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HYBRID MODE CHARACTERISTICS IN MULTILAYERED FARADAY CHIROWAVEGUIDES

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1. Introduction

Anisotropic waveguides play an important role in many applications in the fields of microwave and optical wave, and numerous studies have been concentrated largely on the propagational feature of such complex guided wave structures. For example, multimnode propagating characteristics in anisotropic optical waveguides [1], bi-mode propagation in guided containing gyromagnetic medium [2], and the loss of anisotropic waveguides [3]. Recently, there has been much attention paid to the wave propagation in a new kind of waveguide structure, known as chirowaveguide [4–6]. In 1990, Engheta and Pelet first demonstrated the hybrid mode behavior in parallel-plate and circular metallic chirowaveguides [7,8]; Svedin also examined the mode characteristics in circular chirowaveguides using finite-element method [9]. In 1992, Mahmoud investigated the effect of mode bifurcation in circular chirowaveguides with metallic and constant impedance walls, respectively [10,11], Pelet and Engheta finished the modal analysis for rectangular chirowaveguide with the help of finite-difference technique

[12]. Lindell, Tretyakov and Oksanen examined the wave behaviour in biisotropic layered structure according to the vector transmission-line and circuit theory [13]. Koiviso, Tretyakov and Oksanen considered guided waves in a general class of waveguides filled with biisotropic media [14]. But, to the authors' best knowledge, most of their studies are limited to reciprocal or nonreciprocal isotropic chiral cases.

In the present study, we introduce a new kind of multilayered bianisotropic Faraday chirowaveguide based on the constitutive model suggested by Engheta, Jaggard and Kowarz, et al [15-18]. Then, the general dispersion equation for various cases is derived analytically, solved numerically. Some new phenomena are discovered and compared with the reciprocal chirowaveguides. Also, the lossy effect of material is considered carefully.

2. The Field Expression of the Problem

Figure 1 shows the geometry of the problem. In Fig. 1 (a), it is assumed that the waveguide wall is characterized by impedance Z_0 and admittance Y_0 defined at $\rho = R_N$, while Fig. 1(b) corresponds to the case of circular open chirowaveguide, and the permittivity and permeability in the region of $R > R_{N-1}$ are supposed to be ε_0 and μ_0.

(a) (b)

Figure 1. The cross section of the multilayered Faraday chirowaveguide.

In the frequency domain, the constitutive relations of Faraday chiral materials are described by the following equations for the time harmonic excitation $(e^{-i\omega t})$:

$$\overrightarrow{D}^{(j)} = [\varepsilon^{(j)}]\overrightarrow{E}^{(j)} + i\xi_c^{(j)}\overrightarrow{B}^{(j)} \tag{1}$$

$$\overrightarrow{H}^{(j)} = i\xi_c^{(j)}\overrightarrow{E}^{(j)} + [\mu^{(j)}]^{-1}\overrightarrow{B}^{(j)} \qquad j = 1, 2, \cdots, N-1 \tag{2}$$

where $[\varepsilon^{(j)}], [\mu^{(j)}]$ are the permittivity and permeability tensors represented by 3×3 matrices in any given coordinate system in which the field vectors are 3-element column vectors. Here we pay our attention to matrices of the form

$$[\varepsilon^{(j)}] = \begin{bmatrix} \varepsilon_1^{(j)} & -ig^{(j)} & 0 \\ ig^{(j)} & \varepsilon_1^{(j)} & 0 \\ 0 & 0 & \varepsilon_2^{(j)} \end{bmatrix}, [\mu^{(j)}] = \begin{bmatrix} \mu_1^{(j)} & -i\kappa^{(j)} & 0 \\ i\kappa^{(j)} & \mu_1^{(j)} & 0 \\ 0 & 0 & \mu_2^{(j)} \end{bmatrix} \tag{3}$$

for the chiroferrite media $(g^{(j)} = 0,\ \varepsilon_1^{(j)} = \varepsilon_2^{(j)})$, we have

$$\mu_1^{(j)} = \left[1 + \frac{\omega_0^{(j)}\omega_m^{(j)}}{[\omega_0^{(j)}]^2 - \omega^2}\right]\mu_0 \quad , \kappa^{(j)} = \frac{\mu_0\omega\omega_m^{(j)}}{\omega^2 - [\omega_0^{(j)}]^2}$$

and $\omega_0^{(j)} = |\gamma|H_0^{(j)}, \omega_m^{(j)} = |\gamma|M_s^{(j)}$, where $H_0^{(j)}$ is the magnitude of the internal dc bias field, and its direction is chosen to be along the positive Z-axis; $M_s^{(j)}$ is the saturation magnetization of the chiroferrite, and γ is the gyromagnetic ratio ($= -2.21 \times 10^5$ rad m/C). The lossless character of the Faraday chiral media is implied by the Hermitian nature of the tensors: $[C]^+ = [C^*]^T = [C](C : \varepsilon^{(j)}, \mu^{(j)})$ and $\xi_c^* = \xi_c$, where the embellishments $^{+*T}$ denote the Hermitian adjoint, the complex conjugate, and the transpose, respectively. However, for a lossy medium, $\varepsilon_1^{(j)}, \varepsilon_2^{(j)}, \mu_1^{(j)}$, and $\mu_2^{(j)}$ in (3) should be complex.

Since the gyrotropy and chirality are introduced simultaneously in above waveguide structures, $TE_{n,m}$ and $TM_{n,m}$ modes cannot be supported. The propagating modes along the positive Z-axis are always hybrid and they are commonly classified into $HE_{n,m}$ and $EH_{n,m}$

$(TE_{n,m} \rightarrow HE_{n,m}, TM_{n,m} \rightarrow EH_{n,m})$. Substituting eqs. (1) and (2) into Maxwell's crul equations and considering the z-dependence of all the field components to be of the form $e^{\gamma z}$, the transverse components of hybrid modes can be expressed in terms of the longitudinal field components $E_z^{(j)}$ and $H_z^{(j)}$ for any layer j after tedious mathematical manipulation, given by

$$\begin{bmatrix} E_\rho^{(j)} \\ E_\varphi^{(j)} \\ H_\rho^{(j)} \\ H_\varphi^{(j)} \end{bmatrix} = \begin{bmatrix} A_{22}^{(j)} & A_{21}^{(j)} & A_{42}^{(j)} & A_{41}^{(j)} \\ -A_{21}^{(j)} & A_{22}^{(j)} & -A_{41}^{(j)} & A_{42}^{(j)} \\ A_{24}^{(j)} & A_{23}^{(j)} & A_{44}^{(j)} & A_{43}^{(j)} \\ -A_{23}^{(j)} & A_{24}^{(j)} & -A_{43}^{(j)} & A_{44}^{(j)} \end{bmatrix} \begin{bmatrix} \frac{\partial E_z^{(j)}}{\partial \rho} \\ \frac{1}{\rho}\frac{\partial E_z^{(j)}}{\partial \varphi} \\ \frac{\partial H_z^{(j)}}{\partial \rho} \\ \frac{1}{\rho}\frac{\partial H_z^{(j)}}{\partial \varphi} \end{bmatrix} \tag{4}$$

with

$$A_{21}^{(j)} = [-a_2^{(j)} b_1^{(j)} + a_3^{(j)} b_2^{(j)} + a_4^{(j)} b_3^{(j)}]/D^{(j)}$$

$$A_{22}^{(j)} = [a_1^{(j)} b_1^{(j)} - a_3^{(j)} b_3^{(j)} + a_4^{(j)} b_2^{(j)}]/D^{(j)}$$

$$A_{23}^{(j)} = [-a_1^{(j)} b_2^{(j)} + a_2^{(j)} b_3^{(j)} - a_4^{(j)} b_5^{(j)}]/D^{(j)}$$

$$A_{24}^{(j)} = [-a_1^{(j)} b_3^{(j)} - a_2^{(j)} b_2^{(j)} + a_3^{(j)} b_5^{(j)}]/D^{(j)}$$

$$A_{41}^{(j)} = [a_2^{(j)} b_4^{(j)} + a_3^{(j)} b_7^{(j)} - a_4^{(j)} b_8^{(j)}]/D^{(j)}$$

$$A_{42}^{(j)} = [-a_1^{(j)} b_4^{(j)} + a_3^{(j)} b_9^{(j)} + a_4^{(j)} b_6^{(j)}]/D^{(j)}$$

$$A_{43}^{(j)} = A_{21}^{(j)}$$

$$A_{44}^{(j)} = [a_1^{(j)} b_8^{(j)} - a_2^{(j)} b_6^{(j)} - a_3^{(j)} b_3^{(j)}]/D^{(j)}$$

$$a_1^{(j)} = \gamma - i\omega\xi_c^{(j)}\kappa^{(j)}, a_2^{(j)} = -\omega\xi_c^{(j)}\mu_1^{(j)}, a_3^{(j)} = \omega\kappa^{(j)}, a_4^{(j)} = -i\omega\mu_1^{(j)},$$

$$a_5^{(j)} = -\omega[g^{(j)} + \xi_c^{(j)2}\kappa^{(j)}], a_6^{(j)} = i\omega[\varepsilon_1^{(j)} + \mu_1^{(j)}\xi_c^{(j)2}],$$

$$b_1^{(j)} = a_1^{(j)2} + a_2^{(j)2}, b_2^{(j)} = a_1^{(j)} a_6^{(j)} - a_2^{(j)2} a_5^{(j)}, b_3^{(j)} = a_2^{(j)} a_6^{(j)} + a_1^{(j)} a_5^{(j)},$$

$$b_4^{(j)} = a_2^{(j)} a_4^{(j)} + a_1^{(j)} a_3^{(j)}, b_5^{(j)} = a_5^{(j)2} + a_6^{(j)2}, b_6^{(j)} = a_4^{(j)} a_5^{(j)} - a_1^{(j)} a_2^{(j)},$$

$$b_7^{(j)} = a_1^{(j)} a_2^{(j)} - a_3^{(j)} a_6^{(j)}, b_8^{(j)} = a_1^{(j)2} + a_4^{(j)} a_6^{(j)}, b_9^{(j)} = a_2^{(j)2} + a_3^{(j)} a_5^{(j)}$$

$$\begin{aligned} D^{(j)} &= a_1^{(j)}[a_1^{(j)} b_1^{(j)} - a_3^{(j)} b_3^{(j)} + a_4^{(j)} b_2^{(j)}] \\ &+ a_2^{(j)}[a_2^{(j)} b_1^{(j)} - a_3^{(j)} b_2^{(j)} - a_4^{(j)} b_5^{(j)}] \\ &+ a_3^{(j)}[-a_2^{(j)} b_2^{(j)} - a_1^{(j)} b_3^{(j)} + a_3^{(j)} b_5^{(j)}] \\ &+ a_4^{(j)}[-a_2^{(j)} b_3^{(j)} + a_1^{(j)} b_2^{(j)} + a_4^{(j)} b_5^{(j)}] \end{aligned}$$

where γ is a complex quantity expressed as $\gamma = i\beta - \alpha$, with β and α being real quantities and representing the propagation constant and the attenuation rate of the mode, respectively. In Eq. (4), the transverse components $E_z^{(j)}$ and $H_z^{(j)}$ are coupled with each other, and with respect to the geometry of the problem, $E_z^{(j)}$ and $H_z^{(j)}$ are expressed as

$$E_z^{(j)} = S_+^{(j)} U_+^{(j)} + S_-^{(j)} U_-^{(j)} \tag{5}$$

$$H_z^{(j)} = q_+^{(j)} U_+^{(j)} + q_-^{(j)} U_-^{(j)} \tag{6}$$

with

$$\nabla_t^2 U_\pm^{(j)} + S_\pm^{(j)} U_\pm^{(j)} = 0 \tag{7}$$

$$S_\pm^{(j)} = \frac{-(C_1^{(j)} + C_4^{(j)}) \pm \sqrt{(C_1^{(j)} - C_4^{(j)})^2 - 4C_2^{(j)} C_3^{(j)}}}{2} \tag{8a}$$

$$q_\pm^{(j)} = i\frac{S_\pm^{(j)}(S_\pm^{(j)} + C_1^{(j)})}{C_2^{(j)}} \tag{8b}$$

and

$$\begin{aligned} C_1^{(j)} &= -[A_{21}^{(j)} a_8^{(j)} + iA_{41}^{(j)} a_9^{(j)}]/\Delta^{(j)}, \\ C_2^{(j)} &= -[A_{21}^{(j)} a_7^{(j)} + iA_{41}^{(j)} a_8^{(j)}]/\Delta^{(j)} \\ C_3^{(j)} &= [A_{21}^{(j)} a_9^{(j)} - iA_{23}^{(j)} a_8^{(j)}]/\Delta^{(j)}, C_4^{(j)} = -[A_{21}^{(j)} a_8^{(j)} - iA_{23}^{(j)} a_7^{(j)}]/\Delta^{(j)} \\ \Delta^{(j)} &= A_{21}^{(j)2} - A_{23}^{(j)} A_{41}^{(j)}, \\ a_7^{(j)} &= \omega\mu_2^{(j)}, a_8^{(j)} = \omega\xi_c^{(j)}\mu_2^{(j)}, a_9^{(j)} = \omega[\varepsilon_2^{(j)} + \mu_2^{(j)}\xi_c^{(j)2}] \end{aligned}$$

Thus, using Eqs. (4)–(8), the tangential field components for the waveguides (Fig. 1(a)(b)) are derived, and in layer 1

$$E_z^{(1)} = [D_1^{(1)} S_+^{(1)} J_n(\sqrt{S_+^{(1)}}\rho) + D_2^{(1)} S_-^{(1)} J_n(\sqrt{S_-^{(1)}}\rho)]e^{in\varphi} \tag{9a}$$

$$\begin{aligned} E_\varphi^{(1)} = \Big\{ & D_1^{(1)}[-M_+^{(1)} J_n'(\sqrt{S_+^{(1)}}\rho) + \frac{inN_+^{(1)}}{\rho} J_n(\sqrt{S_+^{(1)}}\rho)] \\ & + D_2^{(1)}[-M_-^{(1)} J_n'(\sqrt{S_-^{(1)}}\rho) + \frac{inN_-^{(1)}}{\rho} J_n(\sqrt{S_-^{(1)}}\rho)]\Big\} e^{in\varphi} \end{aligned} \tag{9b}$$

$$H_z^{(1)} = [D_1^{(1)} q_+^{(1)} J_n(\sqrt{S_+^{(1)}}\rho) + D_2^{(1)} q_-^{(1)} J_n(\sqrt{S_-^{(1)}}\rho)]e^{in\varphi} \tag{9c}$$

$$\begin{aligned} H_\varphi^{(1)} = \Big\{ & D_1^{(1)}[-X_+^{(1)} J_n'(\sqrt{S_+^{(1)}}\rho) + \frac{inY_+^{(1)}}{\rho} J_n(\sqrt{S_+^{(1)}}\rho)] \\ & + D_2^{(1)}[-X_-^{(1)} J_n'(\sqrt{S_-^{(1)}}\rho) + \frac{inY_-^{(1)}}{\rho} J_n(\sqrt{S_-^{(1)}}\rho)]\Big\} e^{in\varphi} \end{aligned} \tag{9d}$$

In layer j $(j = 2, \cdots, N-1)$,

$$\begin{aligned} E_z^{(j)} = & [D_1^{(j)} S_+^{(j)} J_n(\sqrt{S_+^{(j)}}\rho) + D_2^{(j)} S_+^{(j)} N_n(\sqrt{S_+^{(j)}}\rho) \\ & + D_3^{(j)} S_-^{(j)} J_n(\sqrt{S_-^{(j)}}\rho) + D_4^{(j)} S_-^{(j)} N_n(\sqrt{S_-^{(j)}}\rho)]e^{in\varphi} \end{aligned} \tag{10a}$$

$$\begin{aligned} E_\varphi^{(j)} = \Big\{ & D_1^{(j)}[-M_+^{(j)} J_n'(\sqrt{S_+^{(j)}}\rho) + \frac{inN_+^{(j)}}{\rho} J_n(\sqrt{S_+^{(j)}}\rho)] \\ & + D_2^{(j)}[-M_+^{(j)} N_n'(\sqrt{S_+^{(j)}}\rho) + \frac{inN_+^{(j)}}{\rho} N_n(\sqrt{S_+^{(j)}}\rho)] \\ & + D_3^{(j)}[-M_-^{(j)} J_n'(\sqrt{S_-^{(j)}}\rho) + \frac{inN_-^{(j)}}{\rho} J_n(\sqrt{S_-^{(j)}}\rho)] \\ & + D_4^{(j)}[-M_-^{(j)} N_n'(\sqrt{S_-^{(j)}}\rho) + \frac{inN_-^{(j)}}{\rho} N_n(\sqrt{S_-^{(j)}}\rho)]\Big\} e^{in\varphi} \end{aligned} \tag{10b}$$

$$\begin{aligned} H_z^{(j)} = & [D_1^{(j)} q_+^{(j)} J_n(\sqrt{S_+^{(j)}}\rho) + D_2^{(j)} q_+^{(j)} N_n(\sqrt{S_+^{(j)}}\rho) \\ & + D_3^{(j)} q_-^{(j)} J_n(\sqrt{S_-^{(j)}}\rho) + D_4^{(j)} q_-^{(j)} N_n(\sqrt{S_-^{(j)}}\rho)]e^{in\varphi} \end{aligned} \tag{10c}$$

$$H_\varphi^{(j)} = \Big\{ D_1^{(j)} [-X_+^{(j)} J_n'(\sqrt{S_+^{(j)}}\rho) + \frac{inY_+^{(j)}}{\rho} J_n(\sqrt{S_+^{(j)}}\rho)]$$
$$+ D_2^{(j)} [-X_+^{(j)} N_n'(\sqrt{S_+^{(j)}}\rho) + \frac{inY_+^{(j)}}{\rho} N_n(\sqrt{S_+^{(j)}}\rho)]$$
$$+ D_3^{(j)} [-X_-^{(j)} J_n'(\sqrt{S_-^{(j)}}\rho) + \frac{inY_-^{(j)}}{\rho} J_n(\sqrt{S_-^{(j)}}\rho)]$$
$$+ D_4^{(j)} [-X_-^{(j)} N_n'(\sqrt{S_-^{(j)}}\rho) + \frac{inY_-^{(j)}}{\rho} N_n(\sqrt{S_-^{(j)}}\rho)] \Big\} e^{in\varphi} \quad (10d)$$

in which

$$M_\pm^{(j)} = \sqrt{S_\pm^{(j)}}(A_{21}^{(j)} S_\pm^{(j)} + A_{41}^{(j)} q_\pm^{(j)}), \quad N_\pm^{(j)} = A_{22}^{(j)} S_\pm^{(j)} + A_{42}^{(j)} q_\pm^{(j)}$$
$$X_\pm^{(j)} = \sqrt{S_\pm^{(j)}}(A_{23}^{(j)} S_\pm^{(j)} + A_{43}^{(j)} q_\pm^{(j)}), \quad Y_\pm^{(j)} = A_{24}^{(j)} S_\pm^{(j)} + A_{44}^{(j)} q_\pm^{(j)}$$

In Fig. 1(a), the tangential field components in the region of $R_{N-1} \leq \rho \leq R_N$ are

$$E_z^{(N)} = [D_1^{(N)} J_n(\gamma_0 \rho) + D_2^{(N)} N_n(\gamma_0 \rho)] e^{in\varphi} \quad (11a)$$
$$H_z^{(N)} = [D_3^{(N)} J_n(\gamma_0 \rho) + D_4^{(N)} N_n(\gamma_0 \rho)] e^{in\varphi} \quad (11b)$$

$$E_\varphi^{(N)} = [D_1^{(N)} \frac{i\gamma n}{\gamma_0^2 \rho} J_n(\gamma_0 \rho) + D_2^{(N)} \frac{i\gamma n}{\gamma_0^2 \rho} N_n(\gamma_0 \rho)$$
$$- D_3^{(N)} \frac{i\omega\mu_0}{\gamma_0} J_n'(\gamma_0 \rho) - D_4^{(N)} \frac{i\omega\mu_0}{\gamma_0} N_n'(\gamma_0 \rho)] e^{in\varphi} \quad (11c)$$
$$H_\varphi^{(N)} = [D_1^{(N)} \frac{i\omega\epsilon_0}{\gamma_0} J_n'(\gamma_0 \rho) + D_2^{(N)} \frac{i\omega\epsilon_0}{\gamma_0} N_n'(\gamma_0 \rho)$$
$$+ D_3^{(N)} \frac{i\gamma n}{\gamma_0^2 \rho} J_n(\gamma_0 \rho) + D_4^{(N)} \frac{i\gamma n}{\gamma_0^2 \rho} N_n(\gamma_0 \rho)] e^{in\varphi} \quad (11d)$$

and in Fig. 1(b) ($\rho \geq R_{N-1}$), we have

$$E_z^{(N)} = D_1^{(N)} H_n^{(1)}(\gamma_0 \rho) e^{in\varphi}, \ H_z^{(N)} = D_2^{(N)} H_n^{(1)}(\gamma_0 \rho) e^{in\varphi} \quad (12a, b)$$

$$E_\varphi^{(N)} = [D_1^{(N)} \frac{i\gamma n}{\gamma_0^2 \rho} H_n^{(1)}(\gamma_0 \rho) - D_2^{(N)} \frac{i\omega\mu_0}{\gamma_0} H_n^{(1)\prime}(\gamma_0 \rho)] e^{in\varphi} \quad (12c)$$

$$H_\varphi^{(N)} = [D_1^{(N)} \frac{i\omega\varepsilon_0}{\gamma_0} H_n^{(1)\prime}(\gamma_0 \rho) + D_2^{(N)} \frac{i\gamma n}{\gamma_0^2 \rho} H_n^{(1)}(\gamma_0 \rho)] e^{in\varphi} \quad (12d)$$

where $\gamma_0^2 = k_0^2 + \gamma^2, k_0^2 = \omega^2\mu_0\varepsilon_0$. The propagation factor $e^{(\gamma z - i\omega t)}$ is understood and suppressed in (9)-(12). $J_n(\cdot)$ is the Bessel function, $N_n(\cdot)$ is the Neumann function, and $H_n^{(1)}(\cdot)$ is the Hankel function of the first kind and order n, n may be either positive or negative integer. $D_1^{(1)} \sim D_2^{(1)}$, $D_1^{(j)} \sim D_4^{(j)}$, and $D_1^{(N)} \sim D_4^{(N)}$ are the constants which are determined by the boundary conditions at $\rho = R_1, \cdots, R_N$.

3. Dispersion Equation of the Hybrid Modes

For $N-1$ layers of loading in Fig. 1(a), the dispersion equation of modes $HE_{n,m}$ and $EH_{n,m}$ is given by $2(2N+1) \times 2(2N+1)$ matrix equation. In numerically solving the equation, the computer time required become cumbersome as the number of layers increases. An efficient method for formulating the dispersion equation for an arbitrary number of layers can be utilized using the transmission-matrix technique[19–21], in which the boundary matching equations at each medium interface are written as 4×4 matrix equations. At $\rho = R_1$

$$\begin{bmatrix} M_{11}^{(1)} & M_{12}^{(1)} & 0 & 0 \\ M_{21}^{(1)} & M_{22}^{(1)} & 0 & 0 \\ M_{31}^{(1)} & M_{32}^{(1)} & 0 & 0 \\ M_{41}^{(1)} & M_{42}^{(1)} & 0 & 0 \end{bmatrix} \begin{bmatrix} D_1^{(1)} \\ D_2^{(1)} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} N_{11}^{(2)} & N_{12}^{(2)} & N_{13}^{(2)} & N_{14}^{(2)} \\ N_{21}^{(2)} & N_{22}^{(2)} & N_{23}^{(2)} & N_{24}^{(2)} \\ N_{31}^{(2)} & N_{32}^{(2)} & N_{33}^{(2)} & N_{34}^{(2)} \\ N_{41}^{(2)} & N_{42}^{(2)} & N_{43}^{(2)} & N_{44}^{(2)} \end{bmatrix} \begin{bmatrix} D_1^{(2)} \\ D_2^{(2)} \\ D_3^{(2)} \\ D_4^{(2)} \end{bmatrix} \quad (13)$$

At $\rho = R_j \quad (2 \le j \le N-2)$

$$\begin{bmatrix} M_{11}^{(j)} & M_{12}^{(j)} & M_{13}^{(j)} & M_{14}^{(j)} \\ M_{21}^{(j)} & M_{22}^{(j)} & M_{23}^{(j)} & M_{24}^{(j)} \\ M_{31}^{(j)} & M_{32}^{(j)} & M_{33}^{(j)} & M_{34}^{(j)} \\ M_{41}^{(j)} & M_{42}^{(j)} & M_{43}^{(j)} & M_{44}^{(j)} \end{bmatrix} \begin{bmatrix} D_1^{(j)} \\ D_2^{(j)} \\ D_3^{(j)} \\ D_4^{(j)} \end{bmatrix} = \begin{bmatrix} N_{11}^{(j+1)} & N_{12}^{(j+1)} & N_{13}^{(j+1)} & N_{14}^{(j+1)} \\ N_{21}^{(j+1)} & N_{22}^{(j+1)} & N_{23}^{(j+1)} & N_{24}^{(j+1)} \\ N_{31}^{(j+1)} & N_{32}^{(j+1)} & N_{33}^{(j+1)} & N_{34}^{(j+1)} \\ N_{41}^{(j+1)} & N_{42}^{(j+1)} & N_{43}^{(j+1)} & N_{44}^{(j+1)} \end{bmatrix} \begin{bmatrix} D_1^{(j+1)} \\ D_2^{(j+1)} \\ D_3^{(j+1)} \\ D_4^{(j+1)} \end{bmatrix} \tag{14}$$

At $\rho = R_{N-1}$

$$\begin{bmatrix} M_{11}^{(N-1)} & M_{12}^{(N-1)} & M_{13}^{(N-1)} & M_{14}^{(N-1)} \\ M_{21}^{(N-1)} & M_{22}^{(N-1)} & M_{23}^{(N-1)} & M_{24}^{(N-1)} \\ M_{31}^{(N-1)} & M_{32}^{(N-1)} & M_{33}^{(N-1)} & M_{34}^{(N-1)} \\ M_{41}^{(N-1)} & M_{42}^{(N-1)} & M_{43}^{(N-1)} & M_{44}^{(N-1)} \end{bmatrix} \begin{bmatrix} D_1^{(N-1)} \\ D_2^{(N-1)} \\ D_3^{(N-1)} \\ D_4^{(N-1)} \end{bmatrix} = \begin{bmatrix} M_{11}^{(N)} & M_{12}^{(N)} & 0 & 0 \\ M_{21}^{(N)} & M_{22}^{(N)} & 0 & 0 \\ M_{31}^{(N)} & M_{32}^{(N)} & 0 & 0 \\ M_{41}^{(N)} & M_{42}^{(N)} & 0 & 0 \end{bmatrix} \begin{bmatrix} D_1^{(N)} \\ D_2^{(N)} \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

where the boundary equations

$$E_\varphi^{(N)}/H_z^{(N)} = Z_0, \quad H_\varphi^{(N)}/E_z^{(N)} = -Y_0 \tag{16}$$

have been taken into account, and all the matric elements in (13)–(15) are presented in Appendix 1.

Thus, the above boundary equations (13)-(15) may be simplified as

$$\begin{aligned} \text{at} \quad \rho = R_1 : &\qquad M_{11}D^{(1)} = N_{21}D^{(2)} &\quad (17a)\\ \rho = R_2 : &\qquad M_{22}D^{(2)} = N_{32}D^{(3)} &\quad (17b)\\ &\qquad \vdots \\ \rho = R_{N-1} : &\qquad M_{(N-1)(N-1)}D^{(N-1)} = N_{N(N-1)}D^{(N)} &\quad (17c) \end{aligned}$$

Furthermore, Eq. (17) can be expressed as

$$M_{11}D^{(1)} = MD^{(N)} \tag{18}$$

where $M = N_{21}M_{22}^{-1}N_{32}\cdots M_{(N-1)(N-1)}^{-1}N_{N(N-1)}$, and it must have the following form

$$M = \begin{bmatrix} m_{11} & m_{12} & 0 & 0 \\ m_{21} & m_{22} & 0 & 0 \\ m_{31} & m_{32} & 0 & 0 \\ m_{41} & m_{42} & 0 & 0 \end{bmatrix} \tag{19}$$

Combining (13)–(15) with (17)–(19), we find

$$\begin{bmatrix} M_{11}^{(1)} & M_{12}^{(1)} & -m_{11} & -m_{12} \\ M_{21}^{(1)} & M_{22}^{(1)} & -m_{21} & -m_{22} \\ M_{31}^{(1)} & M_{32}^{(1)} & -m_{31} & -m_{32} \\ M_{41}^{(1)} & M_{42}^{(1)} & -m_{41} & -m_{42} \end{bmatrix} \begin{bmatrix} D_1^{(1)} \\ D_2^{(1)} \\ D_3^{(N)} \\ D_4^{(N)} \end{bmatrix} = 0 \tag{20}$$

For a nontrivial solution of (20), the determinant of the 4×4 matrix must be zero. This leads to the following equation for γ of propagating modes:

$$det \begin{bmatrix} M_{11}^{(1)} & M_{12}^{(1)} & -m_{11} & -m_{12} \\ M_{21}^{(1)} & M_{22}^{(1)} & -m_{21} & -m_{22} \\ M_{31}^{(1)} & M_{32}^{(1)} & -m_{31} & -m_{32} \\ M_{41}^{(1)} & M_{42}^{(1)} & -m_{41} & -m_{42} \end{bmatrix} = 0 \tag{21}$$

This is the dispersion equation for hybrid modes in a circular multilayered loaded Faraday chirowaveguide. Following the similar procedure used above, we can also obtain the generalized dispersion equation of the 4×4 matrix for circular multilayered open case. Equation (21) may be solved numerically by using Mueller's method of calculating roots, and naturally, it includes the special case of perfectly conducting wall ($Z_0 = 0$, $Y_0 \to \infty$).

4. The Effects of Constitutive Parameters

Based on the results of the previous section, we have developed a computer code to examine the dispersion characteristics for hybrid modes propagating in various Faraday chirowaveguides, and different phenomena are demonstrated as the constitutive parameters of Faraday chiral media are changed. Since the properties of a Faraday chiral medium have never been completely characterized either at microwave or millimeter wave frequencies, we had to assume the material properties used in the calculations. The values used for the constitutive parameters are related to what are reported in the literature [7,9,11,12,22].

The normalized modal phase constant β/k_0 and attenuation rate α/k_0 for modes $EH_{0,1}$ and $HE_{\pm1,1}$ in a totally filled circular Faraday chirowaveguide with perfectly conducting wall are plotted versus k_0R_1 in Fig. 2.

In Fig. 2, some notable effects can be easily found. At first, both β/k_0 and α/k_0 are enhanced due to the loss of material is increased; for different order of hybrid mode, the attenuation degree is different. Secondly, with the increasing of frequency, the propagation constant β/k_0 increases but normalized attenuation rate α/k_0 decreases. Obviously, for the special lossless case, the value of α is equal to zero. As in a reciprocal biisotropic chirowaveguide, the hybrid modes $HE_{\pm n,1}(n \neq 0)$ propagating in Faraday chirowaveguides are bifurcated. In addition, when the anisotropy of permittivity tensor is introduced in a Faraday chirowaveguide, the cut-off wavelengths of modes $HE_{+n,1}$ and $HE_{-n,1}$ are not equal to each other, although the numerical results have not been presented here.

(a) $\xi_c^{(1)} = 10^{-3}$ **mho** **(b)** $\xi_c^{(1)} = 5 \times 10^{-3}$ **mho**

Figure 2. Dispersion diagrams for modes in a totally filled circular Faraday chirowaveguide with perfectly conducting wall. $g^{(1)} = 0$, $Z_0 = 0$, $Y_0 \to \infty$, $M_s^{(1)}\mu_0 = 0.275T$, $\omega_0^{(1)}/\omega_m^{(1)} = 0.3$, $\varepsilon_1^{(1)} = \varepsilon_2^{(1)} = (12.6 + 0.1i)\varepsilon_0$(- - -); $\varepsilon_1^{(1)} = \varepsilon_2^{(1)} = (12.6 + 0.3i)\varepsilon_0$(——).

(a)

(b)

(c)

Figure 3. Dispersion diagrams for modes in a one-layer central loaded Faraday chirowaveguide, $R_1 = 0.8R_2$. **(a)** $\varepsilon_1^{(1)} = \varepsilon_2^{(1)} = 12.6\varepsilon_0$, $g^{(1)} = 0, M_s^{(1)}\mu_0 = 0.275T$, $\omega_0^{(1)}/\omega_m^{(1)} = 0.3$, $\xi_c^{(1)} = 10^{-3}$mho, $Z_0 = 0$, $Y_0 \to \infty$. **(b) The parameters are the same as (a), and** $\xi_c^{(1)} = 5 \times 10^{-3}$mho. **(c)** $g^{(1)} = 0$, $M_s^{(1)}\mu_0 = 0.275\mathbf{T}$, $\omega_0^{(1)}/\omega_m^{(1)} = 0.3$, $\xi^{(1)} = 5\times10^{-3}$mho, $Z_0 = 0, Y_0 = -1.25\times10^{-3}\mathbf{i}$, $\varepsilon_1^{(1)} = \varepsilon_2^{(1)} = (12.6+0.1i)\varepsilon_0$ **(- - -);** $\varepsilon_1^{(1)} = \varepsilon_2^{(1)} = (12.6+0.3i)\varepsilon_0$ **(—).**

Figure 3 shows the dispersion behaviour of hybrid modes in a one-layer central circular Faraday chirowaveguide, and the lossless case of loaded materials is also given (Fig. 3 (a) (b)).

In Figs. 3 (a), (b), the modes with $n = \pm 1$ are bifurcated into two branches with different β values depending on the sign of n. The bifurcated degree $\Delta\beta_{\pm 1}/k_0 = |\beta_{+1} - \beta_{-1}|/k_0$ is mainly determined by the magnitude of chirality, the thickness of air gap and the intensity of internal bias field, which has no relation to the sign of ξ_c. Compared Fig. 3(b) with 3(a), it is easily found that, the cut-off frequencies of $EH_{0,1}$ and $HE_{\pm 1,1}$ tend to decrease with chirality admittance increasing.

Figure 4 is the dispersion curves of one-layer coated circular Faraday chirowaveguides with perfectly conducting walls.

In Figs. 4 (a), (b), there exist backward waves $(d\omega/d\beta < 0)$ as in general or periodic waveguide structures. The hybrid modes $HE_{\pm 1,1}$ are bifurcated and originate from same cut-off frequency, $\Delta\beta_{\pm 1}/k_0$ depends strongly on the thickness of coated layer, chirality as well as the internal bias field intensity. Fig. 4(c) shows that, the cut-off frequency tends to increase with τ decreasing.

Figure 5 shows the dispersion behaviour of a lossless open circular Faraday chirowaveguide. Here, only the mode $HE_{-1,1}$ is demonstrated, and corresponding to different internal bias field intensity.

The primary change cased by varying the bias field intensity in Fig. 5 is the shift of dispersion curve of mode $HE_{-1,1}$, and the cut-off frequency is proportional to the bias field intensity approximately.

Finally, we depict each field component for the case of Fig. 3(c), and the normalized propagation constant and attenuation rate are chosen to be $\beta/k_0 = 0.7287$, and $\alpha/k_0 = 0.3334$.

Figure 6 shows that, at the boundary $\rho = R_2$, the tangential electric field components E_z and E_φ are indeed equal to zero. At the boundary $\rho = R_1$, the tangential field components E_z, E_φ and H_z, H_φ are continuous, while E_ρ and H_ρ are not continuous. The relative level of $|E|$ and $|H|$ are all the function of magnitude of chirality, the bias field intensity, et al.

(a) $\xi_c^{(2)} = 3 \times 10^{-3}$ **mho**

(b) $\xi_c^{(2)} = 5 \times 10^{-3}$ **mho**

(c) $\xi_c^{(2)} = 5 \times 10^{-3}$ **mho**

Figure 4. Dispersion diagrams for modes $HE_{\pm 1,1}$ in a one-layer coated circular Faraday chirowaveguide. $\varepsilon_1^{(2)} = \varepsilon_2^{(2)} = 12.6\varepsilon_0$, $g^{(2)} = 0$, $M_s^{(2)}\mu_0 =$ 0.275**T**, $\omega_0^{(2)}/\omega_m^{(2)} = 0.3$, $Z_0 = 0$, $Y_0 \to \infty$, **(a) (b)** $\tau = R_2 - R_1 = 0.2R_2$; **(c)** $\tau = 0.3R_2$(——), $0.1R_2$ $(\cdots\cdots)$.

Figure 5. Dispersion diagrams for mode $HE_{-1,1}$ in a open circular Faraday chirowaveguide. $\varepsilon_1^{(1)} = \varepsilon_2^{(1)} = 12.6\varepsilon_0$, $M_s^{(1)}\mu_0 = 0.275\text{T}$, $g^{(1)} = 0$, $\omega_0^{(1)}/\omega_m^{(1)} = 0.3$ (1), 0.7 (2), 1.0 (3), 1.2 (4).

Figure 6. The field distribution of mode $EH_{0,1}$ in a one-layer central loaded Faraday chirowaveguide with perfectly conducting wall. $\varepsilon_1^{(1)} = \varepsilon_2^{(1)} = (12.6 + 0.1i)\varepsilon_0$, $g^{(1)} = 0$, $R_1 = 0.8R_2$, $k_0R_2 = 1.4005$, $M_s^{(1)}\mu_0 = 0.275\text{T}$, $\omega_0^{(1)}/\omega_m^{(1)} = 0.3$, $\xi_c^{(1)} = 10^{-3}\text{mho}$, $Z_0 = 0$, $Y_0 \to \infty$.

5. Conclusion

In this paper we have studied the hybrid mode characteristics in bianisotropic Faraday chirowaveguides theoretically. The general field equations and dispersion equation of the guided waves have been derived in the explicit forms which are also suitable for the circular cylindrical magnetosplasma chirowaveguides. The appearance of the different effects discussed above depends on the combination of many factors: geometrical sizes of the waveguide, constitutive parameters of the Faraday chiral media, et al. Our results provide much insight into the physical properties of the Faraday chirowaveguides. Furthermore, the hybrid mode characteristics in much more complex Faraday chirwavegudies are being investigated and work in this area is in progress.

Appendix

In (13)–(15), the matrix elements are

$$
\begin{aligned}
M^{(1)}_{1^1_2} &= S^{(1)}_{\pm} J_n(\sqrt{S^{(1)}_{\pm}} R_1), M^{(1)}_{2^1_2} = q^{(1)}_{\pm} J_n(\sqrt{S^{(1)}_{\pm}} R_1),\\
M^{(1)}_{3^1_2} &= -M^{(1)}_{\pm} J'_n(\sqrt{S^{(1)}_{\pm}} R_1) + \frac{inN^{(1)}_{\pm}}{R_1} J_n(\sqrt{S^{(1)}_{\pm}} R_1),\\
M^{(1)}_{4^1_2} &= -X^{(1)}_{\pm} J'_n(\sqrt{S^{(1)}_{\pm}} R_1) + \frac{inY^{(1)}_{\pm}}{R_1} J_n(\sqrt{S^{(1)}_{\pm}} R_1),
\end{aligned}
\tag{A1}
$$

$$
\begin{aligned}
M^{(j)}_{1^1_3} &= S^{(j)}_{\pm} J_n(\sqrt{S^{(j)}_{\pm}} R_j), M^{(j)}_{1^2_4} = S^{(j)}_{\pm} N_n(\sqrt{S^{(j)}_{\pm}} R_j),\\
M^{(j)}_{2^1_3} &= q^{(j)}_{\pm} J_n(\sqrt{S^{(j)}_{\pm}} R_j), M^{(j)}_{2^2_4} = q^{(j)}_{\pm} N_n(\sqrt{S^{(j)}_{\pm}} R_j),\\
M^{(j)}_{3^1_3} &= -M^{(j)}_{\pm} J'_n(\sqrt{S^{(j)}_{\pm}} R_j) + \frac{inN^{(j)}_{\pm}}{R_j} J_n(\sqrt{S^{(j)}_{\pm}} R_j),\\
M^{(j)}_{3^2_4} &= -M^{(j)}_{\pm} N'_n(\sqrt{S^{(j)}_{\pm}} R_j) + \frac{inN^{(j)}_{\pm}}{R_j} N_n(\sqrt{S^{(j)}_{\pm}} R_j),\\
M^{(j)}_{4^1_3} &= -X^{(j)}_{\pm} J'_n(\sqrt{S^{(j)}_{\pm}} R_j) + \frac{inY^{(j)}_{\pm}}{R_j} J_n(\sqrt{S^{(j)}_{\pm}} R_j),\\
M^{(j)}_{4^2_4} &= -X^{(j)}_{\pm} N'_n(\sqrt{S^{(j)}_{\pm}} R_j) + \frac{inY^{(j)}_{\pm}}{R_j} N_n(\sqrt{S^{(j)}_{\pm}} R_j)
\end{aligned}
\tag{A2}
$$

$$
\begin{aligned}
N^{(j+1)}_{1^1_3} &= S^{(j+1)}_{\pm} J_n(\sqrt{S^{(j+1)}_{\pm}} R_j), N^{(j)}_{1^2_4} = S^{(j+1)}_{\pm} N_n(\sqrt{S^{(j+1)}_{\pm}} R_j),\\
N^{(j+1)}_{2^1_3} &= q^{(j+1)}_{\pm} J_n(\sqrt{S^{(j+1)}_{\pm}} R_j), N^{(j)}_{2^2_4} = q^{(j+1)}_{\pm} N_n(\sqrt{S^{(j+1)}_{\pm}} R_j),
\end{aligned}
$$

$$N_{3_{3}^{1}}^{(j+1)} = -M_{\pm}^{(j+1)} J_n'(\sqrt{S_{\pm}^{(j+1)}} R_j) + \frac{inN_{\pm}^{(j+1)}}{R_j} J_n(\sqrt{S_{\pm}^{(j+1)}} R_j),$$

$$N_{3_{4}^{2}}^{(j+1)} = -M_{\pm}^{(j+1)} N_n'(\sqrt{S_{\pm}^{(j+1)}} R_j) + \frac{inN_{\pm}^{(j+1)}}{R_j} N_n(\sqrt{S_{\pm}^{(j+1)}} R_j),$$

$$N_{4_{3}^{1}}^{(j+1)} = -X_{\pm}^{(j+1)} J_n'(\sqrt{S_{\pm}^{(j+1)}} R_j) + \frac{inY_{\pm}^{(j+1)}}{R_j} J_n(\sqrt{S_{\pm}^{(j+1)}} R_j),$$

$$N_{4_{4}^{2}}^{(j+1)} = -X_{\pm}^{(j+1)} N_n'(\sqrt{S_{\pm}^{(j+1)}} R_j) + \frac{inY_{\pm}^{(j+1)}}{R_j} N_n(\sqrt{S_{\pm}^{(j+1)}} R_j), \quad \text{(A3)}$$

$$\begin{aligned}
M_{11}^{(N)} &= J_n(\gamma_0 R_{N-1}), \qquad M_{12}^{(N)} = N_n(\gamma_0 R_{N-1}), \\
M_{21}^{(N)} &= \frac{D_1}{D} J_n(\gamma_0 R_{N-1}) + \frac{D_3}{D} N_n(\gamma_0 R_{N-1}), \\
M_{22}^{(N)} &= \frac{D_2}{D} J_n(\gamma_0 R_{N-1}) + \frac{D_4}{D} N_n(\gamma_0 R_{N-1}), \\
M_{31}^{(N)} &= \frac{i\gamma n}{\gamma_0^2 R_{N-1}} J_n(\gamma_0 R_{N-1}) - \frac{i\omega\mu_0}{\gamma_0} J_n'(\gamma_0 R_{N-1}) \frac{D_1}{D} \\
&\quad - \frac{i\omega\mu_0}{\gamma_0} N_n'(\gamma_0 R_{N-1}) \frac{D_3}{D}, \\
M_{32}^{(N)} &= \frac{i\gamma n}{\gamma_0^2 R_{N-1}} N_n(\gamma_0 R_{N-1}) - \frac{i\omega\mu_0}{\gamma_0} N_n'(\gamma_0 R_{N-1}) \frac{D_2}{D} \\
&\quad - \frac{i\omega\mu_0}{\gamma_0} N_n'(\gamma_0 R_{N-1}) \frac{D_4}{D}, \\
M_{41}^{(N)} &= \frac{i\omega\varepsilon_0 J_n'(\gamma_0 R_{N-1})}{\gamma_0} + \frac{i\gamma n}{\gamma_0^2 R_{N-1}} J_n(\gamma_0 R_{N-1}) \frac{D_1}{D} \\
&\quad + \frac{i\gamma n}{\gamma_0^2 R_{N-1}} N_n(\gamma_0 R_{N-1}) \frac{D_3}{D}, \\
M_{42}^{(N)} &= \frac{i\omega\varepsilon_0}{\gamma_0} N_n'(\gamma_0 R_{N-1}) + \frac{i\gamma n}{\gamma_0^2 R_{N-1}} J_n(\gamma_0 R_{N-1}) \frac{D_2}{D} \\
&\quad + \frac{i\gamma n}{\gamma_0^2 R_{N-1}} N_n(\gamma_0 R_{N-1}) \frac{D_4}{D}, \qquad \text{(A4)} \\
D &= G_{13}^N G_{24}^N - G_{14}^N G_{23}^N, \\
D_1 &= G_{14}^N G_{21}^N - G_{11}^N G_{24}^N, D_2 = G_{14}^N G_{22}^N - G_{24}^N G_{12}^N, \\
D_3 &= G_{23}^N G_{11}^N - G_{13}^N G_{21}^N, D_4 = G_{23}^N G_{12}^N - G_{13}^N G_{22}^N, \\
G_{11}^{(N)} &= -\frac{i\gamma n}{{\gamma_0}^2 R_N} J_n(\gamma_0 R_N), G_{12}^{(N)} = -\frac{i\gamma n}{{\gamma_0}^2 R_N} N_n(\gamma_0 R_N),
\end{aligned}$$

$$G_{13}^{(N)} = \frac{i\omega\mu_0}{\gamma_0} J_n'(\gamma_0 R_N) + Z_0 J_n(\gamma_0 R_N),$$
$$G_{14}^{(N)} = \frac{i\omega\mu_0}{\gamma_0} N_n'(\gamma_0 R_N) + Z_0 N_n(\gamma_0 R_N),$$
$$G_{21}^{(N)} = \frac{i\omega\varepsilon_0}{\gamma_0} J_n(\gamma_0 R_N) + Y_0 J_n(\gamma_0 R_N),$$
$$G_{22}^{(N)} = \frac{i\omega\varepsilon_0}{\gamma_0} N_n'(\gamma_0 R_N) + Y_0 N_n(\gamma_0 R_N), G_{23}^{(N)} = -\frac{i\gamma n}{\gamma_0^2 R_N} J_n(\gamma_0 R_N),$$
$$G_{24}^{(N)} = \frac{i\gamma n}{\gamma_0^2 R_N} N_n(\gamma_0 R_N) \ .$$

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