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ROBUST ADAPTIVE BEAMFORMING AGAINST ARRAY CALIBRATION ERRORS

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Abstract—Adaptive beamforming methods degrade in the presence of model mismatch. In this paper, we develop a modified interference covariance matrix reconstruction based beamformer that is robust against large array calibration errors. The calibration errors can come from the element position errors, and/or amplitude and phase errors, etc.. The proposed method is based on the fact that the sample covariance matrix can approximate the interference covariance matrix properly when the desired signal is small, and a reconstructed covariance matrix based on the Capon spectral will be better than the sample covariance matrix when the desired signal is large. A weighted summation of two covariance matrices in references is used to reconstruct the interference covariance matrix. Moreover, a computationally efficient convex optimization-based algorithm is used to estimate the mismatch of the steering vector associated with the desired signal. Several simulation cases are applied to show the superiority of the proposed method over other robust adaptive beamformers.

1. INTRODUCTION

Array beamforming has a wide range of applications in radar, sonar, wireless communications, medical imaging and other fields [1–3]. Compared to data-independent beamformer, the data-dependent beamformers (adaptive beamforming) based on minimum variance distortionless response (MVDR) principle have better resolution and much better interference rejection capability [4]. However, the

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performance of traditional MVDR adaptive beamformers is well-known to be sensitive to model mismatch which can be caused by imprecise knowledge of the array steering vector, small sample size, and the presence of desired signal component in the training data [4–7]. Thus, many techniques have been proposed to improve the robustness of adaptive beamformers in the past decades ([8–23], and many references therein). One popular and widely used approach is the so-called diagonal loading technique [5], where the array covariance matrix is diagonally loaded with a scaled identity matrix. However, the main shortcoming of this technique is that it is not clear how to choose the diagonal loading level based on information about the uncertainty of the array steering vector. The worst-case-based beamforming technique [12], which makes explicit use of an uncertainty set of the signal steering vector, was proposed to improve the robust of adaptive beamforming. Based on the same idea, [9] reported the doubly constrained robust Capon beamforming method. Both the two methods require the steering vector error bound which is not known in some real world applications.

Many state-of-the-art robust adaptive beamformers will degrade their performance greatly when large calibration errors exist. One promising technique used to process the presumed steering vector of the desired signal is to estimate the actual steering vector in a convex formulation [13]. To prevent the steering vector converge to the steering vector associated with one of the interfering sources, a positive definite matrix is built as: $\bar{C} \triangleq \int_{\bar{\Phi}} \mathbf{a}(\varphi)\mathbf{a}^H(\varphi)d\varphi$, where the angular sector $\bar{\Phi}$ is the complement sector of Φ, and Φ is the angular sector in which the desired signal is located. However, when large calibration errors exist, the presumed steering vectors $\mathbf{a}(\theta)$ are mismatch with the real ones. The matrix $\bar{C}$ will be with large errors. A more recently approach based on the interference-plus-noise covariance matrix reconstruction was proposed in [11]. This method requires knowing all the steering vectors associated with the directions in $\bar{\Phi}$. If these steering vectors are not known exactly, the spatial spectrum distribution will be with errors and then the interference covariance matrix cannot be reconstructed properly.

In this paper, we propose a modified interference-plus-noise covariance matrix reconstruction based robust adaptive beamformer against array calibration errors. The steering vector associated with the desired signal is estimated by a convex optimization problem. In this paper we focus on adaptive beamforming with large array calibration errors. If the errors are only on the steering vector associated with the target signal, we may prefer to use the beamformer proposed in [11].

2. THE PROPOSED ROBUST BEAMFORMER

2.1. A Review of Robust Adaptive Beamforming

Let us consider an arbitrary linear array of N sensors that receives signals from multiple narrowband sources. If the array is perfect calibrated, the steering vector will be given as $\mathbf{b}(\varphi)$, $\varphi \in [0, \pi]$. However, because this array is with large calibration errors, the real steering vector is $\mathbf{a}(\varphi)$ which is unknown. The observation signal vector $\mathbf{x}(t)$ at the time instant t is an $N \times 1$ vector which is given as:

$$\mathbf{x}(t) = s(t)\mathbf{a}(\theta_d) + \mathbf{v}(t) \tag{1}$$

where $\mathbf{v}(t)$ denotes the sum of the interferences and the noise, $s(t)$ is the waveform of the desired signal, θ_d is the desired signal direction. The output of beamformer is given as $\mathbf{y}(t) = \mathbf{w}^H\mathbf{x}(t)$, where $\mathbf{w}$ is the $N \times 1$ complex weight vector and $(\cdot)^H$ stands for the Hermitian transpose.

The optimal weight vector $\mathbf{w}$ is to maximize the signal-to-interference-plus-noise ratio (SINR)

$$\text{SNR} = \frac{\sigma_s^2 |\mathbf{w}^H \mathbf{a}(\theta_d)|^2}{\mathbf{w}^H \mathbf{R}_v \mathbf{w}} \tag{2}$$

where $\sigma_s^2 = E\{|s(t)|^2\}$ is the signal power, and $\mathbf{R}_v = E\{\mathbf{v}(t)\mathbf{v}^H(t)\}$ is the interference-plus-noise covariance matrix. The problem of maximizing (2) can be written as the following optimization problem:

$$\min_w \mathbf{w}^H \mathbf{R}_v \mathbf{w} \quad \text{subject to } \mathbf{w}^H \mathbf{a}(\theta_d) = 1, \tag{3}$$

and the solution is the MVDR beamformer

$$\mathbf{w}_{opt} = \frac{\mathbf{R}_v^{-1}\mathbf{a}(\theta_d)}{\mathbf{a}^H(\theta_d)\mathbf{R}_v^{-1}\mathbf{a}(\theta_d)} \tag{4}$$

In practice, $\mathbf{R}_v$ is unavailable. In such case, it is commonly replaced by the sample covariance matrix $\hat{\mathbf{R}}_x$ in many proposed methods [4], which is given by

$$\hat{\mathbf{R}}_x = \frac{1}{T}\sum_{t=1}^{T} \mathbf{x}(t)\mathbf{x}^H(t), \tag{5}$$

where T is the number of training data samples. When $T \to \infty$, the sample covariance matrix $\hat{\mathbf{R}}_x$ will converge to the theoretical covariance matrix $\mathbf{R}_x = \mathbf{R}_v + \sigma_s^2 \mathbf{a}^H(\theta_d)\mathbf{a}(\theta_d)$. The optimization problem (3) is equivalent to solve:

$$\min_w \mathbf{w}^H \mathbf{R}_x \mathbf{w} \quad \text{subject to } \mathbf{w}^H \mathbf{a}(\theta_d) = 1, \tag{6}$$

since the relationship:

$$\mathbf{w}^H\mathbf{R}_x\mathbf{w} = \mathbf{w}^H\mathbf{R}_v\mathbf{w} + \sigma_s^2\left|\mathbf{w}^H\mathbf{a}(\theta_d)\right| = \mathbf{w}^H\mathbf{R}_v\mathbf{w} + \sigma_s^2. \tag{7}$$

However, when the T is small, there is a large gap between $\hat{\mathbf{R}}_x$ and $\mathbf{R}_x$. The beamformer performance degradation due to replacing $\mathbf{R}_v$ with $\hat{\mathbf{R}}_x$ will become significant as SNR increases. Furthermore, the mismatch between the actual steering vector $\mathbf{a}(\theta_d)$ and presumed steering vector $\mathbf{b}(\theta_d)$ associated with the target signal will degrade the performance further.

A promising robust adaptive beamforming based on interference covariance matrix reconstruction is proposed in [11] to avoid the performance degradation due to replacing $\mathbf{R}_v$ with $\hat{\mathbf{R}}_x$. The interference-plus-noise covariance matrix is reconstructed by

$$\hat{\mathbf{R}}_v = \int_{\bar{\Phi}} \frac{\mathbf{b}(\varphi)\mathbf{b}^H(\varphi)}{\mathbf{b}^H(\varphi)\hat{\mathbf{R}}_x^{-1}\mathbf{b}(\varphi)} d\varphi \tag{8}$$

In (8), we can find that $\hat{\mathbf{R}}_v$ collects all the information on interferences and the noise in the angular sector $\bar{\Phi}$, and the effect of the desired signal is removed. This method can provide reasonable estimates of $\mathbf{R}_v$ even in the case of small random steering vector errors. However, the actual steering vectors associated with the angles in $\bar{\Phi}$ are required to be given. If large array calibration errors exist, the presumed steering vectors $\mathbf{b}(\varphi)$ are mismatched with the actual steering vectors $\mathbf{a}(\varphi)$, the information of desired signal will leak into the reconstructed covariance matrix $\hat{\mathbf{R}}_v$ and matrix $\hat{\mathbf{R}}_v$ cannot collect all the information of interferences any more. The reason is that the steering vector $\mathbf{a}(\theta_d)$ is not orthogonal to $\mathbf{b}(\varphi)$, $\varphi \in \bar{\Phi}$. Due to the error of the reconstructed covariance matrix $\hat{\mathbf{R}}_v$, the performance of the beamformer will degrade especially at low SNRs.

2.2. The Proposed Method

At low SNRs, since the desired signal is small, the $\hat{\mathbf{R}}_x$ can approximate $\mathbf{R}_v$ with a small error. At high SNRs, we cannot use $\hat{\mathbf{R}}_x$ to replace $\mathbf{R}_v$ any more. Although $\hat{\mathbf{R}}_v$ cannot approximate $\mathbf{R}_v$ properly when the steering vector errors exist, it will be still better than using $\hat{\mathbf{R}}_x$. Therefore, we may prefer to use $\hat{\mathbf{R}}_x$ to replace $\mathbf{R}_v$ at low SNRs and use $\hat{\mathbf{R}}_v$ to replace $\mathbf{R}_v$ at high SNRs. Based on this idea, we propose a method to estimate the $\mathbf{R}_v$ by

$$\tilde{\mathbf{R}}_v = \alpha\hat{\mathbf{R}}_v \Big/ \left\|\hat{\mathbf{R}}_v\right\|_2 + (1-\alpha)\hat{\mathbf{R}}_x \Big/ \left\|\hat{\mathbf{R}}_x\right\|_2 \tag{9}$$

where

$$\alpha = \frac{\mathbf{b}^H(\theta_d)\hat{\mathbf{R}}_x\mathbf{b}(\theta_d)}{||\hat{\mathbf{R}}_x||_2||\mathbf{b}(\theta_d)||_2^2}. \tag{10}$$

The parameter $\alpha \in [0,1]$ is to reflect the desired signal energy compared with the interference energy. When α is equal to 0 which means that there is no signal from angle θ_d, then $\tilde{\mathbf{R}}_v$ is just composed by the $\hat{\mathbf{R}}_x$. When α is equal to 1 which means that the eigenvector associated with the largest eigenvalue of $\hat{\mathbf{R}}_x$ is equal to $\mathbf{b}(\theta_d)$. That is to say the power of the desired signal is greater than that of interferences. In such case, $\tilde{\mathbf{R}}_v$ is just composed by the matrix $\hat{\mathbf{R}}_v$. A greater value of α reflects a higher SNR of the desired signal and $\tilde{\mathbf{R}}_v$ is mainly composed by $\hat{\mathbf{R}}_v$, and vice versa.

In (10), presumed steering vector $\mathbf{b}(\theta_d)$ is different from the real steering vector $\mathbf{a}(\theta_d)$ due to the calibration errors. Due to the mismatch of $\mathbf{b}(\theta_d)$ and $\mathbf{a}(\theta_d)$, the parameter α calculated with $\mathbf{b}(\theta_d)$ will be different from the real value of α with a relative large error. Since $\hat{\mathbf{R}}_v$ can give a better approximation to $\mathbf{R}_v$ than $\hat{\mathbf{R}}_x$ at high SNR values, a smaller value of α will degrade the beamformer performance. Meanwhile, a greater value of α will degrade the beamformer performance at low SNR values. To solve this problem, we propose a rough estimation method to estimate $\mathbf{a}(\theta_d)$. First, we reconstruct the desired signal covariance matrix as:

$$\hat{\mathbf{R}}_s = \int_{\Phi} \frac{\mathbf{b}(\varphi)\mathbf{b}^H(\varphi)}{\mathbf{b}^H(\varphi)\hat{\mathbf{R}}_x^{-1}\mathbf{b}(\varphi)} d\varphi, \tag{11}$$

and then the estimated $\mathbf{a}(\theta_d)$ is obtained as the eigenvector of $\hat{\mathbf{R}}_s$ that corresponds to its maximal eigenvalue, here marked as $\hat{\mathbf{a}}(\theta_d)$. We note that $\hat{\mathbf{R}}_v$ in (8) collects the information of the interferences and noise in the direction range $\bar{\Phi}$ but here $\hat{\mathbf{R}}_s$ is to collect the information of the desired signal. We then substitute $\hat{\mathbf{a}}(\theta_d)$ for $\mathbf{b}(\theta_d)$ in (10) to obtain α.

To calculate the beamforming weights $\mathbf{w}$ with (4), we are required to know the estimation of $\mathbf{R}_v$ and $\mathbf{a}(\theta_d)$. Now we have obtained the estimation of $\mathbf{R}_v$, given as $\tilde{\mathbf{R}}_v$. In the following, we will estimate the actual steering vector.

Although the error of $\hat{\mathbf{a}}(\theta_d)$ is smaller than that of $\mathbf{b}(\theta_d)$, the vector $\hat{\mathbf{a}}(\theta_d)$ cannot give a good approximation of $\mathbf{a}(\theta_d)$ in many applications. We can decompose the error between $\hat{\mathbf{a}}(\theta_d)$ and $\mathbf{a}(\theta_d)$ into two components, i.e., $\mathbf{e}_\perp$ which is orthogonal to $\hat{\mathbf{a}}(\theta_d)$ and $\mathbf{e}_\parallel$ which is parallel to $\hat{\mathbf{a}}(\theta_d)$. We only search for $\mathbf{e}_\perp$, because any scaling of the steering vector does not impact the SINR [11, 13]. $\mathbf{e}_\perp$ can be solved by the following optimization problem:

$$\min_{\mathbf{e}_\perp} \left(\hat{\mathbf{a}}(\theta_d) + \mathbf{e}_\perp\right)^H \tilde{\mathbf{R}}_v^{-1} \left(\hat{\mathbf{a}}(\theta_d) + \mathbf{e}_\perp\right) \quad \text{subject to } \hat{\mathbf{a}}^H(\theta_d)\mathbf{e}_\perp = 0$$
$$\left(\hat{\mathbf{a}}(\theta_d) + \mathbf{e}_\perp\right)^H \hat{\mathbf{R}}_v \left(\hat{\mathbf{a}}(\theta_d) + \mathbf{e}_\perp\right) \leq \hat{\mathbf{a}}^H(\theta_d)\hat{\mathbf{R}}_v\hat{\mathbf{a}}(\theta_d) \tag{12}$$

If $\tilde{\mathbf{R}}_v$ is a positive semidefinite matrix, (12) will be a convex optimization problem which can be solved by the CVX MATLAB toolbox [24]. Actually, since $\hat{\mathbf{R}}_v$ and $\hat{\mathbf{R}}_x$ are both positive semidefinite matrix, we can easily prove that $\tilde{\mathbf{R}}_v$ is also a positive semidefinite matrix which is proven as: for any vector $\mathbf{x} \in \mathbb{C}^{N\times 1}$:

$$\begin{aligned}\mathbf{x}^T\tilde{\mathbf{R}}_v\mathbf{x} &= \mathbf{x}^T \left(\alpha\hat{\mathbf{R}}_v/\left|\left|\hat{\mathbf{R}}_v\right|\right|_2 + (1-\alpha)\hat{\mathbf{R}}_x/\left|\left|\hat{\mathbf{R}}_x\right|\right|_2\right)\mathbf{x} \\ &= \alpha\mathbf{x}^T\hat{\mathbf{R}}_v\mathbf{x}/\left|\left|\hat{\mathbf{R}}_v\right|\right|_2 + (1-\alpha)\mathbf{x}^T\hat{\mathbf{R}}_x\mathbf{x}/\left|\left|\hat{\mathbf{R}}_x\right|\right|_2 \geq 0\end{aligned} \tag{13}$$

Therefore, $\tilde{\mathbf{R}}_v$ is a positive semidefinite matrix. Now the estimated steering vector associated with the desired signal can be given as:

$$\tilde{\mathbf{a}}(\theta_d) = \hat{\mathbf{a}}(\theta_d) + \mathbf{e}_\perp, \tag{14}$$

Finally, by substituting $\tilde{\mathbf{a}}(\theta_d)$ and $\tilde{\mathbf{R}}_v$ into (4), the proposed beamforming weights can be computed as

$$\mathbf{w}_{pro} = \frac{\tilde{\mathbf{R}}_v^{-1}\tilde{\mathbf{a}}(\theta_d)}{\tilde{\mathbf{a}}^H(\theta_d)\tilde{\mathbf{R}}_v^{-1}\tilde{\mathbf{a}}(\theta_d)} \tag{15}$$

In summary, the steps of the proposed beamformer are given as follows:

1). Calculate the matrix $\hat{\mathbf{R}}_v$ and $\hat{\mathbf{R}}_s$ by (8) and (11), respectively.

2). The rough estimation $\hat{\mathbf{a}}(\theta_d)$ of $\mathbf{a}(\theta_d)$ is calculated as the eigenvector associated with the maximal eigenvalue of $\hat{\mathbf{R}}_s$. Substitute $\hat{\mathbf{a}}(\theta_d)$ for $\mathbf{b}(\theta_d)$ in (10) to obtain α.

3). Obtain $\tilde{\mathbf{R}}_v$ by (9).

4). Estimate $\mathbf{e}_\perp$ by solving the convex optimization problem (12).

5). The weights of the robust adaptive beamformer are given by (15).

Note that the steering vector $\tilde{\mathbf{a}}(\theta_d)$ can also be estimated by other methods, such as the method in [13], and by using the reconstructed interference-plus-noise matrix $\tilde{\mathbf{R}}_v$ can improve the performance of the beamformer in [13] by using $\mathbf{R}_x$. The comparison will be shown in Section 3.

The computational complexity of this proposed method is mainly composed by the calculation of (8), (11), and (12). The computational complexity of (8) and (11) is $o(SN^2)$, where S is the number of sampling points in the angle domain. The convex problem will dominate the computational cost, and the computational complexity of (12) is $o(N^{3.5})$. Therefore, the proposed method has complexity higher than the conventional Capon beamformer ($o(N^3)$) but has comparable complexity with other robust adaptive beamforming algorithms [11, 13].

3. SIMULATIONS

In this section, a uniform linear array (ULA) with 10 omnidirectional sensors spaced a half wavelength apart is considered. In all simulations, we assume two interfering sources with plane wavefronts impinging on the array from 30° and 50°, respectively. And we assume the interference-to-noise ratio in each element is equal to 30 dB. The background noise is assumed to be additive white Gaussian noise. We assume the desired signal is impinging on the array from 3°, and the SNR values range from −20 dB to 40 dB. For each scenario, 200 Monte Carlo simulation runs are used to obtain each simulated point.

In the first example, we consider the array calibration errors caused by gain and phase uncertainties. The error can come from channels uncalibrated, and the error is angle independent. The actual steering vector is given as:

$$\mathbf{a}(\varphi) = (1 + \boldsymbol{\gamma}) \circ \mathbf{b}(\varphi), \quad \varphi \in [0, \pi] \tag{16}$$

where $\circ$ denotes the Schur-Hadamard product, $\boldsymbol{\gamma} \in \mathbb{C}^{N\times 1}$ is the unknown channel gains and phases, $\mathbf{b}(\varphi)$ is the presumed steering vector which is given by $\mathbf{b}(\varphi) = [1, \mathbf{e}^{j\pi \sin(\varphi)}, \ldots, \mathbf{e}^{j9\pi \sin(\varphi)}]^T$. In this simulation, the absolutes and phase values of $\boldsymbol{\gamma}$ are independently and uniformly drawn from the intervals [0, 0.1] and [0, 2π], respectively. The number of snapshots is set to be $T = 30$. We compare the proposed method with the worst-case-based beamformer [8], the diagonally loaded sample matrix inversion (LSMI) beamformer [5], the steering vector estimation based beamformer proposed in [13], and the interference covariance matrix reconstruction based beamformer in [11]. Figure 1 displays the mean output SINRs versus the SNR for different techniques. It can be seen that the proposed beamformer outperforms the beamformer based on interference covariance matrix reconstruction in [11] at low SNRs (from −20 dB to 20 dB), and has nearly the same performances as the beamformer in [11] at high SNRs (from 25 dB to 40 dB). This is because $\tilde{\mathbf{R}}_v$ is a better approximation to the interference covariance matrix than $\hat{\mathbf{R}}_v$. Furthermore, the proposed beamformer can obtain greater SINRs when compared to its other competitors at high SNRs. Figure 2 gives the output SINRs versus the number of snapshots at the condition of SNR = 20 dB. We can find that

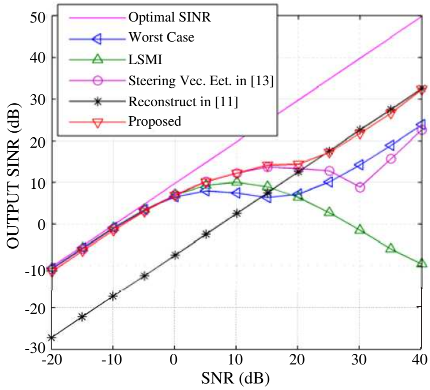

Figure 1. Output SINR versus SNR of the first example, $T = 30$.

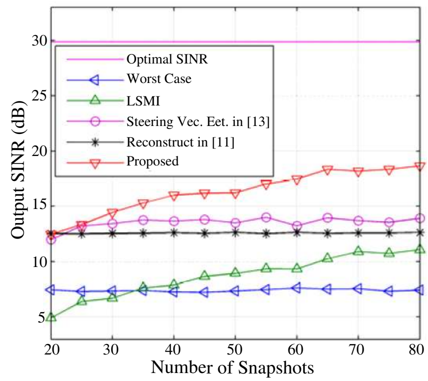

Figure 2. Output SINR versus the number of snapshots of the first example, SNR = 20 dB.

the performance of the proposed method improves with the increasing of the number of snapshots, and the proposed method outperforms other beamformers.

In the following simulation case, we will consider the array calibration error due to array geometry error along with wavefront distortion as discussed in [13] simulation example 3. We suppose that the position error for each element is uniform distributed in the interval $[-0.075\lambda, 0.075\lambda]$ which is greater than the case in [13]. The random position errors change from run to run but remain fixed from snapshot to snapshot. The number of snapshots is set to be $T = 30$. The comparison results are given in Figure 3, and the same conclusion as the previous simulation can be reached. When the SNR is greater than 30 dB which means that the desired signal power is greater than the interference power, the parameter α will approach 1 and then $\tilde{\mathbf{R}}_v$ will approach $\hat{\mathbf{R}}_v$. Therefore, it can be observed that the proposed beamformer can obtain the same results as the beamformer in [11] at SNR $= 40$. $\tilde{\mathbf{R}}_v$ can also be applied into other beamforming methods to improve their performance when large calibration errors exist. Here, $\tilde{\mathbf{R}}_v$ is used as a substitute for $\hat{\mathbf{R}}_x$ in the beamformer [13]. As shown in Figure 3, the beamformer of [13] with $\tilde{\mathbf{R}}_v$ (square line) has higher output SINR than that with $\hat{\mathbf{R}}_x$ (circle line) at SNRs from 25 dB to 40 dB. Figure 4 shows the mean output SINRs versus the number of snapshots at the condition of SNR $=$ 20 dB. The performance of the proposed beamformer outperforms all the other beamformers in Figure 4.

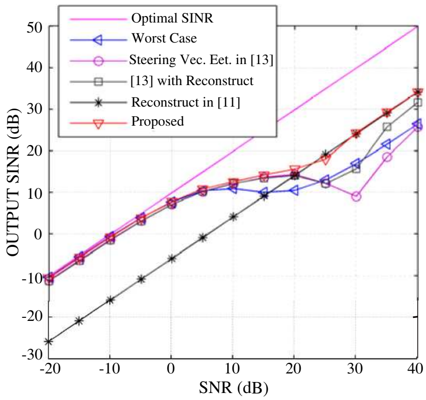

Figure 3. Output SINR versus SNR of the second example, $T = 30$.

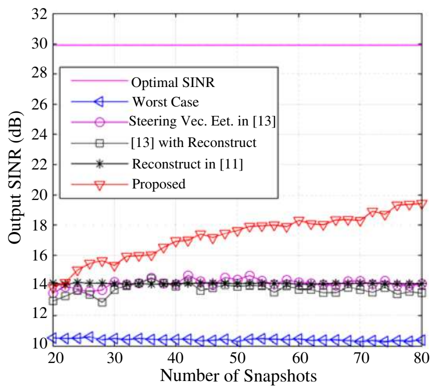

Figure 4. Output SINR versus the number of snapshots of the second example, SNR = 20 dB.

4. CONCLUSION

In this paper, a robust adaptive beamformer against large array calibration errors is proposed. This technique is based on the MVDR beamformer, and both the interference covariance matrix and the steering vector associated with the desired signal are estimated. Simulation results show the robustness of the proposed method against the array steering vector errors.

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