

## THREE-DIMENSIONAL POLYHEDRAL INVISIBLE CLOAK CONSISTING OF HOMOGENEOUS MATERIALS

Yi Hao Yang<sup>1, 2, †</sup>, Shi Sheng Lin<sup>1, 2, †</sup>, Zuo Jia Wang<sup>1, 2</sup>,  
Hong Sheng Chen<sup>1, 2, \*</sup>, Hua Ping Wang<sup>1, 2, 3</sup>, and Er Ping Li<sup>1, 2</sup>

<sup>1</sup>The Electromagnetics Academy at Zhejiang University, Zhejiang University, Hangzhou 310027, China

<sup>2</sup>Department of Information Science and Electronic Engineering, Zhejiang University, Hangzhou 310027, China

<sup>3</sup>Marvell Technology Group Boston, Marlborough, Massachusetts 01752, USA

**Abstract**—Invisible cloak with its amazing functions has been turned into reality due to the advent of transformation optics during the past few years. However, the inhomogeneity and singularity of electromagnetic parameters in cloak are still the main bottlenecks for practical realization. In this paper, we propose a scheme of three-dimensional polyhedral invisible cloak to overcome these shortcomings by using a linear homogeneous transformation method. The constitutive parameters of the polyhedral cloak are homogeneous and anisotropic, which are relatively easy for realization. Numerical simulations demonstrate that good invisibility performance can be achieved for any polarization wave. Our work provides a novel approach to simplify three-dimensional cloak in practice.

### 1. INTRODUCTION

Invisibility has aroused considerable interests owing to Pendry et al.'s and Leonhardt's pioneering theoretical work in 2006 [1, 2]. Ideal isolated cloaks, such as spherical cloak [1], cylindrical cloak [3–5], elliptical cloak [6], and other columnar cloaks [7–13], etc., can work for

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\* Corresponding author: Hong Sheng Chen (hansomchen@zju.edu.cn).

† Yi Hao Yang and Shi Sheng Lin are co-first author of this paper.

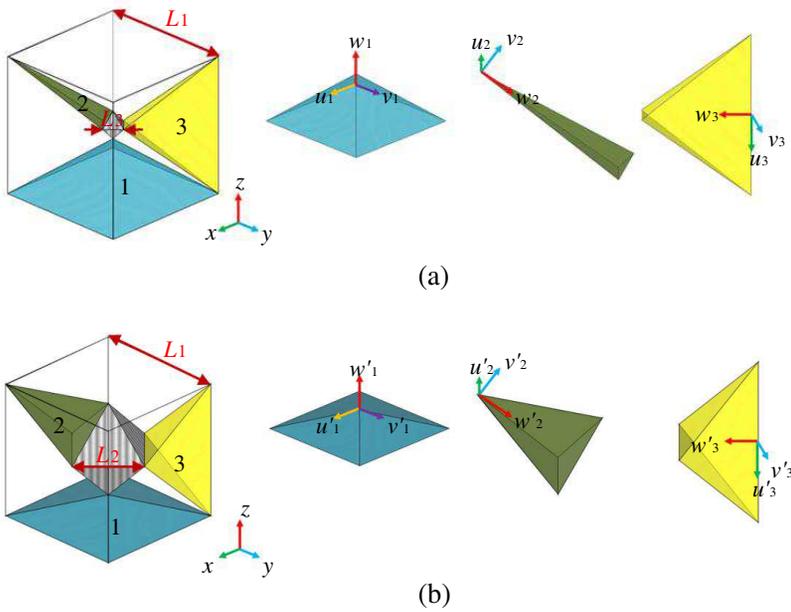
any direction. Compared with the three-dimensional cloaks [1, 13, 14], the columnar cloaks can provide a perfect three-dimensional cloaking performance for wave incident from any direction. However, this requires the columnar cloak to be infinitely long, which is not easy for practical implementation. In this paper, we focus on the design of a three-dimensional cloak and discuss how we can simplify the parameters much more easily.

The main challenges to achieve a perfect isolated cloak are the inhomogeneity and singularity of electromagnetic constitutive parameters in the cloak. The inhomogeneity of the parameters is induced by the inhomogeneous transformation functions. In practical realization, the inhomogeneous regions need to be discretized into a lot of regions, so that in each region the parameters are spatially invariant [3, 15–18]. However, this procedure of discretization degrades the performance of cloaks. To solve this limitation, Xi et al. proposed a linear homogeneous coordinate transformation method [19]. In their work, a diamond is divided into four parts. A linear homogeneous coordinate transformation is applied to each segment, so that the constitutive parameters of each segment are homogeneous. In addition, the singularity of the parameters will be introduced when a point (or a line) in virtual space is transformed into a line (or an area) in physical space [20]. In two-dimensional cloak design, such as carpet cloak [19, 21–27] and elliptical cloak [6], an ordinary strategy to remove the singularity is transforming a line in virtual space into another line in physical space. However, the line-line transformed cloaks sacrifice the omnidirectional invisibility performance of the cloak. To recover the omnidirectional performance and still keep the homogeneous and non-singular parameters, Chen and Zheng proposed a polygonal cloak [7], where a polygonal hidden region is mapped to a smaller one, which is hard to be seen by the naked eyes, and Han et al. [10] and Wei et al. [11] applied twofold spatial compression which expands several lines or a line into a concealed region.

In this paper, we take advantages of the above solutions and extend the ideas to three-dimensional case. We divide a polyhedron into many segments and apply linear homogeneous transformation in each segment to get homogeneous constitutive parameters. A nearly perfect three-dimensional polyhedral cloak without inhomogeneity and singularity is therefore achieved.

## 2. HEXAHEDRAL CLOAK

We firstly take the hexahedron shown in Fig. 1 as an example to demonstrate the design of the three-dimensional cloak. The coordinate



**Figure 1.** Scheme of coordination transformation of a three-dimensional polyhedral cloak. The space between the octahedron (the hidden region marked in gray) and the hexahedron is divided into 26 segments. All of the segments can be grouped into three types which are marked in blue, green, and yellow, respectively. The segments in virtual space (Fig. 1(a)) are transformed linearly and homogenously into the segments in physical space (Fig. 1(b)) along their corresponding coordinate axes.

system is built with the origin at the center of the hexahedron. The space between the hexahedron and the small octahedron (the dual polyhedron of the hexahedron) can be divided into 26 segments. Due to the symmetry of the cloak, we can group the segments into three types. The first type of the segments is composed of a face of the hexahedron and a vertex of the octahedron (Segment 1 marked in blue with local coordinate axes  $(u_1, v_1, w_1)$  and origin at the center of the face of hexahedron) and the quantity of these segments is 6; the second type is composed of a vertex of the hexahedron and a face of the octahedron (Segment 2 marked in green with local coordinate axes  $(u_2, v_2, w_2)$  and origin at the vertex of hexahedron) and the quantity of these segments is 8; the third type is composed of an edge of the hexahedron and an edge of the octahedron (Segment 3

marked in yellow with local coordinate axes ( $u_3, v_3, w_3$ ) and origin at the center of the edge of hexahedron) and the quantity of these segments is 12. In virtual space (Fig. 1(a)), the octahedron region is set to be small so that it is hard to be detected by the detectors. We apply a linear homogeneous transformation in each segment to compress or extend each region, leading to a much bigger octahedron hidden region in physical space (Fig. 1(b)). As long as the octahedron in the virtual system is small enough, the performance of the three-dimensional polyhedral cloak can be very close to perfect. Because of the linear homogenous transformation, all the parameters of the regions are homogeneous, which will facilitate the practical realizations.

The following linear homogeneous transformations are applied in each segment along its local coordinate axes:

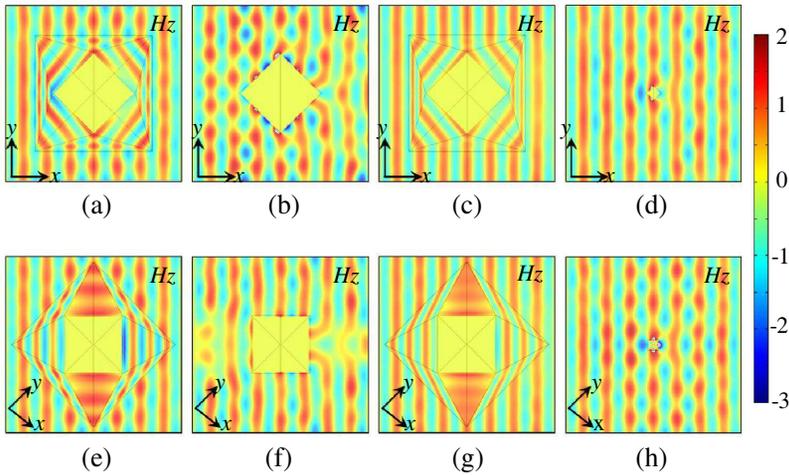
$$\begin{aligned} u'_1 &= u_1, \nu'_1 = \nu_1, w'_1 = \kappa_a w_1, \text{ for segment 1,} \\ u'_2 &= \kappa_b u_2, v'_2 = \kappa_b v_2, w'_2 = \kappa_c w_2, \text{ for segment 2,} \\ u'_3 &= \kappa_b u_3, v'_3 = v_3, w'_3 = \kappa_d w_3, \text{ for segment 3,} \end{aligned}$$

where  $\kappa_a = ((1/2)L_1 - (\sqrt{2}/2)L_2)/((1/2)L_1 - (\sqrt{2}/2)L_3)$ ,  $\kappa_b = L_2/L_3$ ,  $\kappa_c = ((\sqrt{3}/2)L_1 - (\sqrt{6}/6)L_2)/((\sqrt{3}/2)L_1 - (\sqrt{6}/6)L_3)$ ,  $\kappa_d = ((\sqrt{2}/2)L_1 - (1/2)L_2)/((\sqrt{2}/2)L_1 - (1/2)L_3)$ ,  $L_1, L_2, L_3$  are the side lengths of the hexahedron, the octahedron in physical space, the octahedron in virtual space, respectively. Then we can obtain the constitutive parameters of each segment:

$$\begin{aligned} \mu'_{1u} &= \varepsilon'_{1u} = 1/\kappa_a, \mu'_{1v} = \varepsilon'_{1v} = 1/\kappa_a, \mu'_{1w} = \varepsilon'_{1w} = \kappa_a, \text{ for} \\ &\text{segment 1,} \\ \mu'_{2u} &= \varepsilon'_{2u} = 1/\kappa_c, \mu'_{2v} = \varepsilon'_{2v} = 1/\kappa_c, \mu'_{2w} = \varepsilon'_{2w} = \kappa_c/\kappa_b^2, \text{ for} \\ &\text{segment 2,} \\ \mu'_{3u} &= \varepsilon'_{3u} = \kappa_b/\kappa_d, \mu'_{3v} = \varepsilon'_{3v} = 1/\kappa_b\kappa_d, \mu'_{3w} = \varepsilon'_{3w} = \kappa_d/\kappa_b, \text{ for} \\ &\text{segment 3.} \end{aligned}$$

It is interesting that only three kinds of materials are used to design a three-dimensional isolated cloak. Importantly, the constitutive parameters of these materials are homogenous diagonal constant tensors without singularity. Therefore, the complexity for practical realization is reduced.

In the following, we employ the commercial finite element method software, COMSOL Multiphysics 3.5a, to verify the performance of the cloak. We set the octahedron to be PEC and cover it with the proposed cloak, then impose an incident plane wave ( $\lambda = 0.2\text{ m}$ ) with  $H_z$  polarization from left. The cloak is simulated with appropriate boundary conditions. In Fig. 2(a), the left and right boundaries are set as scattering boundary conditions; the top and bottom boundaries are set as perfect electric boundary conditions; the boundaries in



**Figure 2.** (a), (b), (c), (d) The magnetic field distribution when a plane wave with  $H_z$  polarization is incident along the  $x$  direction onto (a) a PEC octahedron covered by the proposed lossless cloak, (b) a bare PEC octahedron, (c) a PEC octahedron covered by the proposed lossy cloak with loss tangent of 0.01 for both permeability and permittivity, and (d) an equivalent PEC octahedron in virtual space. (e), (f), (g), (h) The magnetic field distribution when a plane wave with  $H_z$  polarization is incident along the  $\vec{x}/\sqrt{2} + \vec{y}/\sqrt{2}$  direction onto (e) a PEC octahedron covered by the proposed lossless cloak, (f) a bare PEC octahedron, (g) a PEC octahedron covered by the proposed lossy cloak with loss tangent of 0.01 for both permeability and permittivity, (h) an equivalent PEC octahedron in virtual space.

$z$  direction are set as perfect magnetic boundary conditions. The parameters of the cloak are:  $L_1 = 1$  m,  $L_2 = 0.5$  m,  $L_3 = 0.1$  m,

$$\begin{aligned} \mu'_{1u} = \varepsilon'_{1u} = 2.932, \quad \mu'_{1v} = \varepsilon'_{1v} = 2.932, \quad \mu'_{1w} = \varepsilon'_{1w} = 0.3411, \\ \mu'_{2u} = \varepsilon'_{2u} = 1.247, \quad \mu'_{2v} = \varepsilon'_{2v} = 1.247, \quad \mu'_{2w} = \varepsilon'_{2w} = 0.03208, \\ \mu'_{3u} = \varepsilon'_{3u} = 7.188, \quad \mu'_{3v} = \varepsilon'_{3v} = 0.2875, \quad \mu'_{3w} = \varepsilon'_{3w} = 0.1391. \end{aligned}$$

Figures 2(a)–2(d) show the magnetic field distribution for the lossless cloak, the lossy cloak, the bare PEC octahedron and the equivalent PEC octahedron in virtual space in an  $xy$  plane ( $z = 0$  m) respectively, when an  $H_z$  polarized plane wave is incident along  $\vec{x}$  direction. Comparing Fig. 2(a) with Fig. 2(b), one can see that the perturbation of the wave out of the cloak is so weak that it is hard to be detected and this clearly manifests the cloaking effect.

In Figs. 2(e)–2(g), an  $H_z$  polarized plane wave is incident along  $\vec{x}/\sqrt{2} + \vec{y}/\sqrt{2}$  direction. The magnetic field distribution in an  $xy$  plane ( $z = 0$  m) for the two cases (with and without the lossless cloak) are shown in Fig. 2(e) and Fig. 2(f), respectively. In fact, the perturbation of the wave out of the cloak in Fig. 2(e) (Fig. 2(a)) is equal to the perturbation of the wave obstructed by a PEC octahedron with 0.1-meter-long side in the free space, as shown in Fig. 2(h) (Fig. 2(d)). Due to the geometrical symmetry, the proposed cloak composed of homogeneous and non-singular parameters exhibits prominent omnidirectional performance. In Fig. 2(c) and Fig. 2(g), the proposed cloak is composed of lossy materials (the loss tangent for both permeability and permittivity are 0.01). Obviously, the distortion is small and the cloak can still work well. Although the proposed formalism is independent of operating frequency, the challenge of fabricating a broadband homogeneous non-singular cloak is designing novel materials which exhibit qualified electromagnetic properties in broadband frequency region.

### 3. TETRAHEDRAL CLOAK

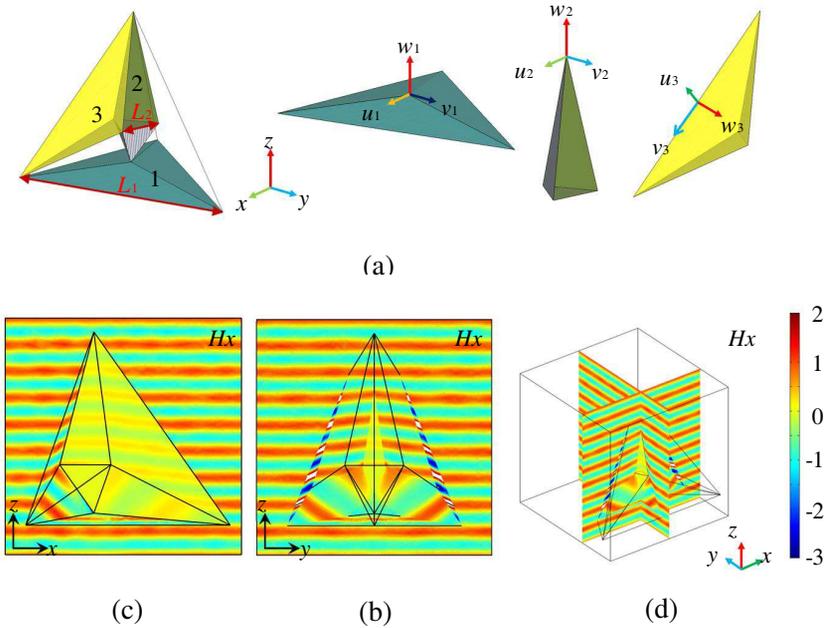
Compared with hexahedral cloak, the tetrahedral cloak is composed of fewer segments and is more convenient for practical implementation. In the following, we will demonstrate how to design a tetrahedral cloak. The tetrahedron is shown in Fig. 3(a), and we build the coordinate system with the origin at the center of the volume. The space between the tetrahedron and its dual polyhedron (a small tetrahedron) is divided into 14 segments. In each segment, we apply the similar coordinate transformation as the above case. The constitutive parameters can be calculated as follows:

$$\mu'_{1u} = \varepsilon'_{1u} = 1/\kappa_a, \quad \mu'_{1v} = \varepsilon'_{1v} = 1/\kappa_a, \quad \mu'_{1w} = \varepsilon'_{1w} = \kappa_a, \quad \text{for segment 1,}$$

$$\mu'_{2u} = \varepsilon'_{2u} = 1/\kappa_c, \quad \mu'_{2v} = \varepsilon'_{2v} = 1/\kappa_c, \quad \mu'_{2w} = \varepsilon'_{2w} = \kappa_c/\kappa_b^2, \quad \text{for segment 2,}$$

$$\mu'_{3u} = \varepsilon'_{3u} = \kappa_b/\kappa_d, \quad \mu'_{3v} = \varepsilon'_{3v} = 1/\kappa_b\kappa_d, \quad \mu'_{3w} = \varepsilon'_{3w} = \kappa_d/\kappa_b, \quad \text{for segment 3,}$$

where  $\kappa_a = ((\sqrt{6}/12)L_1 - (\sqrt{6}/4)L_2)/((\sqrt{6}/12)L_1 - (\sqrt{6}/4)L_3)$ ,  $\kappa_b = L_2/L_3$ ,  $\kappa_c = ((\sqrt{6}/4)L_1 - (\sqrt{6}/12)L_2)/((\sqrt{6}/4)L_2 - (\sqrt{6}/12)L_3)$ ,  $\kappa_d = (L_1 - L_2)/(L_1 - L_3)$ ,  $L_1$ ,  $L_2$ ,  $L_3$  are the side lengths of the cloak, the hidden tetrahedron region in physical space, the hidden tetrahedron region in virtual space, respectively. Simulated magnetic field distribution when a plane wave ( $\lambda = 0.45$  m) with  $H_x$  polarization is incident



**Figure 3.** (a) Scheme of coordination transformation of the tetrahedral cloak. Like the hexahedron, the space between the tetrahedron and its dual polyhedron can be divided into 14 segments. The concealed region is marked in gray, and the three different types of the segments are marked in blue, green, yellow, respectively. (b), (c), (d) Snapshots of magnetic field distribution in (b) a  $yz$  plane ( $x = 0$  m), (c) a  $xz$  plane ( $y = -0.25$  m), and (d) in 3D space, when the cloak is illuminated by a plane wave with  $Hx$  polarization propagating along the  $-z$  direction. The concealed region is PEC core.

along  $-z$  direction are shown in Figs. 3(b)–3(d). The parameters of the cloak are:  $L_1 = 4$  m,  $L_2 = 1$  m,  $L_3 = 0.25$  m,

$$\mu'_{1u} = \varepsilon'_{1u} = 3.25, \mu'_{1v} = \varepsilon'_{1v} = 3.25, \mu'_{1w} = \varepsilon'_{1w} = 0.3077, \text{ for segment 1,}$$

$$\mu'_{2u} = \varepsilon'_{2u} = 1.0682, \mu'_{2v} = \varepsilon'_{2v} = 1.0682, \mu'_{2w} = \varepsilon'_{2w} = 0.0585, \text{ for segment 2,}$$

$$\mu'_{3u} = \varepsilon'_{3u} = 5, \mu'_{3v} = \varepsilon'_{3v} = 0.3125, \mu'_{3w} = \varepsilon'_{3w} = 0.2, \text{ for segment 3.}$$

Based on aforementioned design method, other polyhedrons in geometry, such as octahedron, dodecahedron and icosahedron, can be adopted to project homogeneous and non-singular cloaks

similarly. Among these various geometrical prototypes, tetrahedron-based cloak is most convenient for fabrication with fewest segments and icosahedron-based one is closer to spherical cloak with more complexity. Such geometrical assistant approach opens a different route to realize invisibility. Besides, the main challenge to fabricate our cloaks in reality is to design electromagnetic materials with desired parameters and process them into particular shapes.

#### 4. CONCLUSION

In conclusion, we propose a strategy of a three-dimensional polyhedral cloak designed with linear homogeneous transformation method. This cloak only involves three different homogeneous anisotropic materials without singularity. Numerical simulations of hexahedral and tetrahedral cloak are performed, verifying the effectiveness of our scheme. Our cloaking strategy is very helpful towards realization of the three-dimensional isolated cloak in practice.

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