) Schematic diagram of experimental setup.

$\phi_t = \phi'_t$. Noteworthily, the non-zero R', ϕ'_r, and ϕ_t here are caused by the modal effect, which is not exclusive to the current configuration, but a common effect when there is a geometrical discontinuity. Among the various definitions of delay times [33], the group delay is well-defined as the difference between the time when the peak of the incident wave packet enters the middle section and the time when the peak of output wave packet leaves this section. According to the stationary phase approximation [14], the group delay (τ_g) equals to $d\varphi_T/d\omega$ and is given by

$$\tau_g = \left[\frac{1-R'^2}{1-2R'\cos(2kL+2\phi'_r)+R'^2}\right]\left(\frac{L}{v_g}\right) + \left[\frac{R'\cos(2kL+2\phi'_r)-R'^2}{1-2R'\cos(2kL+2\phi'_r)+R'^2}\right]\left(2\frac{d\phi'_r}{d\omega}\right)+2\frac{d\phi_t}{d\omega}, \quad (3)$$

where $v_g = d\omega/dk$, i.e., the group velocity for an infinitely long middle section. Note that the frequency change rate of the reflection ($dR'/d\omega$) is relatively small as found in the simulation (the frequency response of $\sqrt{R'}$ is nearly a constant as shown in Fig. 1(c)), hence, the term related to $dR'/d\omega$ is omitted in (3).

According to (3), the round-trip phase change ($2kL+2\phi'_r$) plays an important role to control the group delay, and thus τ_g can be discussed at two representative interference conditions:

(I) On-resonant (slow-light) condition: $2kL + 2\phi'_r = 2m\pi$:

$$\tau_g^{\text{on}} = \left(\frac{1+R'}{1-R'}\right)\frac{L}{v_g} + \left(2\frac{d\phi_t}{d\omega}+\frac{2R'}{1-R'}\frac{d\phi'_r}{d\omega}\right). \quad (4)$$

The waves exhibit constructive interference after a round-trip traveling, implying that the total transmission equals unity, but the corresponding group delay is the longest.

(II) Off-resonant (fast-light) condition: $2kL + 2\phi'_r = (2m+1)\pi$:

$$\tau_g^{\text{off}} = \left(\frac{1-R'}{1+R'}\right)\frac{L}{v_g} + \left(2\frac{d\phi_t}{d\omega}-\frac{2R'}{1+R'}\frac{d\phi'_r}{d\omega}\right). \quad (5)$$

Under this condition, all of the multiple-reflection waves destructively interfere with each other, indicating that the corresponding total transmission should be relatively low. According to (5), the group delay can be small enough to meet the superluminal criteria ($\tau_g^{\text{off}} < L/c$) under three conditions with off-resonant operation: the boundary reflection rate (R') is high, the propagating length (L) is short, and the contribution from the last two terms of the right-hand side of (5) is negligible or even negative.

The first parts in Eqs. (4) and (5) have been discussed [20], while the second part (last two terms) is original, which can be considered as the effective time for the signal spending at the two boundaries during multiple-reflection process. These two terms are dominated by modal effect in the present scheme (through R', ϕ'_r, and ϕ_t), and would become crucial as $R' \to 1$ or $L \to 0$ under off-resonance operation.

3. EXPERIMENTAL SETUP

Figure 2(b) schematically depicts the experimental setup. The sinusoidal wave was generated using a frequency-tunable signal generator, and then temporally modulated by the PIN switch (es) to form a Gaussian-like wave packet. The as-generated wave packet was then divided into two identical signals by a power divider. Two divided signals were subsequently sent to the device under test (DUT) and a reference circuit for comparison. The reference circuit is a back-to-back connection of two home-made adaptors (coaxial line to WR-284 rectangular waveguide with total reflection rate less than $-30\,\mathrm{dB}$), while the DUT here is displayed in Fig. 2(a), the regions I and III of which are two adaptors identical to that in the reference circuit. The output signals were led to an oscilloscope with two equal-length coaxial cables to ensure no extra delay. The signals were recorded by an oscilloscope and then analyzed with a computer to find the group delay (τ_g). The frequency response of the group delay can be obtained by varying the carrier frequency step by step.

4. RESULTS AND DISCUSSIONS

Figure 3(a) shows the results of the group delay versus frequency for a DUT of $L = 10\,\mathrm{cm}$. The solid dots are the experimental results measured from the system depicted in Fig. 2. The open triangles are the time-domain simulated results performed using CST (Computer Simulation Technology). The slow-light (subluminal) and fast-light (superluminal) zones are separated with the horizontal dashed line which marks the delay of a wave propagating through L with speed c ($L/c = 0.33\,\mathrm{ns}$). A broadband superluminality ranging from $1.298\omega_c$ to $1.587\omega_c$ (20% bandwidth, centered at $1.467\omega_c$) is presented. Notably, the experimental errors of all measured delays were limited within $0.03\,\mathrm{ns}$ ($30\,\mathrm{ps}$), majorly contributed by the background noise fluctuation and the jitter/bandwidth limitation of the oscilloscope. The measured group delays show in good agreement with the simulated results using CST.

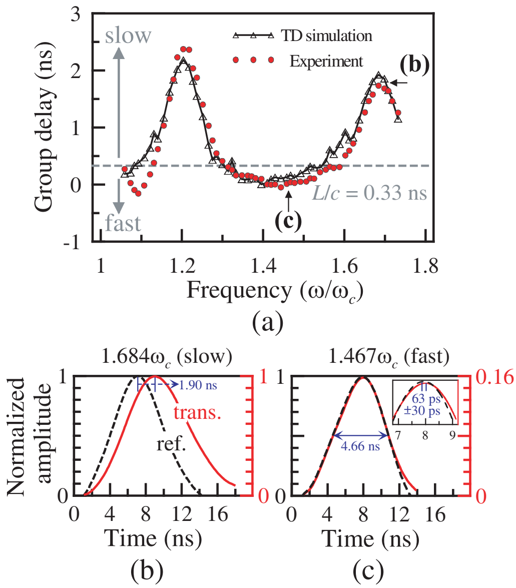

Figure 3. Frequency response of the group delay and two representative profiles for $L = 10\,\text{cm}$. (a) Open triangles are time-domain simulated results (CST); and the solid dots are the measured results. (b) Measured profiles for slow-light case with carrier frequency of $1.683\omega_c$. Solid profile shows the transmitted signal (corresponding to right vertical axis) and dashed profile is the referenced signal (left vertical axis). (c) Profiles of fast-light case at $1.467\omega_c$. The inset displays the close-up view.

The signal profiles of the slow-light and the fast-light cases are shown in Figs. 3(b) and 3(c), respectively. The center frequency of the slow-light signal (Fig. 3(c)) was chosen at $1.684\omega_c$, whose round-trip phase change ($2kL + 2\phi_r'$, shown in Fig. 1(d)) approximately equals to 4π, matching for the on-resonant condition discussed in (4). The slow-light group delay is $1.9 \pm 0.03\,\text{ns}$ corresponding to a group velocity of about $0.17c$. In fast-light case, the center frequency was specified as $1.467\omega_c$, while its round-trip phase becomes 3π (off-resonant interference, (5)), resulting in a superluminal group delay of $63 \pm 30\,\text{ps}$ (much less than $L/c = 0.33\,\text{ns}$) as displayed in the inset of Fig. 3(c). Under this condition, signal propagates with a group velocity of $5.29^{+4.28}_{-1.70}c$.

Since a composite wave packet with a finite spectrum (full width at half maximum (FWHM) $= 0.12\omega_c$, extracted from the profile shown in Figs. 3(b) and (c)) is adopted, its time-domain behavior is sensitive to the frequency response of group delay (manifesting by the group delay dispersion GDD $\equiv d\tau_g/d\omega$). The slow-light range is narrow (i.e., $1.154\omega_c$ to $1.251\omega_c$ and $1.635\omega_c$ to $1.732\omega_c$ in Fig. 3(a)) and thus the group delay dispersion is large, leading to the obvious distortion (broadening) of the transmitted pulse (Fig. 3(b)). On the contrary, the superluminal region ($1.298\omega_c$ to $1.587\omega_c$) is much wider than the signal spectrum (center at $1.467\omega_c$ with FWHM $= 0.12\omega_c$) and therefore the group delay dispersion here is negligible. These two features explain why the envelope of the faster-than-light transmitted signal is almost distortionless (Fig. 3(c)). This wide superluminal region enables us to transmit a relatively short pulse without significant distortion.

Figure 4 plots the group velocity and the transmission versus the height (h) of the middle waveguide. Increasing h allows more energy to be transmitted but would reduce the group velocity. The transmission can be as high as 47% where the corresponding group velocity would approach the border of superluminality ($v_g \approx c$). The fast-light group velocity ($\sim 5.29c$) shown in Fig. 3(c) is obtained at the case of $h/b = 0.09$, in which the transmission approximately equals 3.2%. This undersized waveguide system is not equivalent to a single opaque barrier [9–17], because the signal of fundamental mode does propagate within the middle section, and thus the transmission in superluminal regime is significantly higher than that of single-barrier system.

The broadband characteristic is similar to that of the double-barrier tunneling system, which is generally made of periodic photonic crystals [20–26]. If the double-barrier system is composed by two opaque barriers ($R' \to 1$), the first part in (5) vanishes and the second

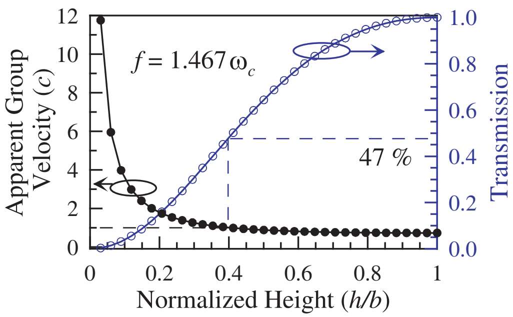

Figure 4. Group velocity and final transmission rate ($|F_T|^2$) versus normalized height. Solid dots and open circles represent the group velocity ($v_g = L/\tau_g$) and transmission, respectively.

part dominates the total group delay. Therefore, in the superluminal region, the total transmission of the double-barrier system is extremely small and the group delay does not change with the opaque barrier width and the barrier separation. This characteristic is renowned as the generalized Hartman effect [11, 20–22], which can be explained as a special case in (5) with extremely high boundary reflection rate ($R' \to 1$). Since the reflection of the boundary in our system is not too high ($R' = 0.55$), the first part in the right hand side of (5) becomes relatively important. The group delay is consequently proportional to the length at the off-resonant conditions (to be verified in Fig. 5).

Figure 5(a) plots the frequency responses of the group delay for five selected lengths ($L = 10$, 30, 50, 70, and 90 cm). A longer length exhibits a narrower superluminal range as shown in Fig. 5(b) due to the dense interference conditions. On the contrast, the system with shorter propagation length L (wider superluminal bandwidth) allows a narrower pulse to accomplish superluminal propagation without distortion. This relation explicitly explains the reason for distortionless-superluminal propagation demonstrated in Fig. 3(c), in which the width of incident pulse (~ 4.66 ns) is longer than the shortest pulse width allowed (1.62 ns, shown in Fig. 5(b) at case of $L = 10$ cm) for superluminal propagation.

The bandwidth of the superluminality can be controlled by

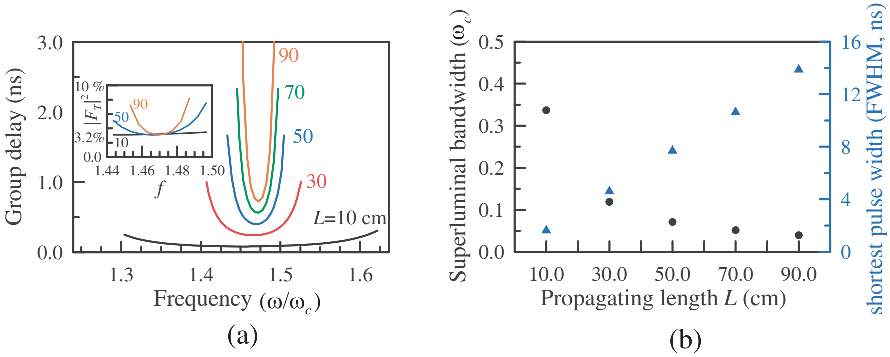

Figure 5. (a) Group delay versus frequency for five selected lengths. The inset in (a) shows the minimum transmission of 3.2% under off-resonant condition ($1.467\omega_c$). (b) Superluminal bandwidth (black solid dots) and the shortest pulse width allowed for distortionless-superluminal propagation (blue solid triangles) versus propagating length. The pulse width is obtained based on the FWHM of a Gaussian profile.

varying the length of the middle section as demonstrated in Fig. 5(a). Moreover, the group delay is proportional to the length at the center frequency of $1.467\omega_c$ (off-resonant condition), which means the group velocity is a constant ($\sim 5.29c$) for all of the selected lengths. The inset in Fig. 5(a) displays the transmission under the off-resonance condition; it is approximately constant (3.2%) and independent of the selected lengths, whereas the transmission in the photonic-band-gap tunneling experiment is inversely proportional to the length [17]. Consequently, these two aforementioned features — constant group velocity and constant transmission with respect to the propagation length suggest that the renowned generalized Hartman effect is valid only for extremely low boundary transmission, e.g., opaque-barrier boundaries, but not applicable to the high transmission case (boundary transmission rate ($T \sim 0.3$) under this study.

5. CONCLUSION

In summary, we have demonstrated a broadband superluminal effect with high transmission in a three-dimensional, non-periodic waveguide system. All of the observed phenomena can be explicitly explained using the given equation set. This study provides a preliminary control of the group delay, transmission and superluminal bandwidth by changing the geometry of the system. Manipulating the boundary reflection and phases, one can reproduce these interesting characteristics in other systems, such as micro-strip and strip-line, which reveal the great application potential in communication, such as the optical wave packet switching or network signal processing for high signal integrity. The proposed approach to control the effective group velocity of EM wave also help to deal with group delay dispersion (GDD) compensation problem by employing the multiple-reflection mechanism, which should benefit the ultra-short pulse generation. Moreover, this investigation clarifies the understanding of the generalized Hartman effect and indicates that the group velocity and the total transmission are independent of the selected lengths for high boundary transmission cases.

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