ZERO-POLE APPROACH TO COMPUTER AIDED DESIGN OF IN-LINE SIW FILTERS WITH TRANSMISSION ZEROS

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Abstract—This paper presents a design of a new type of in-line pseudo-elliptic filters implemented in substrate integrated waveguide (SIW) technology. To realize transmission zeros in in-line topology, frequency-dependent couplings were used. Such dispersive couplings were implemented as shorted stubs. The design process starts with the generation of a suitable starting point. To this end, an approximation of SIW as a rectangular waveguide is used and a fast electromagnetic solver based on mode-matching technique is utilized (\(\mu\)Wave Wizard). The next step is the optimization process of a filter in a full-wave 3D EM simulator Ansoft HFSS. The Optimization routine employs a cost function involving zeros and poles of the rational functions approximating scattering parameters. To increase the speed of convergence, a built-in derivative calculation feature was used and zeros and poles and their derivatives with respect to design parameters were extracted using the vector fitting algorithm. Experimental validation of the method is demonstrated by a third-order filter with asymmetric response and a fifth-order filter with two transmission zeros in addition to an asymmetric response. The experimental results show good agreement between the simulated and measured data.

1. INTRODUCTION

Coupled-resonator microwave filters are an important area of microwave engineering, a field that has been developed over many years. Constant development in this area is motivated by stringent...
Communication system requirements concerning mainly high out-of-band rejection levels. Such requirements can be satisfied by well-established methods, e.g., by using cross-couplings between non-adjacent resonators [1–3]. Cross-couplings may introduce real or imaginary transmission zeros which improve selectivity. The design methodologies of such filters are well described in the literature [4–6]. At an initial step one finds a low-frequency prototype assuming that the couplings between resonators are frequency-independent. However, using cross-coupled resonators is not the only way the transmission zeros can be introduced. In the last decade it has been demonstrated that in-line waveguide filters may exhibit generalized Chebyshev responses if the frequency-dependent couplings are used [7–11]. Such couplings allow for filter implementation in in-line topologies with sharp cut-off skirts and even with an asymmetric response.

Recently a new idea for microwave filter implementation was proposed. It is based on substrate integrated waveguide (SIW), which can be easily manufactured using a standard low-cost PCB process. Filters implemented in SIW technology have many advantages and their practical realizations have been presented in a number of articles [12–16]. In particular, one such an advantage is the possibility of achieving a higher quality factor and better selectivity in comparison to capabilities of planar circuits. However, the implementation of cross-couplings in SIW technology is often a challenge due to the fact that in most cases the cross-coupling has to be of the opposite sign to the main coupling. This requires modification of a top or bottom (or both) metallization layer which, in turn, can produce not negligible radiation effects. It is therefore tempting to use the simplest in-line topology and increase the selectivity by using similar frequency-dependent couplings as in waveguides. Especially shorted stubs proposed in [17, 18] are particularly attractive and simple to implement in SIW. Unfortunately, unless a filter is narrowband, in which case one may use the technique proposed in [7], there is no established way of designing in-line filters with frequency-dependent couplings other than through numerical simulations and full-wave optimization.

State-of-the-art full-wave numerical solvers offer a choice of built-in optimization algorithms including simplex, gradient and genetic algorithms. However, these built-in algorithms are not very flexible in terms of the goal function definitions and may converge very slowly. As a result, the full-wave design by optimization of filters may take a very long time. To mitigate the computational cost, hybrid algorithms [19–21] can be applied; they use both the full-wave and analytical models. The goal of these methodologies is to shorten the design cycle by using fast but less accurate empirical model to get the information
about optimal parameters setting in the electromagnetic model. In that cases the full-wave analysis is used to verify if the solution meets design specifications. The analytic model is optimized to get the satisfactory solution. If the responses of both models are not the same the algorithm reduces the differences to align the empirical model with that of the full-wave one. The success of this approach depends on the availability and the quality of the surrogate model [22, 23].

For CAD of microwave filters one may use a hybrid technique employing analytical models based a lumped element equivalent circuit. The model is very often specified in a form of a coupling matrix. This approach has been proven to be very powerful [24–27] but its success depends on the process of extracting the coupling matrix from electromagnetic simulations. Unfortunately, this CAD methodology cannot be applied for the filters considered in this paper, since all coupling matrix extraction schemes assume that the coupling coefficient is frequency-independent. In our case, the frequency dependence of the coupling is essential for implementing transmission zeros in the in-line configuration.

One should note that the optimization convergence depends also on the set starting point and cost function. In microwave CAD of passive devices one usually uses goal functions based on the scattering parameters evaluated at many [28, 29] or at a few frequency points [30]. Such goal functions, often specified in the form of a frequency mask, compare the desired level of S parameters in a prescribed frequency range with that obtained from the simulation.

For microwave filters with transmission zeros it is more beneficial to chose the frequency points in that goal function definition in such a way that $S_{11}$ and $S_{31}$ are evaluated only at band edges as well as at the transmission and reflection zeros [31–33]. Although, this approach has originally been used for filter prototype synthesis, it is also useful for full-wave optimization as simulations have to carried out at a very few frequency points, which results in a shorter design cycle.

In [34] a new optimization technique, designed for CAD of microwave filters was proposed. In this method, the location zeros and poles of the reflection characteristics, rather than the values of $S$-parameters, are used to construct the goal function. It has been shown [35] that for this new type of goal functions, filters can be designed even if the initial design is very bad.

In this paper we use the zero-pole approach proposed in [34] as a fast way of designing in-line SIW filters with transmission zeros using numerical full-wave optimization. The method allows for the realization of SIW filters with arbitrarily located transmission zeros. Each transmission zero is generated by a frequency-dependent coupling
which is implemented by a stub. This stub is also responsible for the generation of the direct coupling between adjacent resonators. The optimization process is carried out on two levels: the starting point is generated using a rectangular waveguide model, and the final design is obtained by tuning the response in a 3D FEM solver. The design procedure is illustrated by two examples involving three and five-pole generalized Chebyshev filters.

2. DESIGN PROCEDURE

2.1. Preliminary Relations

For the general Chebyshev approximation of filters composed of \( N \) coupled resonators the transfer function \( S_{21}(\omega) \) [36] is defined as

\[
|S_{21}(\omega)| = \frac{1}{1 + \epsilon^2 F_N^2(\omega)}
\]

where \( \epsilon \) is a ripple constant related to the passband return loss, \( F_N(s) \) represents a filtering function and \( \omega \) is a real frequency variable. The filtering function can be defined as a rational function [35, 37], that is

\[
F_N(s) = \frac{R_N(s)}{P_{Nz}(s)}
\]

where \( s = j\omega \) is a complex frequency variable. Transmission and reflection functions of any two-port filtering structure composed of a series of \( N \) coupled resonators may be expressed also as a polynomial [38]

\[
S_{11}(s) = \frac{R_N(s)}{E_N(s)}
\]

\[
S_{21}(s) = \frac{P_{Nz}(s)}{\epsilon E_N(s)}
\]

where \( P_{Nz}(s), R_N(s) \) and \( E_N(s) \) are complex polynomials. \( S_{11} \) and \( S_{21} \) share the common denominator \( E_N(s) \) which is of the \( N \)th degree with \( N \) being the degree of filtering function. The polynomial \( R_N(s) \) is also of the \( N \)th degree and polynomial \( P_{Nz}(s) \), which contains the transfer function transmission zeros, is of degree \( N_z \) where \( N_z \) is a number of finite-position transmission zeros. The roots of \( E_N(s) \) and \( R_N(s) \) correspond to the filters reflections/transmission poles and reflection zeros, respectively.

Assume that \( P_i \) are roots of the denominator, \( Z_i \) are roots of the numerator of an ideal transfer function and \( R_i \) are the roots of the
numerator of the reflection function. The cost function for design of microwave filters [34, 35, 37] is defined as follows

\[ C = \sum_{i=1}^{M} |Z'_i - Z_i|^2 + \sum_{i=1}^{N} |P'_i - P_i|^2 + \sum_{i=1}^{N} |R'_i - R_i|^2 \]  

(5)

where \( N \) is the number of poles and zeros of polynomials \( R_N(s) \) and \( E_N(s) \), and \( M \) is the number of prescribed transmission zeros. \( Z'_i \) and \( P'_i \) are the zeros and poles of the rational function approximation of \( S_{21} \) and \( R'_i \) are the zeros of the numerator of the rational function interpolating \( S_{11} \) for the filter being optimized.

To determine the position of \( Z'_i, P'_i \) and \( R'_i \) from the transmission and reflection characteristics calculated by the full-wave analysis, we interpolate \( S_{11} \) and \( S_{21} \) using the vector fitting [39] procedure. In our technique the values of \( S_{11} \) and \( S_{21} \) are evaluated at a few frequency points, this guarantees good approximation of transmission and reflection functions, and is not time consuming. The minimum number of points which have to be provided is equal to \( 2N + 1 \), where \( N \) is the order of the filter.

2.2. Algorithm

The designing process starts with generation of a suitable starting point. To do this SIW was initially modeled as a rectangular waveguide, and the whole filter was designed using this model, using the cost function presented in the previous section. We applied \( \mu \)Wave Wizard [40] software, which is a very fast tool based on the mode-matching analysis. Moreover, this software has several built-in tools which facilitate the design of filters. For generalized Chebyshev filters a fast optimizer based on the zero-pole approach [34] is provided as an add-on tool. The starting point for design of an in-line waveguide model with stubs was chosen at random within the range of possible dimensions for a given filter. The final solution obtained from \( \mu \)Wave Wizard optimizer yields the initial values for a proper optimization of our target SIW structure.

Using this starting point, the SIW filter was optimized using a gradient-based Quadratic Programming method, available in the Matlab Optimization toolbox [41]. Full-wave simulations were carried-out in Ansoft HFSS v.13. We took the advantage of the fact that this electromagnetic solver has built-in tools for the calculation of first derivatives. This feature was used to compute the gradient of the cost function proposed in the previous section. At each optimization step the structure was analysed at \( 2N + 1 \) frequency points in the desired passband, where the \( N \) is the order of the filter.
In the optimization procedure both $\mu$Wave Wizard (for generating the starting point) and HFSS were invoked from the Matlab environment. As a result, the optimization procedure is fully automated. The tuning process is depicted in Figure 1. It should be emphasized that the generation of the starting point using a waveguide model with initial dimensions selected at random takes much less time than one simulation in 3D EM simulator so it will not be considered.

3. EXPERIMENTAL VERIFICATION

To verify the method outlined in the previous section two bandpass filters have been designed. Both filters have been fabricated on a Taconic RF-35 substrate which has a relative dielectric constant equal to 3.5 and thickness of 0.762 mm. A standard low cost PCB process was used to manufacture both circuits. All metallized vias have the same diameter equal to 1 mm and spacing between their centres is equal to 1.5 mm.

3.1. Third Order in-line SIW Filter with One Transmission Zero

The first example is an in-line third order filter centred at $f_0 = 4.825$ GHz with bandwidth equal to 150 MHz. The filter has a 20 dB return loss and one transmission zero placed on the lower side of the passband at $f_z = 4.678$ GHz. The filter consists of three directly connected resonator cavities with a frequency-dependent coupling
placed between the first and second resonator. The dispersive coupling is implemented via a shorted stub and controls both the proper value of the coupling coefficient at midband frequency and the position of the transmission zero.

The initial filter response is depicted in Figure 2. It can be observed that the return loss has been degraded to the level of 13 dB in the worst case. Additionally, it can be seen that the transmission zero has been moved up by 16.5 MHz in comparison with the desired position. However, such initially dimensioned SIW component, obtained using zero-pole optimization of a rectangular

![Figure 2. Scattering parameters for the initial dimensions ($x_{str}$ in Table 1) and final dimensions ($x_{opt}$ in Table 1) of a 3-pole generalized Chebyshev in-line SIW filter with one transmission zero.](image)

![Figure 3. Convergence of the optimization routine for a 3-pole generalized Chebyshev filter with one transmission zero.](image)

Table 1. Results of the optimization of a third order in-line SIW filter with one transmission zero (the starting point is denoted by $x_{str}$ and is given in the second and fifth column, the final dimensions, denoted as $x_{opt}$ are given in the third and sixth column).

<table>
<thead>
<tr>
<th>x</th>
<th>$x_{str}$</th>
<th>$x_{opt}$</th>
<th>x</th>
<th>$x_{str}$</th>
<th>$x_{opt}$</th>
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<td>$L_1$</td>
<td>4.903</td>
<td>4.495</td>
<td>$w_1$</td>
<td>11.807</td>
<td>13.889</td>
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<td>$L_2$</td>
<td>26.574</td>
<td>28.371</td>
<td>$w_2$</td>
<td>7.478</td>
<td>9.733</td>
</tr>
<tr>
<td>$L_3$</td>
<td>6.418</td>
<td>6.011</td>
<td>$w_3$</td>
<td>7.478</td>
<td>9.733</td>
</tr>
<tr>
<td>$L_4$</td>
<td>19.780</td>
<td>20.859</td>
<td>$w_4$</td>
<td>10.807</td>
<td>12.955</td>
</tr>
<tr>
<td>tap_l</td>
<td>11.869</td>
<td>12.550</td>
<td>tap_w</td>
<td>5.150</td>
<td>4.700</td>
</tr>
</tbody>
</table>
waveguide model is a good starting point for further tuning with a 3D FEM solver.

For a full-wave tuning step, ten independent variables were chosen for the zero-pole optimization routine. To obtain the results satisfying specifications, only thirteen iterations were needed. The final filter characteristics and convergence profile are presented in Figure 2 and Figure 3, respectively. The set of initial values $x_{str}$ and the final solutions $x_{opt}$ are presented in Table 1 (all dimensions in millimetres).

The layout of the proposed filter is shown in Figure 4. At a first sight, it looks as if the filter consisted of two resonators. In fact, the left-hand side of the filter (a SIW section with a stub) consists of two cavities which are produced by loading the long SIW waveguide section with the impedance of the stub (approximately in mid-point between

Figure 4. Layout of an in-line 3-pole generalized Chebyshev filter with one transmission zero using SIW technology. Cavity and coupling dimensions are measured from centre to centre of vias.

Figure 5. Electric field distribution of an in-line 3-pole generalized Chebyshev filter with one transmission zero using SIW technology at centre frequency.
Figure 6. (a) Measured (solid) and simulated (dashed) response of the a 3% FBW 3-pole generalized Chebyshev in-line SIW filter with one transmission zero, (b) photo of the fabricated filter.

irises \( w_1 \) and \( w_3 \). To prove this, the \( E \)-field distribution at centre frequency is presented in Figure 5. Three resonances, similar to the ones which can be expected in the in-line filter are clearly visible.

The comparison between the measured component and the ideal characteristics are presented in Figure 6. As can be seen, the filter return loss performance has been degraded to approximately 17 dB, which is still satisfactory result. The filter band has been slightly narrowed. All mentioned effects are mainly caused by fabrication errors which leads to detuning of the resonators. The transmission zero is placed at desired frequency, which ensures the assumed out-of-band rejection level. The filter insertion loss (IL) is at 2.3 dB level. Such value of IL is the effect of losses included in the dielectric substrate and finite conductivity of the top and bottom metallization layers as well as low quality of via holes metallization. All this elements strongly influence unloaded quality factors of the resonators. Nevertheless, measured characteristics are still acceptable.

### 3.2. Fifth Order in-line SIW Filter with Two Transmission Zeros

The second example is an in-line fifth order filter centred at \( f_0 = 4.95 \) GHz with bandwidth equal to 300 MHz. The filter has a 20 dB return loss level and two transmission zeros located at the lower and upper sides of the passband at \( f_{z_1} = 4.7 \) GHz and \( f_{z_2} = 5.14 \) GHz.

The filter consists of five directly connected resonant cavities with...
frequency-dependent couplings placed between the first and second resonator, and the fourth and fifth. Similarly as in previous case the dispersive couplings were implemented as a shorted stubs.

The initial filter response, obtained by optimizing a waveguide model, is shown in Figure 7. It can be observed that the filter bandwidth, as well as the return loss level, are different from the assumed values. The return loss level has been degraded to the level

![Figure 7. Scattering parameters for the initial dimensions ($x_{str}$ in Table 2) and final dimensions ($x_{opt}$ in Table 2) of a 5-pole generalized Chebyshev in-line SIW filter with two transmission zeros.](image)

![Figure 8. Convergence of the optimization routine for a 5-pole generalized Chebyshev filter with two transmission zeros.](image)

Table 2. Result of the optimization for the fifth order in-line SIW filter with two transmission zeros (the starting point is denoted by $x_{str}$ and is given in the second and fifth column, the final dimensions, denoted as $x_{opt}$ are given in the third and sixth column).

<table>
<thead>
<tr>
<th></th>
<th>$x_{str}$</th>
<th>$x_{opt}$</th>
<th>$x_{str}$</th>
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<tr>
<td>$L_2$</td>
<td>20.174</td>
<td>18.049</td>
<td>19.585</td>
<td>30.089</td>
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<tr>
<td>$L_5$</td>
<td>2.382</td>
<td>3.071</td>
<td>16.344</td>
<td>16.605</td>
</tr>
<tr>
<td>$L_6$</td>
<td>31.828</td>
<td>30.525</td>
<td>14.403</td>
<td>14.993</td>
</tr>
<tr>
<td>$L_7$</td>
<td>1.802</td>
<td>1.301</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$tap_w$</td>
<td>5.150</td>
<td>5.150</td>
<td>11.869</td>
<td>11.869</td>
</tr>
</tbody>
</table>
of 2 dB in the worst case. The transmission zero placed on the upper side of the passband has moved up by about 30 MHz. The second transmission zero has been shifted up by 10 MHz.

In this example fifteen independent variables were chosen for the optimization routine. To achieve characteristics satisfying the specifications twenty nine iterations were needed. The design is completely automated and takes about 6 hours of a CPU time on a workstation. In the near future this time will be further significantly reduced by applying an in-house 3D FEM solver with GPU acceleration [43–46]. The final filter characteristics and convergence profile are presented in Figure 7 and Figure 8, respectively. The set of starting values \( x_{\text{str}} \) and the final solutions \( x_{\text{opt}} \) are presented in Table 2 (all dimensions in millimetres). The layout of the proposed filter is shown in Figure 9. The field distribution shown in Figure 10

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**Figure 9.** Layout of an in-line 5-pole generalized Chebyshev filter with one transmission zeros using SIW technology. Cavity and coupling dimensions are measured from centre to centre of vias.

**Figure 10.** Electric field distribution in an in-line 5-pole generalized Chebyshev filter with two transmission zeros using SIW technology at centre frequency.
clearly shows five electrical resonators generating five poles which clearly demonstrates that stubs load long sections in the middle and act as a zero-generating elements.

Figure 11 presents a comparison between the measured and simulated scattering parameters of the fabricated filter. As can be seen filter return loss performance has been degraded to approximately 15 dB and the filter passband has been narrowed by 40 MHz.

Figure 11. (a) Measured (solid) and simulated (dashed) response of a 6% FBW in-line 5-pole SIW filter with two transmission zeros, (b) photo of the fabricated filter.

Figure 12. Convergence of the optimization routine for a 5-pole in-line SIW filter with two transmission zeros using a quasi-Newton algorithm with cost function defined using values of scattering parameters at position of the reflection and transmission zeros ([32]).
Nevertheless, the return loss performance is still acceptable. Both
transmission zeros are clearly observed on the left and right sides of
the passband. Each of them has been moved down by about 30 MHz
from their designed frequencies. The filter in-band insertion loss level
is approximately equal to 2.3 dB. As in a previous example, it is caused
by the dielectric losses and poor metallization of via-holes.

As a final note we would like to point out that this class of filters
is very difficult to design with other CAD procedures. To give an idea
of the difficulties, a quasi-Newton algorithm with cost function based
on scattering parameters evaluated at zeros and poles of a filtering
function [32] was tested. For the quasi-Newton algorithm with such
a cost function, satisfactory results were obtained after 300 iterations.
The convergence profile for this optimization process is presented in
Figure 12.

4. CONCLUSION

Computer aided design of pseudo-elliptic in-line SIW bandpass
filters with frequency-dependent couplings based on the zero-pole
optimization technique has been presented. The application of the
proposed procedure was demonstrated through the design of a new
type in-line substrate integrated waveguide (SIW) filters with arbitrary
placed transmission zeros. The initial dimensions of the designed
structures were obtained using µWave Wizard, a full-wave simulator
designed for the optimization of microwave structures. The HFSS
software was subsequently applied to achieve the final response of the
filters. Both tools have been integrated via Matlab software, in which
the zero-pole extraction and optimization method were implemented.
The numerical tests showed that to design the 3rd and 5th order filters
fewer than twenty and thirty iterations were required, respectively.
Measured characteristics of the presented examples validate both the
concept of new pseudo-elliptic in-line SIW filters and the proposed
design procedure.

ACKNOWLEDGMENT

This work has been carried out within the framework of the COST
IC0803 RFCSET project and supported by the Polish Ministry of
Science and Higher Education under Contract COST IC0803 618/N-
REFERENCES


33. Amari, S., “Synthesis of cross-coupled resonator filters using analytical gradient-based optimization technique,” *IEEE Transac-


