Progress In Electromagnetics Research, *PIER 14, 317–361, 1996*

WAVE SCATTERING FROM CONDUCTING BODIES EMBEDDED IN RANDOM MEDIA – THEORY AND NUMERICAL RESULTS –

M. Tateiba and Z. Q. Meng

1. Introduction

As well known, the problem of wave scattering from a single or a few bodies has been studied constantly and strongly from over a century ago to now; as a result, various methods for analyzing the problem have been presented and many useful results have been obtained for communication engineering and sensing technology. With the progress of computer, computational techniques have been developed in expectation of a good solution to many problems which have been unsolvable with analytic methods. On the other hand, high frequency techniques also have attracted attention because they give a physical insight of scattering and are applicable to a body of large and complex configuration. In some cases, a few methods have been combined for getting the solution. Here it should be noted that the methods mentioned above are fundamentally based on the assumption that the body is in free space.

In practice, the body is frequently in a random medium: e.g., rain, snow, fog, some kinds of particles, turbulence and so on. The study on wave propagation and scattering in random media also is a subject with a long history. The multiple scattering theory has been developed since 1960 in particular, and applied to many practical cases (e.g., see references [1–6]). Backscattering enhancement of waves in random media has been investigated from an academic point of view during the past decade[7–16]. It has thereby been said to be a fundamental phenomenon in disordered media[14,16] and to be produced by statistical coupling of incident and backscattered waves due to the effect of double passage [8]. When a body is surrounded with a random medium, it may then happen that the backscattering cross-section (BCS) of the body is remarkably different from the BCS in free space. The problem of wave scattering from a body in a random medium has therefore been of great interest in the fields of radar engineering and sensing technology. The problem has not, however, been analyzed as boundary value problems.

Recently an approach to the problem and numerical results based on the approach have been presented for some cases [17–20]. The approach is based on general results of both the independent studies on wave scattering from a conducting body in free space and on wave propagation and scattering in random media. By unifying their results, this paper presents a method for solving the present problem and shows numerically the average of amplitudes and intensities of backscattered waves from a conducting circular cylinder in a turbulent medium.

After the introduction, Section 2 describes why conventional methods developed in free space are not directly applicable to the present scattering problem and how the problem is formulated as boundary value problems. Two operators are introduced: the Green's function in a random medium and the current generator which transforms incident waves into surface currents on the body surface. Here, a representative form of the Green's function is not required but the moments are done for the analysis of the average quantities concerning observed waves, and the current generator is a non-random operator which depends only on the body surface. Construction of the moments and the current generator is discussed and a method for analyzing the problem is presented.

In Section 3, the method presented in Section 2 is applied to the analysis of wave scattering from a conducting circular cylinder in a turbulent medium, and the average of backscattered waves is calculated in the transition region from the Rayleigh to resonance scattering for the cases of E-wave and H-wave incidence. The attenuation coefficient of coherence and the average backscattered cross-section are depicted, as compared with those in free space and in the case where the effect of double passage is not taken into account.

Section 4 is denoted to the summary of this paper and the discussion about forthcoming subjects.

The time factor $\exp(-i\omega t)$ is assumed and suppressed throughout the paper.

2. Scattering Theory

In this section we present a general approach to the problem of wave scattering from a conducting body of arbitrary shape and size in a random medium, by introducing current generators which transform incident waves into surface currents on the body.

2.1 Scattering from a Conducting Body in an Inhomogeneous Medium

When dealing with a realization of a random medium, the present problem may be regarded as wave scattering from a conducting body in an inhomogeneous medium. Geometry of the problem is shown in Fig. 1 where the coordinate system also is done. Assume for simplicity that the dielectric constant of the medium is a function of location: $\varepsilon = \varepsilon(\mathbf{r})$, $\mathbf{r} = (x, y, z)$, the magnetic permeability μ is constant: $\mu = \mu_0$ and the electric conductivity $\sigma = 0$. In addition, it is assumed that $\varepsilon(\mathbf{r})$ is a varying function inside a sphere of radius L around the body, with the body size $a \ll L$, and that $\varepsilon(\mathbf{r}) = \varepsilon_0$, a constant, elsewhere.

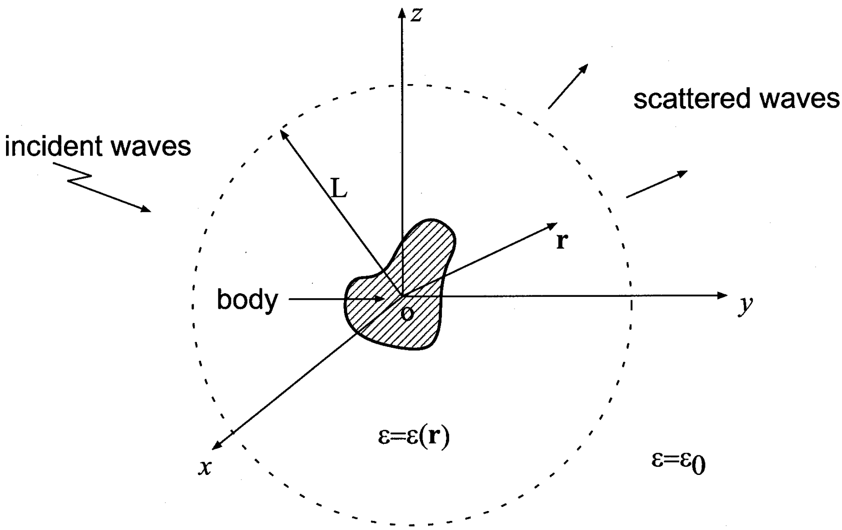

Figure 1. Geometry of the problem of wave scattering from a conducting body in an inhomogeneous medium.

Suppose that $\varepsilon(\mathbf{r})$ is a piecewise smooth function. Then it may be approximately expressed in terms of the Fourier series or the wavelets in the three-dimensional region. Even if $\varepsilon(\mathbf{r})$ is expressed in such a form, it is not easy to obtain wave functions in the medium except for the one-dimensional case. This shows that in the case where a conducting body of arbitrary shape and size is surrounded with an inhomogeneous medium, we have no method useful for analyzing generally the wave scattering as boundary value problems. Consequently, if this is forced to be combined with the fact that an inhomogeneous medium is a realization of a random medium, it may be accepted that when $\varepsilon(\mathbf{r})$ is a random function, it is difficult to find a method for analyzing wave scattering from a body in a random medium as well.

In wave scattering and propagation in random media, we are concerned about not each realization of waves but the moments; and they have been in part obtained for many practical cases. To solve the scattering problem, however, we need to know the moments of surface (electric and magnetic) currents induced by waves or to obtain directly the moments of scattered waves from known incident waves, by fitting the boundary conditions. How to fulfill this requirement will be described in the following subsections.

2.2 Boundary Conditions on a Conducting Body in a Random Medium

Let $\varepsilon(\mathbf{r})$ defined in the previous subsection be a random function throughout the paper from now. It can be expressed as

$$\varepsilon(\mathbf{r}) = \varepsilon_0[1 + \delta\varepsilon(\mathbf{r})] \tag{2.1}$$

Here $\delta\varepsilon(\mathbf{r})$ is a continuous random function with the zero mean: $\langle\delta\varepsilon(\mathbf{r})\rangle = 0$ for a turbulent medium and $\delta\varepsilon(\mathbf{r}) = \sum_{i=1}^{N}\varepsilon_i(\mathbf{r})$ for a discrete random medium, where $\varepsilon_i(\mathbf{r})$ is a random function of position, dielectric constant, shape, size and orientation of the i-th scatterer, and N is the number of random scatterers and is very large. In addition, $\delta\varepsilon(\mathbf{r})$ is assumed to be a bounded function:

$$|\delta\varepsilon(\mathbf{r})| < \infty \tag{2.2}$$

The surface of the body is assumed to be expressed by a smooth function in order to construct operators on the surface in subsection 2.4. Even on the assumption, the surface changes according as physical situations; for example, it may be regarded as a rough surface or a coated surface with a material, when particles stick partly on the surface. In this paper, we assume that an infinitesimal thin layer of free space exists between the surface and the medium and finally the thickness of the layer tends to zero, as shown in Fig. 2. Accordingly, we can assume a smooth surface and impose two types of boundary condition on wave fields on the body: the Dirichlet condition (DC) and the Neumann condition (NC). The former is used for the electric fields tangential to the body surface and for the magnetic field perpendicular to the body surface, and the latter is used for the magnetic field tangential to the surface of an infinite uniform cylinder. They are expressed for the field u as

$$u(\mathbf{r}) = 0, \qquad \text{for DC} \tag{2.3}$$

$$\frac{\partial}{\partial \mathbf{n}} u(\mathbf{r}) = 0, \qquad \text{for NC} \tag{2.4}$$

where $\mathbf{r}$ is on the surface of the body S, and $\partial/\partial\mathbf{n}$ denotes the outward normal derivative at $\mathbf{r}$ on S.

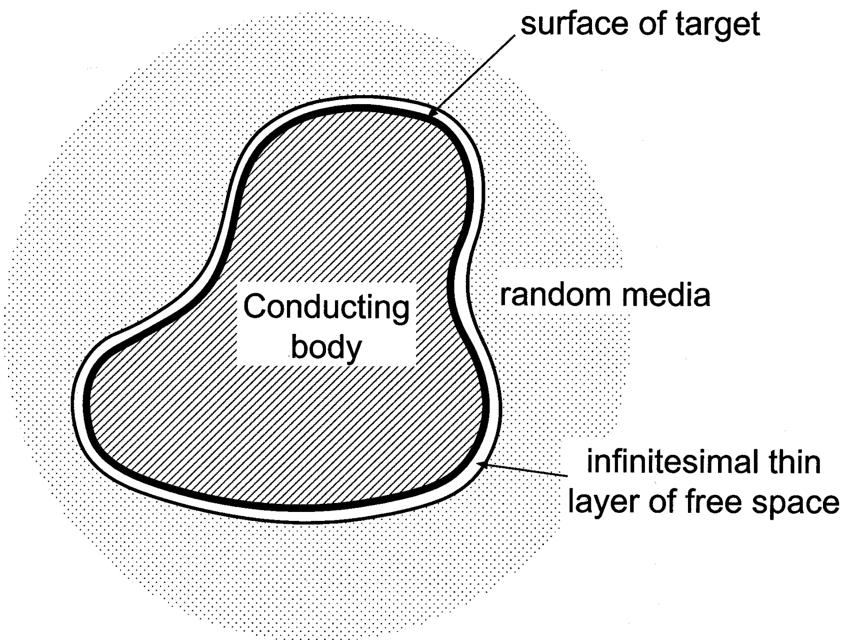

Figure 2. A model of the boundary between a body and a medium.

2.3 A Model of Scattering and its Formulation

According to Appendix A, using the Green's function in the random medium, we can obtain integral equations for surface currents on the surface of the body; and then using the solutions of equations, we can express the scattered waves. However, it is also shown that the methods based on integral equations for surface currents on the body are not applicable to the present scattering problem. The reason is that these surface currents are obtained as the solutions of statistically nonlinear equations constructed by the random incident or re-incident waves and the random Green's function. Instead of obtaining the surface currents directly, we therefore try to express them approximately. Consider the scattering problem qualitatively as follows, referring to Fig. 3. An electromagnetic wave radiated from a source of which the position $\mathbf{r}_t$ is beyond the random medium: $r_t > L$, propagates in the random medium, illuminates the body and induces a surface current on the body. A scattered wave from the body is produced by the surface current and propagates in the random medium; then, a part of the scattered wave is scattered by the random medium in the backward direction toward the body and is re-incident on the body. The re-incident wave produces a new surface current and a new scattered-wave. This iteration leads to a general solution of the scattering problem. Of course, observed waves at an observation point are, in general, obtained as the sum of the scattered wave mentioned above and the wave scattered only by the random medium.

In above scattering process, the surface current is given as the sum of each surface current produced by the n-th re-incident wave where $n = 0, 1, 2, \cdots,$ and $n = 0$ means direct incidence. The transform of the n-th re-incident wave into the surface wave is performed on the surface of the body. The effects of the random medium are included in the n-th re-incident wave and are also done in the surface current only through the transformation. Accordingly, to formulate the scattering process in a solvable form, we introduce a current generator which transforms random incident waves directly into random surface currents on the body and which is a deterministic operator dependent on the body surface. We also introduce the Green's function which transforms the source distribution into the incident wave and also transforms the surface current into the scattered wave. According to Appendix A, the Green's function may be approximately obtained under the condition $L \gg a$ as the Green's function in the random medium where the body is replaced with the same random medium.

Using the Green's function and the current generator, let us formulate the scattering problem. The incident wave expressed in terms of the source distribution and the Green's function is transformed into the surface current by the current generator, and the first scattered wave is expressed in terms of the surface current and the Green's function. From the scattered wave, we may express the first re-incident wave, i.e., the second incident wave (see subsection 2.5). In this way, the n-th scattered wave may be expressed and hence the scattered wave may be obtained as the sum of them. Consequently, an approach to the scattering problem can be described schematically as Fig. 4.

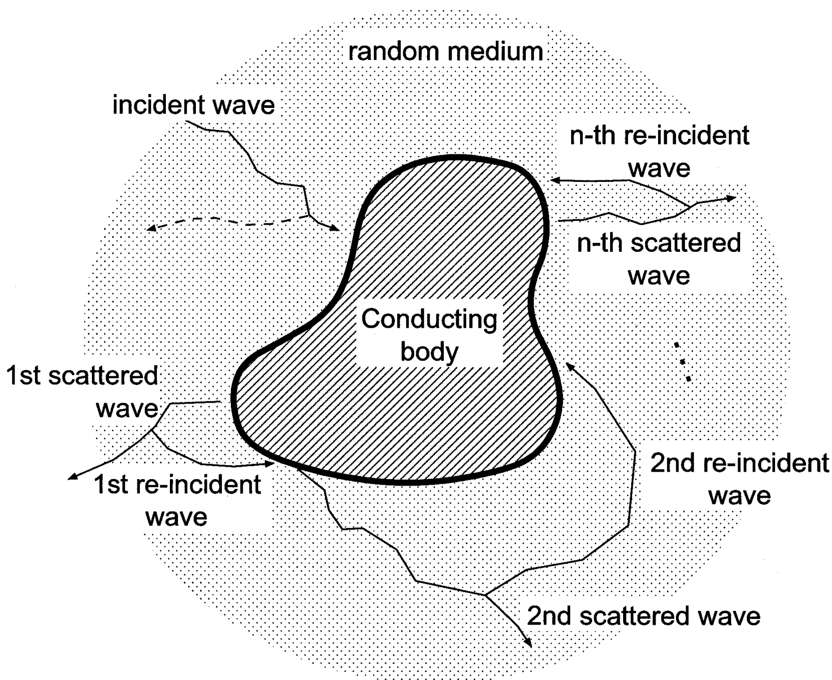

Figure 3. A model of scattering.

As mentioned in subsection 2.1, the moments of the Green's function are required and may be approximately obtained in some practical cases by applying the multiple scattering theory of wave propagation in random media. On the other hand, the current generator must be defined and shown to be constructed. This will be done in the following subsection.

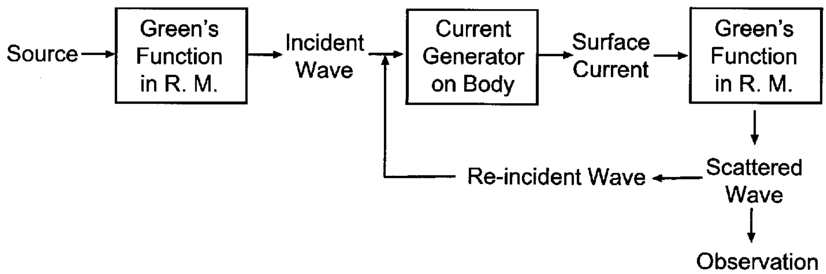

Figure 4. Schematic diagram for solving the scattering problem where a conducting body is surrounded with a random medium.

2.4 Current Generators

As mentioned in the previous subsection, a current generator is an operator which transforms incident waves into surface currents on the body. Here, let us designate the incident wave by u_{in}, the scattered wave by u_s and the total wave by u: $u = u_{in} + u_s$, where u_{in} includes both waves: the incident wave independent of the body and the re-incident wave (see Fig. 3). Surface currents at a point on the body depend on u_{in} on the overall surface. According to the boundary conditions, the current generators, written as Y, may be defined on the body as follows:

$$\frac{\partial u(\mathbf{r})}{\partial \mathbf{n}} = \int_S Y_E(\mathbf{r}|\mathbf{r}')u_{in}(\mathbf{r}')d\mathbf{r}', \quad \text{for DC} \tag{2.5}$$

$$u(\mathbf{r}) = \int_S Y_H(\mathbf{r}|\mathbf{r}')u_{in}(\mathbf{r}')d\mathbf{r}', \quad \text{for NC} \tag{2.6}$$

As mentioned in subsection 2.3, the Y is a deterministic operator which is dependent on the body surface and independent of the random medium and $u_{in}(\mathbf{r})$.

Above description suggests that Y can be constructed in the case where the body is in free space of $\delta\varepsilon(\mathbf{r}) \equiv 0$. Let us try to express Y in an explicit form, which expression can be made in case that the body surface is smooth by applying Yasuura's method [21–23]. It is a general method for analyzing the scattered wave and the surface current, and is simply described in Appendix B from a view point of operator construction.

2.4.1 Expression under the Dirichlet Condition

Let us put $\varepsilon(\mathbf{r}) = \varepsilon_0$ in Fig. 1. According to Yasuura's method, the surface current can be approximated by a truncated modal expansion as follows:

$$\frac{\partial u(\mathbf{r})}{\partial \mathbf{n}} \simeq \sum_{m=1}^{M} b_m(M)\phi_m^*(\mathbf{r}) = \boldsymbol{b}_M \boldsymbol{\phi}_M^{*T} = \boldsymbol{\phi}_M^* \boldsymbol{b}_M^T \tag{2.7}$$

where the basis functions ϕ_m are called the modal functions and constitute the complete set of wave functions satisfying the Helmholtz equation in free space and the radiation condition (A.1). Here the asterisk denotes the complex conjugate, $\boldsymbol{\phi}_M = [\phi_1, \phi_2, \cdots, \phi_M]$ and $\boldsymbol{\phi}_M^T$ denotes the transposed vector of $\boldsymbol{\phi}_M$, where $M = 2N+1$. The coefficient vector $\boldsymbol{b}_M$, defined as $[b_1, b_2, \cdots, b_M]$, can be obtained by the ordinary mode-matching method as shown below.

Let us minimize the mean square error

$$\Omega_E(M) = \int_S \left| \sum_{m=1}^{M} b_m(M)\phi_m^*(\mathbf{r}) - \frac{\partial u(\mathbf{r})}{\partial \mathbf{n}} \right|^2 d\mathbf{r} \tag{2.8}$$

by the method of least squares. That is, we partially differentiate (2.8) with respect to b_m^* and obtain the algebraic equation

$$\sum_{m=1}^{M} b_m(M) \int_S \phi_n(\mathbf{r})\phi_m^*(\mathbf{r}) d\mathbf{r} = \int_S \phi_n(\mathbf{r}) \frac{\partial u(\mathbf{r})}{\partial \mathbf{n}} d\mathbf{r}, \quad n = 1 \sim M. \tag{2.9}$$

Because of $u(\mathbf{r}) = u_{in} + u_s = 0$ on S, the right-hand side of (2.9) can be written as

$$\int_S \left(\phi_m \frac{\partial u}{\partial \mathbf{n}} - \frac{\partial \phi_m}{\partial \mathbf{n}} u \right) d\mathbf{r} = \int_S \left(\phi_m \frac{\partial u_{in}}{\partial \mathbf{n}} - \frac{\partial \phi_m}{\partial \mathbf{n}} u_{in} \right) d\mathbf{r} + \int_S \left(\phi_m \frac{\partial u_s}{\partial \mathbf{n}} - \frac{\partial \phi_m}{\partial \mathbf{n}} u_s \right) d\mathbf{r}$$

Using Green's theorem for $\phi_m,\ u_s$ in the region surrounded by S and infinity, and using the radiation condition for $\phi_m,\ u_s$, we obtain

$$\int_S \left(\phi_m \frac{\partial u_s}{\partial \mathbf{n}} - \frac{\partial \phi_m}{\partial \mathbf{n}} u_s \right) d\mathbf{r} = 0$$

and hence the right-hand side of (2.9) can be given as the reaction of ϕ_m and u_{in} :

$$\int_S \phi_m \frac{\partial u}{\partial \mathbf{n}} d\mathbf{r} = \int_S \ll \phi_m(\mathbf{r}),\ u_{in}(\mathbf{r}) \gg d\mathbf{r} \tag{2.10}$$

where $\ll , \gg$ means

$$\ll \phi_m(\mathbf{r}),\ u_{in}(\mathbf{r}) \gg \equiv \phi_m(\mathbf{r}) \frac{\partial u_{in}(\mathbf{r})}{\partial \mathbf{n}} - \frac{\partial \phi_m(\mathbf{r})}{\partial \mathbf{n}} u_{in}(\mathbf{r}) \tag{2.11}$$

We can therefore write (2.9) as

$$\mathbf{A}_E \boldsymbol{b}_M^T = \int_S \ll \boldsymbol{\phi}_M^T(\mathbf{r}),\ u_{in}(\mathbf{r}) \gg d\mathbf{r} \tag{2.12}$$

where $\mathbf{A}_E$ is a positive definite Hermitian matrix of $M \times M$ except for the internal resonance frequencies, and is given by

$$\mathbf{A}_E = \begin{bmatrix} (\phi_1,\ \phi_1) & \cdots & (\phi_1,\ \phi_M) \\ \vdots & \cdots & \vdots \\ (\phi_M,\ \phi_1) & \cdots & (\phi_M,\ \phi_M) \end{bmatrix} \tag{2.13}$$

in which its $m,\ n$ elements are the inner products of ϕ_m and ϕ_n :

$$(\phi_m,\ \phi_n) \equiv \int_s \phi_m(\mathbf{r}) \phi_n^*(\mathbf{r}) d\mathbf{r} \tag{2.14}$$

From (2.12), the $\boldsymbol{b}_m^T$ is given by

$$\boldsymbol{b}_m^T = \mathbf{A}_E^{-1} \int_s \ll \phi_M(\mathbf{r}'),\ u_{in}(\mathbf{r}') \gg d\mathbf{r}' \tag{2.15}$$

Substituting (2.15) into (2.7) and comparing it with (2.5), we can approximately express the current generator as follows:

$$Y_E(\mathbf{r}|\mathbf{r}') \simeq \phi_m^*(\mathbf{r}) \mathbf{A}_E^{-1} \ll \boldsymbol{\phi}_M^T(\mathbf{r}'), \tag{2.16}$$

where $\ll \boldsymbol{\phi}_M^T ,$ denotes the operation (2.11) of each element of $\boldsymbol{\phi}_M^T$ and the function u_{in} to the right of the $\boldsymbol{\phi}_M^T$. Equation (2.7) has been proved to converge in the mean sense as $M \to \infty$ [21,23]. Therefore (2.16) converges to the true operator in the same sense.

Finally we touch on the set of $\phi_m,\ m = 1, 2, 3, \cdots$. In the case of scattering from the body of finite size, it is chosen from the sets of which each set consists of solutions of the Helmholtz equation with the radiation condition, solutions which are obtained by separation of variables[24]. We usually use the spherical Bessel functions-spherical harmonics $h_n^{(1)}(kr)P_n^m(\cos\theta)\exp(im\phi)$ for three dimensional problems and the Hankel functions $H_m^{(1)}(k\rho)\exp(im\theta)$ for two dimensional problems, because they are well known and tractable to computation.

2.4.2 Expression under the Neumann Condition

Similarly, the surface current can be approximately expressed as

$$u(\mathbf{r}) \simeq \sum_{m=1}^{M} b_m(M)\frac{\partial \phi_m^*(\mathbf{r})}{\partial \mathbf{n}} = \boldsymbol{b}_M \frac{\partial \boldsymbol{\phi}_M^{*T}}{\partial \mathbf{n}} = \frac{\partial \boldsymbol{\phi}_M^*}{\partial \mathbf{n}} \boldsymbol{b}_m^T \tag{2.17}$$

Consider the mean square error

$$\Omega_H(N) = \int_S \left| \sum_{m=1}^{M} b_m(M)\frac{\partial \phi_m^*(\mathbf{r})}{\partial \mathbf{n}} - u(\mathbf{r}) \right|^2 d\mathbf{r} \tag{2.18}$$

and minimize it by the method of least squares. The same procedure as that taken for the Dirichlet condition yields

$$\mathbf{A}_H \boldsymbol{b}_M^T = \int_S \ll \boldsymbol{\phi}_M^T(\mathbf{r}),\ u_{in}(\mathbf{r}) \gg d\mathbf{r} \tag{2.19}$$

where $\mathbf{A}_H$ is $\mathbf{A}_E$ of (2.13) with $(\phi_m,\ \phi_n)$ replaced by $(\partial\phi_m/\partial\mathbf{n},\ \partial\phi_n/\partial\mathbf{n})$.

From (2.19), the $\boldsymbol{b}_M^T$ is given by

$$\boldsymbol{b}_M^T = \mathbf{A}_H^{-1} \int_S \ll \boldsymbol{\phi}_M^T(\mathbf{r}'),\ u_{in}(\mathbf{r}') \gg d\mathbf{r}' \tag{2.20}$$

Substituting (2.20) into (2.17) and comparing it with (2.6), we can approximately obtain the current generator Y_H for the Neumann condition as

$$Y_H(\mathbf{r}|\mathbf{r}') \simeq \frac{\partial \boldsymbol{\phi}_M^*(\mathbf{r})}{\partial \mathbf{n}} \mathbf{A}_H^{-1} \ll \boldsymbol{\phi}_M^T(\mathbf{r}'), \tag{2.21}$$

Here Y_H also converges in the same sense as Y_E does, when $M \to \infty$.

2.4.3 Examples

In the case of special bodies, Y_E and Y_H can be expressed explicitly in terms of known functions. As an illustrative example, let us show Y_E and Y_H for a conducting circular cylinder of radius a and of infinite length in the case of normal incidence to the axis of the cylinder. In this case, when we chose $H_m^{(1)}(k\rho)\exp(im\theta)$, $m = -N \sim N$, as ϕ_m, then they form an orthogonal set on the surface of the cylinder; that is, $(\phi_m,\ \phi_n) = 0$, for $m \neq n$ and hence A_E and A_H become diagonal matrix. Consequently, as $N \to \infty$, we can obtain

$$Y_E(\mathbf{r}|\mathbf{r}_0) = \frac{i}{\pi^2 a^2} \sum_{n=-\infty}^{\infty} \frac{\exp[in(\theta_0 - \theta)]}{J_n(ka) H_n^{(1)}(ka)} \tag{2.22}$$

$$Y_H(\mathbf{r}|\mathbf{r}_0) = \frac{i}{\pi^2 ka^2} \sum_{n=-\infty}^{\infty} \frac{\exp[in(\theta_0 - \theta)]}{J_n(ka) \dfrac{\partial}{\partial(ka)} H_n^{(1)}(ka)} \tag{2.23}$$

where J_n is the Bessel function of order n and $J_n(ka) \neq 0$; that is, the internal resonance frequencies are excepted.

The solution to the scattering problem is well known for the case of plane wave incidence on the cylinder[25]. When using the solution, (2.5) and (2.6), we can also obtain (2.22) and (2.23).

2.5 Re-Incident Waves

Referring to Fig. 4, we need to show how to describe the re-incident wave explicitly. Assume that the random medium is in the region of $-L < z < L$ as shown in Fig. 5. In order to show shortly an idea of the description, we deal with the scalar wave equation:

$$[\nabla^2 + k^2(1 + \delta\varepsilon(\mathbf{r}))]u = 0$$

Then we can obtain the following equation:

$$u = u_{in} + Hu \tag{2.24}$$

where H is the operator in which all effects of the random medium are included; $H = 0$ for $\delta\varepsilon(\mathbf{r}) \equiv 0$ and H includes the integral with respect to z from $-L$ to L. Let us divide H into two parts: $H = H_f + H_b$ where H_f includes the integral from $-L$ to z, called the forward scattering operator for convenience, and H_b does one from z to L, called the backward scattering operator. Of course, H_f and H_b can be given explicitly.

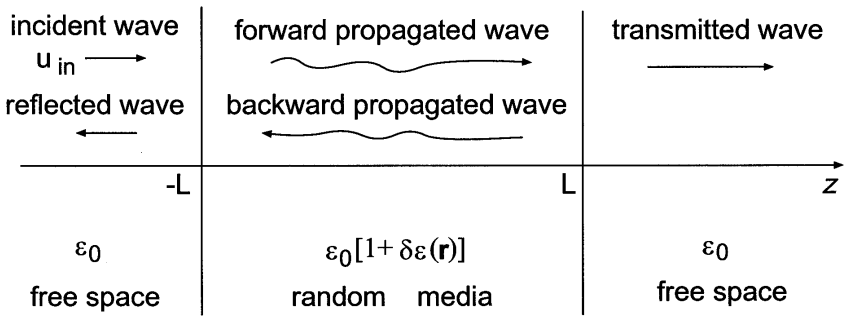

Figure 5. Geometry of the propagation problem in a random medium.

Because (2.24) is deformed as $(I - H_f)u = u_{in} + H_b u$ where I is the identity operator, we have

$$u = (I - H_f)^{-1} u_{in} + (I - H_f)^{-1} H_b u \tag{2.25}$$

where $(I - H_f)^{-1}$ is the inverse operator of $(I - H_f)$ and is expressed in terms of an ordered exponential function[26]. Because (2.25) is a Volterra's integral equation with respect to z, we may express its solution formally as follows:

$$u = u_0 + \sum_{n=1}^{\infty} [(I - H_f)^{-1} H_b]^n u_0 \tag{2.26}$$

where

$$u_0 = (I - H_f)^{-1} u_{in} \tag{2.27}$$

Here u_n represents a wave scattered n times in the backward direction, and u_0 may be called a successively forward-scattered wave of which the moments satisfy so-called moment equations [26, 28]. By replacing u_{in}, u with G_0, G, respectively, we can express the re-incident waves but it is not easy to express the higher order moments in analytic forms because we obtain only a few analytic expressions even for the moments of u_0 .

3. Numerical Results

This section shows numerical results of backscattered waves from a conducting circular cylinder surrounded with a turbulent medium, by applying the theory presented in Section 2 and computing the first and second moments of the waves in the transition region from the Rayleigh to resonance scattering.

3.1 Formulation

Assume that $\delta\varepsilon(\mathbf{r})$ is a continuous random function with

$$\langle \delta\varepsilon(\mathbf{r}) \rangle = 0 \quad , \qquad \langle \delta\varepsilon(\mathbf{r}_1)\delta\varepsilon(\mathbf{r}_2) \rangle = B(\mathbf{r}_1 \ , \ \mathbf{r}_2) \tag{3.1}$$

and

$$B(\mathbf{r},\mathbf{r}) \ll 1 \quad , \qquad kl(\mathbf{r}) \gg 1 \tag{3.2}$$

where the angular brackets denote the ensemble average, $B(\mathbf{r},\mathbf{r})$, $l(\mathbf{r})$ are the local intensity and scale size of turbulence, respectively, and k is the wavenumber in free space: $k = \omega\sqrt{\varepsilon_0\mu_0}$. Under the condition (3.2), depolarization of electromagnetic waves due to the turbulence can be neglected; and the scalar approximation is valid. In addition, the small scattering-angle approximation is also valid[3, 27]; and re-incident waves are negligible at the first stage of analysis. Then the wave equation for an electromagnetic field component is given as

$$[\nabla^2 + k^2(1 + \delta\varepsilon(\mathbf{r}))]v(\mathbf{r}) = 0 \tag{3.3}$$

in the turbulent medium, where v denotes each component.

Suppose that a conducting circular cylinder of radius a and infinite length is surrounded with above turbulent medium. Geometry of the scattering problem is shown in Fig. 6 where the intensity of turbulence is depicted along the z axis. As shown in Fig. 6, when an incident wave propagated along the z axis is scattered and observed at a point close to the z axis, we can approximately express (3.1) under the condition (3.2) as follows:

$$B(\mathbf{r}_1, \mathbf{r}_2) = B(\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2, z_+, z_-) \tag{3.4}$$

where $\mathbf{r} = (\boldsymbol{\rho}, z)$, $\boldsymbol{\rho} = \mathbf{i}_x x + \mathbf{i}_y y$, $z_+ = (z_1 + z_2)/2$ and $z_- = z_1 - z_2$.

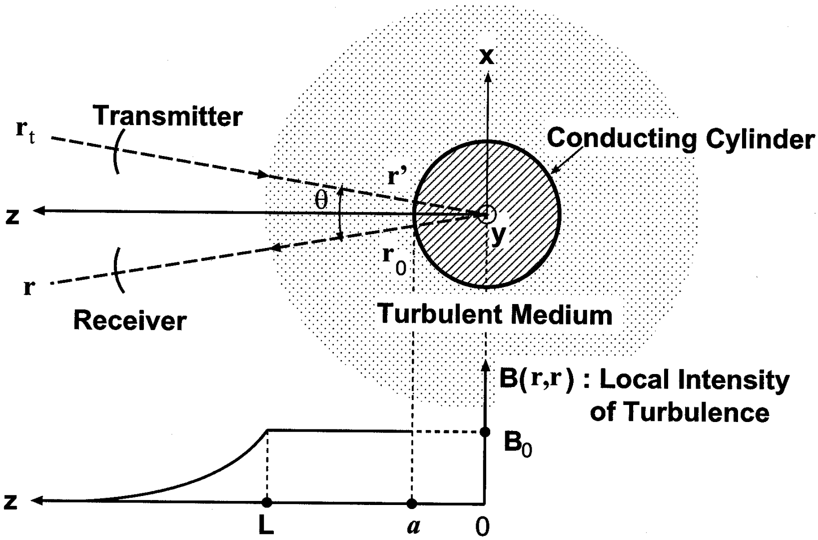

Figure 6. Geometry of the scattering problem from a conducting circular cylinder, the coordinate system and the local intensity of turbulence.

Consider the case where a directly incident wave is produced by a line source distributed uniformly along the y axis. Then we can deal with this scattering problem two-dimensionally under the condition (3.2) and use $\mathbf{r}$ even for this case although $\mathbf{r} = (x, z)$. According as polarizations of incident waves: E_y or H_y, where E_y, H_y are the y components of electric and magnetic fields, respectively, the boundary condition becomes (2.3) or (2.4).

From the above mentioned, the incident wave can, in general, be expressed as

$$u_{in}(\mathbf{r}) = \int_V G(\mathbf{r}|\mathbf{r}_t) f(\mathbf{r}_t) d\mathbf{r}_t \tag{3.5}$$

where $G(\mathbf{r}|\mathbf{r}')$ is the Green's function in the turbulent medium and $f(\mathbf{r})$ is the source distribution. By referring to Fig. 4, the scattered wave can be given by

$$\begin{aligned} u_s(\mathbf{r}) &= -\int_S G(\mathbf{r}|\mathbf{r}_1) \frac{\partial}{\partial \mathbf{n}_1} u(\mathbf{r}_1) d\mathbf{r}_1 \\ &= -\int_S G(\mathbf{r}|\mathbf{r}_1) \int_S Y_E(\mathbf{r}_1|\mathbf{r}_2) u_{in}(\mathbf{r}_2) d\mathbf{r}_2 d\mathbf{r}_1 \end{aligned} \tag{3.6}$$

for the Dirichlet condition and

$$\begin{aligned} u_s(\mathbf{r}) &= \int_S \left[\frac{\partial}{\partial n_1} G(\mathbf{r}|\mathbf{r}_1)\right] u(\mathbf{r}_1) d\mathbf{r}_1 \\ &= \int_S \left[\frac{\partial}{\partial n_1} G(\mathbf{r}|\mathbf{r}_1)\right] \int_S Y_H(\mathbf{r}_1|\mathbf{r}_2) u_{in}(\mathbf{r}_2) d\mathbf{r}_2 d\mathbf{r}_1 \end{aligned} \tag{3.7}$$

for the Neumann condition, where Y_E, Y_H are given by (2.22) and (2.23), respectively, $\partial/\partial \mathbf{n}_i = \partial/\partial \mathbf{r}_i$, $d\mathbf{r}_i = a d\theta_i$, $i = 1, 2$, and the surface integral is performed with respect to θ_i from 0 to 2π. From (3.6) and (3.7), the average scattered wave can be expressed as

$$\langle u_s \rangle = -\int_S d\mathbf{r}_1 \int_S d\mathbf{r}_2 \int d\mathbf{r}_t Y_E(\mathbf{r}_1|\mathbf{r}_2) \langle G(\mathbf{r}|\mathbf{r}_1) G(\mathbf{r}_2|\mathbf{r}_t)\rangle f(\mathbf{r}_t) \tag{3.8}$$

for the Dirichlet condition and

$$\langle u_s \rangle = \int_S d\mathbf{r}_1 \int_S d\mathbf{r}_2 \int d\mathbf{r}_t Y_H(\mathbf{r}_1|\mathbf{r}_2) \left\langle \frac{\partial}{\partial \mathbf{n}_1} G(\mathbf{r}|\mathbf{r}_1) G(\mathbf{r}_2|\mathbf{r}_t) \right\rangle f(\mathbf{r}_t) \tag{3.9}$$

for the Neumann condition. The average intensity of scattered waves is given by

$$\begin{aligned} \langle |u_s|^2 \rangle = &\int_S d\mathbf{r}_1 \int_S d\mathbf{r}_2 \int_S d\mathbf{r}_1' \int_S d\mathbf{r}_1'' \int_V d\mathbf{r}_t \int_V d\mathbf{r}_t' [Y_E(\mathbf{r}_1|\mathbf{r}_2) Y_E^*(\mathbf{r}_1'|\mathbf{r}_2') \\ &\langle G(\mathbf{r}|\mathbf{r}_1) G(\mathbf{r}_2|\mathbf{r}_t) G^*(\mathbf{r}|\mathbf{r}_1') G^*(\mathbf{r}_2'|\mathbf{r}_t')\rangle f(\mathbf{r}_t) f(\mathbf{r}_t')] \end{aligned} \tag{3.10}$$

for the Dirichlet condition and

$$\begin{aligned} \langle |u_s|^2 \rangle = &\int_S d\mathbf{r}_1 \int_S d\mathbf{r}_2 \int_S d\mathbf{r}_1' \int_S d\mathbf{r}_1'' \int_V d\mathbf{r}_t \int_V d\mathbf{r}_t' [Y_H(\mathbf{r}_1|\mathbf{r}_2) Y_H^*(\mathbf{r}_1'|\mathbf{r}_2') \\ &\left\langle \frac{\partial}{\partial \mathbf{n}_1} G(\mathbf{r}|\mathbf{r}_1) G(\mathbf{r}_2|\mathbf{r}_t) \frac{\partial}{\partial \mathbf{n}_1'} G^*(\mathbf{r}|\mathbf{r}_1') G^*(\mathbf{r}_2'|\mathbf{r}_t') \right\rangle f(\mathbf{r}_t) f(\mathbf{r}_t')] \end{aligned} \tag{3.11}$$

for the Neumann condition.

3.2 Coherent Scattered Waves

The scattered wave given in subsection 3-1 can be divided into two parts:

$$u_s = \langle u_s \rangle + \Delta u_s \tag{3.12}$$

where $\langle u_s \rangle$, Δu_s are called the coherent and the incoherent scattered waves, respectively. The coherent scattered waves are given by (3.8) and (3.9). To analyze them, we need to obtain the second moment of the Green's function: $M_{20} = \langle G(\mathbf{r}|\mathbf{r}_1)G(\mathbf{r}_2|\mathbf{r}_t)\rangle$. Here it is assumed that $\delta\varepsilon(\mathbf{r})$ is a smooth random function and the order of averaging procedure and differentiation are exchangeable to each other. This moment is approximately expressed as the product of $\langle G(\mathbf{r}|\mathbf{r}_1)\rangle$ and $\langle G(\mathbf{r}_2|\mathbf{r}_t)\rangle$ if the angle between $\mathbf{r}$ and $\mathbf{r}_t$, shown in Fig. 6, is not very small. In this case, $\langle G(\mathbf{r}|\mathbf{r}')\rangle$ is given in a well known form and hence the second moment also is done. If the angle is quite small, then $G(\mathbf{r}|\mathbf{r}_1)$ and $G(\mathbf{r}_2|\mathbf{r}_t)$ are statistically coupled and the double passage effect[8] plays a leading role in analyzing M_{20} .

Let us assume that the coherence of waves is kept almost complete in propagation of distance $2a$ equal to the diameter of the cylinder. This assumption is acceptable in practical cases under the condition (3.2). On the assumption, we can satisfactorily substitute the turbulence effect in propagation from the source — the plane at $z = a$ — the receiver for that from the source — the cylinder — the receiver. When the source and the receiver are on the same plane perpendicular to the z axis, then M_{20} in $z > a$ can therefore be given as a solution of the following second moment equation[28].

$$\left[\frac{\partial}{\partial z} - i\frac{1}{2k}(\nabla^2 + \nabla_t^2) - i2k\right] M_{20} = \left\{-\frac{k^2}{2}\int_a^z \left[B\left(0, z - \frac{z'}{2}, z'\right) + B\left(\boldsymbol{\rho} - \boldsymbol{\rho}_t, z - \frac{z'}{2}, z'\right)\right] dz'\right\} M_{20} \tag{3.13}$$

and

$$M_{20}|_{z=a} = G_0(\boldsymbol{\rho}, a \mid \boldsymbol{\rho}_1, z_1)G_0(\boldsymbol{\rho}_t, a \mid \boldsymbol{\rho}_2, z_2) \tag{3.14}$$

where

$$\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2, \quad \nabla_t^2 = \partial^2/\partial x_t^2 + \partial^2/\partial y_t^2$$

and $G_0(\mathbf{r}|\mathbf{r}')$ is the Green's function in free space. Although $\boldsymbol{\rho} = \mathbf{i}_x x$, $\boldsymbol{\rho}_t = \mathbf{i}_x x_t$ and $\partial/\partial y = \partial/\partial y_t = 0$ in this case, we use these symbols for convenience.

It is difficult to obtain the solution of (3.13) analytically for a general from of $B(\boldsymbol{\rho}, z_+, z_-)$. Equation (3.14) can be solved, however, on the assumption that $B(\boldsymbol{\rho}, z_+, z_-)$ is approximately expressed in a quadratic form with respect to $\boldsymbol{\rho}$, which assumption leads to the solution valid in the neighborhood of $\boldsymbol{\rho} - \boldsymbol{\rho}_t \simeq 0$. That is, let us assume that

$$B(\boldsymbol{\rho}, z_+, z_-) = B(z_+)\left[1 - \frac{\rho^2}{l^2(z_+)}\right]\exp\left[-\frac{z_-^2}{l^2(z_+)}\right] \tag{3.15}$$

where,

$$B(z_+) = \begin{cases} B_0 & , a \le z \le L \\ B_0(z/L)^{-m} & , L \le z \end{cases} \tag{3.16}$$

and $l(z_+) = l_0$, a constant, as shown in Fig. 6. Then (3.13) can be solved; i.e., according to the Appendix C, the final form of M_{20} is given by

$$\begin{aligned} M_{20}(\mathbf{r}, \mathbf{r}_1 \ : \ \mathbf{r}_2, \mathbf{r}_t) =& G_0(\mathbf{r}|\mathbf{r}_1)G_0(\mathbf{r}_2|\mathbf{r}_t) \\ & \exp\left[-\frac{\sqrt{\pi}}{2}k^2 B_0 l_0\left(\frac{m}{m-1}L - a\right)\right] M(\boldsymbol{\rho}_d, z) \end{aligned} \tag{3.17}$$

$$\begin{aligned} M(\boldsymbol{\rho}_d, z) =& \frac{4\sin(\nu\pi)}{\pi^2 Q_0 Q_2(L) P_3(z)}\left(\frac{z}{a}\right)^{1/2} \\ & \exp\left\{-\left(i\sqrt{\pi}\frac{k^3}{16 l_0}B_0\right)^{1/2}\left(\frac{z}{L}\right)^{-m/2}\frac{P_4(z)}{P_3(z)}\rho_d^2\right. \\ & - i\frac{k}{2z}\left[1 - \frac{4\sin(\nu\pi)}{\pi^2 Q_1(a) Q_2(L) P_3(z)}\left(\frac{z}{a}\right)^{1/2}\right]\boldsymbol{\rho}_d\cdot\boldsymbol{\rho}_{dc} \\ & - i\frac{k}{4}\left[\int_a^L \frac{1}{(z'-a)^2}\left\{1 - \left(\frac{2}{\pi Q_1(a) P_1(z')}\right)^2\frac{z'}{a}\right\}dz'\right. \\ & \left.\left. + \int_L^z \frac{1}{(z'-a)^2}\left\{1 - \left(\frac{4\sin(\nu\pi)}{\pi^2 Q_1(a) Q_2(L) P_3(z')}\right)^2\frac{z'}{a}\right\}dz'\right]\rho_{dc}^2\right\} \end{aligned} \tag{3.18}$$

where $m > 1$, $m \neq 2$, $z \gg L$, $\nu = 1/(2-m)$, $\boldsymbol{\rho}_d = \boldsymbol{\rho} - \boldsymbol{\rho}_t$, $\boldsymbol{\rho}_{dc} = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2$,

$$Q_1(z) = i\sqrt{\pi}kB_0 z/l_0$$

$$Q_2(z) = i\nu\sqrt{\pi}kB_0(z/L)^{-m/2}z/l_0$$

$$P_1(z) = J_{3/2}[Q_1(a)]J_{-1/2}[Q_1(z)] + J_{-3/2}[Q_1(a)]J_{1/2}[Q_1(z)]$$

$$P_2(z) = J_{3/2}[Q_1(a)]J_{-3/2}[Q_1(z)] + J_{-3/2}[Q_1(a)]J_{3/2}[Q_1(z)]$$

$$P_3(z) = P_1(L)\{J_{\nu+1}[Q_2(L)]J_{-\nu}[Q_2(z)] + J_{-\nu-1}[Q_2(L)]J_{\nu}[Q_2(z)]\} + P_2(L)\{J_{\nu}[Q_2(L)]J_{-\nu}[Q_2(z)] - J_{-\nu}[Q_2(L)]J_{\nu}[Q_2(z)]\}$$

$$P_4(z) = P_1(L)\{J_{\nu+1}[Q_2(L)]J_{-\nu-1}[Q_2(z)] + J_{-\nu-1}[Q_2(L)]J_{\nu+1}[Q_2(z)]\} + P_2(L)\{J_{\nu}[Q_2(L)]J_{-\nu-1}[Q_2(z)] - J_{-\nu}[Q_2(L)]J_{\nu+1}[Q_2(z)]\}$$

Let us assume $z_t = z$ and a single point source: $f(\mathbf{r}_t) = \delta(\mathbf{r}-\mathbf{r}_t)$. Then we can calculate the coherent backscattered wave from (3.8) and (3.9) by using (2.22), (2.23) and (3.17). Figure 7 shows the normalized amplitude $|\langle u_s\rangle|$ to that in free space in the case of $ka = 0.1 \sim 1.0$, $m = 8/3$, $B_0 = 1.0\times 10^{-7}$, $kl_0 = 20\pi$, $kz = 2\pi\times 10^4$, $kL = 6\pi\times 10^3$, where the difference of the normalized amplitude between the Dirichlet and Neumann conditions is negligible. In this figure, the broken line shows the normalized amplitude calculated on the assumption that incident and scattered waves are statistically independent of each other: $\langle G(\mathbf{r}|\mathbf{r}_1)G(\mathbf{r}_2|\mathbf{r}_t)\rangle = \langle G(\mathbf{r}|\mathbf{r}_1)\rangle\langle G(\mathbf{r}_t|\mathbf{r}_2)\rangle$. The effect of double passage causes the difference between the solid and the broken lines. The solid line shows inaccurate values in the neighborhood of $\theta = 0.5\times 10^{-2}$ [rad] because of the assumption of (3.15). Here the turbulence parameters are chosen for convenience of computation. In practice, B_0 should be very smaller and kB_0L gives dominantly turbulence effect on the coherent wave; on the other hand, large kL takes much computation time. In this paper therefore we chose B_0 to be large and kL to be small, keeping kB_0L in effective values.

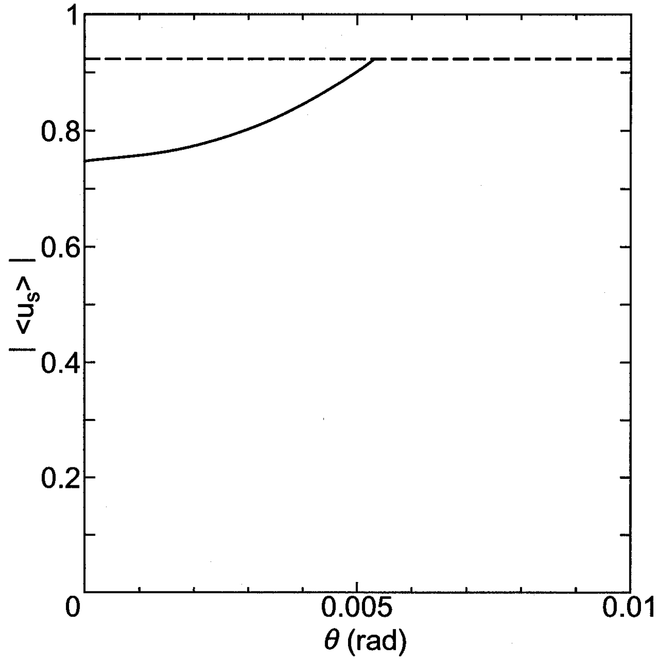

Figure 7. Bistatic scattering characteristics of the coherent wave normalized to the scattered wave in free space for $ka = 0.1 \sim 10.0$, where the broken line shows that the double passage effect is not taken into account.

The average backscattering cross-section, written as σ, depends on $\langle |u_s|^2 \rangle$ and can be divided into two parts, following to (3.12).

$$\sigma = \sigma_0 + \sigma_{in} \tag{3.19}$$

where σ_0 depends on $|\langle u_s \rangle|^2$ and σ_{in} on $\langle |\Delta u_s|^2 \rangle$. Figure 8 shows the change of σ_0 with ka in both cases of the E-wave and H-wave incidence, where the dotted line shows the σ in free space. As expected from Fig. 7, the σ_0 is about 0.6 times the σ in free space.

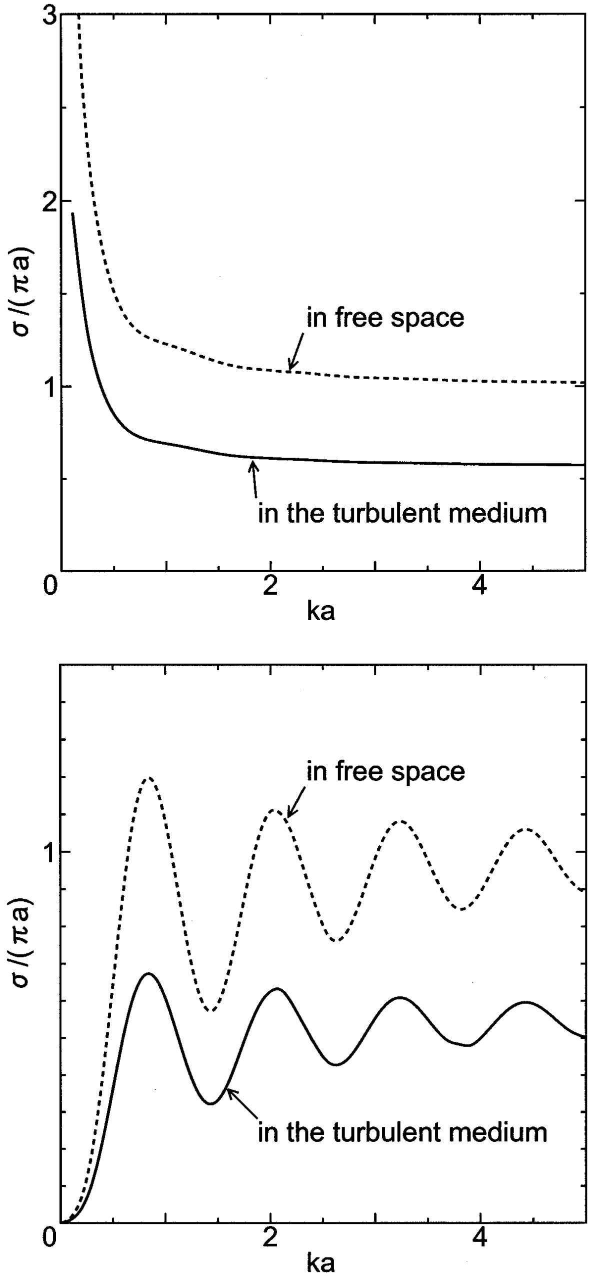

Figure 8. Backscattering cross-sections vs. cylinder size, calculated from the coherent scattered waves. (a) E-wave incidence case. (b) H-wave incidence case.

The coefficient of coherence attenuation in the turbulent medium is well known to be proportional to $B_0 l_0$. When we change B_0 only in the previous parameters, keeping $B_0 l_0$ constant, the broken line in Fig. 7 does not change but $|\langle u_s \rangle|$ changes as Fig. 9 because of the effect of double passage.

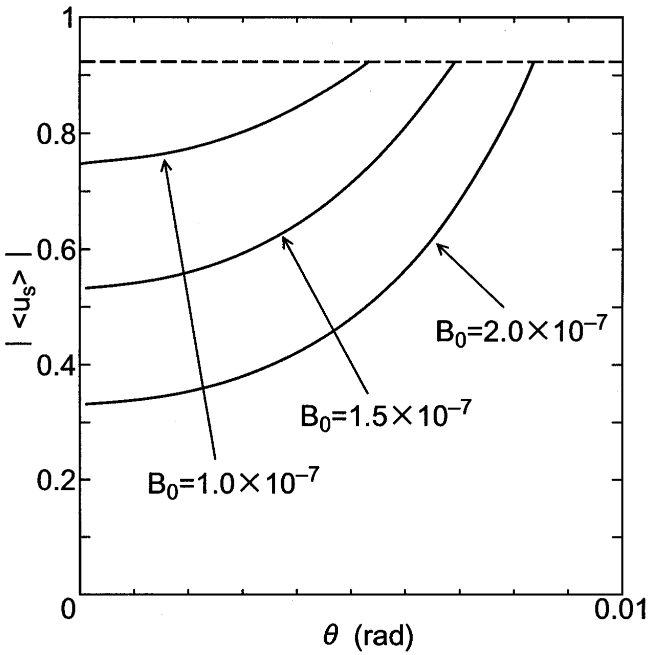

Figure 9. Bistatic scattering characteristics of the coherent wave under the condition of $B_o l_o$ constant, where the broken line shows that the effect of double passing is not included in $|\langle u_s \rangle|$.

3.3 Backscattering Cross-Sections

Equations (3.10) and (3.11) show that the analysis of the average intensity of scattered waves requires the fourth moment of the Green's functions. At a general situation, it is difficult to express the fourth moment in an analytic form. We concentrate on the state of $u_s \simeq \Delta u_s$ in (3.12) and the backscattering, so that B_0 is chosen as larger values than those used in the previous subsection. In wave propagation through a strong turbulent medium, we may assume that the Green's function becomes approximately complex Gaussian random and the fourth moment in the backward direction is expressed as the product of the second moments.

$$\begin{aligned}&\langle G(\mathbf{r}|\mathbf{r}_1)G(\mathbf{r}_2|\mathbf{r}_t)G^*(\mathbf{r}|\mathbf{r}_1')G^*(\mathbf{r}_2'|\mathbf{r}_t')\rangle\\ &\simeq \langle G(\mathbf{r}|\mathbf{r}_1)G^*(\mathbf{r}|\mathbf{r}_1')\rangle\langle G(\mathbf{r}_2|\mathbf{r}_t)G^*(\mathbf{r}_2'|\mathbf{r}_t')\rangle\\ &+\langle G(\mathbf{r}|\mathbf{r}_1)G^*(\mathbf{r}_2'|\mathbf{r}_t')\rangle\langle G(\mathbf{r}_2|\mathbf{r}_t)G^*(\mathbf{r}|\mathbf{r}_1')\rangle\end{aligned}\tag{3.20}$$

where $\mathbf{r}_t = \mathbf{r}_t' = \mathbf{r}$ on the assumptions of backscattering and a single point source.

The second moments in (3.20) have been given[29-31]: for instance,

$$\langle G(\mathbf{r}|\mathbf{r}_1)G^*(\mathbf{r}|\mathbf{r}_1')\rangle = G_0(\mathbf{r}|\mathbf{r}_1)G_0^*(\mathbf{r}|\mathbf{r}_1')$$

$$\exp\left\{-\frac{k^2}{4}\int_a^z dz_1\int_a^{z-z_1} dz_2\ D\left[\frac{z-a-z_2}{z-a}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2), z-z_2-\frac{z_1}{2}, z_1\right]\right\}\tag{3.21}$$

where

$$D(\boldsymbol{\rho}, z_+, z_-) = 2[B(0, z_+, z_-) - B(\boldsymbol{\rho}, z_+, z_-)]\tag{3.22}$$

which is called the structure function of turbulence. Without the approximation (3.15), we may calculate (3.20) for a general form of $D(\boldsymbol{\rho}, z_+, z_-)$; here we assume

$$D(\boldsymbol{\rho}, z_+, z_-) = 2B(z_+)\left\{1-\exp\left[-\left(\frac{\rho}{l(z_+)}\right)^2\right]\right\}\exp\left[-\left(\frac{z}{l(z_+)}\right)^2\right]$$

for computation below, where $B(z_+)$ is given by (3.16) and $l(z) = l_0$, a constant, as assumed in the previous subsection.

When we express the coherent Green's function as

$$\langle G(\mathbf{r}|\mathbf{r}_1)\rangle = G_0(\mathbf{r}|\mathbf{r}_1)\exp[-\alpha(L)]$$

then $\alpha(L) > 2$ is required in order that (3.20) holds. In this case,

$$\begin{aligned}\alpha(L) &= \frac{k^2}{4}\int_a^z dz_1 \int_a^z dz_2 B\left(0, z_1 - \frac{z_2}{2}, z_2\right) \\ &\simeq \frac{\sqrt{\pi}}{5} B_0 \times kl_0 \times kL\end{aligned} \tag{3.23}$$

and hence it is assumed that $B_0 = 5 \times 10^{-\nu}$, $\nu = 5, 6$ and other parameters are the same as those used previously. Although the incident wave becomes sufficiently incoherent, we should pay attention to spatial coherence of the incident wave because the wave scattering from the cylinder in the turbulent medium is expected to depend largely on the coherence length of the incident wave about the cylinder. The degree of spatial coherence is defined by

$$\Gamma(\boldsymbol{\rho}, z) = \frac{\langle G(\mathbf{r}_1|\mathbf{r}_t) G^*(\mathbf{r}_2|\mathbf{r}_t)\rangle}{\langle |G(\mathbf{r}_0|\mathbf{r}_t)|^2\rangle} \tag{3.24}$$

where $\mathbf{r}_1 = (\boldsymbol{\rho}, 0)$, $\mathbf{r}_2 = (-\boldsymbol{\rho}, 0)$, $\mathbf{r}_0 = (0, 0)$, $\mathbf{r}_t = (0, z)$.

Figure 10 shows the degree of spatial coherence calculated from (3.24) and that the coherence length of the incident wave is sufficiently larger than the diameter of the cylinder. In this situation, Figures 11 and 12 show the average backscattering cross-sections (BCS) for the E-wave and H-wave incidences, respectively, compared with those in free space. Their BCS in the turbulent medium become nearly twice as large as those in free space except the internal resonance frequencies: $J_n(ka) = 0$, $n = 0, 1, 2, \cdots$. A part of Fig. 12 enlarged about the zero points of J_0 and J_1 is shown in Figs. 13 (a) and (b), respectively.

These figures are obtained by substituting (3.20), (2.22) or (3.20), (2.23) into (3.10) or (3.11) according as polarization of incident waves and by carrying out directly the quadruple integrals with respect to θ_i, θ_i', $i = 1, 2$. For the E-wave incidence, the BCS computed above is similar in change with $ka = 0.1 \sim 5.0$ to that in free space, so that these is not any abnormal change of the BCS in the neighborhood of the internal resonance frequencies. Consequently, it may be concluded that the BCS is nearly twice as large as that in free space in the overall region of $ka = 0.1 \sim 5.0$ in the case of E-wave incidence.

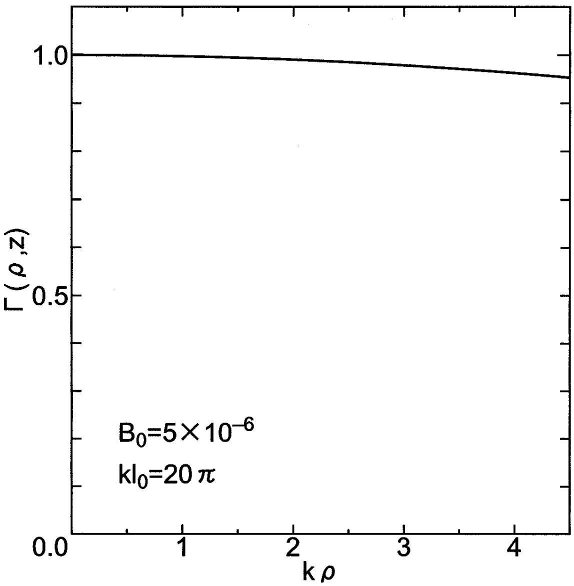

Figure 10. The degree of spatial coherence of incident waves about the cylinder.

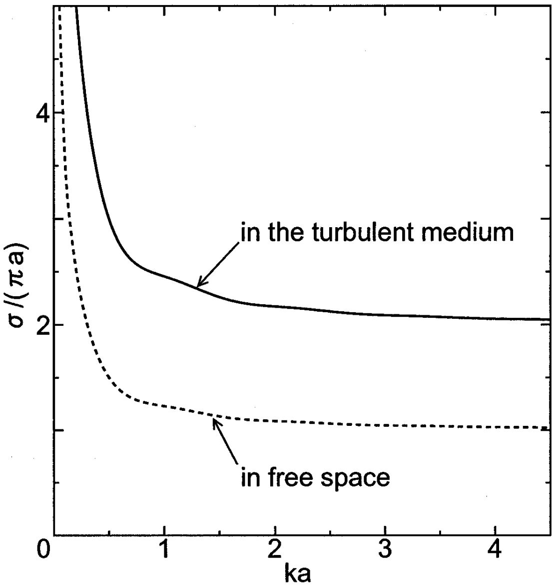

Figure 11. The average backscattering cross-section in the case of E-wave incidence, where the coherence length of the incident wave is shown in Figure 10.

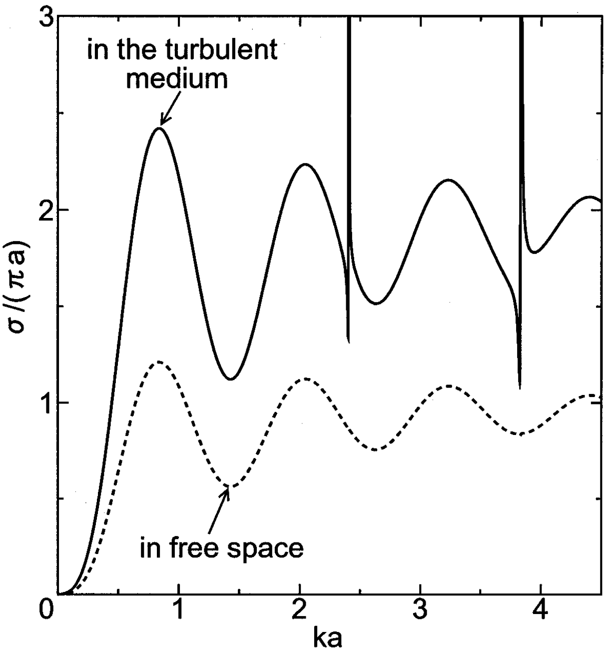

Figure 12. The average backscattering cross-section in the case of H-wave incidence, where the coherence length of the incident wave is shown in Figure 10.

On the other hand, in the case of H-wave incidence, the change of BCS is different from that in free space about the internal resonance frequencies, as shown in Fig. 13. This difference causes the abnormal change of BCS. The current induced by H-wave incidence flows circularly along the surface of the cylinder and hence the BCS of a conducting cylinder coated with a thin dielectric layer changes remarkably about the internal resonance frequencies. An illustrative example is shown in Fig. 14. In the present case, however, such a phenomenon does not occurs and the abnormal change is considered to be caused by the low accuracy of computation. Although the value of the quadruple integral is expected to take the same order as each value of the Bessel functions near the zero points of the Bessel functions in the dominator of the current generator, it is difficult virtually to carry out the integral with high accuracy so as to do that, which difficulty we do not have for the E-wave incidence.

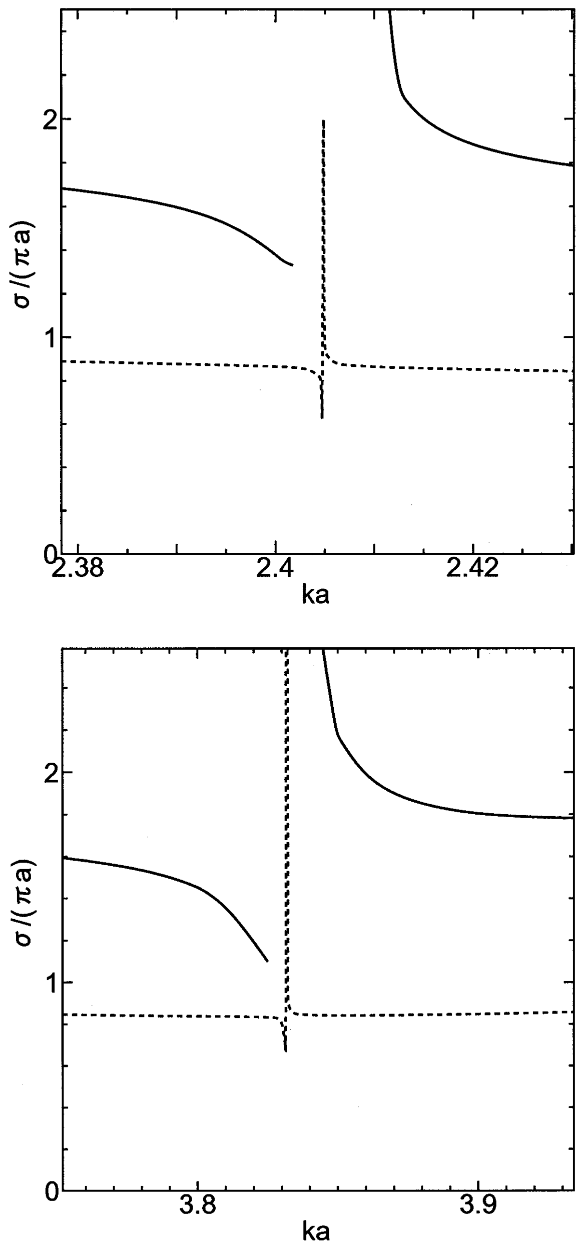

Figure 13. Enlargement of Fig. 12 about the internal resonance frequencies of the cylinder. (a) In the neigborhood of the first zero point of $J_o(ka_o) : ka_o = 2.40482\cdots$ (b) In the neighborhood of the second zero point of $J_1(ka_o) : ka_o = 3.83171\cdots$.

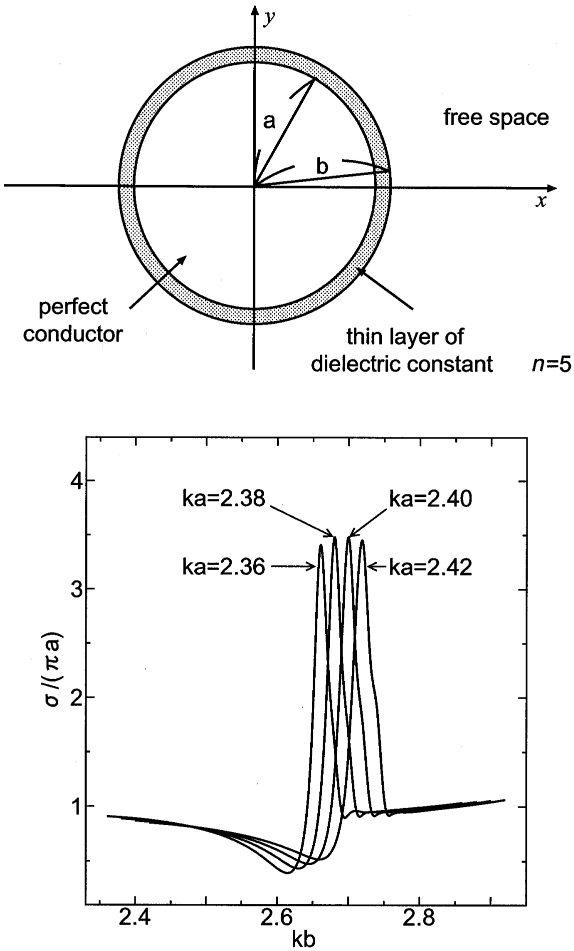

Figure 14. Backscattering cross-sections of the cylinder coated with a thin dielectric layer.

On the assumption that the average intensity of backscattered waves is finite at the internal resonance frequencies, we carry out the computation. We rewrite (2.23) as

$$Y_H(\mathbf{r}|\mathbf{r}_0) = \frac{X_l}{J_l(ka)} + \Delta Y_H \tag{3.25}$$

$$X_l = \frac{i}{\pi^2 ka^2} \frac{\exp[il(\theta_0 - \theta)]}{\dfrac{\partial}{\partial(ka)} H_l^{(1)}(ka)} \tag{3.26}$$

$$\Delta Y_H = \frac{i}{\pi^2 ka^2} \sum_{n=-\infty}^{\infty} \frac{\exp[in(\theta_0 - \theta)]}{J_n(ka)\dfrac{\partial}{\partial(ka)} H_n^{(1)}(ka)} \quad (n \neq l) \tag{3.27}$$

Using the above, (3.11) can be expressed as follows:

$$\langle |u_s|^2 \rangle = \alpha(ka) J_l^{-2}(ka) + \beta(ka) J_l^{-1}(ka) + \gamma(ka) \tag{3.28}$$

$$\alpha = \int_s d\mathbf{r}_1 \int_s d\mathbf{r}_2 \int_s d\mathbf{r}_1' \int_s d\mathbf{r}_2' \int_v d\mathbf{r}_t \int_v d\mathbf{r}_t' X_l X_l^* Z \tag{3.29}$$

$$\beta = \int_s d\mathbf{r}_1 \int_s d\mathbf{r}_2 \int_s d\mathbf{r}_1' \int_s d\mathbf{r}_2' \int_v d\mathbf{r}_t \int_v d\mathbf{r}_t' (X_l \Delta Y_Z^* + X_l^* \Delta Y_Z) Z \tag{3.30}$$

$$\gamma = \int_s d\mathbf{r}_1 \int_s d\mathbf{r}_2 \int_s d\mathbf{r}_1' \int_s d\mathbf{r}_2' \int_v d\mathbf{r}_t \int_v d\mathbf{r}_t' \Delta Y_Z \Delta Y_Z^* Z \tag{3.31}$$

where

$$Z = \left\langle \frac{\partial}{\partial \mathbf{n}_1} G(\mathbf{r}|\mathbf{r}_1) G(\mathbf{r}_2|\mathbf{r}_t) \frac{\partial}{\partial \mathbf{n}_1'} G^*(\mathbf{r}|\mathbf{r}_1') G^*(\mathbf{r}_2'|\mathbf{r}_t') \right\rangle f(\mathbf{r}_t) f(\mathbf{r}_t') \tag{3.32}$$

Let us $J_l(ka_0) = 0$, $l = 0, 1, 2, \cdots$, and expand α and β in the Taylor series about $ka = ka_0$:

$$\alpha = \sum_{m=0}^{\infty} \alpha_m (ka - ka_0)^m \tag{3.33}$$

$$\beta = \sum_{m=0}^{\infty} \beta_m (ka - ka_0)^m \tag{3.34}$$

Then we have from the above assumption

$$\alpha_0 = \alpha(ka_0) = 0, \quad \alpha_1 = \partial\alpha(ka_0)/\partial(ka) = 0 \tag{3.35}$$

$$\beta_0 = \beta(ka_0) = 0 \tag{3.36}$$

According to the computation of (3.29) and (3.30), the coefficients α_0, α_1 and β_0 are order of 10^{-6}, 10^{-5} and 10^{-3}, respectively, and α_2, β_1 are order of 1 at the first zero point of $J_0(ka_0) = 0$ and the second zero point of $J_1(ka_0) = 0$, when $J_0(ka_0)$ and $J_1(ka_0)$ at each zero point are order of 10^{-7} and 10^{-6}, respectively. This result shows the validity of the assumption from a numerical analysis point of view and also does no good accuracy of direct computation of (3.11) about the resonance frequencies. Using (3.33) and (3.34), we can express (3.28) about $ka = ka_0$ as

$$\langle |u_s|^2 \rangle = \frac{\displaystyle\sum_{m=2}^{\infty} \frac{1}{m!} \frac{\partial^m \alpha(ka_0)}{\partial (ka)^m} (ka - ka_0)^m}{\displaystyle\sum_{m=2}^{\infty} \frac{1}{m!} \frac{\partial^m J_l^2(ka_0)}{\partial (ka)^m} (ka - ka_0)^m} + \frac{\displaystyle\sum_{m=1}^{\infty} \frac{1}{m!} \frac{\partial^m \beta(ka_0)}{\partial (ka)^m} (ka - ka_0)^m}{\displaystyle\sum_{m=1}^{\infty} \frac{1}{m!} \frac{\partial^m J_l(ka_0)}{\partial (ka)^m} (ka - ka_0)^m} + \gamma(ka) \tag{3.37}$$

The numerical results of the average BCS calculated from (3.37) are shown by the solid lines in Figs. 15(a) and (b) where the dotted lines show the BCS calculated directly from (3.11). These figures show clearly that there is not any abnormal change of the BCS about the internal resonance frequencies of the cylinder in both the turbulent medium and free space. Consequently, in the case of $kl_c \gg 5$, the BCS is nearly twice as large as that in the free space in the overall region of $ka = 0.1 \sim 5.0$ for the incidence of H-waves as well as E-waves.

On the other hand, for $l_c \leq a$, the spatial coherence of the incident wave is not kept on the overall cylinder and hence the BCS is expected to change largely, compared with that for $l_c \gg a$. This is shown in Fig. 16 for the E-wave incidence and Fig. 17 for the H-wave incidence, where these BCS are obtained in the spatial coherence situation shown in Fig. 18. In the case of $l_c \leq a$, the BCS for the H-wave incidence changes anomalously and is diminished in some cases, because the direct and creeping backscattered-waves interact statistically with each other in addition to the double passage effect; and the BCS for the E-wave incidence is reduced as l_c becomes small, because the cylinder surface on which the spatial coherence of the wave is kept enough is limited, but it does not change so anomalously as that for the H-wave does.

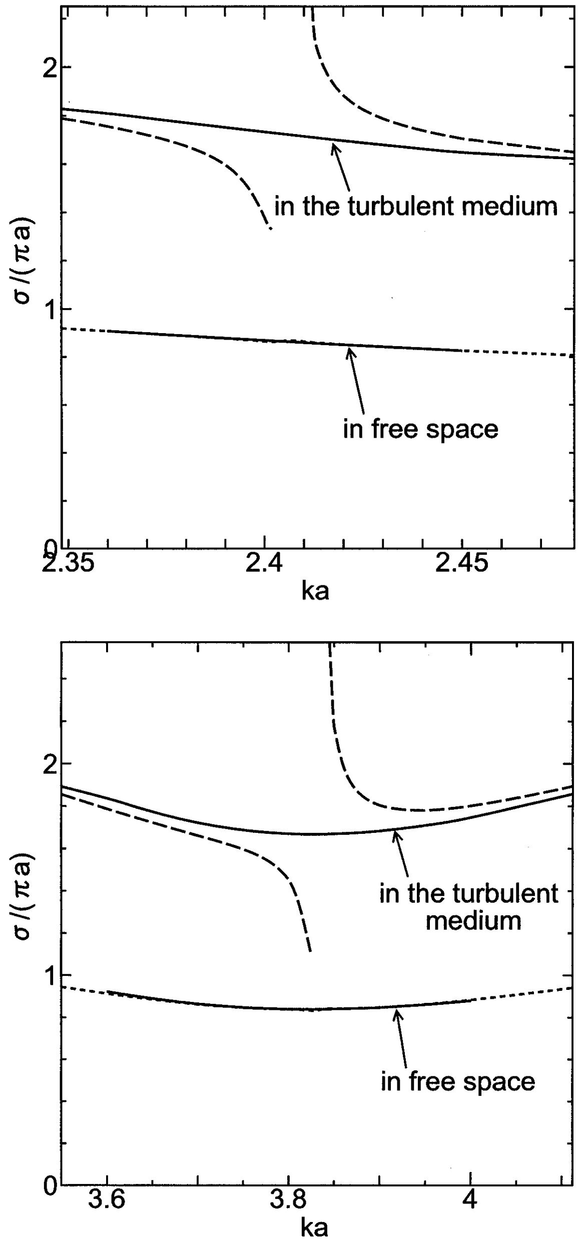

Figure 15. The average backscattering cross-sections about the internal resonance frequencies in the case of H-wave incidence. (a) In the neighborhood of the first zero point of $J_o(ka_o) : ka_o = 2.40482\cdots$ (b) In the neighborhood of the second zero point of $J_1(ka_o) : ka_o = 3.83171\cdots$.

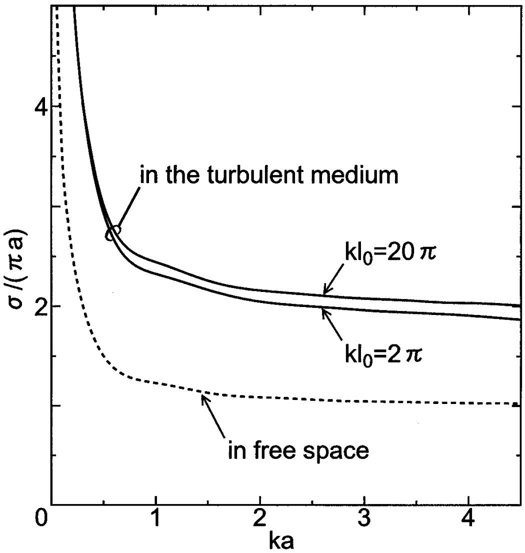

Figure 16. The average backscattering cross-section in the case of E-wave incidence, where the coherence length of the incident wave is shown in Figure 18.

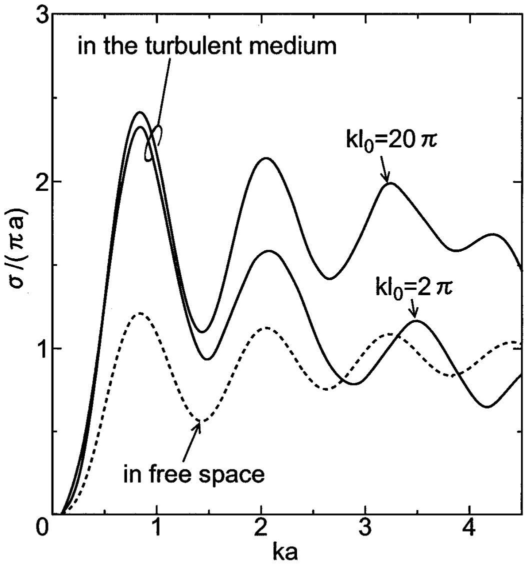

Figure 17. The average backscattering cross-section in the case of H-wave incidence, where the coherence length of the incident wave is shown in Figure 18.

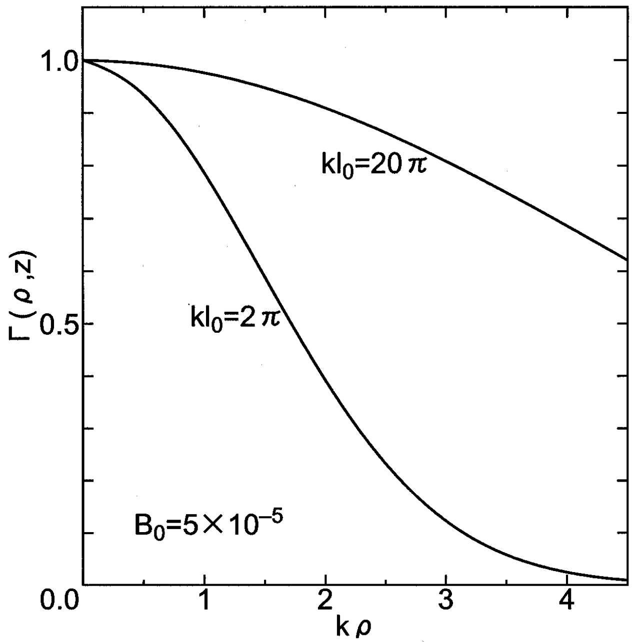

Figure 18. The degree of spatial coherence of incident waves about the cylinder. This figure shows that the coherence length of the incident wave becomes short, compared with Figure 10.

4. Concluding Remarks

We have presented a method for analyzing wave scattering from a conducting body in a random medium as boundary value problems. In doing that, we have introduced the current generators which transform any incident wave into surface currents on the body, and shown that the generators are constructed by Yasuura's method. The introduction of current generators makes the analysis of the scattering problem separated: that is, the analysis of wave propagation in random media and that of surface currents on the body. The former is based on the multiple scattering theory in random media and is to obtain the moments of the Green's functions. The latter is based on the surface integral and the inversion of the matrix of which each element is the inner product of basis functions in the complete set of wave functions.

By applying the method to the analysis of wave scattering from a circular cylinder surrounded with a turbulent medium, we have calculated the first and second moments of backscattered waves in the transition region from the Rayleigh to resonance scattering. First, in the case where the coherent and incoherent backscattered waves exist both appreciably, it has been depicted that the amplitude of the coherent backscattered wave decreases still more owing to the effect of double passage and in what degree it changes with the cylinder radius, the observation angle and the turbulence parameters. Second, in the case where the coherent backscattered wave is negligible and the backscattered wave becomes almost incoherent, the average backscattering cross-section (BCS) has been computed carefully and shown to be nearly twice as large as one in free space, if the coherence length of an incident wave about the cylinder is larger enough than the diameter of the cylinder. Above first and second results are valid for both the cases of E-wave and H-wave incidence. Third, the degree of spatial coherence of the incident wave has significant effect on the BCS if the coherence length is not larger than the diameter of the cylinder; as a result, the BCS for the H-wave incidence changes anomalously.

In this computation, the body is a circular cylinder and the curvature of the surface is constant at all points. To make clear the characteristics of the BCS, we need to compute the BCS of an elliptic cylinder and in addition the BCS of a body with concave surfaces. To obtain the BCS and other physical quantities in many practical cases, it is necessary that the moments of Green's functions in each random medium are expressed in analytic forms. The accuracy of computation for the physical quantities depends mainly on the analytic forms although the computation includes the multiple surface-integral.

Acknowledgments

The authors thank Mr. E. Tomita (NASDA), Mr. Y. Kakura (NEC) and Mr. H. Koga (NTT) for their helpful assist to calculation. This work was supported in part by the Scientific Research Grant-in-Aid (02650248, 1991; 04555087, 1992; 05452223, 1993, 1994) from the Ministry of Education, Science and Culture, Japan.

Appendices

A. Integral Equations for Random Surface Currents

Consider the scattering problem shown in Fig. 1 where $\varepsilon(\mathbf{r})$ is the random function defined by (2.1), (2.2) and the boundary conditions (2.3), (2.4) are valid. For simplicity we deal with scalar waves. To formulate the problem, we introduce the Green's function which satisfies the radiation condition

$$\lim_{r\to\infty} r\left(\frac{\partial G}{\partial r} - ikG\right) = 0 \quad ; \quad k = \omega\sqrt{\varepsilon_0\mu_0} \tag{A.1}$$

and the equation

$$[\nabla^2 + k^2(1 + \delta\varepsilon(\mathbf{r})]G(\mathbf{r}|\mathbf{r}') = -\delta(\mathbf{r} - \mathbf{r}') \ , \quad \text{for any } \mathbf{r} \tag{A.2}$$

where the body is replaced by the random medium with the same property.

When a source distribution is $f(\mathbf{r}_t)$, $r_t > L$, then the incident wave is defined by

$$u_{in}(\mathbf{r}) = \int G(\mathbf{r}|\mathbf{r}_t) f(\mathbf{r}_t) d\mathbf{r}_t \tag{A.3}$$

which means the wave propagated in the random medium without the body. In order that (A.3) is the incident wave independent of the body, it is necessary that $G(\mathbf{r}|\mathbf{r}_t)$ is hardly affected by the random medium replaced instead of the body; that is,

$$L \gg a \tag{A.4}$$

is required. Strictly speaking, it is required that $G(\mathbf{r}|\mathbf{r}_t)$ satisfies (A.2) where the body is replaced with free space and that the boundary between free space and the random medium is matched; that is, nonreflection is assumed. Some moments of $G(\mathbf{r}|\mathbf{r}_t)$ in this case, however, are approximately obtainable at present under the condition (A.4): i.e., on the assumption that the effect of free space is neglected. Consequently, under the condition (A.4), we can use the Green's function in the random medium where the body is replaced with the same random medium.

The wave produced by the body under the existence of u_{in} is called here the scattered wave and is designated by u_s. Then u_s satisfies the homogeneous equation of (A.2) in the random medium and the radiation condition at infinity. When the total wave is designated by u: $u = u_{in} + u_s$, then according to subsection 2.2, boundary conditions on the body are specified by (2.3) and (2.4).

Using Green's theorem, the radiation condition and the boundary conditions, we can obtain the integral representations of u_s:

$$u_s(\mathbf{r}) = -\int_s G(\mathbf{r}|\mathbf{r}_0)\frac{\partial}{\partial \mathbf{n}_0}u(\mathbf{r}_0)d\mathbf{r}_0 \qquad \text{for DC} \tag{A.5}$$

$$u_s(\mathbf{r}) = \int_s \left(\frac{\partial}{\partial \mathbf{n}_0}G(\mathbf{r}|\mathbf{r}_0)\right)u(\mathbf{r}_0)d\mathbf{r}_0 \qquad \text{for NC} \tag{A.6}$$

It can be shown from (2.2) that the singularities of

$$\lim_{r\to r_0} G(\mathbf{r}|\mathbf{r}_0) \quad \text{and} \quad \lim_{r\to r_0} \frac{\partial}{\partial \mathbf{n}_0}G(\mathbf{r}|\mathbf{r}_0)$$

are the same as these in free space[18]. This fact leads to the Fredholm integral equations of the second kind on the surface:

$$\frac{1}{2}\frac{\partial u(\mathbf{r})}{\partial \mathbf{n}} + \int_s \frac{\partial G(\mathbf{r}|\mathbf{r}_0)}{\partial \mathbf{n}}\frac{\partial u(\mathbf{r}_0)}{\partial \mathbf{n}_0}d\mathbf{r}_0 = \frac{\partial u_{in}(\mathbf{r})}{\partial \mathbf{n}}\,, \qquad \text{for DC} \tag{A.7}$$

$$\frac{1}{2}u(\mathbf{r}) + \int_s \frac{\partial G(\mathbf{r}|\mathbf{r}_0)}{\partial \mathbf{n}}u(\mathbf{r}_0)d\mathbf{r}_0 = u_{in}\,, \qquad \text{for NC} \tag{A.8}$$

For the Dirichlet condition case, substituting the solution $\partial u/\partial \mathbf{n}$ of (A.7) into (A.5), we can obtain the scattered wave. However, G and $\partial u/\partial \mathbf{n}$ in (A.5) are statistically coupled, and $\partial G/\partial \mathbf{n}$ and $\partial u/\partial \mathbf{n}_0$ in (A.7) are also done, so that it is difficult to express the moments of u_s in a closed form from (A.5) and (A.7). This holds also for the Neumann condition case. That is, the integral equation method including the boundary element method is not applicable to the problem of wave scattering from a conducting body in the random medium if we want to obtain the moments of u_s directly.

B. A Construction of the Scattering Problem by Yasuura's Method

Consider the scattering problem shown in Fig. 1 where $\varepsilon(\mathbf{r}) = \varepsilon_0$. When Yasuura's method is applied under the Dirichlet condition on the body, the scattered wave can be approximately expressed in terms of the modal functions defined in subsection $2 \cdot 4 \cdot 1$ as follows:

$$u_s(\mathbf{r}) \simeq \sum_{m=1}^{M} a_m(M)\phi_m(\mathbf{r}) = \boldsymbol{a}_M \boldsymbol{\phi}_M^T = \boldsymbol{\phi}_M \boldsymbol{a}_M^T \tag{B.1}$$

where $\boldsymbol{a}_M = [a_1, a_2, \ldots, a_M]$, given by

$$\boldsymbol{a}_M = -\mathbf{A}_E^{-1}(\boldsymbol{\phi}_M^T,\ u_{in}) \tag{B.2}$$

in which $\mathbf{A}_E$ is given by (2.13) and $(\boldsymbol{\phi}_M^T,\ u_{in})$ denotes the column vector of which each element is the inner product of ϕ_m and u_{in}, defined as (2.14). Equation (B.2) is obtained by minimizing the mean square error of the boundary value

$$\| u_s + u_{in} \| = \int_S \left| \sum_{m=1}^{M} a_m(M)\phi_m(\mathbf{r}) + u_{in}(\mathbf{r}) \right|^2 d\mathbf{r}$$

and the deviation procedure is the same as that used to obtain (2.9).

Let us now introduce the scattering operator S_E defined by

$$u_s(\mathbf{r}) = \int_S S_E(\mathbf{r}|\mathbf{r}')u_{in}(\mathbf{r}')d\mathbf{r}', \qquad \mathbf{r}' \text{ on } S \tag{B.3}$$

where u_{in} is any incident wave satisfying the Helmholtz equation. Substitution of (B.2) into (B.1) and comparison of it with (B.3) lead to

$$S_E(\mathbf{r}|\mathbf{r}') \simeq -\boldsymbol{\phi}_M \mathbf{A}_E^{-1}(\boldsymbol{\phi}_M^T, \tag{B.4}$$

where $(\boldsymbol{\phi}_M^T$ means the operation (3.2) of $\boldsymbol{\phi}_M^T$ and the function u_{in} to the right of the $\boldsymbol{\phi}_M^T$.

Under the Neumann condition, $\boldsymbol{a}_M^T$ in (B.1) can be obtained by

$$\boldsymbol{a}_M^T = -\mathbf{A}_H^{-1}\left(\frac{\partial \boldsymbol{\phi}_M^T}{\partial \mathbf{n}},\ \frac{\partial u_{in}}{\partial \mathbf{n}}\right) \tag{B.5}$$

where $\mathbf{A}_H$ is given by (2.19), and then the error of the boundary value

$$\| \frac{\partial u_s}{\partial \mathbf{n}} + \frac{\partial u_{in}}{\partial \mathbf{n}} \| = \int_S \left| \sum_{m=1}^{M} a_m(M) \frac{\partial \phi_n(\mathbf{r})}{\partial \mathbf{n}} + \frac{\partial u_{in}(\mathbf{r})}{\partial \mathbf{n}} \right|^2 d\mathbf{r}$$

has been minimized in the sense of mean squares. In the same way of (B.3), let us define the scattering operator in this case as follows:

$$u_s(\mathbf{r}) = \int_S S_H(\mathbf{r}|\mathbf{r}') u_{in}(\mathbf{r}') d\mathbf{r}', \qquad \mathbf{r}' \text{ on} S \tag{B.6}$$

Then it can be approximately given by

$$S_H(\mathbf{r}|\mathbf{r}') \simeq -\boldsymbol{\phi}_M \mathbf{A}_H^{-1} \left(\frac{\partial \boldsymbol{\phi}_M^T}{\partial \mathbf{n}} \right), \tag{B.7}$$

Here it should be noted that as $M \to \infty$, the scattering operators S_E and S_H converge uniformly to true operators, respectively, because (B.1) and (B.4) do so[21].

Using the scattering operators defined here and the current generators defined in subsection 2.4, we can schematically describe an approach to the problem of wave scattering from a conducting body in free space as Fig. A-1. That is, under the Dirichlet condition,

$$u_s = S_E \bullet u_{in} \qquad \text{(Direct Type)} \tag{B.8}$$

$$= -G_0 \bullet \frac{\partial u}{\partial n} \; ; \quad \frac{\partial u}{\partial n} = Y_E \bullet u_{in} \qquad \text{(Indirect Type)} \tag{B.9}$$

and under the Neumann condition,

$$u_s = S_H \bullet u_{in} \qquad \text{(Direct Type)} \tag{B.10}$$

$$= \frac{\partial G_0}{\partial n} \bullet u \; ; \quad u = Y_H \bullet u_{in} \qquad \text{(Indirect Type)} \tag{B.11}$$

where G_0 is Green's function in free space and the dot $\bullet$ denotes the integration on the surface of the body. We should pay attention to the fact that S and Y are constructed by using the same matrix $\mathbf{A}$ which depends only on the body surface, although it must be satisfied from a physical point of view.

When dealing with the scattering problem methodically as shown in Fig. A-1, operators S and Y must be well defined and mathematically constructed. Yasuura's method has definitely constructed them. In general, the scattering problem has been analyzed without getting S or Y, even if Yasuura's method is applied. The reason is that the computation is simple and effective, so that it seems that the idea of S and Y has not been specially required. If you try to apply Yasuura's method to some practical cases and to obtain numerical results, you should refer to the article[32]. As obvious from comparison of Fig. A-1 and Fig. 4, however, an operator construction by Yasuura's method is important for analyzing wave scattering from a conducting body in a random medium as boundary value problems. This appendix puts emphasis on an aspect of Yasuura's method, the aspect which has not usually been paid attention to but may be useful for the development of approaches to some problems.

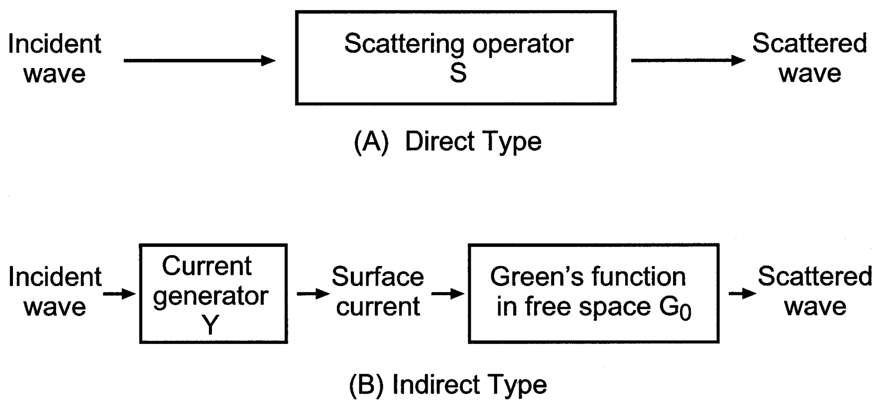

Figure A-1. Schematic diagram for solving the scattering problem where a conducting body is in free space.

C. A Solution of the Second Moment Equation

Assuming that a solution is expressed as

$$M_{20} = G_0(\mathbf{r}|\mathbf{r}_1)G_0(\mathbf{r}_t|\mathbf{r}_2) \times \exp\left[-k^2 \int_a^z dz_1 \int_a^{z_1} dz_2 B\left(0, z_1 - \frac{z_2}{2}, z_2\right)\right] M(\boldsymbol{\rho}_d, z) \tag{C.1}$$

where $\boldsymbol{\rho}_d = \boldsymbol{\rho} - \boldsymbol{\rho}_t$, and substituting it into (3.13), we have

$$\left[\frac{\partial}{\partial z} - i\frac{1}{k}\nabla_d^2 - \frac{1}{z-a}(\boldsymbol{\rho}_d - \boldsymbol{\rho}_{dc})\cdot\right.$$

$$\left.\nabla_d - \frac{k^2}{4}\int_a^z D(\boldsymbol{\rho}_d, z - \frac{z'}{2}, z')dz'\right]M(\boldsymbol{\rho}_d, z) = 0 \quad \text{(C.2)}$$

$$M(\boldsymbol{\rho}_d, a) = 1 \quad \text{(C.3)}$$

where

$$\begin{aligned} D(\boldsymbol{\rho}, z_+, z_-) &\equiv 2[B(0, z_+, z_-) - B(\boldsymbol{\rho}, z_+, z_-)] \\ &= B(z_+)\left(\frac{\boldsymbol{\rho}}{l(z_+)}\right)^2 \exp\left[-\left(\frac{z_-}{l(z_+)}\right)^2\right] \end{aligned} \quad \text{(C.4)}$$

Considering $D(\boldsymbol{\rho}, z_+, z_-) \propto \rho^2$, we assume that $M(\boldsymbol{\rho}_d, z)$ is expressed in the following form.

$$M(\boldsymbol{\rho}_d, z) = \frac{1}{g(z)}\exp[h(z)\rho_d^2 + f(z)\boldsymbol{\rho}_d \cdot \boldsymbol{\rho}_{dc}] \quad \text{(C.5)}$$

where $\boldsymbol{\rho}_{dc} = \boldsymbol{\rho}_1 - \boldsymbol{\rho}_2$. The substitution of (C.5) into (C.2) leads to the equations for $g(z)$, $h(z)$ and $f(z)$:

$$-\frac{1}{g(z)}\frac{dg(z)}{dz} + i\frac{4}{k}h(z) + \left[i\frac{1}{k} - \frac{f(z)}{z-a}\right]\rho_d^2 = 0 \quad \text{(C.6)}$$

$$\frac{df(z)}{dz} + \left[\frac{1}{z-a} + i\frac{4}{k}h(z)\right]f(z) - \frac{2}{z-a}h(z) = 0 \quad \text{(C.7)}$$

$$\frac{dh(z)}{dz} + i\frac{12}{k}h^2(z) + \frac{2}{z-a}h(z) - \frac{k^2}{4}\rho_d^{-2}\int_a^z D\left(\boldsymbol{\rho}_d, z - \frac{z'}{2}, z'\right)dz' = 0 \quad \text{(C.8)}$$

In the case where the turbulence intensity is characterized by

$$\frac{B(z)}{l(z)} = \left(\frac{z}{L}\right)^n \frac{B_0}{l_0} \quad \text{(C.9)}$$

we assume

$$h(z) = -i\frac{k}{4}\frac{1}{p(z)}\frac{dp(z)}{dz} \quad \text{(C.10)}$$

and substitute it into (C.8). Then we obtain the Riccati-type equation for $p(z)$.

$$\frac{d^2p(\xi)}{d\xi^2} - i\sqrt{\pi}\left(k\frac{B_0}{l_0}\right) L^{-n}\xi^{-(n+4)}p(\xi) = 0 \tag{C.11}$$

where $\xi = 1/z$. If $n \neq z$, the solution $p(z)$ is given by[33]

$$p(z) = z^{-1/2}\{C_1 J_{-\nu}[Q(z)] + C_2 J_\nu[Q(z)]\} \tag{C.12}$$

where $a \ll z$ is assumed, $Q(z) = i\nu\sqrt{\pi}kB_0(z/L)^{n/2}z/l_0$, $\nu = 1/(n+2)$, and C_1, C_2 are constant. Substituting (C.10) with (C.12) into (C.6) and (C.7), we can readily obtain $g(z)$ and $f(z)$ as follows:

$$g(z) = \frac{p(z)}{p(a)}\exp\left\{-i\frac{k}{4}\int_a^z dz'\frac{1}{(z'-a)^2}\left[1-\left(\frac{p(a)}{p(z)}\right)^2\right]\rho_{dc}^2\right\} \tag{C.13}$$

$$f(z) = -i\frac{k}{2}\frac{1}{z-a}\left[1-\frac{p(a)}{p(z)}\right] \tag{C.14}$$

Consequently, the substitution of (C.10), (C.13) and (C.14) into (C.5) yields

$$\begin{aligned} M(\boldsymbol{\rho}_d, z) &= \frac{p(a)}{p(z)}\exp\left\{-i\frac{k}{4}\frac{1}{p(z)}\frac{dp(z)}{dz}\rho_d^2 - i\frac{k}{2}\frac{1}{z-a}\left[1-\frac{p(a)}{p(z)}\right]\boldsymbol{\rho}\cdot\boldsymbol{\rho}_{dc}\right.\\ &\quad\left.+\int_a^z dz'\frac{1}{z'-a}\left[1-\left(\frac{p(a)}{p(z)}\right)^2\right]\rho_{dc}^2\right\} \end{aligned} \tag{C.15}$$

In the case of

$$\frac{B(z)}{l(z)} = \begin{cases} \dfrac{B_0}{l_0}, & a < z < L \\ \left(\dfrac{z}{L}\right)^{-m}\dfrac{B_0}{l_0}, & L < z \end{cases}$$

which corresponds to the turbulence in subsection 3.2, we need to connect the waves continuously at the boundary $z = L$. When we determine the constants C_1 and C_2 in (C.12) under the conditions of (C.3) and the continuity of $M(\boldsymbol{\rho}_d, L)$, then (3.18) can be obtained.

References

1. Frish, U., "Wave propagation in random media," *Probabilistic Methods in Applied Mathematics*, Vol. 1, 76–198, (ed. by A. T. Bharucha-Reid) Academic Press, New York, 1968.
2. Tatarski, V. I., "The effects of the turbulent atmosphere on wave propagation," *U. S. Dept. Commerce, NTIS, Spring-field,* 1971.
3. Ishimaru, A., *Wave Propagation and Scattering in Random Media*, Vol. 2, Academic Press, Orlando, Fla., 1978.
4. Strohbehn, J. W. (ed.), *Laser Beam Propagation in the Atmosphere*, Springer-Verlag, Berlin, 1978.
5. Furutsu, K., *Wave Propagation in Random Media*, Iwanami, Tokyo, 1982, (in Japanese).
6. Rytov, S. M., Yu. A. Kravtsov, and V. L. Tatarskii, *Principles of Statistical Radio physics*, Springer-Verlag, Berlin Heidelberg, 1987.
7. de Wolf, D. A., "Electromagnetic reflection from an extended turbulent medium: cumulative forwardscatter single-backscatter approximation," *IEEE Trans.*, Vol. AP-19, No. 2, 254–262, 1971.
8. Kravtsov, Yu. A., and A. I. Saichev, "Effect of double passage of waves in randomly inhomogeneous media," *Sov. Phys. Usp.*, Vol. 25, 7, 494-508 (1982).
9. Kuga, Y., and A. Ishimaru, "Retroreflectance from a dense distribution of spherical particles," *J. Opt. Soc. Am. A*, Vol. 1, No. 8, 831–835, 1984.
10. Tatarskii, V. I., "Some new aspects in the problem of wave and turbulence," *Radio Sci.*, Vol. 22, No. 6, 859–865, 1987.
11. Jakeman, E., "Enhanced backscattering through a deep random phase screen," *J. Opt. Soc. Am. A*, Vol. 5, No. 10, 1638–1648, 1988.
12. Mandt, C. E., L. Tsang, and A. Ishimaru, "Copolarized and depolarized backscattering enhancement of random discrete scatterers of large size based on second-order ladder and cyclical theory," *J. Opt. Soc. Am. A*, Vol. 7, No. 4, 585–592, 1990.
13. Mazar, R., "High-frequency propagators for diffraction and backscattering in random media," *J. Opt. Soc. Am. A*, Vol. 7, No. 1, 34–46, 1990.
14. John, S., Localization of Light, *Physics Today*, Vol. 45, No. 5, 32–40, 1991.

15. Mishchenko, M. I., "Enhanced backscattering of polarized light from discrete random media: calculations in exactly the backscattering direction," *J. Opt. Soc. Am. A*, Vol. 9, No. 6, 978–982, 1992.
16. John, S., "The localization of light," *Photmic band gaps and localization*, 1–22, (ed. by C. M. Soukoulis) Plenum Press, New York, 1993.
17. Tateiba, M., and E. Tomita, "Coherent electromagnetic waves scattered from a conducting cylinder surrounded by turbulent Media," *IEICE Trans.*, Vol. E-74, No. 5, 1288–1292, 1991.
18. Tateiba, M., and E. Tomita, "Theory of scalar wave scattering from a conducting target in random media," *IEICE Trans.*, Vol. E75-C, No. 1, 101–106, 1992.
19. Tateiba, M., and Y. Kakura, "Analysis of scattering from a conducting circular cylinder in turbulent media," *Proc. URSI Symp. on EMT*, Sydney, 78–80, 1992.
20. Tateiba, M., H. Koga, and Z. Q. Meng, "Numerical analysis of wave scattering from a conducting cylinder in turbulent media," *Proc. IGARSS*, Vol. 1, 90–93, 1994.
21. Yasuura, K., and T. Itakura, "Approximation method for wave functions (I), (II), (III)," *Tech. Rep. of Kyushu Univ.*, Vol. 38, 72–77, 1965; Vol. 38, 378–385, 1966; Vol. 39, 51–56, 1966, respectively (in Japanese).
22. Yasuura, K., "A view of numerical method in diffraction problem," *Progress in Radio Science 1966–1969*, 257–270, (ed. by W. V. Tilson and M. Sauzade) URSI, Brussels, 1971.
23. Yasuura, K., "Numerical methods for solving boundary value problems," *Modern Methods for Analyzing Electromagnetic Fields*, 57–117, (ed. by T. Izima) Inst. Electro. Comm. Erg., Tokyo, 1976, (in Japanese).
24. Morse, P. M., and H. Freshbach, *Methods of Theoretical Physics*, Part I, McGraw-Hill, New York, 1953.
25. Bowman, J. I., T. B. A. Senior, and P. L. E. Uslenghi, *Electromagnetic and Acoustic Scattering by Simple Shape*, North-Holland, Chap. 2, Amsterdam, 1969.
26. Tateiba, M., "Moment equation of a wave propagating through random media," *Mem. Fac. Eng. Kyushu Univ.*, Vol. 33, No. 4, 129–137, 1974.

27. Clifford, S. F., "The classical theory of wave propagation in a turbulent medium," *Laser beam propagation in the atmosphere*, 26–27, (ed. by J. W. Strohbehn) Springer-Verlag, Berlin,1978.
28. Tateiba, M., "Multiple scattering analysis of optical wave propagation through inhomogeneous random media," *Radio Sci.*, Vol. 17, No. 1, 205–210, 1982.
29. Tateiba, M., "Some useful expressions for spatial coherence functions propagated through random media," *Radio Sci.*, Vol. 20, No. 5, 1019–1024, 1985.
30. Tateiba, M., "Numerical analysis of non reciprocity for spatial coherence and spot dancing in random media," *Radio Sci.*, Vol. 17, No. 6, 1531–1535, 1982.
31. Tateiba, M., "The Lorentz reciprocity in random media — the derivation from Maxwell's equations," *Radio Sci.*, Vol. 26, No. 2, 499–503, 1991.
32. Okuno, Y., "Mode matching method," *Basic Analytical Methods in Electromagnetic Wave Problems*, 93–129, (ed. by E. Yamashita) Inst. Electro. Infor. Comm. Eng., Tokyo, 1987, (in Japanese).
33. Terasawa, K., *An Outline of Mathematics — Application Part*, Iwanami, Tokyo, 1960, (in Japanese).