MULTIBAND MULTIMODE ARCHED BOW-SHAPED FRACTAL HELIX ANTENNA

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Abstract—A novel circular arc fractus named Arched Bow-shaped Fractal Curve (ABFC) is originally proposed. Four ABFCs are connected end-to-end, forming so called Arched Bow-shaped Fractal Loop (ABFL). The loop antenna peculiarly presents multiband multimode characteristics with resonance compression. The normal mode, which is pertinent to the loop area and circumference, is found improved with the iterative procedure. Thus, an eight-turned wire helix of small pitch angle ($\alpha = 3^\circ$) with a circular disc ground called Arched Bow-shaped Fractal Helix (ABFH) antenna is shaped from $K_2$ ABFLs. It can unprecedentedly operate in multiband of axial and off-axial modes with dual-sensed circular polarizations and high gain. Four matched bands ($|S_{11}| \leq -10 \text{ dB}$) are obtained within 2 GHz–8 GHz, of which $f_1 = 2.34 \text{ GHz}$ (400 MHz, 17.09%; $G = 10.63 \text{ dBi}$; RHCP), $f_2 = 4.24 \text{ GHz}$ (770 MHz, 18.16%; $G = 12.43 \text{ dBi}$; LHCP), $f_3 = 5.48 \text{ GHz}$ (300 MHz, 5.47%; $G = 8.13 \text{ dBi}$; RHCP), and $f_4 = 6.98 \text{ GHz}$ (960 MHz, 13.75%; $G = 15.89 \text{ dBi}$; RHCP). The unique multiband multimode property has been theoretically analyzed with illustrations and can be attributed to existence of the fractal boundary, which particularly encloses multiple equivalent loops with considerable areas. These peculiarities make $K_2$ ABFH antenna a very attractive candidate for multiband circularly polarized antennas, especially for space applications, such as spacecrafts communication, remote sensing, and telemetry, where reduction of quantity, height and weight of antennas are urgently wanted. It can also be configured into large array for higher gain service like radars and radio astronomy.
1. INTRODUCTION

Maxwell Equations can be used to explain all macro electromagnetic phenomena. The equations indicate that all electromagnetism are temporal-spatial problems. As one of the most important applications of Electromagnetics, antenna embodies the viewpoint very well. It is a spatial electromagnetic problem, which is closely correlated with the geometry, materials, and space environment. Unfortunately, there are usually no definite analytical expressions for precise calculation of the problem. Especially, fractal-shaped antenna is just such a typical type, of which performance is highly dependent upon the geometry. In essence, multiband and miniaturization of a fractal antenna are tightly related to its geometrical characteristics of self-similarity and space-filling, respectively. Fractal antenna is a breakthrough of antenna theory and technology in recent years. It had drawn much attention since it was formally put forth by Cohen [1, 2] 12 years after Mandelbrot established Fractal Geometry in 1983 [3]. Many particular attributes had been discovered for fractal antennas during extensive researches and applications in the past years [4–8].

Mathematically, fractal or fractus is a highly complicated, irregular or broken geometry [3, 9]. Just as its name implies, it consists of similar-shaped or affine-transformed pieces of different scales, which usually cannot be represented with a continuous function. With these exquisite and self-similar structures, a fractal antenna can yield similar, diversiform even optimum current distribution. So, it might have better radiation properties than Euclidean counterparts. So far, almost all fractal antennas utilizing fractal geometries that consist of straight segments, such as Koch curve series [10–12], Minkowski curve [13], Hilbert/Peano curve [14–17] or linearly-edged blocks like Sierpinski Carpet series [18–22]. Naturally, we conceive the idea of fabricating a fractal with curved elements, such as circular or elliptical arc with the hope of further enhancing the antenna’s performances, like bandwidth, reflection coefficient, directivity, gain, and polarization.

Cylindrical helix antenna (CHA) was invented by Kraus in 1946 [23–25]. It has two operation modes of circular polarization: normal mode [26] and axial mode. The former is narrow band and the latter is wideband. Polarization sense of both modes is accordant with winding direction of the helix. CHA has been used for various applications like FM radio receiver, satellite communications, GPS [27], electronic reconnaissance and radio astronomy. In principle, CHA is equivalent to a series of identical loop antennas and dipole antennas cascaded one by one. Intuitively, geometry of the loops should bear crucial influence on performance of the antenna. With circular
equivalent loops, CHA has approximately consistent impedance and radiation properties within a broad band, whose perimeter-to-wavelength ratio ranges from 3/4 to 4/3 [28]. While the loops have other arbitrary Euclidean geometry like congruent triangle, square, and regular polygon, CHA also presents the wideband trait. And if the loop radius changes continuously, an ultra-wideband will be obtained, such as conical helix antenna [29] and spherical helix antenna. However, it’s difficult to obtain multiband helix antenna through coalescing several loops of different radii in series. Heretofore, we would naturally conceive the idea of creating a multiband helix with fractal geometry. Unluckily, it’s quite difficult to fancy out a loop with fractal contour and considerable area for a helix antenna. For example, Minkowski Loop [30] is not a good candidate for helix antenna owing to its diminishing area with iteration procedure.

In this paper, we originally propose a novel circularly arched bow-shaped fractal curve and construct a loop with four such fractal curves cascaded end-to-end. For convenience and simplicity, full names of the curve and the loop are written as ABFC and ABFL respectively. We modeled and simulated ABFL of each iterative with Ansoft HFSS™ v.13. Simulation results indicate that electromagnetic properties of the fractal loop are tightly associated with its geometrical characteristics. Then, we designed and fabricated a circularly arched bow-shaped fractal helix (ABFH) antenna from 2-iterated ABFL. Good agreement is acquired from simulation and measurement. Quad-bands, which comprise dual senses of circular polarization and axial/off-axial modes with high gain, are obtained within band 2 GHz–8 GHz. In these bands, AR (Axial Ratio) has low level (≤ 2 dB) and good flatness (≤ 0.05 dB) both in frequency and space. Compared with CHA ($K_0$ ABFH), $K_2$ ABFH has a little higher frequency, smaller bandwidth, and comparable radiation properties at fundamental resonance $f_1$; small pitch angle and more geometrical complexity. The peculiarities manifest circularly curved fractal’s superiority and ingenious potentiality in performance enhancement of antennas. Meanwhile, these advantages make $K_2$ ABFH a very attractive candidate for multiband circular polarization applications, such as spacecraft-to-ground-station communications, where several helices for separate bands can be superseded by such one helix. By this way, quantity and weight of antennas, which are significant to spacecrafts, will be reduced greatly.
A fractal-shaped antenna, no matter comprises a fractal curve or a 2-D Euclidean sheet with or without such a fractal edge or side, manifests multiband with shrunk dimension. So, conceiving of fractal curve is a foremost task for developing fractal antennas. The iteration procedure of the proposed fractal curve begins with a circle initiator $K_0$ of radius $d_0 = 0.5 \cdot L_0 = 15.5 \text{ mm}$. Then, $K_0$ changes into $K_1$ by replacing its four quarter-arcs with four circular arcs of $1.5 \cdot \pi$ radian and radius $d_1 = d_0/3$ then cascaded end-to-end with another four arc segments of $1 \cdot \pi$ radian and the same radius. From $K_2$ iteration on, the procedure is implemented both on iterative arcs and connecting arcs. The coalescing way must ensure smooth transition of all the circular arcs with identical radii of $d_i = d_0/3^i$. The procedure proceeds to form the ABFC and ABFL of each iterative. The $K_i$-iterated fractal curves are circumscribed by a common contour square with side length of $L_0 = 2 \cdot d_0 = 31 \text{ mm}$, as shown in Figure 1. For better explanation of ABFL’s traits, Minkowski Loop (ML), which derives from square Koch curve, is also presented for comparison. Here, $K_i \ (i = 1,2,\ldots,n)$ denotes the iterative sequence for convenience. The ABFL and ML are modeled with Ansoft HFSS™ v.13. All shape parameters have close relationship during iterative, as depicted in Formulas (1)–(15) in Appendix A.

As depicted in Figures 1(a)–(d), geometrical difference between ABFL and ML is great. First, initiator of ABFL and ML is a circle and a square respectively. Second, inner loop of ABFL and ML extends outwards and dents inwards separately. The relationships between circumference $l_i$ and area $A_i$ of the loops and iteration $K_i$ are illustrated in Figure 2.

As depicted in Figure 2, enclosed area $A_i$ and $A'_i$ diminish into zero as the fractal contours $l_i$ and $l'_i$ approach infinitude when $K_i$ grows infinitely. For distinct showing discrepancies among these curves, only $K_i \leq 5$ are presented. $A'_i$ has larger initial value than $A_i$, but it decreases more quickly and gets smaller than $A_i$ when $K_i \geq 2$, which means ML encircles fewer area than ABFL in large iteration. In contrast, $l'_i$ also has bigger initial value than $l_i$, but it increases more slowly and becomes smaller than $l_i$ when $K_i \geq 18$, which means ML contains shorter circumference than ABFL in high iteration. Consequentially, there will be some electrical discrepancy between the two loops. When $K_i > 5$, values of $l_i$ and $l'_i$ can be calculated from Equations (5) and (7) in the appendix.

Radius $r_i$ (or width $w_i$) of the two loop wires (or stripes) of each
Figure 1. $K_i$ ABFL vs. ML (red, green, blue, and black denotes $K_3$, $K_2$, $K_1$, $K_0$ of ABFL and black, blue, green, and red denotes $K_3$, $K_2$, $K_1$, $K_0$ of ML; red spot-feed gap of $g = 0.025$ mm). (a) $K_0$ (black-ABFL, red-ML). (b) $K_1$ (blue-ABFL, green-ML). (c) $K_2$ (green-ABFL, blue-ML). (d) $K_3$ (red-ABFL, black-ML).

Iterative is:

$$r_i |w_i = \frac{r_0 |w_0}{2^i-1} \quad (i = 1, 2, \ldots, n; \quad r_0 |w_0 = 0.375 \text{ mm})$$

Form Formula (1) and Figure 1, we can see radius/width of wire/strip decreases with iterative number $K_i$. And this is the main reason why the maximum iterative time of a fractal wire or stripe antenna is usually chosen as a small natural number, like 2 to 5 times at most, or else the fractal wire or stripe will be self-overlapped.
or infinitesimal [10, 14]. Of course, the radius relationship between iterative doesn’t have the only form like Formula (1), but no matter what form it is, radius surely diminishes with iterative.

3. SIMULATION AND DISCUSSION OF ABFL ANTENNAS

3.1. Simulation of ABFL Antennas

Electrical properties of a fractal antenna can be revealed from its geometrical traits in ideal circumstances. CHA can be analyzed through the equivalent loops, so ABFH antenna will be evaluated from fractal electrical property of the $K_0–K_3$ ABFL antenna in free space without matching. For displaying discrepancy of ABFL and ML, $K_2$ ML is chosen for comparison. Radii of the loop and copper wire are $d_i = d_0/3^i$ and $r_i = r_0/2^i$ ($i = 0, 1, 2, 3$; $d_0 = 15.5$ mm; $r_0 = 0.375$ mm), respectively. The fractal loops are fed with a fixed gap of $g = 0.025$ mm centered at the starting points in $+X$-axis, as shown in Figure 1, and analyzed with FEM (Finite Element Method) solver of Ansoft HFSS™ v.13 in band 1 GHz–16 GHz. For perfect matching, the impedance of resonance $f_{ij}$ ($i–K_i$, $j$–$j$th resonant frequency of $K_i$) is chosen as port impedance, so that electrical properties of the fractal loops can be better unveiled. During iteration, ABLFs show distinct fractal electrical properties such as input impedance $Z_{in}$, bandwidth $BW$ and radiation pattern $G$, as shown in Figures 3–8. The resonant properties are summarized in Table 1.
Table 1. Simulated resonant properties of $K_i$ ABFL and $K_2$ ML antennas.

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<tr>
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<th>$f_i$ (GHz)</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
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<tbody>
<tr>
<td>$K_0$</td>
<td>3.383</td>
<td>2.01</td>
<td>6.791</td>
<td>-</td>
<td>-</td>
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<tr>
<td>$K_1$</td>
<td>152.25</td>
<td>237.69</td>
<td>15.31%</td>
<td>16.56%</td>
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<tr>
<td></td>
<td>560,</td>
<td>1040,</td>
<td>15.31%</td>
<td>16.56%</td>
<td>-</td>
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<tr>
<td></td>
<td>3.38</td>
<td>4.18</td>
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<tbody>
<tr>
<td>$K_2$</td>
<td>2.135</td>
<td>3.776</td>
<td>7.63</td>
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<tr>
<td></td>
<td>108,</td>
<td>125,</td>
<td>260,</td>
<td>5.06%</td>
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<td></td>
<td>2.54</td>
<td>3.89</td>
<td>4.94</td>
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<tr>
<td>$K_2$</td>
<td>1.593</td>
<td>2.835</td>
<td>6.715</td>
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<td></td>
<td>40,</td>
<td>30,</td>
<td>100,</td>
<td>5.06%</td>
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<td></td>
<td>2.12</td>
<td>3.92</td>
<td>9.14</td>
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<tr>
<td>$K_2$</td>
<td>1.316</td>
<td>2.445</td>
<td>5.689</td>
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<td></td>
<td>17.8%</td>
<td>9.5%</td>
<td>102,</td>
<td>5.06%</td>
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<tr>
<td></td>
<td>2.01</td>
<td>3.86</td>
<td>6.80</td>
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<td>$K_3$</td>
<td>1.413</td>
<td>2.492</td>
<td>5.661</td>
<td>11.927</td>
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<tr>
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<td>30,</td>
<td>45,</td>
<td>71,</td>
<td>2530,</td>
<td>-</td>
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<tr>
<td></td>
<td>2.19</td>
<td>4.21</td>
<td>7.83</td>
<td>6.68</td>
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(Bold-low resistance, underscored-high resistance).
3.2. Discussion and Conclusions

As shown in Figures 3–4, the input impedance $Z_{in}(f)$ and reactance $X_{in}(f)$ of $K_i$ ABFL and $K_2$ ML have remarkable multi-resonance

Figure 3. Input resistance $R_{in}(f)$ of $K_i$ ABFL ($K_0$-black, $K_1$-blue, $K_2$-green, $K_3$-red, $K_2$ ML-cyan).

Figure 4. Input reactance $X_{in}(f)$ of $K_i$ ABFL ($K_0$-black, $K_1$-blue, $K_2$-green, $K_3$-red, $K_2$ ML-cyan).

Figure 5. Gain patterns of $K_i$ ABFL and $K_2$ ML at Mode #1 (bare solid-Phi = 0°, XOZ, H-plane; circle marked solid-Phi = 90°, YOZ, E-plane, box marked solid-Theta = 90°, XOY; V-polarization; blue-$K_1$-$f_1$, green-$K_2$-$f_1$, red-$K_3$-$f_1$, cyan-$K_2$ ML-$f_1$, as shown in Table 1).

Figure 6. Gain patterns of $K_i$ ABFL and $K_2$ ML at Mode #2 (bare solid-Phi = 0°, XOZ, H-plane; circle marked solid-Phi = 90°, YOZ, H-plane; box marked solid-Theta = 90°, XOY, E-plane; H-polarization; black-$K_0$-$f_2$, blue-$K_1$-$f_2$, green-$K_2$-$f_2$, red-$K_3$-$f_2$, cyan-$K_2$ ML-$f_2$, as shown in Table 1).
during iterative process. $R_{in}(f)$ fluctuates between high-low impedances, and $X_{in}(f)$ fluctuates around 0Ω more frequently when $K_i$ grows, which means more resonant frequencies within the band. High and low resistances emerge alternately as reactance swings between inductance and capacitance quickly. Nevertheless, as shown

![Diagram](image_url)

**Figure 7.** Gain patterns of $K_i$ ABFL and $K_2$ ML at Mode #3 (bare solid-Phi = 0°, XOZ-H-plane; circle marked solid-Phi = 90°, YOZ, E-plane, box marked solid-Theta = 90°, XOY; V-polarization; black-\(K_0-f_1\), blue-\(K_1-f_3\), green-\(K_2-f_3\), red-\(K_3-f_3\), cyan-\(K_2\) ML-\(f_3\), as shown in Table 1. (a) Phi = 0°, XOZ. (b) Phi = 90°, YOZ. (c) Theta = 90°, XOY.
in Table 1, number of multi-resonance of $K_i$ never shows the usual relationship of $N = i + 1$ [11,21]. The initiator $K_0$ has two resonant frequencies, $f_1$ and $f_2$. When it iterates to $K_1$, a new lower resonant frequency occurs. The resonance number is unchanged when, $K_1$ evolves into $K_2$, but these resonances all have lower frequency. When $K_2$ iterates to $K_3$, another new resonance emerges in upper band. Impedance difference between $K_2$ (green) and $K_2$ ML (cyan) is conspicuous. $K_2$ ML has lower resonances and larger resistance, reactance in upper band. Resonances $f_{ij}$ of each iterative $K_i$ are categorized by means of putting those who have similar radiation patterns together. Thus, gain patterns of resonant frequencies of identical mode are merged into a pattern for relevance and discrepancy analysis, as depicted in Figures 5–8. In these patterns, bare solid, circle marked, box marked represents Phi = 0°, Phi = 90°, and Theta = 90° principle cut-planes respectively; and red, green, blue, black, and cyan denotes $K_3$, $K_2$, $K_1$, $K_0$, and $K_2$ ML in sequence. For more distinct display, variable scale range ($−100++20$, $−40++10$) is chosen for these patterns.

Figure 5 shows that Mode #1 has ideal dipole-like gain pattern with $V$-polarization, which is omnidirectional in $XOZ$ ($H$-plane, Phi = 0°) and doughnut-shaped in $YOZ$ ($E$-plane, Phi = 90°, $V$-polarization) and $XOY$ (Theta = 90°). Figure 6 shows that Mode #2

**Figure 8.** Gain patterns of $K_i$ ABFL at Mode #4 (bare solid-Phi = 0°, $XOZ$, $H$-plane; circle marked solid-Phi = 90°, $YOZ$, $E$-plane, box marked solid-Theta = 90°, $XOY$; $V$-polarization; red-$K_3-f_4$, as shown in Table 1).
has clove dipole-like radiation pattern with H-polarization, which is doughnut-shaped in \( XOZ \) (\( H \)-plane, \( \Phi = 0^\circ \)), \( YOZ \) (\( H \)-plane, \( \Phi = 90^\circ \)), and red cross-shaped in \( XOY \) (\( E \)-plane, \( \Theta = 90^\circ \), \( H \)-polarization). Figure 7 shows that Mode #3 has bi-directional radiation pattern in normal (\( Z \)-axis) with \( V \)-polarization, which is peanut-shaped in \( XOZ \) (\( H \)-plane, \( \Phi = 0^\circ \)) and \( YOZ \) (\( E \)-plane, \( \Phi = 90^\circ \), \( V \)-polarization) with nulls in \( X/Y \)-axis (\( \Theta = 90^\circ \)). Figure 8 shows that Mode #4 also has normal bi-directional radiation patterns but with side lobes in \( X/Y \)-axis, which are stub-crossed in \( XOZ \) (\( H \)-plane, \( \Phi = 0^\circ \)), cross-shaped in \( YOZ \) (\( E \)-plane, \( \Phi = 90^\circ \), \( V \)-polarization) and \( XOY \) (\( \Theta = 90^\circ \)). \( K_2 \) ML and \( K_2 \) ABFL have almost the same gain patterns in Mode #1, 2. But \( K_2 \) ABFL (green) has more ideal peanut-shaped pattern, and its gain is 2.34 dBi larger than \( K_2 \) ML (cyan) in Mode #3.

\( f_1 \) and \( f_2 \) of \( K_0 \) ABFL are with Mode #3 and Mode #2; \( f_1 \), \( f_2 \), and \( f_3 \) of \( K_1 \), \( K_2 \) ABFL, and \( K_2 \) ML are with Mode #1, Mode #2, and Mode #3; \( f_1 \), \( f_2 \), \( f_3 \), and \( f_4 \) of \( K_3 \) ABFL are with Mode #1, Mode #2, Mode #3, and Mode #4. Resonant frequency of Mode #3 first shifts upwards then shifts downwards with \( K_i \), because loop area diminishes but loop circumference grows with \( K_i \). Meanwhile, gain pattern gradually involves into ideal peanut-shaped with \( K_i \). Resonant frequencies of Mode #2 and Mode #1 always shift downwards with \( K_i \) due to constant growth of loop circumference. However, they maintain cloven dipole-like and ideal dipole-like gain pattern respectively with \( K_i \).

Surface current density \( J_s \) distribution at the resonant frequencies of \( K_3 \) ABFL antenna is illustrated in Figures 9–12. Intuitively, we will unveil the fractal traits in essence with the distribution. For better comparison, identical scale is chosen for all plots.

Figure 9. \( f_1 = 1.413 \text{ GHz} \ (1 \cdot \lambda_1) \). Figure 10. \( f_2 = 2.492 \text{ GHz} \ (2 \cdot \lambda_2) \).
Figure 11. $f_3 = 5.661 \, \text{GHz (} 5 \cdot \lambda_3 \text{)}$.

Figure 12. $f_4 = 11.927 \, \text{GHz (} 15 \cdot \lambda_4 \text{)}$.

From the illustration of $J_s$ in Figures 9–12, we can see that $K_3$ ABFL antenna operates in integral-timed wavelength, namely the electrical size at $f_1$ is $1 \cdot \lambda$, at $f_2$ is $2 \cdot \lambda$, at $f_3$ is $5 \cdot \lambda$, and at $f_4$ is $15 \cdot \lambda$. $f_1$ is dipole-like pattern, $f_2$ quadrifoliate dipole-like pattern, $f_3$ normal peanut-shaped pattern, and $f_4$ normal peanut-shaped pattern with stub-crossed side lobes in X/Y axis. Comparably, all conventional wire electrical loops antenna of various Euclidean shapes only have the normal mode. This phenomenon indicates that ABFL antenna has distinctive peculiarity of multi-resonance and multimode.

We can draw some conclusions from the aforementioned analysis as follows:

1. The number of resonant frequencies $N_i$ and iterative times $K_i$ has the following relationship, which is different from other fractal antenna counterparts like Koch/Koch-like [11] dipoles:

$$N_i = n_{\text{max}} = \text{mod}(i, 2) + \text{rem}(i, 2) + 2 \quad (i = 0, 1, 2 \ldots)$$

(mod- modul, rem- remain; $i = k \cdot 2 + k', \text{ mod}(i, 2) = k, \text{ rem}(i, 2) = k', k, k' \in \mathbb{N}$)

2. Ratio of adjacent resonant frequencies $\delta_n$ can be derived from $J_s$ distribution of $K_3$, which is very approximate to fractal scale ratio $\sigma_n$ when $K_i$ is large enough [11, 21]:

$$C_{\text{loop}} = N \lambda_{n+1} \cdot \frac{c_0}{f_{n+1}} = N \lambda_n \cdot \frac{c_0}{f_n};$$

$$\lim_{n \to n_{\text{max}}} \delta_n = \lim_{n \to n_{\text{max}}} \frac{f_{n+1}}{f_n} = \lim_{n \to n_{\text{max}}} \frac{N \lambda_{n+1}}{N \lambda_n} = 3 \approx \sigma_n, \quad (2)$$

where $C_{\text{loop}}$ is circumference of the fractal loop, $c_0$ is light speed in free space; $N \lambda_n$ is wavelength number of $f_n$ on the loop.
(3) Resonant frequency $f_n$, resistance $R_{in}$, and impedance bandwidth of each mode always decreases with iterative times $K_i$ except Mode #3. However, for each iterative case $K_i$, percentage bandwidth of resonant frequency $f_n$ doesn’t monotonously diminish with $n$.

(4) Resonant frequency of Mode #3 increases first and then decreases with $K_i$, because loop area $A_i$ diminishes but loop circumference $l_i$ grows more quickly with $K_i$. Meanwhile, the gain pattern gradually involves into ideal peanut-shaped with $K_i$.

(5) ABFL antenna of each iteration commonly has two fundamental radiation modes, namely Mode #2 ($H$-polarization) and Mode #3 ($V$-polarization). When $K_i \geq 1$, an iteratively induced mode, namely Mode #1 ($V$-polarization) yields; while $K_i \geq 3$, Mode #4 ($V$-polarization), which is a variant mode of Mode #3 emerges. The larger $K_i$, the more radiation modes. Gain patterns of Mode #1 and Mode #2 don’t significantly change with iteration $K_i$, but Mode #3 has remarkable improvement in gain patterns with growth of $K_i$.

(6) $K_i$-iterated ($i \geq 1$) ABFL antenna operates in multiband of integral wavelength.

(7) Compared with $K_1$, $K_2$, and $K_0$ counterparts, $K_3$ ABFL antenna presents 33.82%, 63.31% size shrinkage and 67.34% size increment at Mode #1, Mode #2, and Mode #3 respectively.

(8) $K_2$ ABFL (green) has larger bandwidth, more uniform impedances, and higher gain in Mode #1, Mode #2, and Mode #3 than $K_2$ ML (cyan). However, it has higher resonances than $K_2$ ML because $K_2$ ML covers larger area and circumference due to its initiator is bigger than that of ABFL.

Hereunto, we can conclude that ABFL is a more desirable fractal loop than Minkowski Loop due to the fact that it keeps considerable area when perimeter grows infinitely. This property makes it a better choice for multiband and multimode operation, especially for the normal mode, which is relevant to axial mode of helix antenna. Therefore, we bring forth the ideal of designing a high gain multiband helix antenna with this fractal loop in the next Section.

4. $K_2$ ABFH ANTENNA

4.1. Physical Design of $K_2$ ABFH Antenna

As discussed in Introduction, helix antenna is a very useful antenna type. However, it is wide but has single band in axial mode operation. If it is made into multiple wideband of this mode, that would bring
a great upgrade in performance and more attractions to applications. Therefore, we preferably modify the ABFC into helix antenna rather than loop antenna. In addition, in view of design and fabrication complexity, we choose $K_1 = 2$ as the antenna solution. Thus, a $K_2$ ABFH antenna is designed and fabricated for the previous assumption. It comprises eight turns of wire-wound $K_2$ ABFL with a small helix pitch angle $\theta$. Due to periodicity of the helix configuration, equation of a $K_2$ ABFH can be derived from that of a single turn as follows:

$$\begin{align*}
\theta & \in [\theta_0, \theta_0 + \gamma_i \cdot \pi]; \ \varphi \in [0, 2 \cdot \pi]; \ d_2 = \frac{d_0}{3^2} = \frac{17.5}{9} \text{ mm}; \ r_0 = 0.625 \text{ mm}; \\
(N_{x1} = N_{x2} = N_{x3} = N_{x9} = N_{x10} = N_{x11} = 4; \\
N_{x4} = 6; N_{x5} = N_{x6} = N_{x7} = 8; N_{x12} = 2; N_{x13} = 0) \\
N_x (12 \cdot k + 1 + i) &= -N_x (12 \cdot k + 1 - i) ; \\
N_y [12 \cdot (k - 1) + i] &= -N_x [12 \cdot k + 2 - i] \\
(\gamma_{12 \cdot k+1} = \gamma_{12 \cdot k+2} = \gamma_{12 \cdot k+4} = \gamma_{12 \cdot k+6} = \gamma_{12 \cdot k+8} = \gamma_{12 \cdot k+10} = \gamma_{12 \cdot k+12} = 1) \\
(\gamma_{12 \cdot k+3} = \gamma_{12 \cdot k+11} = 0.5; \gamma_{12 \cdot k+5} = \gamma_{12 \cdot k+7} = \gamma_{12 \cdot k+9} = 1.5) \\
(k = 1, 2, 3; \ i = 1, 2, \ldots, 12)
\end{align*}$$

$$ai = (-1)^{i+1}, \quad (i \in \psi = \{1, 2, 3, \ldots, 48\});$$

$$bi = \begin{cases} 
1, & i \in \phi = \{4, 8, 12, 14, 18, 22, 28, 32, 36, 38, 42, 46\} \\
0, & i \in \psi - \phi
\end{cases}, \quad (3)$$

where $\theta, \theta_0,$ and $\alpha$ are winding angle, initial winding angle, and pitch angle of the helix, respectively; $\varphi$, $\beta$, and $r_0$ are section azimuth angle, taper angle, and radius of the wire in several; $d_0$ and $d_2$ are radius of the initiator and the circular arcs of $K_2$ ABFL, respectively; $i = 1, 2, \ldots, 48$ denotes the $i$th arc; $N_{xi}$ and $N_{yi}$ are the centre coordinate; $a_i$ and $b_i$ are coefficients of winding direction; $\gamma_i$ is coefficient of ending angle of the $i$th arc. From Equation (3), it is clear that ABFH cannot be described with a continuous function but a series of piecewise functions, just as we stated in Section 1. Then, the unit helix is replicated eight times along the axial direction ($+Z$-axis) with a turn-to-turn spacing $S = (50 \cdot \pi \cdot d_2) \cdot \tan \alpha$. So, the overall height and total wire length of the $K_2$ ABFH are $H = 8 \cdot S$ and $L_T = 8 \cdot (S \cdot \csc \alpha) = (400 \cdot \pi \cdot d_2) \cdot \sec \alpha$, respectively. Likewise, the same solution is adopted in fabrication. First, eight unit wire helices are fabricated with a customized mould, as shown in Figure 13, then soldered end-to-end to form a complete ABFH. The design parameters for the helix antenna are $\alpha = 3^\circ$, $\beta = 0^\circ$, $\theta$,
**Figure 13.** Simple mould for fabrication of $K_2$ ABFH. ((a) 48 circular holes, which are concentric with the arcs, are drilled in a wood block, then 48 stainless steel posts are inserted into these holes. (b) A unit helix can been fabricated by winding a copper wire of radius $r_0 = 0.625$ mm around these posts).

**Figure 14.** Geometry of $K_2$ ABFH antenna ((a) Top view, red dash-circumcircle, blue dash-initiator $K_0$. (b) Front view, cyan-helix, yellow-ground plate; green-coaxial cable; unit: mm).

$d_0 = 17.5$ mm ($0.12 \cdot \lambda_1$, $\lambda_1$-wavelength of $f_1 = 2.34$ GHz in free space), $d_2 = 2.94$ mm, $r_0 = 0.625$ mm; diameter and thickness of the circular copper ground plate $D_g = 105$ mm ($0.819 \cdot \lambda_1$), $T_g = 1$ mm; distance of the helix above the ground $h_s = 1$ mm. So the circumradius is $D_s = 49$ mm ($0.382 \cdot \lambda_1$); element spacing is $S = 16$ mm ($0.11 \cdot \lambda_1$); overall height is $H = 128$ mm ($0.998 \cdot \lambda_1$), as depicted in Figure 14.
It is directly fed by a 50 Ω flanged SMA connector with a coaxial stub. The inner conductor of the coaxial stub is soldered with the helix wire and the flange fastened on the ground plate with screw nuts respectively. Owing to considerable weight of copper wire, the helix antenna is sustained in the diagonal corners by four dried bamboo posts of radius $r_p = 2.8$ mm and height $h_p = 138$ mm. A photo of the antenna prototype is shown in Figure 15. Because of inaccuracy of the fabrication, geometrical precision of the antenna is not satisfactory.

![Figure 15. Photos of the prototype of $K_2$ ABFH antenna (SMA with 4-holed square flange, 4 bamboo posts for support). (a) Top view. (b) Front view.](image)

4.2. Results and Analysis

Full-wave EM analysis was performed with FEM solver of ANSYS HFSS™ v.13 for $K_2$ ABFH antenna in band 2 GHz–8 GHz. Then, the antenna’s prototype was measured for reflection coefficient $|S_{11}|$ and gain patterns in an in-house anechoic chamber. Gain by contrast, peculiarity and superiority of $K_2$ ABFH antenna will reveal themselves to CHA ($K_0$ ABFH) with comparison. So, identical simulation analysis is also made for $K_0$ ABFH antenna, which has the same helix parameters ($r_0 = 0.625$ mm, $d_0 = 17.5$ mm, $\alpha = 3^\circ$, $N = 8$, $H = 128$ mm, $h_s = 1$ mm) and ground configuration ($D_g = 105$ mm, $T_g = 1$ mm). The simulated and measured results of $K_2$ and $K_0$ are merged into corresponding plots for discrepancy comparison and redundancy.
avoidance. In these plots, red, black, blue denote simulated, measured results of $K_2$, and simulated results of $K_0$ in sequence, as shown in Figures 16, 17 and Figures 19–29.

**Figure 16.** Simulated input impedance $Z_{in}$ of $K_i$ ABFH antenna (red-$K_2$, blue-$K_0$; solid-$R_{in}$, dash-$X_{in}$).

**Figure 17.** Reflection coefficient $|S_{11}|$ of $K_i$ ABFH antenna (red-simulated of $K_2$, black-measured of $K_2$, blue-simulated of $K_0$ at 25Ω port impedance).

Multi-resonance property manifests itself remarkably in input impedance $Z_{in}$. Low and high resonant impedances emerge alternately, as shown in Figure 16. The low resonant impedances (solid-$R_{in} = 35\Omega–65\Omega$, dash-$X_{in} = -20\Omega–+20\Omega$) are very approximate to 50Ω in quad-bands, which means four matched bands can be obtained easily with 50Ω coaxial feeding. Compared with the empirical formula $Z_{in} = R_{in} \approx 140 \cdot C\lambda$ given by Kraus [24] for a side-fed CHA, $K_2$ ABFH antenna has lower value due to impedance transformation of fractal geometry and coaxial cable. $Z_{in}$ can be further tuned to 50Ω with an extra quarter wavelength of impedance transformer beneath the ground plate or tapering initial segment of the fractal helix wire. Accordingly, $|S_{11}|$ presents quad-bands ($|S_{11}| \leq -10\, \text{dB}$), of which $f_1 = 2.34\, \text{GHz}$ (S-band, 2.02 GHz–2.42 GHz, BW = 400 MHz, 17.09%), $f_2 = 4.24\, \text{GHz}$ (C-band, 3.81 GHz–4.58 GHz, BW = 770 MHz, 18.16%), $f_3 = 5.48\, \text{GHz}$ (C-band, 5.35 GHz–5.65 GHz, BW = 300 MHz, 5.47%), and $f_4 = 6.98\, \text{GHz}$ (C-band, 6.93 GHz–7.89 GHz, BW = 960 MHz, 13.75%), as shown in Figure 17 and Table 2. For convenience, the quad-band denotes band #1, band #2, band #3, and band #4 separately. Comparably, the simulated (red) and measured (black) results of $|S_{11}|$ are largely accordant with each other though the latter shows narrower bandwidth and conspicuous shifting. This could be mainly imputed to the fabrication tolerance owing to the crude mould and process, as well as inherent error of simulation and the measurement systems. In contrast, $K_0$ ABFH gets close to 50Ω in a single band, so it
has only one matched band but with larger width \((f_c = 3.5 \text{ GHz}, 2.91 \text{ GHz} \text{–} 4.13 \text{ GHz}, \text{ BW} = 1200 \text{ MHz}, 34.26\%)\). The bandwidth can also be further widened out with those methods proposed for \(K_2\) ABFH antenna.

The multi-resonance properties of \(K_2\) ABFH antenna are summarized in Table 2 for highlighting the fractal features.

**Table 2.** Resonant properties of \(K_2\) ABFH antenna.

<table>
<thead>
<tr>
<th>Item</th>
<th>(f_1) (GHz)</th>
<th>(f_2) (GHz)</th>
<th>(f_3) (GHz)</th>
<th>(f_4) (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>sim</td>
<td>2.34</td>
<td>4.24</td>
<td>5.48</td>
<td>6.98</td>
</tr>
<tr>
<td>mea</td>
<td>2.50</td>
<td>4.05</td>
<td>5.44</td>
<td>6.72</td>
</tr>
</tbody>
</table>

\[ \delta_n = \frac{f_{n+1}}{f_n} \]

| Item          | \(\delta_n\) | \(\frac{f_{n+1}}{f_n}\) |
|--------------|--------------|----------------|----------------|
| sim          | -            | 1.812          | 1.292          | 1.274          |
| mea          | -            | 1.620          | 1.343          | 1.235          |

<table>
<thead>
<tr>
<th>Item</th>
<th>(R_i) (Ω)</th>
<th>(\sim)</th>
<th>(\text{mea})</th>
</tr>
</thead>
<tbody>
<tr>
<td>sim</td>
<td>38.46</td>
<td>51.72</td>
<td>47.66</td>
</tr>
<tr>
<td>mea</td>
<td>35.01</td>
<td>52.37</td>
<td>53.89</td>
</tr>
</tbody>
</table>

| Item          | \(|S_{11}|\) (dB) | \(\sim\) | \(\text{mea}\) |
|--------------|-------------------|---------|---------------|
| sim          | -25.02            | -29.61  | -24.67        | -31.58        |
| mea          | -22.97            | -32.44  | -31.57        | -26.36        |

<table>
<thead>
<tr>
<th>Item</th>
<th>(\text{BW}) (-10 dB)</th>
<th>(\sim)</th>
<th>(\text{mea})</th>
</tr>
</thead>
<tbody>
<tr>
<td>sim</td>
<td>400 MHz, 770 MHz, 300 MHz, 960 MHz, 17.09% 18.16% 5.47% 13.75%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mea</td>
<td>198 MHz, 300 MHz, 255 MHz, 383 MHz, 7.92% 7.41% 4.69% 5.69%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(\(\sim\)-simulated, \(\text{mea}\)-measured).

As presented in Table 2, adjacent frequency ratio \(\delta_n\) of \(K_2\) ABFH antenna is not equal to the fractal scale ratio \(\sigma_n\) of ABFL but varies from 1.274 to 1.812, because the four matched bands, as shown in Figure 17, are not identical modes engendered by the fractal iteration. However, ratio of \(f_4\) to \(f_1\delta\) is just close to the fractal scale ratio \(\sigma\). \(f_1 = 2.34 \text{ GHz}, f_2 = 4.24 \text{ GHz}, \) and \(f_4 = 6.98 \text{ GHz}\) are axial modes induced by fractal’s space-filling and self-similarity while \(f_3 = 5.48 \text{ GHz}\) is off-axial mode yielded by fractal’s impedance uniformity for multi-resonance, as shown in Figure 27. \(K_0\) ABFH antenna also has off-axial mode at \(f = 6.3 \text{ GHz}\), but the impedance is far different from that of axial mode at \(f = 3.5 \text{ GHz}\). The fact manifests fractal’s various functions in antenna design. The condition for axial mode operation is given in Equation (4) [26].

\[
\frac{3}{4} \leq C_{\lambda_i} \leq \frac{4}{3} \Rightarrow \frac{C_{\text{loop}}}{\lambda_i} = \frac{2 \cdot \pi \cdot d_i'}{300 f_i} = \frac{2 \cdot \sqrt{\pi \cdot A_i}}{300 f_i} \approx 1.1, \quad (4)
\]

where \(d_i'\) and \(A_i\) are equivalent radius and area of the loop respectively, and \(f_i\) is frequency in GHz, as shown in Figure 27. Considering
complexity, only qualitative analysis will be performed for the modes. Regardless of wave attenuation and turn-to-turn electromagnetic coupling, we can illustrate the equivalent current distributions of the bands with schematics, as shown in Figures 18(a)–(e). For $f_{1p} = 2.34\text{ GHz}$, $f_{2p} = 4.20\text{ GHz}$, $f_{3p} = 5.40\text{ GHz}$, and $f_{4p} = 7.34\text{ GHz}$, the equivalent loop radius is $d_1' = 20.4\text{ mm}$, $d_2' = 11.36\text{ mm}$, $d_3' = 17.24\text{ mm}$, and $d_4' = 8.62\text{ mm}$, respectively, as shown in Figures 18(b)–(e). Just like CHAs, $K_2$ ABFH operates in axial mode also with an equivalent loop circumference of about $1.1 \cdot \lambda$. According to Formula (4), $C_{\lambda 1} = 1.1$ corresponds to $f_{1p} \approx 3.0\text{ GHz}$ for $K_0$ ABFH with radius $d_0 = 17.5\text{ mm}$. In contrast, $K_2$ ABFH operates in $C_{\lambda 1} = 1.1$ at $f_{1p} = 2.34\text{ GHz}$ with an overall diameter of $D_s = 49\text{ mm}$, which means 16.73\% size increase at fundamental resonant frequency, as depicted in Figure 18(b). In view of overall circumference of the ABFH loop, we can infer that there is another axial-mode band $f_0 \approx 1\text{ GHz} (C_{\lambda 0} = 1)$. The equivalent loop radius is $d_0' = 48.6\text{ mm}$, but diameter of the circum-circle is $D_s = 49\text{ mm}$, which means 49.59\% of size shrinkage, as shown in Figure 18(a). However, this band’s impedance is different from that of the four upper matched bands. Furthermore, the size of the ground plate is electrically small $(0.343 \cdot \lambda_0)$, which will result in low gain. Therefore, the fundamental axial mode band $f_0$ is not presented.

As shown in Figures 18(a)–(e), currents of $f_{0p} = 1.0\text{ GHz}$, $f_{1p} = 2.34\text{ GHz}$, $f_{2p} = 4.20\text{ GHz}$, $f_{3p} = 5.40\text{ GHz}$, and $f_{4p} = 7.34\text{ GHz}$ turn counter-clockwise and clockwise alternately on the ABFH loop with about $1 \cdot \lambda_0$, $2 \cdot \lambda_1$, $4 \cdot \lambda_2$, $6 \cdot \lambda_3$, and $8 \cdot \lambda_4$, respectively. The $K_2$ ABFH loop can be regards as a complex of four sub-loops in the corners ($K_1$ ABFH loop) and one main-loop in the center. Just like a CHA, $f_{0p} = 1.0\text{ GHz}$ has a circularly symmetrical current distribution, which is equal in amplitude, identical in direction but opposite in phase, as depicted in Figure 18(a). Fields in $+Z$-axis are superposed and strengthened. Phase progresses anti-clockwise along the winding direction of the helix. So, RHCP axial mode forms and high gain will be obtained with a larger ground disc. Current distributions of $f_{1p} = 2.34\text{ GHz}$ on the sub-loops are identical but rotationally reverse, forming a four-element circular array. Such array effect for fractal antenna is firstly reported in [22]. In this way, current of opposite phase and orthogonal direction emerges in pairs on the loops. So, fields of these currents are cancelled with each other. Current on the main-loop turns counter-clockwise and clockwise alternately. However, it is electrical small without the current pairs, which are identical phase and orthogonally oriented, so axial fields cannot be cancelled mutually. Therefore, $f_{1p}$ is RHCP axial mode with high gain, as
depicted in Figure 18(b). Current distributions of $f_{2p} = 4.20$ GHz on the sub-loops are identical and rotationally symmetrical, forming a four-element circular array. In this way, current of same phase and orthogonal direction emerges on the array in pairs. Furthermore, clockwise currents predominate on the sub-loops. Thus, the array operates in axial mode with LHCP. However, current on the main-loop is uniformly counter-clockwise with even amplitude. So, it behaves like an electrically small loop and yields axial nulls. Therefore, $f_{2p}$ is LHCP axial mode with considerable gain, as depicted in Figure 18(c). Sense reversion of circular polarization is the most noteworthy peculiarity of $K_2$ ABFH antenna. Current distribution of $f_{3p} = 5.40$ GHz mainly resembles that of $f_{1p}$. The sub-loops also form a four-element array with identical but rotationally reverse current distributions, so the axial fields are cancelled. The main-loop also carries counter-clockwise and clockwise currents. However, it is electrically large with the current pairs, so axial fields are also cancelled. Therefore, $f_{3p}$ is RHCP off-
Figure 18. Bird’s eye on current distribution of $f_{ip}$ on the helix wire (green curve-outer boundary of helix wire, blue curve-schematic current path; red arrow-counterclockwise, blue arrow-clockwise; black ring-null of current, black spot-peak of current; outer dash circle with arrow-equivalent circular loop of $f_{ip}$). (a) $f_{0p} = 1$ GHz (The overall $K_2$ ABFH loop, which carries two currents of circular symmetry, engenders RHCP axial mode, just as $K_0$ CHA dose). (b) $f_{1p} = 2.34$ GHz (Current distribution on the four sub-loops are identical but rotationally reverse, so axial fields are cancelled with each other. Main-loop is electrically small with counter-clockwise and clockwise current, so yields RHCP axial mode). (c) $f_{2p} = 4.20$ GHz (Current distribution on the four sub-loops are identical and rotationally symmetrical, so forming 4-element array with LHCP axial mode. Main-loop is electrically small with counter-clockwise current, so yields axial null. Therefore, overall is LHCP axial mode). (d) $f_{3p} = 5.40$ GHz (Current distribution on the four sub-loops are identical but rotationally reverse, so axial fields are cancelled with each other. Main-loop is electrically large with counter-clockwise and clockwise current, so yields RHCP off-axial mode). (e) $f_{4p} = 7.34$ GHz (Current distribution on the four sub-loops are identical and rotationally symmetrical, so forming 4-element array with RHCP axial mode. Main-loop is electrically large with counter-clockwise current, so yields RHCP axial mode. The overall gain is as high as 17.14 dBi).

axial mode with medium gain, as depicted in Figure 18(d). Current distribution of $f_{4p} = 7.34$ GHz resembles that of $f_{3p}$ in the main. The sub-loops also form a four-element array with identical and rotationally symmetrical current distributions. The main-loop also carries counter-clockwise current. But it has large electrical size and the current pairs,
so yields RHCP axial mode. Furthermore, the sub-loop array also operates in RHCP axial mode. Therefore, $f_{4p}$ is RHCP axial mode with unusual high gain, as depicted in Figure 18(e). In conclusion, main-loop has and sub-loop hasn’t contribution to axial-mode of $f_{1p}$, main-loop hasn’t and sub-loop has contribution to axial-mode of $f_{2p}$, neither main-loop nor sub-loop has contribution to axial-mode of $f_{3p}$, both main-loop and sub-loop have contribution to axial-mode of $f_{4p}$.

$K_2$ ABFH antenna achieves multiband multimode operation on trade-off of bandwidth decrease and size increment at fundamental band. However, its superiority and advantages over $K_0$ ABFH (CHA) are significant, such as axial-mode in multiband, dual senses of CP, usual high gain in upper band, small pitch angle. Fundamental bandwidth as wide as CHA is our major target in the next stage of study.

Hereunto, we can draw a significant conclusion that axial-mode wideband helix antenna can been made into multiband multimode with a special loop, which should form several relatively independent sub-loops surrounding considerable area. Obviously, ML is not such an ideal loop for the helix antenna.

Gain can be approximately calculated with the empirical formula in [28]:

$$G_{\text{max}} \text{ (dB)} = 10.25 + \zeta_1 \cdot \left( \frac{L' T}{\lambda_p} \right) - \zeta_2 \cdot \left( \frac{L' T}{\lambda_p} \right)^2;$$
\[ L'_T = N \cdot \pi \cdot D' \cdot \tan \alpha = N \cdot \pi \cdot \left( 2 \cdot \sqrt{\frac{A'}{\pi}} \right) \cdot \tan \alpha; \quad (5) \]

where \( \zeta_1 \) and \( \zeta_2 \) are constant coefficients; \( L'_T \) and \( N \) are total length and turns of the equivalent cylindrical helix respectively; \( D' \) and \( A' \)

**Figure 21.** Gain patterns of \( K_i \) ABFH antenna at \( f_3 \) (\( G = 8.1 \) dBi; red-simulated of \( K_2 \) at \( f_3 = 5.40 \) GHz, black-measured of \( K_2 \) at \( f_3 = 5.44 \) GHz, blue-simulated of \( K_0 \) at \( f_3 = 6.4 \) GHz; solid-\( \Phi = 0^\circ \)-\( XOZ \), dash-\( \Phi = 90^\circ \)-\( YOZ \); bare line-RHCP, marked line-LHCP).

**Figure 22.** Gain patterns of \( K_2 \) ABFH antenna at \( f_4 \) (\( G = 17.1 \) dBi; red-simulated of \( K_2 \) at \( f_4 = 7.34 \) GHz, black-measured of \( K_2 \) at \( f_4 = 6.72 \) GHz; solid-\( \Phi=0^\circ\)-\( XOZ \), dash-\( \Phi= 90^\circ\)-\( YOZ \); bare line-RHCP, marked line-LHCP).

**Figure 23.** Axial ratio of \( K_i \) ABFH antenna at \( f_1 \) (red-simulated of \( K_2 \) at \( f_1 = 2.34 \) GHz, black-measured of \( K_2 \) at \( f_1 = 2.50 \) GHz; blue-simulated of \( K_0 \) at \( f_1 = 3.50 \) GHz; solid-\( \Phi = 0^\circ \)-\( XOZ \), dash-\( \Phi = 90^\circ \)-\( YOZ \)).

**Figure 24.** Axial ratio of \( K_2 \) ABFH antenna at \( f_2 \) (red-simulated of \( K_2 \) at \( f_2 = 4.20 \) GHz, black-measured of \( K_2 \) at \( f_2 = 4.05 \) GHz; solid-\( \Phi = 0^\circ \)-\( XOZ \), dash-\( \Phi = 90^\circ \)-\( YOZ \)).
are diameter and area of the equivalent loop. As shown in Figure 27, when \( f_p = 2.34 \text{ GHz}, 4.20 \text{ GHz}, 5.40 \text{ GHz}, \) and \( 7.34 \text{ GHz}, \) \( \zeta_2 = 0.1468, 0.1185, 0.2121, \) and \( 0.0542, \) respectively. Besides band \#1, high axial gain is also observed in bands \#2, \#3, and \#4, as shown in Figure 27. In contrast, \( K_0 \) ABFH has high axial gain only in the fundamental band, also as shown in Figure 27. Gain patterns of the quad-bands in Phi = 0\(^\circ\) and Phi = 90\(^\circ\) cut-planes are illustrated in Figures 19–22. In these plots, solid and dash lines represent Phi = 0\(^\circ\) (XOZ) and Phi = 90\(^\circ\) (Y OZ), respectively; red, black, and blue lines stand for simulated, measured \( K_2 \) ABFH, simulated \( K_0 \) ABFH in sequence, and bare and marked lines denotes co-polarization and cross-polarization component respectively. For distinct comparison, identical dynamic range (−52 dBi−+18 dBi) is chosen for these plots.

As shown in Figures 19–22, \( K_2 \) ABFH antenna has ideal axial radiation patterns, which are high gain dual-sensed circularly polarized (RHCP and LHCP) radiation at zenith (+Z-axis) with quasi-symmetry in cut-planes Phi = 0\(^\circ\) (XOZ) and Phi = 90\(^\circ\) (Y OZ). The peak gains of the quad-bands are \( G = 10.63 \text{ dBi}, 12.43 \text{ dBi}, 8.13 \text{ dBi}, \) and \( 17.14 \text{ dBi}, \) which occur at \( f_{1p} = 2.34 \text{ GHz}, f_{2p} = 4.20 \text{ GHz}, f_{3p} = 5.54 \text{ GHz}, \) and \( f_{4p} = 7.34 \text{ GHz}, \) respectively. Higher gain can be obtained with the ground plate of larger dimension or reverse truncated conical-cup shape. The good polarization purity can be seen from spatial patterns of axial ratio, as shown in Figures 23–26, which degrade with Theta off the axial direction. Furthermore, beamwidth of axial ratio patterns is consistent with that of the gain patterns, which means good circular polarization within whole half-power beamwidth.

**Figure 25.** Axial ratio of \( K_i \) ABFH antenna at \( f_3 \) (red-simulated of \( K_2 \) at \( f_3 = 5.40 \text{ GHz}, \) black-measured of \( K_2 \) at \( f_3 = 5.44 \text{ GHz}, \) blue-simulated of \( K_0 \) at \( f_3 = 6.4 \text{ GHz}; \) solid-Phi = 0\(^\circ\)-XOZ, dash-Phi = 90\(^\circ\)-Y OZ).

**Figure 26.** Axial ratio of \( K_2 \) ABFH antenna at \( f_4 \) (red-simulated of \( K_2 \) at \( f_4 = 7.34 \text{ GHz}, \) black- measured of \( K_2 \) at \( f_4 = 6.72 \text{ GHz}; \) solid-Phi = 0\(^\circ\)-XOZ, dash-Phi = 90\(^\circ\)-Y OZ).
In addition, gain patterns have little discrepancy between the two cut-planes. Axial gain and front-to-back-ratio (FTBR) versus frequency in the quad-bands are shown in Figures 27 and 28. According to the formula in [24], axial AR in decibel (dB) can be calculated as below.

$$\text{AR (dB)} = \left| L\lambda \cdot \left( \sin \alpha - \frac{1}{p} \right) \right| = \sin \alpha \cdot \frac{(2 \cdot N + 1)}{2 \cdot N};$$

$$p = \frac{L\lambda}{S\lambda + \frac{(2 \cdot N + 1)}{2 \cdot N}} = \frac{1}{\sin \alpha + \frac{(2 \cdot N + 1)}{2 \cdot N} \cdot \cos \alpha},$$

where $L\lambda$, $S\lambda$, and $C\lambda$ are wire length, turn-to-turn spacing, and circumference of the helix in wavelength, respectively; $p = v/c$ is relative phase velocity; $N$ and $\alpha$ have identical definition to the previous formulas. The smallest AR around $+Z$-axis of the quad-bands is $3.23 \text{ dB}$, $1.72 \text{ dB}$, $0.11 \text{ dB}$ ($\pm 30^\circ$), $0.35 \text{ dB}$ in sequence, which is approximate to the theoretical value $0.056$ from Formula (6). The

Figure 27. Simulated axial gain vs. $f$ of $K_i$ ABFH antenna (red-$K_2$, blue-$K_0$; solid-RHCP, dash-LHCP).

Figure 28. Simulated axial FTBR vs. $f$ of $K_i$ ABFH antenna (red-$K_2$, blue-$K_0$).

Figure 29. Simulated axial AR vs. $f$ of $K_i$ ABFH antenna (red-$K_2$, blue-$K_0$).
Table 3. Radiation properties of $K_2$ ABFH antenna.

<table>
<thead>
<tr>
<th>Item</th>
<th>$f_1$ (GHz)</th>
<th>$f_2$</th>
<th>$f_3$</th>
<th>$f_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>sim</td>
<td>2.34</td>
<td>4.20</td>
<td>5.40</td>
<td>7.34</td>
</tr>
<tr>
<td>mea</td>
<td>2.50</td>
<td>4.05</td>
<td>5.44</td>
<td>6.72</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Gain (dBi)</th>
<th>sim</th>
<th>10.63</th>
<th>12.43</th>
<th>8.13</th>
<th>17.14</th>
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<tbody>
<tr>
<td>mea</td>
<td></td>
<td>9.43</td>
<td>12.01</td>
<td>6.92</td>
<td>16.41</td>
</tr>
</tbody>
</table>

| Pol | sim | RHCP | LHCP | RHCP | RHCP |
|     | mea  |      |      |      |      |

| AR (dB) | sim | 3.23 | 1.72 | 0.11 | 1.35 |
|         | mea  | 2.68 | 1.41 | 1.64 | 0.91 |

| HPWB ($^\circ$) | sim | 53.62, | 47.72, | 25.63, | 31.56, |
|                 |     | 52.26, | 46.56, | 23.67, | 32.94 |
|                 | mea | 59.41, | 52.02, | 24.45, | 35.53 |
|                 |     | 60.62 | 50.61 | 22.18 | 36.24 |

| BWFN ($^\circ$) | sim | 64.12, | 48.06, | 0, 120.21, | 51.21 |
|                 |     | 58.03, | 46.11, | 0, 131.11, | 54.82 |
|                 | mea | 113.24, | 52.92, | 0, 96.21; | -39.92 |
|                 |     | 75.16 | 52.93 | 0, 110.23 | 47.04 |

| FSLL (dB) | sim | -1.78, | -2.09, | 0.46, | -7.49, |
|           | mea  | -5.05, | -1.62, | -0.03, | -7.11 |

|           | sim | -19.51, | -5.26, | -11.74, | -6.71, |
|           | mea  | -9.45, | -5.82, | -11.22, | -5.92 |

(axial AR versus frequency also manifests expected low value within the quad-bands, as depicted in Figure 29. The overall radiation characteristics of the quad-bands are presented in Table 3.  

5. CONCLUSION

A novel circularly arched fractal named Arched Bow-shaped Fractal Curve (ABFC) is originally proposed. Then, a loop is constructed with four ABFCs and called Arched Bow-shaped Fractal Loop (ABFL). Fractal dimension $D_F$, physical scale ratio $\zeta_i$, and fractal scale ratio $\sigma_i$, which dominate geometrical properties of ABFL, are thoroughly formulated. ABFL antenna peculiarly presents multiband multimode traits with resonance compression just when $K_i \geq 1$ ($i \geq 1$). Ratio of adjacent resonant frequencies $\delta_n(f_{n+1}/f_n)$ is found approximate to fractal scale ratio $\sigma_i$ just when $K_i$ is large and $f_n$ high. The normal mode (Mode $\neq 3$) is discovered pertinent to the loop area and circumference simultaneously, which diminishes and increases with $K_i$, respectively. So, the resonant frequency $f_{ni}$ of $K_i$-iterated ($i \geq 1$) ABFL is higher than that of the initiator $K_0$ ABFL (circular loop) but...
also decreases with $K_i$.

$K_2$ ABFL is then configured into a wire helix called Arched Bow-shaped Fractal Helix (ABFH) antenna with a copper disc ground. It can unprecedentedly operate in multiband of axial/off-axial mode with dual-sensed circular polarizations (CP) and high gain, which is almost impossible for a conventional helix antenna with any Euclidean geometry. Quad-bands ($|S_{11}| \leq -10$ dB) are obtained within 2 GHz–8 GHz. Compared with $K_0$ counterpart, $K_2$ ABFH has some negligible drawbacks, such as slight size increment and bandwidth shrinkage at fundamental resonance, and fabrication complexity. However, it has approximate fundamental band gain, higher upper-band gain, dual senses of circular polarization and more ideal axial ratio with smaller pitch angle ($\alpha = 3^\circ$), which mean significant height reduction ($\alpha = 10^\circ$–15$^\circ$). This should be a big breakthrough for helix antenna since it was invented by J. D. Kraus early in 1946. The peculiar multiband multimode property has been theoretically analyzed. This trait can be attributed to the existence of the fractal boundary, which particularly encloses multiple equivalent loops with considerable areas. The unique properties makes $K_2$ ABFH antenna a very valuable candidate for multiband circularly polarized antennas, especially for space applications, such as spacecrafts communication, remote sensing, and telemetry, where reduction of quantity and weight of antennas are urgently wanted. It can also be configured into parabolic reflector’s feed or large array for higher gain service like radar and radio astronomy. The problem of narrower bandwidth of fundamental resonance can be solved by some impedance matching techniques, using precise mould and accurate manufacture process, such as 3D printing technology.

ACKNOWLEDGMENT

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APPENDIX A. MATHEMATICAL TRAITS OF ABFL AND ML

The total number of arc segments of ABFL and rectilinear segments of ML of successive iteration is:

$$N_{i+1} = 4 \cdot \left[ (N_i + Q_i) + (i + 1) \cdot i^2 + 1 \right]$$

$$i = 0, 1, 2, \ldots, n; \quad N_0 = 1; \quad Q_0 = 0; \quad Q_{i \geq 1} = 1$$

(A1)
and
\[ N'_i + 1 = 5 \cdot N'_i = 5^{i+1} \quad (i = -1, 0, 1, \ldots, n) \tag{A2} \]
respectively, where \( N_i \) and \( N'_i \) is the number of segments of \( K_i \)-iterated \textbf{ABFL} and \textbf{ML} separately. Radius \( d_i \) and total radian \( p_i \) of the arcs of \textbf{ABFL} of each iterative is:
\[ d_i = \frac{d_0}{3^i} \quad (d_0 = 15.5 \text{ mm}; \ i = 0, 1, 2, \ldots, n) \tag{A3} \]
and
\[ p_{i+1} = 5 \cdot p_i = 5^{i+1} \cdot p_0; \quad (p_0 = 2 \cdot \pi; \ i = 0, 1, 2, \ldots, n) \tag{A4} \]
From Equations (3) and (4), we can get overall arc length of \textbf{ABFL} of each iterative easily as follows:
\[ l_{i+1} = p_{i+1} \cdot d_{i+1} = \left( \frac{5^{i+1} \cdot p_0}{3^{i+1}} \right) \cdot \left( \frac{5 \cdot p_0 \cdot d_0}{3} \right)^{i+1} \quad (i = 0, 1, 2, \ldots, n) \tag{A5} \]
The length ratio of middle indentation square to side segments of \textbf{ML} is \( e \), as shown in Figures 1(b)–(d), so we obtain the minimum scale \( l_0 \):
\[ l_0 = \frac{L_0}{(2 + e)^i} \quad (i = 0, 1, 2, \ldots, n) \tag{A6} \]
and total circumference \( l'_i \):
\[ l'_{i+1} = \left( \frac{2 + 3 \cdot e}{2 + e} \right) \cdot l'_i = \left( \frac{2 + 3 \cdot e}{2 + e} \right)^i \cdot l'_0 \]
\[ = 4 \cdot \left( \frac{2 + 3 \cdot e}{2 + e} \right)^{i+1} \cdot L_0 \quad (i = -1, 0, 1, \ldots, n) \tag{A7} \]
of \textbf{ML} of \( K_i \) iteration.

Obviously, \( l_i \) and \( l'_i \) both grow infinitely during iterative process, but \( l_i \) increases more quickly. According to definition of box dimension [3, 9], we obtain fractal dimensions of \textbf{ABFL} and \textbf{ML} as follows:
\[ D_F = - \lim_{i \to \infty, \delta \to 0} \frac{\log M_{i+1}}{\log \delta_{i+1}} = - \lim_{i \to \infty, \delta \to 0} \frac{\log \left( \frac{5^{i+1} \cdot (p_0 \cdot d_0)}{3^{i+1}} \right)}{\log \delta_{i+1}} \]
\[ = - \lim_{i \to \infty, \delta \to 0} \frac{\log \left( \frac{5 \cdot p_0 \cdot d_0}{3^{i+1} \cdot 3 \cdot \pi} \right)}{\log \left( \frac{\pi}{2} \cdot \frac{d_0}{3^i \cdot \pi} \right)} + 1 \]
\[ = 1.465 \quad (p_0 = 2 \cdot \pi, \ d_0 = 15.5 \text{ mm}; \ i = 0, 1, \ldots, n) \tag{A8} \]
\[ D'_F = - \lim_{i \to \infty, \delta \to 0} \log \frac{M'_{i+1}}{\log \delta'_{i+1}} = \log \frac{(e \cdot l_{i+1})}{e \cdot l_{0i+1}} - \lim_{i \to \infty, \delta \to 0} \frac{\log(2 + 3 \cdot e)}{\log (2 + e)} < 1.465 \]

\[ (e \in [0, 1], \; L_0 = 2 \cdot d_0; \; i = 0, 1 \ldots, n) \]

where \( M_i, M'_i \) is equivalent segment numbers of the \( i \)th-iterated ABFL and ML measured with unit scale of \( \pi/4 \) radian and \( e \cdot l_{0i} \) respectively.

Then, we obtain physical scale ratios \( \zeta_i \):

\[ \zeta_{i+1} = \lim_{i \to \infty} \frac{M_{i+1} \cdot \delta_{i+1}}{M_i \cdot \delta_i} = \frac{l_{i+1}}{l_i} = \frac{(5/3)^{i+1} \cdot (p_0 \cdot d_0)}{(5/3)^i \cdot (p_0 \cdot d_0)} = \frac{5}{3} \approx 1.667 \] (A10)

and fractal scale ratios \( \sigma_i \):

\[ \lim_{i \to \infty} \sigma_i = \lim_{i \to \infty} \frac{r^i}{r^{i-1}} = \lim_{i \to \infty} \left[ \frac{(3^i - 1) \cdot \sqrt{2} + 1}{(3^{i-1} - 1) \cdot \sqrt{2} + 1} \cdot \frac{d_0}{3^{i-1}} \right] \approx 3 \] (i=1, 2, \ldots, n) (A12)

\[ \lim_{i \to \infty} \sigma'_i = \lim_{i \to \infty} \frac{l_{0i}}{l_{0i-1}} = 2 < 2 + e < 3 (e \in [0, 1]; \; i = 1, 2, \ldots, n) \] (A13)

of ABFL and ML.

where \( r^i \) is diagonal radii of the \( i \)th-iterated ABFL. As shown in formula (8)–(13), fractal dimension \( D_F \), physical scale ratio \( \zeta_i \), and fractal scale ratio \( \sigma_i \) of the two fractal loops are both all different. This circumstance is very frequent with almost all fractals. And this property might be intimately associated with the antenna’s performance. Besides the quantities shown above, another crucial parameter for ABFL and ML is area \( A_i \), which is:

\[ \lim_{i \to \infty} A_i = 4 \cdot d_0^2 - (b_i - \pi) \cdot d_i^2 \]

\[ = 4 \cdot d_0^2 - \left( \frac{\sum_{k=0}^{2i} a_k \cdot 3^k}{3^{2i} - \frac{\pi}{3^{2i}}} \right) \cdot d_0^2 \]

\[ = \left\{ 4 - \left[ \frac{\beta \cdot (1 - 3^{1+2i})}{3^{2i} - \frac{\pi}{3^{2i}}} \right] \right\} \cdot d_0^2 \]
\[= \left[4 - \beta \cdot \frac{3}{2} + \frac{(0.5 \cdot \beta + \pi)}{3^2} \right] \cdot d_0^2 \approx \left(4 - \beta \cdot \frac{3}{2}\right) \cdot d_0^2\]

\[0 = \left(4 - \frac{8}{3} \cdot \frac{3}{2}\right) \cdot d_0^2 \leq A_i \leq \left(4 - 1 \cdot \frac{3}{2}\right) \cdot d_0^2 = 2.5 \cdot d_0^2\]

\[(\beta \in \Phi = \left[1, \frac{8}{3}\right]) \quad \text{(A14)}\]

and

\[
\lim_{i \to +\infty} A_i' = \left\{1 - 4 \cdot \sum_{j=0}^{i-1} \left[\frac{2 + 3 \cdot e^2}{(2 + e)^2}\right]^j \cdot \left(\frac{e}{2 + e}\right)^2\right\} \cdot A_0'
\]

\[
(A_i' = L_0^2; \quad e \in [0, 1]; \quad i = 1, 2 \ldots n), \quad \text{(A15)}
\]

respectively. In Equation (A14), when \(i = 0, b_0 = a_0 = 4; i = 1, b_1 = 20, a_2 = 1, a_1 = 3, a_0 = 2; i = 2, b_2 = 196, a_4 = 1, a_3 = a_2 = 3, a_1 = 2, a_0 = 1; i = 3, b_3 = 1844, a_6 = 2, a_5 = a_4 = a_3 = 1, a_2 = 3, a_1 = a_0 = 2. There is an approximate expression for terms of series \(b_i (i = 0, 1, \ldots, n): b_{i+2} = 9 \cdot b_{i+1} + 4 \cdot b_i\), which tends to grow exponentially.

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