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THE ANALYTIC EXTRACTION OF THE COMPLEX-VALUED COUPLING MATRIX AND ITS APPLICATION IN THE MICROWAVE FILTER MODELING

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Abstract—The idea behind the coupling matrix identification is to find the coupling matrix corresponding to the measured or designed scattering characteristics of the microwave filter. The typical attitude towards coupling matrix parameter extraction is to use some optimization methods to minimize the appropriate cost function. In this paper, we concentrate on the analytic solutions — how they may be found and their application in further optimization processes. In general case, the suggested method generates complex-valued coupling matrix. For a special case of the filter without cross-couplings we give fast and simple recursive method of finding such complex-valued coupling matrix. The method is based on Laplace's formula for expanding the determinant. The complex-valued coupling matrix is used as a good starting point for the optimization methods to find the regular coupling matrix. The examples are presented showing that the optimization arrives to global minimum starting from real parts of complex-valued entries considerably more often than when the starting point is selected randomly.

1. INTRODUCTION

The coupling matrix model of microwave filters introduced by Atia and Williams [1, 2] was widely applied (for latest references and ideas see e.g., [3–5]) — but mainly to the microwave filter synthesis (see [6] for an overview). The methodologies for designing microwave filters are known for a long time but a lot of researchers are still introducing new solutions (cf. [7–14]). In this paper, we are focusing on the related problem of coupling coupling matrix extraction. Assuming

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$(N+2) \times (N+2)$ coupling matrix model (see especially [6] and [15] for an overview) the problem may be described as: given the reflection characteristics S_{11} find the matrix M that satisfies:

$$S_{11}(\lambda) = 1 + 2j[\lambda I_N - J + M]^{-1}_{1,1} \tag{1}$$

where J has all entries equal to 0 except $J_{1,1} = J_{N+2,N+2} = j$, while I_N is a diagonal matrix with main diagonal given by $(0, 1, 1, \ldots, 1, 0)$. There are at least two important applications of the coupling matrix model, where the problem described above appears. The first one is the filter synthesis: given the designed reflection characteristics we search for the matrix M that satisfies the Equation (1). In this case, we have the $S_{11}(\lambda)$ given directly as the rational function having the desired properties. When we find the coupling matrix corresponding to the reflection characteristics, we can translate it into the physical properties of the device that we are planning to develop. Usually the time is not critical in the designing process, so we can use different optimization techniques, test different solutions and if we have many solutions we can pick the best one. The other important problem that we have to consider is the filter topology. When we know the filter topology, we can specify assumptions on matrix M, i.e., point out the nonzero entries in the matrix. Based on this we can translate our optimization result into the right configuration, which may be achieved by some rotations, that do not change eigenvalues of the matrix (see [15] and [16] for more details). And even in this case, there are plenty of matrices generating the same reflection characteristics.

The other application is to use the coupling matrix in the filter tuning process (cf. [3] and [17, 18]) — in this case, we start with given, real device, with measured reflection characteristics. And based on the characteristics we find the matrix M. At first glance it looks like the problem identical to the one discussed in the previous paragraph, but there are important differences. First of all, we do not have functions $S_{11}(\lambda)$ and $S_{12}(\lambda)$ given directly by some theoretical methods, but rather approximated (or interpolated) based on the discrete set of measured samples. Moreover, the time becomes an issue: the filter tuning process performed in the real production environment should be as quick as possible. And last but not least, we can usually assume we know the filter topology, so when looking for the matrix we should impose some assumptions on the shape of the matrix. We need to mention that there are plenty of different microwave filter tuning methods and new solutions are still introduced (see e.g., [19–21]).

Among different attitudes towards the microwave filter tuning the one that refers to the coupling matrix describing the filter is powered by the simple idea: generally speaking, we want to have the ideal coupling matrix M_0 of the correctly tuned filter and, based on the

measured reflection characteristics, the detuned coupling matrix M. Based on the difference $M - M_0$ we can see which tuning element (cavity or coupling) must be changed — and how much. The idea looks simple but there are numerous problems that appear in practical implementations of this attitude. One of more important is probably the multitude of matrices that may be found. Depending on the filter topology it appears that there are many matrices that satisfy the Equation (1) for the given function $S_{11}(\lambda)$ — see [22] for some examples. Having multiple matrices we need to pick the right one — we have to decide which one should be chosen to be compared to the reference matrix M_0. And there is no obvious criterion to select one of them. We cannot control this selection process when we use the optimization techniques to generate M — usually we cannot be sure if we arrived to some local minimum of the cost function, and which minimum was chosen by the method we use. Among the different optimization methods the special role of Levenberg-Marquardt algorithm should be mentioned (as an example see [23] and [24]). On the other hand, different cost functions may be used (see e.g., [19, 23, 24]).

The idea of solving Equations (1) analytically looks quite appealing in this context. First of all having the closed formula usually means we have a quick computational method. Additionally, by understanding the structure of the solution set, we control the number of solutions and there is a chance we are able to arrange the solutions somehow, so it is possible to find the solution for the measured characteristics *corresponding* to the ideal matrix M_0.

The rest of the paper is devoted mainly to analytic formulas for the coupling matrix identification. At first, we concentrate on the simplest case: filters without cross-couplings and show that we can always find the complex-valued coupling matrix corresponding to the given reflection characteristics. For some filters with more complex topology the complex-valued coupling matrices corresponding to reflection and transmission characteristics may also be found analytically. Later we show how the analytical methods of finding the complex-valued matrix may be practically applied. The application we point out is to use the matrix we find as a starting point for the minimalization methods (the Levenberg-Marquardt and quasi-Newton are considered as examples) when we search for the regular coupling matrix. We present some simulations showing how the method works.

2. FILTERS WITHOUT CROSS-COUPLINGS

When we have the filter with N cavities and no cross-coupling we may develop very quick recursive method of coupling matrix identification. First of all we assume, that the matrix is given in the following form

$$M = \begin{pmatrix} -j & R_1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ R_1 & y_1 & x_1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & x_1 & y_2 & x_2 & \cdots & 0 & 0 & 0 \\ 0 & 0 & x_2 & y_3 & \cdots & 0 & 0 & 0 \\ \vdots & & & & \ddots & & & \vdots \\ 0 & 0 & 0 & 0 & \cdots & y_{N-1} & x_{N-1} & 0 \\ 0 & 0 & 0 & 0 & \cdots & x_{N-1} & y_N & R_2 \\ 0 & 0 & 0 & 0 & \cdots & 0 & R_2 & -j \end{pmatrix} \tag{2}$$

On the other hand, we have reflection characteristics given as the rational function

$$S_{11}(\lambda) = \frac{\lambda^N + a_{N-1}\lambda^{N-1} + \ldots + a_1\lambda + a_0}{\lambda^N + b_{N-1}\lambda^{N-1} + \ldots + b_1\lambda + b_0} = \frac{A(\lambda)}{B(\lambda)} \tag{3}$$

As we can see Equation (1) defines the map

$$F(R_1, R_2, x_1, \ldots, x_{N-1}, y_1, \ldots, y_N) = (a_0, \ldots, a_{N-1}, b_0, \ldots, b_{N-1}) \tag{4}$$

Actually we are interested in inverting the map, i.e., for a given set of coefficients $(a_0, \ldots, a_{N-1}, b_0, \ldots, b_{N-1})$ we would like to find out the set of coupling matrix attributes $(R_1, R_2, x_1, \ldots, x_{N-1}, y_1, \ldots, y_N)$. Moreover, to keep the physical interpretations of the values, we should assume that all coefficients are real. On the other hand, there are no clear assumptions on the coefficients $(a_0, \ldots, a_{N-1}, b_0, \ldots, b_{N-1})$, so we start on assuming that all of them are just complex numbers. Looking at the problem more formally, we have the map $F : \mathbb{R}^{2N+1} \to \mathbb{C}^{2N}$. The map is clearly polynomial, so its image is a residual subset of $\mathbb{C}^{2N}$. That is why for almost all coefficients $(a_0, \ldots, a_{N-1}, b_0, \ldots, b_{N-1}) \in \mathbb{C}^{2N}$ the Equation (4) will have no solution at all.

This general setting is quite different from the specific cases investigated in Chapter 14.1 of [6]. There is given the recursive formula to generate the coupling matrix without any cross-coupling matching the desired reflection characteristic. But the formula refers to the special class of S_{11} functions being Chebyshev functions. In this case, we create the coupling matrix matching the required function, but the function is a result of a theoretical model having requested properties. In the general case, we cannot assume the rational function is given by a Chebyshev model. Unfortunately inverting the map F in general case is not possible, mainly because its image may not be identified

precisely. We could expect it is (at least almost everywhere) $2N+1$ dimensional real manifold $\mathcal{M}$, but there is not general description of this manifold.

We will try to attack the problem from some other perspective — we are going to increase the dimension of the domain of the map F, by assuming that all of the numbers $(R_1, Q_2, x_1, \ldots, x_{N-1}, y_1, \ldots, y_{N-1})$ are complex, where $Q_2 = -jy_N - R_2^2$. With such attitude we have the map $F : \mathbb{C}^{2N} \to \mathbb{C}^{2N}$ and we can expect the Equation (4) has the finite and nonempty set of solutions (at least for "almost all" right-hand values). Below we will present the description of the effective method of finding the solutions of this equation. The solutions will be given by complex numbers in general. We call this generalization of the regular coupling matrix the *complex-valued coupling matrix*. Such complex version of the coupling matrix may possibly be used in certain applications — especially when we are interested in the differences between coupling matrix entries for different measurements (what may be useful in the filter tuning process). The solutions are not unique, but the values $(R_1^2, Q_2, x_1^2, \ldots, x_{N-1}^2, y_1, \ldots, y_{N-1})$ are. So any two solutions differ in sign only, and only for some of the variables.

Let us further denote $M_0(\lambda) = \lambda I - jR + M$. The value in the first column and the first row of the matrix $(M_0(\lambda))^{-1}$ is given by

$$\frac{\det M_{11}(\lambda)}{\det M_0(\lambda)} \tag{5}$$

where $M_{11}(\lambda)$ denotes the submatrix of $M_0(\lambda)$, with first row and column removed. Let $A_i(\lambda)$ be the determinant of the submatrix of $M_0(\lambda)$ with first i rows and columns removed. So, following this convention, $A_0(\lambda) = \det M_0(\lambda)$, $A_1(\lambda) = \det M_{11}(\lambda)$ and so on. We can see that

$$A_0(\lambda) = -jA_1(\lambda) - R_1^2 A_2(\lambda) \tag{6}$$

Continuing the process we see that for $i = 1, 2, 3, \ldots, N-1$ the recursive formula may be used:

$$A_i(\lambda) = (y_i + \lambda)A_{i+1}(\lambda) - x_i^2 A_{i+2}(\lambda) \tag{7}$$

We arrive to the recursive formulas (6) and (7) by applying Laplace's formula for expanding determinant along its columns. Let us take a look at the sample matrix

$$M_i = \begin{pmatrix} y_i + \lambda & x_i & 0 & \cdots & 0 & 0 \\ x_i & y_{i+1} + \lambda & x_{i+1} & \cdots & 0 & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & y_N & R_2 \\ 0 & 0 & 0 & \cdots & R_2 & -j \end{pmatrix} \tag{8}$$

Then $\det M_i = (y_i + \lambda) \det M_{i+1} - x_i \det \tilde{M}_{i+1}$, where

$$\tilde{M}_{i+1} = \begin{pmatrix} x_i & x_{i+1} & \cdots & 0 & 0 \\ 0 & y_{i+2} + \lambda & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & y_N & R_2 \\ 0 & 0 & \cdots & R_2 & -j \end{pmatrix} \tag{9}$$

Determinant of the matrix $\tilde{M}_{i+1}$ may be again expanded along its first column according to Laplace's formula what leads to $\det \tilde{M}_{i+1} = x_{i+1} \det M_{i+2}$. This finally leads to the formula (7). Now we have $S_{11}(\lambda) = 1 + 2j\frac{A_1(\lambda)}{A_0(\lambda)}$. Both $A_0(\lambda)$ and $A_1(\lambda)$ are polynomials of the degree N and we can see that coefficients by λ^N in both polynomials are -1 and $-j$ respectively. This implies that highest degree coefficient of the $S_{11}(\lambda)$ numerator must be equal to 1 while the denominator must equal to -1. Hence we need to assume this when approximating rational function $S_{11}(\lambda)$. With this assumption the algorithm described below may be used:

1. Based on the $S_{11}(\lambda)$ the function $A_0(\lambda)$ is identified as the denominator of $S_{11}(\lambda)$. Let us denote the numerator of $S_{11}(\lambda)$ as $B(\lambda)$ and then we calculate

$$A_1(\lambda) = \frac{B(\lambda) - A_0(\lambda)}{2j}.$$

2. Now we have the formula resulting from the decomposition of $A_0(\lambda)$

$$A_0(\lambda) = -jA_1(\lambda) - R_1^2 A_2(\lambda).$$

We know that the coefficient by λ^{N-1} of $A_2(\lambda)$ equals to $-j$, and we already know both polynomials $A_0(\lambda)$ and $A_1(\lambda)$. Hence by comparing the coefficients by λ^{N-1} on both sides we can find R_1^2, and the polynomial

$$A_2(\lambda) = \frac{A_0(\lambda) + jA_1(\lambda)}{-R_1^2}.$$

3. Now we are in position of knowing two polynomials $A_i(\lambda)$ and $A_{i+1}(\lambda)$, for $i = 1, 2, \ldots, N$ where $A_i(\lambda)$ is of degree $N - i + 1$. Based on the recursive formula

$$A_i(\lambda) = (y_i + \lambda)A_{i+1}(\lambda) - x_i^2 A_{i+2}(\lambda) \tag{10}$$

and knowing that highest degree coefficients of all $A_i(\lambda)$ equal to $-j$, we can compare coefficients of λ^{N-i} on both sides of (10). This gives us the value of y_i. Knowing y_i we can compare coefficients by λ^{N-i-1}

to find the value of x_i^2. Then we get the polynomial $A_{i+2}(\lambda)$ and may proceed to the next value of i.

4. The last step is slightly different because we have the following formulas:

$$A_{N-1}(\lambda) = (y_{N-1} + \lambda)A_N(\lambda) - x_{N-1}^2(-j) \tag{11}$$

$$A_N(\lambda) = (y_N + \lambda)(-j) - R_2^2 = -j\lambda + Q_2. \tag{12}$$

Actually, we calculate values of y_{N-1} and x_{N-1}^2 as in step 3 of this algorithm but there is no need to calculate $A_{N+1}(\lambda)$ from (11). On the other hand as we look at the formula (12) we can see that $A_{N-1}(\lambda) = -j\lambda + Q_2$ and $R_2^2 = -\mathrm{Re}(Q_2)$ while $y_N = -\mathrm{Im}(Q_2)$. The case of $N \times N$ coupling matrix is similar — it just leads to slightly different algorithm. The recursive formula is basically the same but its first and last steps are slightly different. We will not discuss it is details here.

3. CROSS-COUPLINGS AND REFLECTION CHARACTERISTICS

The case discussed in the previous section is the simplest possible. When the cross couplings are added to the coupling matrix the recursive formulas are not valid any more. We can follow them up to some value of i, but when the row or column with nonzero cross-coupling is encountered, the recursive formula may not be used any more. With the reflection characteristics only, there is not enough information to uniquely recreate the coupling matrix, as it is performed in the no-cross couplings case.

In order to handle such more complex case the two attitudes may be suggested:

(1) in case the transmission characteristics S_{12} is known, the additional equations may be used and solutions may be searched analytically, what finally allows to find the coupling matrix (or matrices);

(2) find the tridiagonal coupling matrix by means of the method described in the previous section and treat it as the starting point of some approximation process leading to the coupling matrix of the right format.

The example of the first method will be given in this section, while the second idea will be presented and tested in the next section. There is no general method to find the right coupling matrix analytically but some solutions of certain specific cases may be suggested. These will be discussed below.

The transmission characteristics S_{12} is related to the coupling matrix M by the formula

$$S_{12}(\lambda) = -2j[\lambda I_N - J + M]^{-1}_{N+2,1} \tag{13}$$

On the other hand the transmission characteristics is known as the rational function of the form

$$S_{12}(\lambda) = \frac{c_M\lambda^M + c_{M-1}\lambda^{M-1} + \ldots + c_1\lambda + c_0}{\lambda^N + b_{N-1}\lambda^{N-1} + \ldots + b_1\lambda + b_0} = \frac{C(\lambda)}{B(\lambda)} \tag{14}$$

where M depends on the number of transmission zeroes. Let us observe how this idea may be applied to the sample matrix ($N = 4$) as given in the Fig. 10.33(a) on page 394 of [6], i.e.,

$$M_0 = \begin{pmatrix} -j & 1.1506 & 0 & 0 & 0 & 0 \\ 1.1506 & 0.053 & 1.0394 & 0 & 0 & 0 \\ 0 & 1.0394 & 0.949 & 0.7128 & 0.353 & 0 \\ 0 & 0 & 0.7128 & -0.4198 & 0.9777 & 0 \\ 0 & 0 & 0.353 & 0.9777 & 0.053 & 1.1506 \\ 0 & 0 & 0 & 0 & 1.1506 & -j \end{pmatrix} \tag{15}$$

The matrix is rewritten with the input and output reversed, in order to have the cross-coupling more distanced from the filter's entry. This allows for more iterations of the recursive formula (7). The formula given by the model (1) is derived from the matrix

$$\begin{pmatrix} -j & R_1 & 0 & 0 & 0 & 0 \\ R_1 & y_1 + \lambda & x_1 & 0 & 0 & 0 \\ 0 & x_1 & y_2 + \lambda & x_2 & k_1 & 0 \\ 0 & 0 & x_2 & y_3 + \lambda & x_3 & 0 \\ 0 & 0 & k_1 & x_3 & y_4 + \lambda & R_2 \\ 0 & 0 & 0 & 0 & R_2 & -j \end{pmatrix} \tag{16}$$

Following the methods used in the previous section we may create the following recursive formulas

$$A_0(\lambda) = -jA_1(\lambda) - R_1^2A_2(\lambda) \tag{17}$$

$$A_1(\lambda) = (y_1 + \lambda)A_2(\lambda) - x_1^2A_3(\lambda) \tag{18}$$

Following the algorithm described above the values R_1^2, y_1 and x_1^2 may be found as well as polynomials $A_2(\lambda)$ and $A_3(\lambda)$. Additionally we know that

$$A_2(\lambda) = \begin{pmatrix} y_2 + \lambda & x_2 & k_1 & 0 \\ x_2 & y_3 + \lambda & x_3 & 0 \\ k_1 & x_3 & y_4 + \lambda & R_2 \\ 0 & 0 & R_2 & -j \end{pmatrix} \tag{19}$$

$$A_3(\lambda) = \begin{pmatrix} y_3+\lambda & x_3 & 0 \\ x_3 & y_4+\lambda & R_2 \\ 0 & R_2 & -j \end{pmatrix} \tag{20}$$

The coefficients of polynomials

$$A_2(\lambda) = -j\lambda^3 + p_2\lambda^2 + p_1\lambda + p_0 \tag{21}$$

$$A_3(\lambda) = -j\lambda^2 + q_1\lambda + q_0 \tag{22}$$

are known. This leads to the system of equations

$$\begin{cases} R_2^2x_2^2 - 2jk_1x_2x_3 + jx_3^2y_2 + jk_1^2y_3 - R_2^2y_2y_3 + jx_2^2y_4 - jy_2y_3y_4 = p_0 \\ jk_1^2 + jx_2^2 + jx_3^2 - R_2^2y_2 - R_2^2y_3 - jy_2y_3 - jy_2y_4 - jy_3y_4 = p_1 \\ -R_2^2 - jy_2 - jy_3 - jy_4 = p_2 \\ jx_3^2 - R_2^2y_3 - jy_3y_4 = q_0 \\ -R_2^2 - jy_3 - jy_4 = q_1 \end{cases} \tag{23}$$

This system contains too many unknown values to be uniquely solved. In order to solve it analytically it seems to be a good idea to add to this system more Equations coming from the transmission characteristics. We know that the numerator of the reflection characteristics S_{21} is given by

$$C(\lambda) = -2jR_1x_1 \det \begin{pmatrix} x_2 & k_1 & 0 \\ y_3+\lambda & x_3 & 0 \\ x_3 & y_4+\lambda & R_2 \end{pmatrix} \tag{24}$$

On the other hand the coefficients of $C(\lambda)$ are known

$$C(\lambda) = r_1\lambda + r_0 \tag{25}$$

what leads to additional equations

$$\begin{cases} -2jR_1x_1(x_2x_3R_2 - R_2k_1y_3) = r_0 \\ 2jR_1x_1R_2k_1 = r_1 \end{cases} \tag{26}$$

The following system of polynomial equations may be solved for all right-hand side values

$$\begin{cases} R_2^2x_2^2 - 2jk_1x_2x_3 + jx_3^2y_2 + jk_1^2y_3 - R_2^2y_2y_3 + jx_2^2y_4 - jy_2y_3y_4 = p_0 \\ jk_1^2 + jx_2^2 + jx_3^2 - R_2^2y_2 - R_2^2y_3 - jy_2y_3 - jy_2y_4 - jy_3y_4 = p_1 \\ -R_2^2 - jy_2 - jy_3 - jy_4 = p_2 \\ jx_3^2 - R_2^2y_3 - jy_3y_4 = q_0 \\ -R_2^2 - jy_3 - jy_4 = q_1 \\ -2jR_1x_1(x_2x_3R_2 - R_2k_1y_3) = r_0 \\ 2jR_1x_1R_2k_1 = r_1 \end{cases} \tag{27}$$

We may treat it as the nonlinear equation $G(x_2, x_3, y_2, y_3, y_4, k_1, R_2) = (p_0, p_1, p_2, q_0, q_1, r_0, r_1)$, where $G : \mathbb{R}^7 \to \mathbb{R}^7$ is given by the left side of the system (27). The values of x_1 and R_1 may be treated as known parameters.

Let us review how this system may be solved analytically:

(i) From Equations (27.3) and (27.5) the value of y_2 may be found;

(ii) From the last equation we can have the value of $k_1 R_2$, which may be substituted to Equation (27.6);

(iii) From Equation (27.5) we may have R_2^2 as the function of y_3 and y_4. It may be substituted to all other equations;

(iv) Equations (27.6) and (27.7) let us represent $x_2 x_3 = -\alpha k_1 + \beta k_1 y_3$ for some constants α, β. This may be applied to Equation (27.1);

(v) Moreover Equation (27.6) may be replaced by the previous Equation squared, i.e., $x_2^2 x_3^2 = (\beta k_1 y_3 - \alpha k_1)^2$. This may introduce additional solutions that should be verified against the original system, but now x_2 and x_3 appear as x_2^2 and x_3^2 only;

(vi) Similarly as in (v) we may replace the Equation (27.7) with $k_1^2 R_2^2 = \text{const}$.

Now denoting $x_2^2 = s$, $x_3^2 = t$, $R_2^2 = v$, $y_3 = b$, $y_4 = c$, $k_1^2 = u$ we arrive to the following system:

$$\begin{cases} a_1 t + a_2 bu + a_3 b^2 + a_4 s + a_5 bs + a_6 b + a_6 u = \alpha_1 \\ jt + js + a_7 b + ju + jb^2 = \alpha_2 \\ jt + a_8 b + jb^2 = \alpha_3 \\ -v - jc - jb = \alpha_4 \\ a_9 st - a_{10} u + a_{11} bu - a_{12} b^2 u = \alpha_5 \\ uv = \alpha_6 \end{cases} \tag{28}$$

for some constant values a_i and α_j. Now from Equations (28.1), (28.2) and (28.3) we may find t, u, s as a rational function of b. These values introduced into Equation (28.5) give us the polynomial equation on b (of degree not exceeding 4). When the Equation (28.5) is solved for each solution we have the value of s, t, u, v and c. Following the procedure described above and assuming the values p_i, q_i, r_i are taken from the matrix (15) we have the solutions listed below. The values $y_2 = 0.949, y_3 = -0.4198, y_4 = 0.053$ appear in each solution. The rest of variables takes the following values:

$$\begin{array}{llll} x_2 = -0.7128, & x_3 = 0.9777, & R_2 = -1.1506, & k_1 = -0.353 \\ x_2 = 0.7128, & x_3 = -0.9777, & R_2 = -1.1506, & k_1 = -0.353 \\ x_2 = -0.7128, & x_3 = -0.9777, & R_2 = 1.1506, & k_1 = 0.353 \\ x_2 = 0.7128, & x_3 = 0.9777, & R_2 = 1.1506, & k_1 = 0.353 \end{array}$$

It should be observed that all of the solutions listed here generate the same scattering characteristics. Additionally, it may be observed that for the point corresponding to the investigated matrix M_0 the Jacobian of the map G is nonzero. Similarly for each of the above solutions what means that in the neighborhood of each of these solutions the system (27) may be solved uniquely.

4. THE COUPLING MATRIX EXTRACTION

Let us start with the more formal definition of the coupling matrix extraction problem. Assume the transmission characteristics is given as a sequence of points (λ_i, S_i), $i = 1, 2, \ldots, N$. We would like to find entries of the coupling matrix M such that the function given by the formula (1) matches the discrete set of samples in an optimal way. Starting from this point we may select different strategies. So here is what we can do:

(i) Choose the appropriate cost function, such that its minimum corresponds to coupling matrix coefficients that we are looking for. Between different choices of the cost functions probably the most natural is

$$J = \sum_{i=1}^{N} |S_{11}(\lambda_i) - S_i|^2 \tag{29}$$

as used in [16]. The cost function J that we are going to minimize depends on the coupling matrix coefficients. The function is smooth so we may use different optimization procedures including Levenberg-Marquardt algorithm, that we used in tests described below.

(ii) One may also choose different strategy. In the first step, we find the rational function that matches the discrete sample data. This may be achieved by means of different methods including least squares/total least squares modification of Cauchy interpolation schema. Having the rational function we may look for coupling matrix coefficients that match the rational function according to the formula (1). Here we may use procedures described in previous sections but we can see that the coupling matrix calculated according to this procedure may have nonreal entries. But when we have the function that matches the data being the result of the measurement it may happen that the coupling matrix has nonreal entries and we are losing the physical interpretation of its values. So when we choose the rational function we almost always select the function that does not have the corresponding real-valued coupling matrix. With complex-valued coupling matrix we do not have this problem — as we can see in calculations given in the previous section, for any rational function we find at least one complex-valued coupling matrix. In case we simplify the model to the no-cross couplings case the values of matrix coefficients are given by simple recursive formulas, so the calculations are very quick.

We can see that both attitudes have some drawbacks. Optimization methods may appear to be time consuming and do not guarantee that we arrive to the global minimum (the one we are looking for). On

the other hand the analytical methods may be specified by precise formulas (at least in some cases), so are quick and accurate, but lead us to some complex-valued coupling matrices that do not correspond directly to physical properties of the microwave filter. Below the method merging the two attitudes is described.

We will have a look at several examples of different filters, starting from relatively simple ones — with small order ($N = 4$) and only one cross-coupling — to more complex cases. We will show that our methods will be of great help in the optimization process in these cases. It always does better then random selection of the starting point, and when filter order increases the method gets even better (relative to random starting point selection).

We perform a series of tests. Each test starts with certain coupling matrix. We use matrices reported in several papers — as described below. We use them to simulate the situations that might be met in the filter tuning process. Here we cannot assume that we have filter described by ideally tuned coupling matrix but rather by a matrix with some entries distorted. For some tests we will need a set of 1000 randomly selected matrices M_i ($i = 1, 2, \ldots, 1000$) differing from M by random distortion, independently on each nonzero entry of M with normal distribution of mean value 0 and standard deviation equal to 0.1. This distortion simulates the detuned filter. Fig. 1 shows the sample distortion for one of the filters used in our tests.

Reference test. We take the given coupling matrix, then generate $S_{11}(\lambda)$ characteristics, sample it at 512 points. Based on these points we build the cost function and call the nonlinear optimization method with randomly selected starting points. Two methods are used: Levenberg-Marquardt and quasi-Newton. For each method and given starting point we perform 300 iterations of the method. Starting points are selected from $[-1, 1]$ interval according to the uniformly distributed random variable.

The following test cases are performed as reference tests:

(A) the cost function corresponding to the reference (ideal) coupling matrix is concerned with starting point selected randomly form $[-1, 1]$ interval;

(B) for each matrix M_i we create the separate cost function with starting point is selected randomly from $[-1, 1]$ interval.

Method test. For each distorted matrix M_i we create set of 512 sample values, recreate rational function $S_{11}(\lambda)$, find complex-valued coupling matrix and then use its real parts as starting point of the Levenberg-Marquardt or quasi-Newton approximation. We check how often we arrive to the coupling matrix that is close to the one that we started from.

4.1. Test Case 1: $N = 4$

Now we will focus on the same sample matrix as given in Section 3 above, i.e., matrix given in the Fig. 10.33(a) on page 394 of [6]:

$$M_0 = \begin{pmatrix} -j & 1.1506 & 0 & 0 & 0 & 0 \\ 1.1506 & 0.053 & 1.0394 & 0 & 0 & 0 \\ 0 & 1.0394 & 0.949 & 0.7128 & 0.353 & 0 \\ 0 & 0 & 0.7128 & -0.4198 & 0.9777 & 0 \\ 0 & 0 & 0.353 & 0.9777 & 0.053 & 1.1506 \\ 0 & 0 & 0 & 0 & 1.1506 & -j \end{pmatrix} \tag{30}$$

This matrix is the starting point of the experiment. It is used to generate the discrete set of samples of the function $S_{11}(\lambda)$, as given by the formula (1). We treat this discrete set of complex numbers as the representation of the reflection characteristics. Starting from this reflection characteristics the coupling matrix is recreated. The expected result is the original coupling matrix, of course, but it is not always the case.

The complex-valued matrix given by the procedure described in

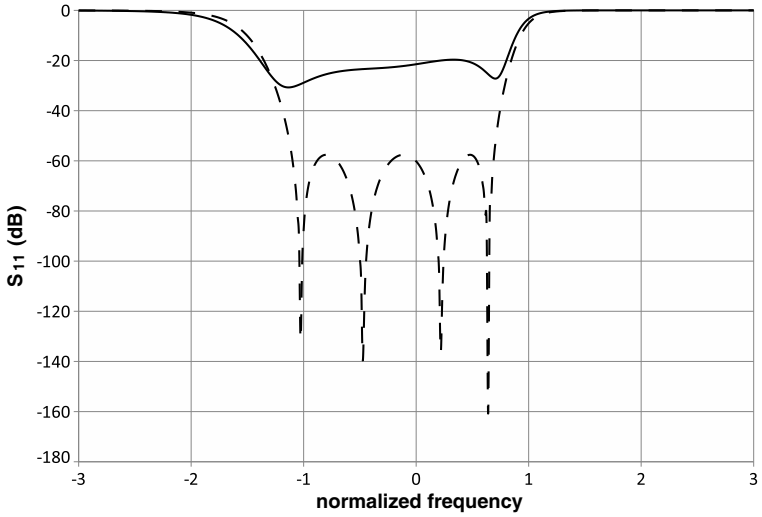

Figure 1. Sample S_{11} distortion for randomly generated matrix. Example used in test case 1. Dashed line corresponds to the basic characteristic, solid to distorted.

Table 1. Percentage of successful optimization processes for $N = 4$.

Test	Levenberg-Marquardt	Quasi-Newton
A	12.1%	86.0%
B	10.4%	84.0%
Method test	79.6%	92.9%

the previous section is as follows

$$\begin{pmatrix} -j & 1.1506 & 0 & 0 & 0 & 0 \\ 1.1506 & 0.053 & 1.039 & 0 & 0 & 0 \\ 0 & 1.0394 & 0.949 & 0.795 & 0 & 0 \\ 0 & 0 & 0.795 & 0.4451 - 0.261j & 0.781 - 0.526j & 0 \\ 0 & 0 & 0 & 0.781 - 0.526j & -0.818 & 1.031 \\ 0 & 0 & 0 & 0 & 1.031 & -j \end{pmatrix} \tag{31}$$

Now starting from the model (1) the appropriate formula is derived from the matrix (16). To write the formula we need to know the filter topology. Then the cost function $J(x_1, x_2, x_3, y_1, y_2, y_3, y_4, k_1, R_1, R_2)$ as given by (29) is used. The real parts of the complex-valued coupling matrix are used as a starting point of the minimization process. The optimization should arrive to expected result, i.e., the real-valued coupling matrix that was our starting point. The number of successful attempts is presented in the Table 1.

Let us present the results of our tests in the Table 1. We can see that the chance to arrive to a right minimum is highest when we start from the complex-valued matrix. When we start from the randomly selected values (even in the neighborhood of the ideal matrix) the chance is always lower.

4.2. Test Case 2: $N = 6$

The initial matrix M_0 is taken from [17] given by

$$\begin{pmatrix} -j & 1.037 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.037 & 0 & 0.869 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.869 & 0 & -0.614 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.614 & 0 & 0.555 & 0 & -0.168 & 0 \\ 0 & 0 & 0 & 0.555 & 0 & 0.723 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.723 & 0 & 0.826 & 0 \\ 0 & 0 & 0 & -0.168 & 0 & 0.826 & 0 & 1.037 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1.037 & -j \end{pmatrix} \tag{32}$$

And the corresponding complex valued matrix is given by

Table 2. Percentage of successful optimization processes for $N = 6$.

Test	Levenberg-Marquardt	Quasi-Newton
A	1.5%	12.0%
B	0.2%	12.5%
Method test	28.0%	49.2%

$$\begin{pmatrix} -j & 1.037 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.037 & 0 & 0.869 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.869 & 0 & 0.614 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.614 & 0 & 0.58 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.58 & -0.09j & 0.341 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.341 & 3.09j & 3.687 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.687 & 0 & 2.019 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2.019 & -j \end{pmatrix} \tag{33}$$

In this case, we can also see the chance to arrive to a right minimum is highest when we start from the complex-valued matrix (Table 2).

4.3. Test Case 3: $N = 8$

The initial matrix M_0 is taken from [25]. It is given by

$$\begin{pmatrix} -j & 1.0283 & 0 & 0 & 0 \\ 1.0283 & 0.2256 & 0.7541 & 0 & 0 \\ 0 & 0.7541 & -0.024 & 0.5371 & 0 \\ 0 & 0 & 0.5371 & 0.0499 & 0.4741 \\ 0 & 0 & 0 & 0.4741 & -0.0049 \\ 0 & 0 & 0 & 0 & 0.6531 \\ 0 & 0 & 0 & -0.1581 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -0.1581 & 0 & 0 & 0 \\ 0.6531 & 0 & 0 & 0 & 0 \\ -0.0042 & 0.4745 & 0 & 0 & 0 \\ 0.4745 & 0.0509 & 0.5377 & 0 & 0 \\ 0 & 0.5377 & -0.0238 & 0.7543 & 0 \\ 0 & 0 & 0.7543 & 0.2244 & 1.0284 \\ 0 & 0 & 0 & 1.0284 & -j \end{pmatrix} \tag{34}$$

The corresponding complex-valued matrix entries are given by:

$$\begin{aligned} M_{1,2} &= 1.028 & M_{2,2} &= 0.226 \\ M_{2,3} &= 0.754 & M_{3,3} &= -0.024 \\ M_{3,4} &= 0.537 & M_{4,4} &= 0.049 \\ M_{4,5} &= 0.5 + 0.001j & M_{5,5} &= 0.005 \\ M_{5,6} &= 0.502 - 0.006j & M_{6,6} &= -0.061 - 0.018j \\ M_{6,7} &= 0.482 + 0.019j & M_{7,7} &= 0.135 - 0.159j \\ M_{7,8} &= 0.125 - 0.323j & M_{8,8} &= -1.595 - 3.172j \\ M_{8,9} &= 2.849 - 1.617j & M_{9,9} &= 0.871 + 1.156j \\ M_{9,10} &= 0.382 + 1.141j \end{aligned} \tag{35}$$

The results of this test case are presented in Table 3. As we can see, the chance to arrive to a right minimum is highest when we start from the complex-valued matrix. In all other cases, the chance is practically zero.

4.4. Test Case 4: Physical Device $N = 6$

The coupling matrix extraction method was tested for the physical filter with $N = 6$ cavities and topology presented in Fig. 2. This is 900 MHz filter being the RX part of the combiner.

Table 3. Percentage of successful optimization processes for $N = 8$.

Test	Levenberg-Marquardt	Quasi-Newton
A	0%	1.9%
B	0%	1.8%
Method test	12.8%	34.4%

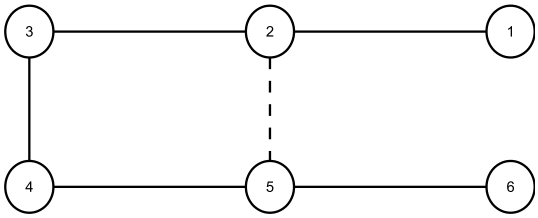

Figure 2. Topology of the filter used in the experiment. The dashed line shows the cross-coupling between cavities 2 and 5.

Table 4. Complex-valued coupling matrix entries.

	Tuned	**Detuned**
M_{22}	$-1.73 + 2.2j$	$-1.32 + 2.23j$
M_{33}	$-0.359 + 0.197j$	$-0.501 + 0.263j$
M_{44}	$2.61 - 0.651j$	$2.58 - 0.704j$
M_{55}	$2.06 - 0.521j$	$2.1 - 0.475j$
M_{66}	$2.95 + 0.169j$	$2.88 + 0.157j$
M_{77}	1.69	1.65

	Tuned	**Detuned**
M_{12}	$1.43 - 0.255j$	$1.38 - 0.251j$
M_{23}	$3.72 - 0.794j$	$3.44 - 0.853j$
M_{34}	$1.07 + 0.173j$	$1.14 + 0.204j$
M_{45}	$0.0694 - 0.367j$	$0.056 - 0.36j$
M_{56}	$0.753 - 0.214j$	$0.771 - 0.196j$
M_{67}	$0.187 + 0.182j$	$0.26 + 0.173j$
M_{78}	0.357	0.409

The reflection characteristics S_{11} was measured in 256 frequency points for two settings of tuning screws: for the tuned filter and the filter with the tuning screw corresponding to the first cavity moved up by 360°. The two characteristics are presented in Fig. 3.

The complex valued coupling matrices for the tuned and detuned filter are presented in the Table 4.

The regular real-valued coupling matrix (after the optimization

process) for the tuned filter is

$$\begin{pmatrix} -i & 1.56 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.56 & -1.93 & 3.88 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.88 & -0.198 & 0.642 & 0 & -1.01 & 0 & 0 \\ 0 & 0 & 0.642 & 2.1 & 0.467 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.467 & 3.12 & 0.756 & 0 & 0 \\ 0 & 0 & -1.01 & 0 & 0.756 & 4.42 & 1.42 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.42 & 3.32 & 0.941 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.941 & -i \end{pmatrix} \tag{36}$$

The regular real-valued coupling matrix (after the optimization process) for the detuned filter is

$$\begin{pmatrix} -j & 1.54 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.54 & -1.89 & 3.72 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.72 & -0.362 & 0.682 & 0 & -0.977 & 0 & 0 \\ 0 & 0 & 0.682 & 2.11 & 0.479 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.479 & 3.09 & 0.704 & 0 & 0 \\ 0 & 0 & -0.977 & 0 & 0.704 & 4.21 & 1.39 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.39 & 3.36 & 0.957 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.957 & -j \end{pmatrix} \tag{37}$$

It may be observed that the coupling matrix retrieved from the

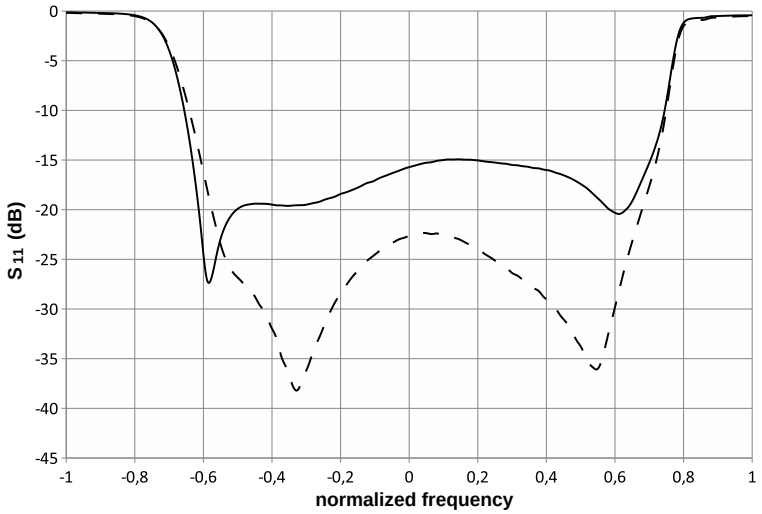

Figure 3. S_{11} measured for the $N = 6$ RX filter discussed in test case 4. Dashed line corresponds to the basic characteristic, solid to distorted.

measured data is changed mainly in the neighbourhood of the element M_{22}, that corresponds to the detuned element.

5. CONCLUSION

In the paper the quick, recursive and analytical method of coupling matrix extraction is introduced. It is shown that for a given reflection characteristics we can find many coupling matrices M satisfying Equation (1). All matrices have the same main diagonal but in the subdiagonal values differing in sign. In certain situations the method may be extended to filters with more complex topologies. The coupling matrix that is extracted may be used in general complex-valued but we may use it as a starting point of the optimization methods that lead to the extraction of regular real-valued coupling matrix, corresponding to the measured reflection characteristics. This starting point selection does much better then random selection, as we show in examples presented above.

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