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EFFECT OF BEAMFORMING ON MULTI-ANTENNA TWO HOP ASYMMETRIC FADING CHANNELS WITH FIXED GAIN RELAYS

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Abstract—In this paper, we investigate the impact of beamforming (BF) on a multi-antenna two hop amplify-and-forward (AF) fixed gain relay network over Rician-Rayleigh and Rayleigh-Rician asymmetric fading channels, respectively. The network consists of a relay with single antenna used to assist the signal transmission from the source to the destination, both of which are equipped with multiple antennas. By using the channel state information (CSI), the maximal output signal-to-noise ratio (SNR) with optimal beamforming is first obtained. Then, the novel analytical expressions for the outage probability (OP), probability density function (PDF) and generalized moments of the maximal output SNR are derived. Moreover, the theoretical formulas of the Ergodic capacity and average symbol error rates (ASERs) with various modulation formats are also developed. To gain further insights, the asymptotic ASERs at high SNR are presented to reveal the diversity order and array gain of the multi-antenna relay network. Finally, computer simulations confirm the validity of the theoretical analysis and indicate the influence of antenna number and Rican factor on the system performance.

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1. INTRODUCTION

It is well-known that the application of multi-antenna, or multiple-input multiple-output (MIMO) technology in wireless communications has received considerable attention for many years, since it can increase the spectrum and energy efficiencies through transmit-receive beamforming (BF), and/or achieve high data rates through spatial diversity [1–7]. More recently, due to the ability of increasing the capacity, reliability and coverage of wireless networks with a given power or bandwidth, cooperative transmission such as relay technology [8–10] and network coding [11, 12] has become an active research topic in various wireless systems (see, e.g., [8] and the references therein). Among the commonly used relay protocols, including amplify-and-forward (AF), decode-and-forward (DF) and compress-and-forward (CF), AF scheme is of particular interest due to its less-required processing power and simplicity. Under this situation, the combination of two hop AF relay networks with multiple antennas has been well studied to achieve both advantages and provide significant performance benefits. Although a lot of multi-antenna relay schemes, including beamforming (BF) [13–15], joint precoding and limited feedback [16], antenna selection [17], and spatial multiplexing technology [18] have been proposed by far, BF is considered as a more practical strategy [14]. When the channel state information (CSI) is available, it has been found that the optimal BF based on maximizing the output SNR is equivalent to implement the maximal-ratio-transmission (MRT) and the maximal-ration-combining (MRC) at the source and destination, respectively [14, 15]. Besides, it is also widely recognized that performance analysis has been an important and active research topic in wireless communications and radar, since it can provide useful guidance for the engineers for the purpose of system design in practice. Until now, the performances of AF relay networks with optimal BF have been well investigated in a rich body of the related papers. For example, the outage probability (OP), probability density function (PDF), and average symbol error rate (ASER) of multi-antenna relaying over Rayleigh fading channels have been derived in [13, 15, 19]. Besides the most extensively studied Rayleigh fading distributions, the application of BF in AF relay systems and its performance analysis have also been presented in Nakagami-m fading environments [20].

In the aforementioned works, it is noteworthy that the fading conditions of the two hop links associated with the relay systems are all assumed to be symmetric. In practical wireless environments, however, asymmetric fading channel is considered as a more general and effective model to describe the wireless environments, since different links involved in a relay system are often subject to different fading distributions [21–23]. Unfortunately, quite few works have studied the two hop AF relaying with BF in asymmetric fading environments until now. Among them, taking a variable gain AF relay network with all nodes having a single antenna into account, the authors of [24] have investigated the performance of the relay systems over Rayleigh-Rician asymmetric fading channels. The extended works of [24] to multi-antenna BF relaying have been presented in [25, 26]. In [27], the author has also studied the performance of two hop asymmetric fading channels with fixed gain relaying. However, only the simplest case with all nodes equipped with a single antenna is considered, while no results on the multi-antenna fixed gain relaying with BF over Rician-Rayleigh and Rayleigh-Rician asymmetric fading channels have been reported by far. These observations motivate the work presented in this paper.

In this paper, we analyze the effect of beamforming on a multi-antenna two hop relay network with fixed gain, where the source-relay and relay-destination links experience asymmetric fading conditions. Specifically, we derive the analytical expressions for cumulative distributed functions (CDF), probability density function, outage probability, generalized moments, Ergodic capacity and average symbol error rates. Moreover, in order to gain variable insights, besides the complex derivation of analytical expressions, we have also obtained the asymptotic ASERs at high SNR, which are straightforward enough to reveal the diversity order and array gain of the multi-antenna relay network, and verify the benefits of beamforming in state-of-the-art wireless systems. Finally, we confirm the validity of the analytical results and investigate the effects of various parameters on the system performance through computer simulations.

The rest of the paper is outlined as follows: In Section 2, we first introduce the system model, and then obtain the maximal output SNR with optimal BF. In Section 3, we derive the analytical expressions for the OP, PDF and generalized SNR moments. The Ergodic capacity and ASERs of the relay network are developed in Section 4. Section 5 provides computer simulations to confirm the validity of the analysis. Finally, conclusions are drawn in Section 6.

Notation: Boldfaced letters represent matrices or vectors, $(\cdot)^H$ the Hermitian transpose, and $\|\cdot\|_F$ the Frobenius norm of a matrix. $\mathrm{E}(\cdot)$ denotes the expectation, $\exp(\cdot)$ the exponential function, $Q(x)$ the Gaussian Q-function defined as $Q(x) = (1/\sqrt{2\pi})\int_0^{\infty} e^{-y^2/2}dy$. $\mathbb{N}_C(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ stands for the complex Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$.

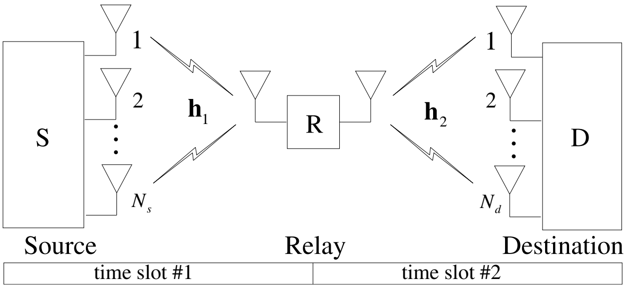

Figure 1. The model of two hop multi-antenna relay network.

2. SYSTEM DESCRIPTION

As shown in Fig. 1, we consider a two hop multi-antenna AF relay network, where a source S having N_s antennas communicates with a destination D having N_d antennas through a relay R with a single antenna. Such scenario can be easily found in various practical wireless communication systems, such as LTE-Advanced and WiMAX mobile systems. For example, when the link quality of two multi-antenna base stations degrades severely due to fading, shadowing and path loss, a relay deployed between the two base stations with a single antenna can be used to assist the signal transmission, conforming the feasibility, operability and prospect of our proposed scenario [7].

In this network, we assume that the direct link between the source and destination is unavailable, and the complete communication between S and D occurs in two time slots. During the first time slot, S performs transmit BF with weight vector $\mathbf{w}_1$ obeying $\|\mathbf{w}_1\|_F = 1$, and sends the signal to R through the fading channel $\mathbf{h}_1 \in \mathbb{C}^{N_s \times 1}$. The received signal at R can be expressed as

$$y_r(t) = \sqrt{P_1}\mathbf{h}_1^H \mathbf{w}_1 x(t) + n_1(t) \tag{1}$$

where P_1 denotes the transmit power at the source, $x(t)$ the transmitted signal, and $n_1(t)$ the zero mean additive white Gaussian noise (AWGN) with $\mathrm{E}[|n_1(t)|^2] = \sigma_1^2$.

During the second time slot, R amplifies the received signal with a fixed gain G as

$$G^2 = \frac{1}{P_s \mathrm{E}\left[\|\mathbf{h}_1\|_F^2\right] + \sigma_1^2} \tag{2}$$

and then transmits the signal to D through the fading channel $\mathbf{h}_2 \in \mathbb{C}^{N_d \times 1}$. By applying the received BF at D, the output signal can be written as

$$y_d(t)=\mathbf{w}_2^H\left[\sqrt{P_2}G\mathbf{h}_2 y_r(t)+\mathbf{n}_2(t)\right]$$
$$=\sqrt{P_1P_2}G\mathbf{w}_2^H\mathbf{h}_2\mathbf{h}_1^H\mathbf{w}_1 x(t)+\sqrt{P_2}G\mathbf{w}_2^H\mathbf{h}_2 n_1(t)+\mathbf{w}_2^H\mathbf{n}_2(t) \quad (3)$$

where P_2 represents the transmit power at R, $\mathbf{w}_2$ the normalized received BF weight vector satisfying $\|\mathbf{w}_2\|_F = 1$ and $\mathbf{n}_2(t) \sim \mathbb{N}_c(0, \sigma_2^2\mathbf{I}_{N_d})$.

After some necessary algebraic manipulations, it is straightforward to obtain the output instantaneous SNR as

$$\gamma_{eq}=\frac{P_1P_2G^2\mathbf{w}_2^H\mathbf{h}_2\mathbf{h}_1^H\mathbf{w}_1\mathbf{w}_1^H\mathbf{h}_1\mathbf{h}_2^H\mathbf{w}_2}{P_2G^2\mathbf{w}_2^H\mathbf{h}_2\mathbf{h}_2^H\mathbf{w}_2\sigma_1^2+\sigma_2^2} \quad (4)$$

It has been shown that when CSI is available, MRT and MRC are the widely used strategies to maximize the output SNR [14]. Thus, the BF weights at S and D are respectively given by

$$\mathbf{w}_1^{opt}=\mathbf{h}_1/\|\mathbf{h}_1\|_F \quad \text{and} \quad \mathbf{w}_2^{opt}=\mathbf{h}_2/\|\mathbf{h}_2\|_F \quad (5)$$

Consequently, the maximal output SNR is obtained as

$$\gamma=\frac{P_1\|\mathbf{h}_1\|_F^2 P_2\|\mathbf{h}_2\|_F^2}{P_2\sigma_1^2\|\mathbf{h}_2\|_F^2+\sigma_2^2/G^2}=\frac{\bar{\gamma}_1\|\mathbf{h}_1\|_F^2\bar{\gamma}_2\|\mathbf{h}_2\|_F^2}{\bar{\gamma}_2\|\mathbf{h}_2\|_F^2+1/\left(\sigma_1^2G^2\right)}\triangleq\frac{\gamma_1\gamma_2}{\gamma_2+C} \quad (6)$$

where $\gamma_1=\bar{\gamma}_1\|\mathbf{h}_1\|_F^2$ and $\gamma_2=\bar{\gamma}_2\|\mathbf{h}_2\|_F^2$ with $\bar{\gamma}_i=P_i/\sigma_i^2$ being the per hop average SNR, and $C=1/(\sigma_1^2G^2)$.

Interestingly, when both of the source and destination are equipped with one antenna, namely, $N_s=N_d=1$, $\|\mathbf{h}_1\|_F^2$ and $\|\mathbf{h}_2\|_F^2$ equals to the fading amplitudes α_1 and α_2 in [27, Eq. (1)], respectively. Thus the work of [27] is only a special case of our research.

In this paper, we focus on the following two scenarios, namely,

- Scenario (a): the S-R link follows independent and identically distributed (i.i.d.) Rician fading distribution while the R-D link experiences i.i.d. Rayleigh fading distribution, which is also defined as Rician-Rayleigh fading environments.
- Scenario (b): the S-R link follows i.i.d. Rayleigh fading while the R-D link experiences i.i.d. Rician fading, which is also defined as Rayleigh-Rician fading environments.

and study the performance of the considered AF relay network.

3. STATISTICAL PROPERTIES OF THE OUTPUT SNR

In this section, based on (6), the statistical properties of the output SNR for the considered relay network are investigated, by deriving the analytical expressions for the OP, PDF and generalized moments of the output SNR.

3.1. CDF and PDF of the Output SNR

In the case of scenario (a), the channel vector of the S-R link $\mathbf{h}_1 \in \mathbb{C}^{N_s\times 1}$ can be expressed as the sum of a line-of-sight component $\mathbf{h}_L$ and a scatter component $\mathbf{h}_S$, namely,

$$\mathbf{h}_1 = \sqrt{\frac{K}{K+1}}\mathbf{h}_L + \sqrt{\frac{1}{K+1}}\mathbf{h}_S \tag{7}$$

where K represents the Rician factor defined as the ratio of the powers of the line-of-sight (LoS) component to the scatter components. Thus, $\gamma_1 = \bar{\gamma}_1\|\mathbf{h}_1\|_F^2$ follows a noncentral Chi-square distribution with $2N_s$ degrees of freedom, whose PDF is given by

$$\begin{aligned} f_{\gamma_1}^a(x) = &\exp\left(-KN_s - \frac{1+K}{\bar{\gamma}_1}x\right)\left(\frac{x}{KN_s}\right)^{\frac{N_s-1}{2}}\left(\frac{1+K}{\bar{\gamma}_1}\right)^{\frac{N_s+1}{2}} \\ &I_{N_s-1}\left(2\sqrt{\frac{N_sK(1+K)}{\bar{\gamma}_1}x}\right) \end{aligned} \tag{8}$$

where $I_v(\cdot)$ denotes the mth-order modified Bessel function of the first kind, whose infinite-series representation can be expressed as [28, Eq. (8.445)]

$$I_m(x) = \sum_{n=0}^{\infty}\frac{(x/2)^{m+2n}}{n!\Gamma(m+n+1)} \tag{9}$$

where $\Gamma(\cdot)$ is the gamma function. In addition, since the channel vector of the R-D link satisfies $\mathbf{h}_2(t) \sim \mathbb{N}_c(0, \mathbf{I}_{N_d})$, $\gamma_2 = \bar{\gamma}_2\|\mathbf{h}_2\|_F^2$ follows a Chi-square distribution with $2N_d$ degrees of freedom, whose PDF is written as [24]

$$f_{\gamma_2}^a(x) = \frac{1}{\bar{\gamma}_2(N_d-1)!}\exp\left(-\frac{x}{\bar{\gamma}_2}\right)\left(\frac{x}{\bar{\gamma}_2}\right)^{N_d-1} \tag{10}$$

Theorem 1: The CDF of γ is given by

$$\begin{aligned} F_{\gamma}^a(x) = 1-2\exp\left(-KN_s-\frac{1+K}{\bar{\gamma}_1}x\right)\sum_{i=0}^{N_d-1}\sum_{n=0}^{\infty}\sum_{j=0}^{N_s+n-1}\binom{N_s+n-1}{j} \\ \frac{(KN_s)^n}{i!n!\Gamma(N_s+n)}\times\left(\frac{1+K}{\bar{\gamma}_1}x\right)^{\frac{2N_s+2n+i-j-1}{2}}\left(\frac{C}{\bar{\gamma}_2}\right)^{\frac{i+j+1}{2}} \\ K_{j-i+1}\left(2\sqrt{\frac{C(1+K)}{\bar{\gamma}_1\bar{\gamma}_2}x}\right) \end{aligned} \tag{11}$$

Proof: See appendix A.

Differentiating (11) with respect to x, and applying the following equality [28, Eq. (8.486.12)]

$$x\frac{d}{dx}K_v\left(x\right)+vK_v\left(x\right)=-xK_{v-1}\left(x\right) \tag{12}$$

after some necessary mathematical calculation, one can obtain

$$\begin{aligned} f_{\gamma}^{a}\left(x\right)=&2\exp\left(-KN_s-\frac{\left(K+1\right)x}{\bar{\gamma}_1}\right)\sum_{i=0}^{N_d-1}\sum_{n=0}^{\infty}\sum_{j=0}^{N_s+n-1}\binom{N_s+n-1}{j} \\ &\frac{\left(KN_s\right)^n}{i!n!\Gamma\left(N_s+n\right)}\left(\frac{\left(K+1\right)x}{\bar{\gamma}_1}\right)^{\frac{2N_s+2n+i-j-1}{2}}\left(\frac{C}{\bar{\gamma}_2}\right)^{\frac{i+j+1}{2}} \\ &\times\left[\sqrt{\frac{C\left(1+K\right)}{\bar{\gamma}_1\bar{\gamma}_2}}xK_{j-i}\left(2\sqrt{\frac{C(1+K)}{\bar{\gamma}_1\bar{\gamma}_2}x}\right)\right. \\ &\left.-\left(\frac{i}{x}-\frac{\left(K+1\right)}{\bar{\gamma}_1}\right)K_{j-i+1}\left(2\sqrt{\frac{C(1+K)}{\bar{\gamma}_1\bar{\gamma}_2}x}\right)\right] \end{aligned} \tag{13}$$

In the case of scenario (b), by using the similar procedures, the CDF and PDF of the output SNR are respectively given by

$$\begin{aligned} F_{\gamma}^{b}\left(x\right)=&1-2\exp\left(-KN_d-\frac{x}{\gamma_1}\right)\sum_{i=0}^{N_s-1}\sum_{n=0}^{\infty}\sum_{j=0}^{i}\binom{i}{j}\left(\frac{C\left(K+1\right)}{\bar{\gamma}_2}\right)^{\frac{N_d+n+j}{2}} \\ &\times\frac{\left(KN_d\right)^n}{i!n!\Gamma\left(N_d+n\right)}\left(\frac{x}{\bar{\gamma}_1}\right)^{\frac{N_d+n+2i-j}{2}}K_{N_d+n-j}\left(2\sqrt{\frac{C\left(1+K\right)}{\bar{\gamma}_1\bar{\gamma}_2}x}\right) \end{aligned} \tag{14}$$

$$\begin{aligned} f_{\gamma}^{b}\left(x\right)=&2\exp\left(-KN_d-\frac{x}{\gamma_1}\right)\sum_{i=0}^{N_s-1}\sum_{n=0}^{\infty}\sum_{j=0}^{i}\binom{i}{j}\left(\frac{C\left(K+1\right)}{\bar{\gamma}_2}\right)^{\frac{N_d+n+j}{2}} \\ &\frac{\left(KN_d\right)^n}{i!n!\Gamma\left(N_d+n\right)}\left(\frac{x}{\bar{\gamma}_1}\right)^{\frac{N_d+n+2i-j}{2}}\times\left[\sqrt{\frac{C\left(1+K\right)}{\bar{\gamma}_1\bar{\gamma}_2x}}K_{N_d+n-j-1}\right. \\ &\left.\left(2\sqrt{\frac{C\left(1+K\right)}{\bar{\gamma}_1\bar{\gamma}_2}x}\right)-\left(\frac{i}{x}-\frac{1}{\bar{\gamma}_1}\right)K_{N_d+n-j}\left(2\sqrt{\frac{C(1+K)}{\bar{\gamma}_1\bar{\gamma}_2}x}\right)\right] \end{aligned} \tag{15}$$

Although the analytical expressions of CDF and PDF appear complicated, it is worth-mentioning that the infinite-series representations can be truncated to finite terms without sacrificing the numerical accuracy, thus providing a more computationally efficient alternative method to evaluate the performance of the multi-antenna AF relay network in comparison with Monte Carlo simulations.

3.2. Outage Probability

In wireless communications, OP is often adopted as an important measurement for the quality-of-service (QoS), which is defined as the probability that the output SNR γ falls below a threshold γ_{th}, namely,

$$P_o = \Pr\left(\gamma \leq \gamma_{th}\right) = F_\gamma\left(\gamma_{th}\right) \tag{16}$$

Replacing x with γ_{th} in (11) and (14) respectively, the outage probability of the multi-antenna AF fixed gain relay network over Rician-Rayleigh and Rayleigh-Rician asymmetric fading channels can be obtained as

$$P_o^a\left(\gamma\right) = F_\gamma^a(x)|_{x=\gamma_{th}} = F_\gamma^a(\gamma_{th}) \tag{17a}$$

$$P_o^b\left(\gamma\right) = F_\gamma^b(x)|_{x=\gamma_{th}} = F_\gamma^b(\gamma_{th}) \tag{17b}$$

3.3. Generalized SNR Moments

The definition of the mth-order SNR moments is given by

$$M\left(\gamma^m\right) = \int_0^\infty x^m f_\gamma\left(x\right)dx = m\int_0^\infty x^{m-1}\left[1 - F_\gamma\left(x\right)\right]dx \tag{18}$$

As for scenario (a), substituting (11) into (18) yields

$$M^a\left(\gamma^m\right) = 2me^{-KN_s}\sum_{i=0}^{N_d-1}\sum_{n=0}^{\infty}\sum_{j=0}^{N_s+n-1}\binom{N_s+n-1}{j}$$

$$\frac{\left(KN_s\right)^n}{i!n!\Gamma\left(N_s+n\right)}\left(\frac{1+K}{\bar{\gamma}_1}\right)^{\frac{2N_s+2n+i-j-1}{2}}\left(\frac{C}{\bar{\gamma}_2}\right)^{\frac{i+j+1}{2}}$$

$$\times\underbrace{\int_0^\infty x^{\frac{2N_s+2n+2m+i-j-3}{2}}\exp\left(-\frac{1+K}{\bar{\gamma}_1}x\right)K_{j-i+1}\left(2\sqrt{\frac{C\left(1+K\right)}{\bar{\gamma}_1\bar{\gamma}_2}x}\right)dx}_{I_1} \tag{19}$$

To solve the integral term I_1 in (19), we apply the following identity [28, Eq. (6.643.3)]

$$\int_0^\infty x^{\mu-\frac{1}{2}}\exp\left(-\alpha x\right)K_{2v}\left(2\beta\sqrt{x}\right)dx$$

$$= \exp\left(\frac{\beta^2}{2\alpha}\right)\alpha^{-\mu}\frac{\Gamma\left(\mu+v+\frac{1}{2}\right)\Gamma\left(\mu-v+\frac{1}{2}\right)}{2\beta}W_{-\mu,v}\left(\frac{\beta^2}{\alpha}\right) \tag{20}$$

and the relationship [29, p505]

$$W_{k,m}\left(z\right) = z^{m+\frac{1}{2}}\exp\left(-\frac{z}{2}\right)U\left(m-k+\frac{1}{2}, 2m+1; z\right) \tag{21}$$

where $W_{k,m}(\cdot)$ and $U(\cdot, \cdot; \cdot)$ denotes the Whittaker function and the confluent hypergeometric function of the second kind [28], respectively, and obtain

$$\begin{aligned} I_1 &= \frac{1}{2}\Gamma(N_s+n+m)\,\Gamma(N_s+n+m+i-j-1) \\ &\quad \frac{\beta_c^{j-i+1}}{\alpha_c^{N_1+n+m}} U\left(N_s+n+m, j-i+2; \frac{\beta_c^2}{\alpha_c}\right) \end{aligned} \tag{22}$$

where $\alpha_c = (1+K)/\bar{\gamma}_1$, $\beta_c = \sqrt{(C(1+K))/(\bar{\gamma}_1\bar{\gamma}_2)}$. Furthermore, using (19) and (22) yields

$$\begin{aligned} M^a(\gamma^m) &= m e^{-KN_s} \sum_{i=0}^{N_d-1} \sum_{n=0}^{\infty} \sum_{j=0}^{N_s+n-1} \binom{N_s+n-1}{j} \frac{(KN_s)^n}{i!n!} \left(\frac{1+K}{\bar{\gamma}_1}\right)^{-m} \\ &\quad \left(\frac{C}{\bar{\gamma}_2}\right)^{j+1} \times \frac{\Gamma(N_s+n+m)\,\Gamma(N_s+n+m+i-j-1)}{\Gamma(N_s+n)} \\ &\quad U\left(N_s+n+m, j-i+2; \frac{C}{\bar{\gamma}_2}\right) \end{aligned} \tag{23}$$

Similar to scenario (a), one can also obtain the analytical expression of the generalized SNR moments in scenario (b), giving

$$\begin{aligned} M^b(\gamma^m) &= m e^{-KN_d} \sum_{i=0}^{N_s-1} \sum_{n=0}^{\infty} \sum_{j=0}^{i} \binom{i}{j} C^{\frac{N_d+n+j}{2}} \bar{\gamma}_1^{m+i-j} \left(\frac{K+1}{\bar{\gamma}_2}\right)^{N_d+n} \\ &\quad \frac{(KN_d)^n}{i!n!\Gamma(N_d+n)} \times \frac{\Gamma(N_d+m+n+i-j)\,\Gamma(m+i)}{\Gamma(N_d+n)} \\ &\quad U\left(N_d+m+n+i-j, N_d+n-j+1; \frac{C(1+K)}{\bar{\gamma}_2}\right) \end{aligned} \tag{24}$$

4. PERFORMANCE ANALYSIS

4.1. Ergodic Capacity

According to the information theory, the Ergodic capacity is defined as the expectation of the instantaneous mutual information between the source and destination, namely

$$C_{Erg} = \frac{1}{2}\mathrm{E}\left[\log_2(1+\gamma)\right] = \frac{1}{2}\int_0^{\infty} \log(1+x) f_\gamma(x)\,dx \tag{25}$$

where $f_\gamma(x)$ is the PDF of the output SNR. Substituting (11) into (25), we can obtain the Ergodic capacity of scenario (a) as

$$C_{Erg}^{a} = e^{-KN_d} \sum_{i=0}^{N_d-1} \sum_{n=0}^{\infty} \sum_{j=0}^{N_s+n-1} \binom{N_s+n-1}{j} \frac{(KN_s)^n}{i!n!\Gamma(N_s+n)}$$
$$\left(\frac{(K+1)}{\bar{\gamma}_1}\right)^{\frac{2N_s+2n+i-j-1}{2}} \left(\frac{C}{\bar{\gamma}_2}\right)^{\frac{i+j+1}{2}} \times \left\{ \int_0^\infty \frac{(K+1)}{\bar{\gamma}_1} x^{\frac{2N_s+2n+i-j-1}{2}} \right.$$
$$\exp\left(-\frac{(K+1)x}{\bar{\gamma}_1}\right) \log(1+x) K_{j-i+1}\left(2\sqrt{\frac{C(1+K)}{\bar{\gamma}_1\bar{\gamma}_2}x}\right) dx$$
$$+ \int_0^\infty \sqrt{\frac{C(1+K)}{\bar{\gamma}_1\bar{\gamma}_2}} x^{\frac{2N_s+2n+i-j-2}{2}} \exp\left(-\frac{(K+1)x}{\bar{\gamma}_1}\right) \log(1+x)$$
$$K_{j-i}\left(2\sqrt{\frac{C(1+K)}{\bar{\gamma}_1\bar{\gamma}_2}x}\right) dx - \int_0^\infty i x^{\frac{2N_s+2n+i-j-3}{2}}$$
$$\left. \exp\left(-\frac{(K+1)x}{\bar{\gamma}_1}\right) \log(1+x) K_{j-i+1}\left(2\sqrt{\frac{C(1+K)}{\bar{\gamma}_1\bar{\gamma}_2}x}\right) dx \right\} \quad (26)$$

However, due to the mathematical complexity of the integral terms, it is impossible to obtain a closed-form expression for (26). To overcome this problem, we expand the function $\log_2(1+\gamma)$ with Taylor series, and thus obtain the second-order approximation of the Ergodic capacity, giving [15]

$$C_{Erg}^{a} \approx \frac{1}{2}\log_2(e)\left[\ln(1+M^a(\gamma)) - \frac{M_a(\gamma^2) - [M^a(\gamma)]^2}{2[1+M^a(\gamma)]^2}\right] \quad (27)$$

where $M^a(\gamma)$ and $M^a(\gamma^2)$ are the first- and second-order moments of the output SNR, respectively, both of which can be calculated through (23). Similarly, the approximate Ergodic capacity of the considered relay network in scenario (b) can also be obtained by using (24), namely,

$$C_{Erg}^{b} \approx \frac{1}{2}\log_2(e)\left[\ln\left(1+M^b(\gamma)\right) - \frac{M_b(\gamma^2) - \left[M^b(\gamma)\right]^2}{2[1+M^b(\gamma)]^2}\right] \quad (28)$$

where both $M^b(\gamma)$ and $M^b(\gamma^2)$ can be calculated through (24) with $m=1$ and $m=2$, respectively.

4.2. Average Symbol Error Rate

It is well-known that the ASERs of wireless systems in terms of various modulation formats over fading channels can be expressed as [30]

$$P_s = \mathrm{E}\left[aQ\left(\sqrt{2b\gamma}\right)\right] = \int_0^{\infty} aQ\left(\sqrt{2bx}\right) f_\gamma\left(x\right) dx \tag{29}$$

where a and b are the specific modulations constants, such as $(a = 1, b = 1)$ for BPSK and $(a = 2(M-1)/M,\ b = 3/(M^2-1))$ for M-PAM. Meanwhile, (29) can also provide approximate ASER evaluation for other modulations, e.g., $(a = 2, b = \sin^2(\pi/M))$ for M-PSK $(M \geq 4)$), and the modulation formats whose ASER can be expressed as a finite weighted summation of the Q- and Q^2-function, e.g., M-QAM [7]. After integration by parts, (29) can be rewritten with a single integral as

$$P_s = \frac{a}{2}\sqrt{\frac{b}{\pi}} \int_0^{\infty} \frac{e^{-bx}}{\sqrt{x}} F_\gamma\left(x\right) dx \tag{30}$$

We first consider the asymmetric fading channel of scenario (a). By substituting the analytical CDF expression of (11) into (30), one can obtain

$$\begin{aligned} P_s^a\left(\gamma\right) = {} & \frac{a}{2} - a\sqrt{\frac{b}{\pi}} \exp\left(-KN_s\right) \sum_{i=0}^{N_d-1} \sum_{n=0}^{\infty} \sum_{j=0}^{N_s+n-1} \begin{pmatrix} N_s+n-1 \\ j \end{pmatrix} \\ & \frac{\left(KN_s\right)^n}{i!n!\Gamma\left(N_s+n\right)} \left(\frac{1+K}{\bar{\gamma}_1}\right)^{\frac{2N_s+2n+i-j-1}{2}} \left(\frac{C}{\bar{\gamma}_2}\right)^{\frac{i+j+1}{2}} \\ & \underbrace{\times \int_0^{\infty} x^{\frac{2N_s+2n+i-j-2}{2}} \exp\left(-bx - \frac{1+K}{\bar{\gamma}_1} x\right) K_{j-i+1}\left(2\sqrt{\frac{C(1+K)}{\bar{\gamma}_1\bar{\gamma}_2}x}\right) dx}_{I_2} \end{aligned} \tag{31}$$

With the help of (20) and (21), the integral term required to evaluate the ASER is given by

$$\begin{aligned} I_2 \ = \ & \frac{1}{2}\Gamma\left(N_s+n+\frac{1}{2}\right)\Gamma\left(N_s+n+i-j-\frac{1}{2}\right) \frac{\beta_p^{j-i+1}}{\alpha_p^{N_1+n+1/2}} \\ & U\left(N_s+n+\frac{1}{2}, j-i+2; \frac{\beta_p^2}{\alpha_p}\right) \end{aligned} \tag{32}$$

where $\alpha_p = b + (1+K)/\bar{\gamma}_1$, $\beta_p = \sqrt{(C\left(1+K\right))/(\bar{\gamma}_1\bar{\gamma}_2)}$. Using (31) and (32) yields

$$P_s^a(\gamma) = \frac{a}{2} - \frac{a}{2}\sqrt{\frac{b}{\pi}}\exp(-KN_s)\sum_{i=0}^{N_d-1}\sum_{n=0}^{\infty}\sum_{j=0}^{N_s+n-1}\binom{N_s+n-1}{j}$$
$$\frac{(KN_s)^n}{i!n!\Gamma\left(N_s+n\right)}\left(\frac{1+K}{\bar{\gamma}_1}\right)^{\frac{2N_s+2n+i-j-1}{2}}\left(\frac{C}{\bar{\gamma}_2}\right)^{\frac{i+j+1}{2}}$$
$$\times\Gamma\left(N_s+n+\frac{1}{2}\right)\Gamma\left(N_s+n+i-j-\frac{1}{2}\right)$$
$$\frac{\beta_p^{j-i+1}}{\alpha_p^{N_s+n+1/2}}U\left(N_s+n+\frac{1}{2}, j-i+2; \frac{\beta_p^2}{\alpha_p}\right) \quad (33)$$

Next, as for scenario (b), by substituting (14) into (30), and using the similar derivation as obtaining (33), the ASER of the considered AF relay network can be calculated as

$$P_s^b\left(\gamma\right) = \frac{a}{2} - a\sqrt{\frac{b}{\pi}}e^{-KN_d}\sum_{i=0}^{N_s-1}\sum_{n=0}^{\infty}\sum_{j=0}^{i}\binom{i}{j}\frac{(KN_d)^n}{i!n!\Gamma\left(N_d+n\right)}$$
$$\left(\frac{1}{\bar{\gamma}_1}\right)^{\frac{N_d+n+2i-j}{2}}\left(\frac{C\left(1+K\right)}{\bar{\gamma}_2}\right)^{\frac{N_d+n+j}{2}}$$
$$\times\Gamma\left(N_d+n+i-j+\frac{1}{2}\right)\Gamma\left(i+\frac{1}{2}\right)$$
$$\frac{\beta_q^{N_d+n-j}}{\alpha_q^{N_d+n+i-j+\frac{1}{2}}}U\left(N_d+n+i-j+\frac{1}{2}, N_d+n-j+1; \frac{\beta_q^2}{\alpha_q}\right) \quad (34)$$

where $\alpha_q = 1/\bar{\gamma}_1$, $\beta_q = \sqrt{(C\left(1+K\right))/(\bar{\gamma}_1\bar{\gamma}_2)}$.

4.3. Asymptotic Expressions at High SNR

Although (33) and (34) are valid and accurate, it is too complicated to provide a clear insight. Thus, in what follows, we derive the asymptotic ASER expressions at high SNR to reveal the diversity and array gain of the multi-antenna relay network.

First of all, to obtain the asymptotic CDF at high SNR over Rician-Rayleigh fading channels, we apply the series expansion to the exponential function $e^{-(1+K)x/\bar{\gamma}_1}$ and the modified Bessel function

$K_n(x)$ as [28, Eq. (8.446)]

$$K_n(x) = \frac{1}{2}\sum_{k=0}^{n-1}(-1)^k \frac{(n-k-1)!}{k!}\left(\frac{x}{2}\right)^{2k-n} + (-1)^{n+1} \sum_{k=0}^{\infty}\frac{1}{k!\,(n+k)!}\left(\frac{x}{2}\right)^{n+2k}\left[\ln\frac{x}{2} - \frac{1}{2}\psi(k+1) - \frac{1}{2}\psi(n+k+1)\right] \quad (35)$$

where $\psi(z)$ is the digamma function [28]. With the help of the even property of the modified Bessel function as $K_n(x) = K_{-n}(x)$, it is not difficult to find that x^n ($n < N_s$) terms sum to zero for $N_s < N_d$, x^n ($n < N_d$) terms sum to zero for $N_s < N_d$, and x^n ($n < N_{Eq}$) terms sum to zero for $N_s = N_d = N_{Eq}$. Hence, the asymptotic $F_r^a(x)$ in high SNR region can be derived without lower order terms as

$$F_\gamma^a(x) = \begin{cases} \beta_{N_s}^a x^{N_s} + O\left(x^{N_s+1}\right), & N_s < N_d \\ \beta_{N_{Eq}}^a x^{N_{Eq}} + O\left(x^{N_{Eq}+1}\right), & N_s = N_d = N_{Eq} \\ \beta_{N_d}^a x^{N_d} + O\left(x^{N_d+1}\right), & N_s > N_d \end{cases} \quad (36)$$

where

$$\beta_N^a = e^{-KN_s}\sum_{i=0}^{N_d-1}\sum_{n=0}^{\infty}\sum_{j=0}^{N_s+n-1}\binom{N_s+n-1}{j}\frac{(KN_s)^n}{i!n!\Gamma(N_s+n)}\left(\frac{(1+K)}{\bar{\gamma}_1}\right)^N \Theta_a \quad (37)$$

In (37), the closed-from expression of Θ_a can be obtained in the following three scenarios, namely for $j-i+1>0$

$$\Theta_a = \sum_{k=0, N\geq N_s+n+i-j+k-1}^{j-i-2}\frac{(-1)^{N-N_s-n-i+j}(j-i-k-2)!}{k!\,(N-N_s-n-i+j-k+1)!}\left(\frac{C}{\bar{\gamma}_2}\right)^{k+i} + \sum_{k=0}^{N-N_s-n-1}\frac{(-1)^{N-N_s-n-k+j-i}}{k!\,(j-i+k+1)!\,(N-N_s-n-k-1)!}\left(\frac{C}{\bar{\gamma}_2}\right)^{k+j}\Upsilon_{j-i-1} \quad (38a)$$

for $j-i+1<0$

$$\Theta_a = \sum_{k=0, N\geq N_s+n+k-1}^{i-j}\frac{(-1)^{N-N_s-n}(i-j-k)!}{k!\,(N-N_s-n-k+1)!}\left(\frac{C}{\bar{\gamma}_2}\right)^{k+j} + \sum_{k=0}^{N-N_s-n-i+j+1}\frac{(-1)^{N-N_s-n-k}}{k!\,(k-j+i-1)!\,(N-N_s-n-i+j-k+1)!}\left(\frac{C}{\bar{\gamma}_2}\right)^{k+j}\Upsilon_{i-j+1} \quad (38b)$$

for $j-i-1=0$

$$\Theta_a = \sum_{k=0}^{N-N_s-n+1}\frac{(-1)^{N-N_s-n-k+1}}{k!k!\,(N-N_s-n-k+1)!}\left(\frac{C}{\bar{\gamma}_2}\right)^{k+i+1}\Upsilon_0 \quad (38c)$$

where

$$\Upsilon_z = \ln\left(\frac{C(1+K)x}{\bar{\gamma}_1\bar{\gamma}_2}\right) - \psi(k+1) - \psi(z+k+1) \quad (39)$$

Thus, substituting (36) into (30), we can derive the asymptotic ASER for the scenario (a) with respect to diversity order G_D^a and array gain G_A^a as

$$P_s^{a,\infty} = (G_A^a \bar{\gamma}_1)^{-G_D^a} + O\left(\bar{\gamma}_1^{-(G_D^a+1)}\right) \quad (40)$$

where G_D^a and G_A^a are respectively given by

$$G_D^a = \min(N_s, N_d) \quad (41)$$

$$G_A^a = \begin{cases} \left(\frac{a\beta_N^a \Gamma\left(N_s+\frac{1}{2}\right)}{2\sqrt{\pi}b^{N_s}}\right)^{-1/N_s}, & N_s < N_d \\ \left(\frac{a\beta_N^a \Gamma\left(N_{Eq}+\frac{1}{2}\right)}{2\sqrt{\pi}b^{N_{Eq}}}\right)^{-1/N_{Eq}}, & N_s = N_d = N_{Eq} \\ \left(\frac{a\beta_N^a \Gamma\left(N_d+\frac{1}{2}\right)}{2\sqrt{\pi}b^{N_d}}\right)^{-1/N_d}, & N_s > N_d \end{cases} \quad (42)$$

Similarly, the asymptotic $F_r^b(x)$ at high SNR can also be derived without lower order terms as

$$F_\gamma^b(x) = \begin{cases} \beta_{N_s}^b x^{N_s} + O\left(x^{N_s+1}\right), & N_s < N_d \\ \beta_{N_{Eq}}^b x^{N_{Eq}} + O\left(x^{N_{Eq}+1}\right), & N_s = N_d = N_{Eq} \\ \beta_{N_d}^b x^{N_d} + O\left(x^{N_d+1}\right), & N_s > N_d \end{cases} \quad (43)$$

where

$$\beta_N^b = e^{-KN_d} \sum_{i=0}^{N_s-1} \sum_{n=0}^{\infty} \sum_{j=0}^{i} \binom{i}{j} \frac{(KN_d)^n}{i!n!\Gamma(N_d+n)} \left(\frac{1}{\bar{\gamma}_1}\right)^N \Theta_b \quad (44)$$

In (44), the closed-from expression of Θ_b can also be obtained in the following three scenarios, namely for $N_d + n - j > 0$

$$\Theta_b = \sum_{k=0, N\geq i+k}^{N_d+n-j-1} \frac{(-1)^{N-i+1}(N_d+n-j-k-1)!}{k!(N-i-k)!} \left(\frac{C(1+K)}{\bar{\gamma}_2}\right)^{k+j}$$
$$+ \sum_{k=0}^{N-N_d-n-i+j} \frac{(-1)^{N-k-i}}{k!(N_d+n-j+k)!(N-N_d-k-n-i+j)!}$$
$$\left(\frac{C(1+K)}{\bar{\gamma}_2}\right)^{N_d+k+n} \Upsilon_{N_d+n-j} \quad (45a)$$

for $N_d + n - j < 0$

$$\Theta_b = \sum_{k=0, N \geq N_d+n+k+i-j}^{j-n-N_d-1} \frac{(-1)^{N-N_d-n-i+j+1} (j - N_d - n - k - 1)!}{k! (N - N_d - k - n - i + j)!} \left(\frac{C(1+K)}{\bar{\gamma}_2}\right)^{N_d+k+n} + \sum_{k=0}^{N-i} \frac{(-1)^{N-N_d-n-k-i+j}}{k! (k + j - N_d - n)! (N - i - k)!} \left(\frac{C(1+K)}{\bar{\gamma}_2}\right)^{k+j} \Upsilon_{j-N_d-n} \quad (45b)$$

for $N_d + n - j = 0$

$$\Theta_b = \sum_{k=0}^{N-i} \frac{(-1)^{N-i-k}}{k!k! (N - i - k)!} \left(\frac{C(1+K)}{\bar{\gamma}_2}\right)^{k+j} \Upsilon_0 \quad (45c)$$

Finally, by substituting (43) into (30), the asymptotic ASER for the scenario (b) with respect to diversity order G_D^b and array gain G_A^b can be expressed as

$$P_s^{b,\infty} = \left(G_A^b \bar{\gamma}_1\right)^{-G_D^b} + O\left(\bar{\gamma}_1^{-(G_D^b+1)}\right) \quad (46)$$

where G_D^b and G_A^b are respectively given by

$$G_D^b = \min(N_s, N_d) \quad (47)$$

$$G_A^b = \begin{cases} \left(\frac{a\beta_N^b \Gamma\left(N_s+\frac{1}{2}\right)}{2\sqrt{\pi} b^{N_s}}\right)^{-1/N_s}, & N_s < N_d \\ \left(\frac{a\beta_N^b \Gamma\left(N_{Eq}+\frac{1}{2}\right)}{2\sqrt{\pi} b^{N_{Eq}}}\right)^{-1/N_{Eq}}, & N_s = N_d = N_{Eq} \\ \left(\frac{a\beta_N^b \Gamma\left(N_d+\frac{1}{2}\right)}{2\sqrt{\pi} b^{N_d}}\right)^{-1/N_d}, & N_s > N_d \end{cases} \quad (48)$$

It is revealed in (42) and (48) that due to the employment of a single antenna at R, the diversity order of the considered network is determined by the minimum number of the antennas at S and D, justifying the maximum diversity order can be achieved in a multi-antenna two hop AF relay network through implementing BF at the source and destination. Since all of the nodes are equipped with a single antenna, this interesting property was unfortunately neglected in [27].

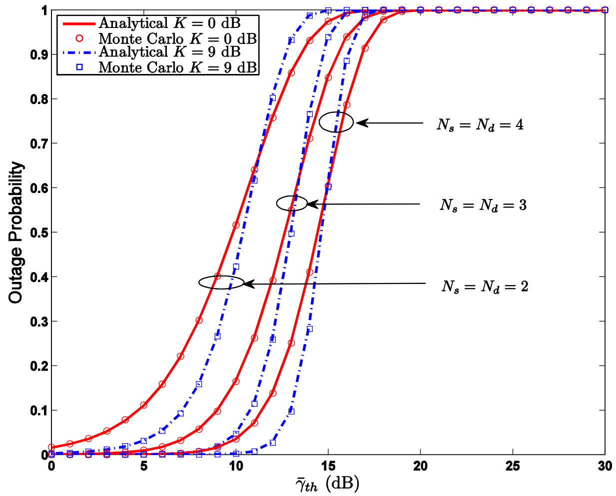

Figure 2. Outage probability of the relay network over Rician-Rayleigh fading channels with $\bar{\gamma}_1 = \bar{\gamma}_2 = 10\,\text{dB}$.

5. COMPUTER SIMULATIONS

In this section, considering two asymmetric fading channels, namely, Rician-Rayleigh and Rayleigh-Rician fading channels, we carry out computer simulations to demonstrate the validity of the presented analytical expressions, and show the superiority of BF strategies in the two hop AF relay networks. Moreover, the impact of Rician factor on the system performance is also studied. In all the plots, the label (N_s, N_d) represents the combination of antenna number at the source and destination.

Figures 2 and 3 depict, respectively, the OP curves against SNR with various antenna configurations and Rician factors. It is found that the OP curves are highly consistent with Monte Carlo simulations irrespective of different channel parameters, justifying the effectiveness of the derived analytical expressions given by (11) and (14). In addition, one can also observe that the OP performance is improved with the increase of antenna number and Rician factor, which confirms the benefits of employing multiple antennas as well as enhancing the LOS components in the fading channels. From these two figures, we can find that the impact of Rician factor K on the relay network in Rician-Rayleigh fading is greater than that in Rayleigh-Rician fading. This is because the Rician factor K only affect the numerator term of (6) over the former fading channels, while it affects both the numerator and denominator terms of (6) over the latter fading channels.

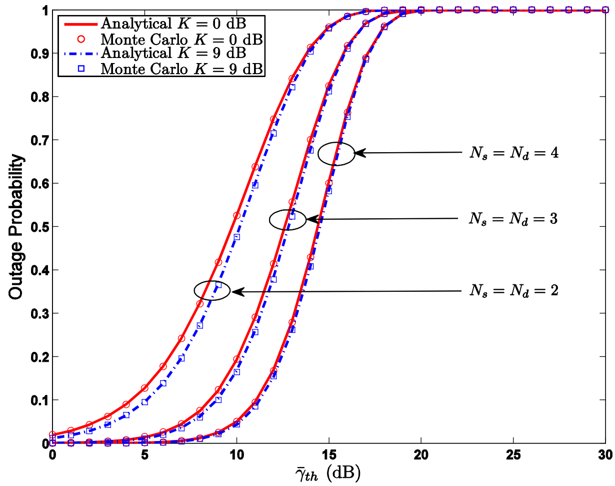

Figure 3. Outage probability of the relay network over Rayleigh-Rician fading channels with $\bar{\gamma}_1 = \bar{\gamma}_2 = 10\,\text{dB}$.

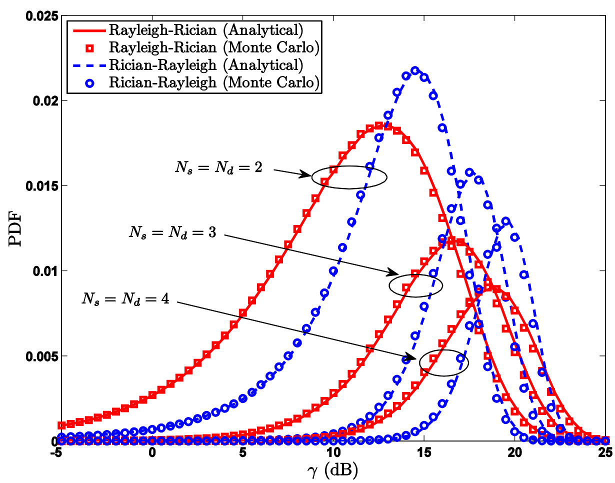

Figure 4. PDF of the output SNR for different antenna configurations over asymmetric fading channels with $\bar{\gamma}_1 = \bar{\gamma}_2 = 15\,\text{dB}$ and $K = 6\,\text{dB}$.

Figure 4 gives the PDF curves in the case of different antenna configurations and fading channels. We see that the PDF curves match well with the Monte Carlo experiments, confirming the validity of the theoretical analysis. Furthermore, we find that with the fixed antenna number and K factor, the PDF curves of the fixed gain AF relay network over Rician-Rayleigh fading channels shift to

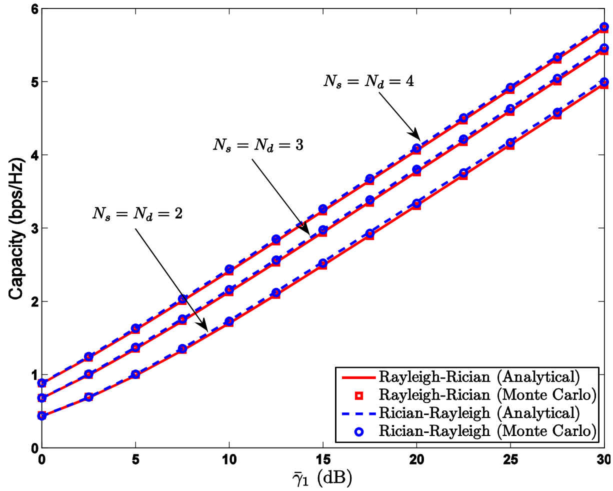

Figure 5. Ergodic capacity of the relay network over asymmetric fading channels with $\bar{\gamma}_1 = \bar{\gamma}_2$ and $K = 6\,\text{dB}$.

higher SNR values than those over Rayleigh-Rician fading channels. Since most performance measures monotonically with the output SNR, the scenario implies that Rician-Rayleigh fading channels exhibit an improved performance in comparison with Rayleigh-Rician fading channels. Thus, in practice, the relay should be deployed close to the source, so that better channel condition in the first hop (Rician fading) can be used to improve the system performance with a given transmit power.

Figure 5 illustrates the Ergodic capacity of the AF relay network for various antenna combinations. Apparently, the good match between the analytical and simulation results shows the satisfied accuracy of the approximate expressions giving by (27) and (28). Meanwhile, according to the figure, we can conclude that the more antennas are employed, the higher Ergodic capacity of the relay network can be achieved.

Figures 6 and 7 show, respectively, the theoretical and asymptotic ASERs versus SNR with different antenna configurations and Rician factors in terms of QPSK modulation format. Just as we expect, the ASER curves calculated through theoretical formulas are highly consistent with Monte Carlo experiments, and all of them coincide well with the asymptotic results at high SNR, where the fact that the diversity order equals to min (N_s, N_d) can also be found. This scenario confirms the validity of the presented error performance analysis. Meanwhile, with the increase of antenna number, the ASER

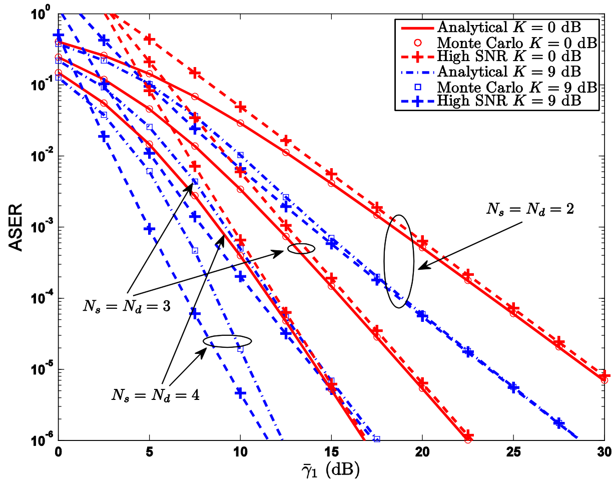

Figure 6. ASER versus SNR for QPSK modulation format over Rician-Rayleigh fading channels with $\bar{\gamma}_1 = \bar{\gamma}_2$.

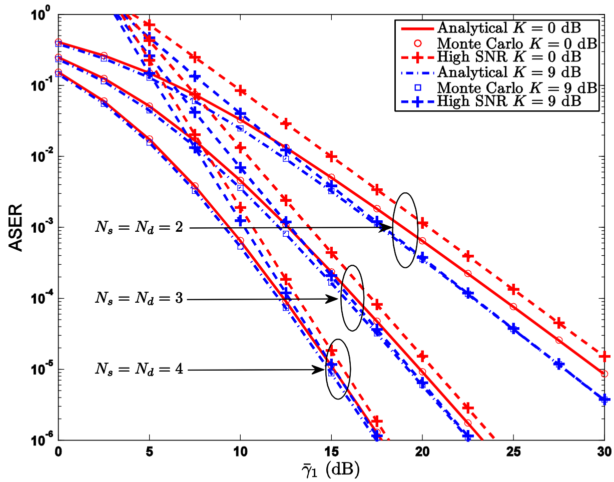

Figure 7. ASER versus SNR for QPSK modulation format over Rayleigh-Rician fading channels with $\bar{\gamma}_1 = \bar{\gamma}_2$.

performance is significantly enhanced due to the higher diversity gain. For example, in the case of $K = 0$ and $P_s = 10^{-4}$, the antenna configuration of $(N_s, N_d) = (4, 4)$ yields a gain of about 10 dB and 8 dB than that of $(N_s, N_d) = (2, 2)$ for the Rician-Rayleigh and Rayleigh-Rician fading channels, respectively. Furthermore, when the antenna configuration is fixed as $(N_s, N_d) = (4, 4)$ and the ASER equals to 10^{-4}, the AF relay network can obtain a gain of about 2 dB and 1 dB over the Rician-Rayleigh and the Rayleigh-Rician fading channels, respectively, provided that the Rician factor K increases from 0 dB to 9 dB. The scenario again confirms that the system performance can be improved with more antennas and better channel conditions.

6. CONCLUSIONS

In real wireless relay networks, it has been found that different links are often subject to different fading distributions. In this paper, we have investigated the performance of a multi-antenna two hop fixed gain AF relay system in Rician-Rayleigh and Rayleigh-Rician asymmetric fading environments, respectively. Specifically, we have first derived the analytical expressions of the OP, PDF and generalized SNR moments. Then, we have developed the Ergodic capacity and ASERs for the AF relay network. Finally, we have examined the new analytical results and the superiority of the multi-antenna schemes through computer simulations. Since Rayleigh fading is a special case of Rician fading, and Rayleigh/Rician asymmetric fading channel is considered as a more general and accurate model, our work provides useful reference and guidance to the engineers for the purpose of system design and performance evaluation in practice.

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APPENDIX A. PROOF OF THEOREM 1

After performing integral calculation of (10), the CDF of γ_2 can be expressed as

$$F_{\gamma_2}^{a}(x) = 1 - \exp\left(-\frac{x}{\bar{\gamma}_2}\right)\sum_{i=0}^{N_d-1}\frac{1}{i!}\left(\frac{x}{\bar{\gamma}_2}\right)^i \tag{A1}$$

Then, applying the definition of $F^a_\gamma(x)$ as

$$F^a_\gamma(x)=\Pr\left[\frac{\gamma_1\gamma_2}{\gamma_2+C}\leq x\right]=1-\int_u^\infty\left[1-F_{\gamma_1}\left(\frac{Cx}{u-x}\right)\right]f_{\gamma_2}(u)\,du \quad \text{(A2)}$$

and using (8) and (A1) along with (10), one can obtain

$$\begin{aligned}
F^a_\gamma(x) &= 1-\exp(-KN_s)\left(\frac{1}{KN_s}\right)^{\frac{N_s-1}{2}}\left(\frac{1+K}{\bar{\gamma}_1}\right)^{\frac{N_s+1}{2}}\sum_{i=0}^{N_d-1}\frac{1}{i!}\left(\frac{Cx}{\bar{\gamma}_2}\right)^i \\
&\quad \int_u^\infty \exp\left(-\frac{1+K}{\bar{\gamma}_1}u-\frac{Cx}{\bar{\gamma}_2(u-x)}\right) \\
&\quad \times u^{\frac{N_s-1}{2}}(u-x)^{-i}I_{N_s-1}\left(2\sqrt{\frac{N_sK(1+K)}{\bar{\gamma}_1}u}\right)du \\
&= 1-e^{-KN_s}\sum_{i=0}^{N_d-1}\sum_{n=0}^{\infty}\frac{(KN_s)^n}{i!n!\Gamma(N_s+n)}\left(\frac{1+K}{\bar{\gamma}_1}\right)^{N_s+n}\left(\frac{Cx}{\bar{\gamma}_2}\right)^i \\
&\quad \underbrace{\int_u^\infty u^{N_s+n-1}(u-x)^{-i}\exp\left(-\frac{1+K}{\bar{\gamma}_1}u-\frac{Cx}{\bar{\gamma}_2(u-x)}\right)du}_{I_A} \quad \text{(A3)}
\end{aligned}$$

To solve the integral term in (A3), we perform a variable replacement as $t=u-x$ and rewrite the integral term as

$$\begin{aligned}
I_A &= \exp\left(-\frac{1+K}{\bar{\gamma}_1}x\right)\int_0^\infty(t+x)^{N_s+n-1}t^{-i}\exp\left(-\frac{1+K}{\bar{\gamma}_1}t-\frac{Cu}{\bar{\gamma}_2}\frac{1}{t}\right)dt \\
&= \exp\left(-\frac{1+K}{\bar{\gamma}_1}x\right)\sum_{j=0}^{N_s+n-1}\begin{pmatrix}N_s+n-1\\ j\end{pmatrix}x^{N_s+n-j-1}\int_0^\infty t^{j-i} \\
&\quad \exp\left(-\frac{1+K}{\bar{\gamma}_1}t-\frac{Cu}{\bar{\gamma}_2}\frac{1}{t}\right)dt \quad \text{(A4)}
\end{aligned}$$

Then, by using the following identity [28, Eq. (3.471.9)]

$$\int_0^\infty x^{v-1}\exp\left(-\frac{\alpha}{x}-\beta x\right)dx=2\left(\frac{\alpha}{\beta}\right)^{\frac{v}{2}}K_v\left(2\sqrt{\alpha\beta}\right) \quad \text{(A5)}$$

it is straightforward to yield

$$\begin{aligned}
I_A &= 2\exp\left(-\frac{1+K}{\bar{\gamma}_1}x\right)\sum_{j=0}^{N_s+n-1}\begin{pmatrix}N_s+n-1\\ j\end{pmatrix}\left(\frac{1+K}{\bar{\gamma}_1}\right)^{\frac{i-j-1}{2}} \\
&\quad \left(\frac{C}{\bar{\gamma}_2}\right)^{\frac{j-i+1}{2}}x^{\frac{2N_s+2n-j-i-1}{2}}K_{j-i+1}\left(2\sqrt{\frac{C(1+K)}{\bar{\gamma}_1\bar{\gamma}_2}x}\right) \quad \text{(A6)}
\end{aligned}$$

where $K_v(\cdot)$ denotes the vth-order modified Bessel function of the second kind [28]. Furthermore, by Substituting (A6) into (A3), the CDF of γ is directly obtained as

$$F_{\gamma}^{a}(x) = 1-2\exp\left(-KN_s-\frac{1+K}{\bar{\gamma}_1}x\right)\sum_{i=0}^{N_d-1}\sum_{n=0}^{\infty}\sum_{j=0}^{N_s+n-1}\binom{N_s+n-1}{j}\frac{(KN_s)^n}{i!n!\Gamma(N_s+n)} \times\left(\frac{1+K}{\bar{\gamma}_1}x\right)^{\frac{2N_s+2n+i-j-1}{2}}\left(\frac{C}{\bar{\gamma}_2}\right)^{\frac{i+j+1}{2}}K_{j-i+1}\left(2\sqrt{\frac{C(1+K)}{\bar{\gamma}_1\bar{\gamma}_2}x}\right) \quad \text{(A7)}$$

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