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SPARSITY OF THE FIELD SIGNAL-BASED METHOD FOR IMPROVING SPATIAL RESOLUTION IN ANTENNA SENSOR ARRAY PROCESSING

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Abstract—The goal of array processing is to gather information from propagating radio-wave signals, as their Direction Of Arrival (DOA). The estimation of the DOA can be carried out by extracting the information of interest from the steering vector relevant to the adopted antenna sensor array. Such task can be accomplished in a number of different ways. However, in source estimation problems, it is essential to make use of a processing algorithm which feature not only good accuracy under ideal working conditions, but also robustness against non-idealities such as noise, limitations in the amount of collectible data, correlation between the sources, and modeling errors. In this work particular attention is devoted to spectrum estimation approaches based on sparsity. Conventional algorithms based on Beamforming fail wherein the radio sources are not within Rayleigh resolution range which is a function of the number of sensors and the dimension of the array. DOA estimation techniques such as MUSIC (Multiple Signal Classifications) allow having a larger spatial resolution compared to Beamforming-based procedures, but if the sources are very close and the Signal to Noise Ratio (SNR) level is low, the resolution turns to be low as well. A better resolution can be obtained by exploiting sparsity: if the number of sources is small, the power spectrum of the signal with respect to the location is sparse. In this way, sparsity can enhance the accuracy of the estimation. In this paper, an estimation procedure based on the sparsity of the radio signals and useful to improve the conventional MUSIC method is presented and analyzed. The sparsity level is set in order to focus the signal energy only along the actual direction of arrival. The obtained numerical results have shown an improvement of the spatial resolution as well as a reduced error in DOA estimation with respect to conventional techniques.

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1. INTRODUCTION

The estimation of spatial parameters is of crucial importance in many applications related to source localization by means of sensors and antenna sensors. The signal of interest can be picked up by using one or more sensors. However, compared to the individual sensor configuration, only a sensor array actually allows taking advantage of the spatial processing, because it naturally samples the incoming signals and in this way can filter the signal impinging from a desired direction, while attenuating the signals from other ones [1, 2]. In fact, a plurality of signals originating from different spatial directions but overlapping with each one, both in time and in frequency, is often received by an antenna sensor array. The goal is to estimate, as accurately as possible, a particular signal coming from a certain direction. When the desired signal and the interfering signals occupy simultaneously the same frequency band, the only temporal filtering is certainly not able to isolate the useful signal. However, the signal of interest and those interfering have usually origin from different spatial regions, so it is possible to make use of spatial diversity for the aforementioned purpose using a spatial filter in reception [3]. This is the principle on which array processing techniques such as Beamforming are based [4–6]. However the directions of arrival (DOA) are not always known, and so the beam pattern cannot be pointed in the desired direction. In this case the space could be inspected in blind manner to search possible sources by implementing multiple filters which point, in turn, in a certain direction to pass the relative information and to lock those associated with the other ones. A less cumbersome but more accurate solution is to solve, in a preliminary way, the DOA estimation problem according to Figure 1 [7].

In practice, the estimation of DOAs is made difficult by the fact that there is usually an unknown number of signals simultaneously impinging on the array, each from unknown direction and with unknown amplitude. In addition the received signals are always corrupted by noise. Nevertheless, there are several methods to estimate the number of signals and their directions.

In all these methods the DOA is a parameter estimated from the received data. The minimum variance in this estimate is given by the Cramer-Rao bound (CRB), which provides a measure of the fact that estimating parameters from noisy data necessarily results in a noisy estimation. In particular, the CRB quantifies the minimal residual noise achievable in unbiased estimates [8, 9].

The Maximum likelihood estimation technique achieves the Cramer-Rao bound but it requires a significant amount of computa-

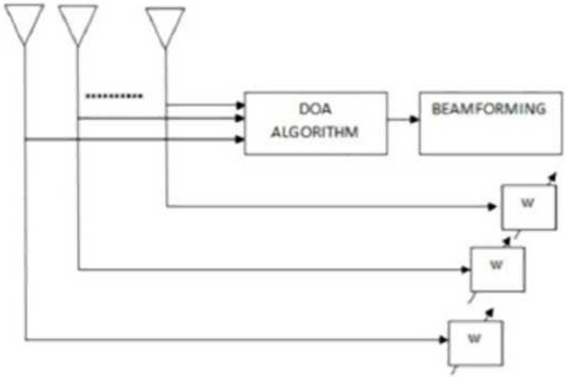


Figure 1. DOA estimation and Beamforming process for antenna sensor array.

tions to derive an estimate [1]. Computationally less demanding estimation procedures such as the Capon Beamformer [10] and the MUSIC [11] methods allow evaluating the DOAs of incoming radio signals by computing the location of peaks in the energy spectrum, but they feature limitations in the DOA estimation accuracy that are related to spatial resolution. Spatial resolution, i.e., the minimum angular distance such that two distinct signals coming from diverse directions can be distinguishable, is related to the parameter Half Power Beam Width (HPBW) [1]. It depends on the geometry of the array and the number of elements, but large-size arrays are often not desirable in many applications. In [12] we have presented a technique for improving the accuracy in the estimate of the DOAs in order to retrieve more information about the scanning objects, such as their shape. The concept of spatial sparsity is useful in DOA estimation problem because it allows to hold only the information of interest and to set to zero unnecessary one. This sparsity must be set *a priori* in the field signal, that is the signal incident on the array. The method presented here is an enhanced version of the method proposed in [12]. The estimate approach is optimized by an iterative method based on the minimization of two costs, one related to goodness of data fitting, and other dealing with maintenance of the spatial sparsity of the signal. So, for this scope, Tikhonov regularization method [13, 14] is used in combination with the MUSIC method in order to converge to the desired solution in a shorter time and with a lower computational effort. The key point in the development of a method based on the sparsity of the field signal is the possibility to represent the spectrum of the signal in sparse way because the signal energy is concentrated only along the directions of interest, i.e., those along which the signals impinging on the sensor array actually propagate. This results in an improvement of the Signal to Noise Ratio (SNR) and, hence, of the accuracy in the estimation of the DOAs even wherein the data are corrupted by a higher level of noise.

The paper is organized in this way. In Section 2 limitations known in literature of the classical Capon Beamformer method and MUSIC are discussed. In Section 3 the proposed method is presented. In Section 4 final comments are reported.

2. BEAMFORMING AND MUSIC LIMITATIONS

The design of the sensor array in order to achieve specific performances is the trade-off among the array geometry, the number of sensors, SNR, as well as a number of other factors. We consider a linear array consisting of n sensors equally spaced at a distance d as in Figure 2 [1].

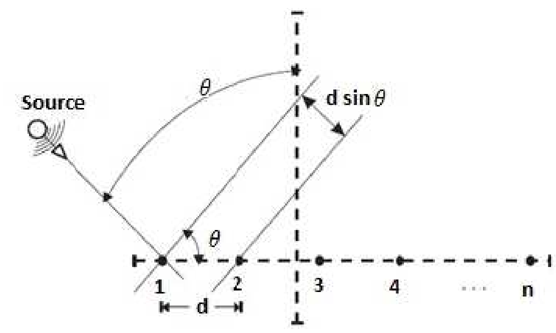


Figure 2. Linear array of n antenna sensors.

So $d_n = (n-1)d$ indicates the distance between the n-th sensor and the first taken as reference in the array, and we suppose that a source placed at a great distance from the sensor array and coming from direction ϑ is a signal of frequency f_0 and amplitude s_0:

$$s_0(t) = s_0 e^{j2\pi f_0 t} \tag{1}$$

The source is in the far field with respect to the array, so that the impinging wave fronts on the sensors can be considered plane. Hence, the received signal by the n-th sensor is equal to that received by the first one, except for a time delay τ_n given by:

$$\tau_n = \frac{d_n}{c}\sin(\theta_0) = \frac{d_n}{\lambda f_0}\sin(\theta_0) \tag{2}$$

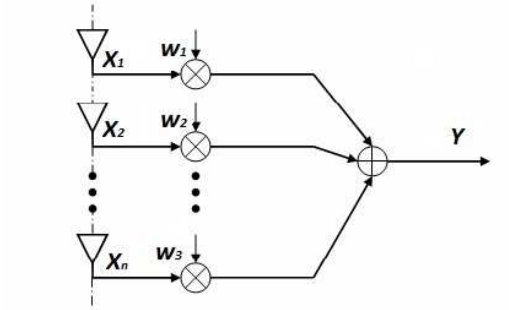


Figure 3. Narrowband Beamformer.

where c is the speed of light and $\lambda = \frac{c}{f_0}$ is the wavelength of the signal. So, the signal $x_n(t)$ received by the n-th sensor is:

$$x_n(t) = s_0 e^{j2\pi f_0 t} e^{j2\pi f_0 \tau_n} = s_0(t) e^{j2\pi(n-1)\frac{d}{\lambda}\sin(\theta_0)} \tag{3}$$

Omitting for simplicity the time index t and representing with the vector $X = [X_1\ X_2\ X_3\ \ldots\ X_n]^T$ the signals received by the n sensors, it has:

$$X = s_0 v_0 \tag{4}$$

where v_0 represents the response of the array to narrowband signal coming from the direction ϑ, known as steering vector:

$$v_0 = v(\theta_0) = \left[1 \quad e^{j2\pi\frac{d}{\lambda}\sin(\theta_0)} \quad e^{j2\pi 2\frac{d}{\lambda}\sin(\theta_0)} \quad \ldots \quad e^{j2\pi(N-1)\frac{d}{\lambda}\sin(\theta_0)}\right] \tag{5}$$

Let us assume that there are L uncorrelated radio signals propagating along different directions and impinging on the considered sensor array. A steering vector for each direction of arrival can be defined and they can be grouped in a matrix, namely the array manifold, as follows:

$$C(\vartheta) = [v(\vartheta_0) \quad v(\vartheta_1) \quad v(\vartheta_2) \quad \ldots \quad v(\vartheta_L)] \tag{6}$$

A DOA estimation problem is defined considering that there are several (D) signals impinging on a linear array with n equispaced elements, each coming from a direction ϑ_i, $i = 1, \ldots, D$. The goal is to use the data received at the array to estimate ϑ_i.

In a first moment we take in consideration the concept of Beamforming to solve the DOA estimation problem. So considering a narrowband Beamfomer (Figure 3), the signals received by the n sensors are linearly combined by the Beamformer, according to the coefficients w_n^* (weights), so the output Y is:

$$Y = W^H X \tag{7}$$

where W is the weights vector, H represents the Hermitian operator and $X = [X_1(k)\ X_2(k)\ X_3(k)\ \ldots\ X_n(k)]^T$ is the vector of the signals sampled and received by the n sensors at the time instant k.

Capon Beamformer belongs to the class of the so-called Quadratic Algorithms [1], where the steering vector $v(\vartheta)$ is varied over *a priori* range of the sources defined in ϑ-space. The spatial spectrum is obtained and its value is plotted where the D largest values are selected as the estimate of $\vartheta_1, \vartheta_2, \ldots, \vartheta_D$. It is assumed that D, the number of sources, is known.

The spatial spectrum is defined as:

$$P_{Capon} = \frac{1}{k}\sum_{k=1}^{K}\left|v^H(\vartheta)wX(k)\right|^2 \quad -\frac{\pi}{2} \le \vartheta \le \frac{\pi}{2} \tag{8}$$

where w is calculated in accordance with Minimum Variance Distorsionless Response (MVDR) Beamformer algorithm:

$$w = \frac{R_x^{-1}v(\vartheta)}{v^H(\vartheta)R_x^{-1}v(\vartheta)} \tag{9}$$

R_x is the sample covariance matrix, i e., $R_x = \frac{1}{k}\sum_{k=1}^{K} X(k)X(k)^T$.

So, by replacing (9) in (8) and by performing the calculations, the Capon spectrum is:

$$P_{Capon}(\vartheta) = \frac{1}{v^H(\vartheta)R_x^{-1}v(\vartheta)} \tag{10}$$

For a comparison with the proposed method, three different cases of study are reported. In the first test case, we consider an operating frequency of 32.8 kHz, in accordance to ultrasonic sensor presented in [12]. The linear array consists of 10 sensors, and we suppose that two signals coming from different spatial directions are picked up by the sensor array. Although the spatial position of an object is completely defined by azimuth and elevation angles, the adopted linear array allows estimating only the azimuth angle ϑ. The results are shown in Figure 4, where the considered DOAs and SNR are specified for each case.

It is possible to see that in Figure 4(a) the peaks related to the two DOAs are distinguishable. In fact, the angular separation between the two DOAs is equal to 15°, i.e., this value is greater than the value HPBW (Half Power Beam Width) that is for a linear array of 10 sensors about 10° [1]. In Figure 4(b) the angular separation between the DOAs is 5°, but the SNR is such that the two peaks are slightly distinguishable. In Figure 4(c) the spatial resolution is low due to the small angular separation and reduced SNR. So the spectrum contains

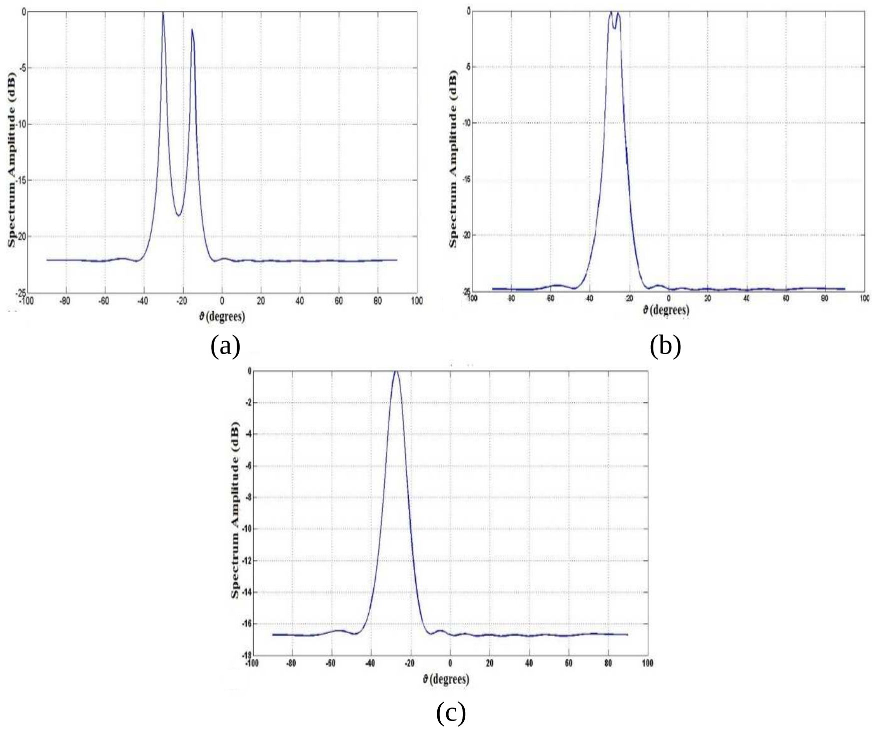


Figure 4. (a) $\text{DOA}_1 = -30°$, $\text{DOA}_2 = -15°$; $\text{SNR} = 15\,\text{dB}$. (b) $\text{DOA}_1 = -30°$, $\text{DOA}_2 = -25°$; $\text{SNR} = 15\,\text{dB}$. (c) $\text{DOA}_1 = -30°$, $\text{DOA}_2 = -25°$; $\text{SNR} = 5\,\text{dB}$.

Table 1. Estimate angles by Capon Beamformer.

Case/Estimate	ϑ_1	ϑ_2	$\hat{\vartheta}_1$	$\hat{\vartheta}_2$
Case 1	$-30°$	$-15°$	$-30.41°$	$-15.52°$
Case 2	$-30°$	$-25°$	$-30.27°$	$-25.85°$
Case 3	$-30°$	$-25°$	/	$-26.97°$

a single peak in the region of two signals. In Table 1 the respective estimates are reported.

MUSIC is a method which belongs to family of Subspace Algorithms. It is assumed that the received waveform consists of D plane wave signals plus uncorrelated noise, and so it is possible to reduce the problem from a K-dimensional problem to a D-dimensional problem which defines the signal subspace. Generally, the signal subspace can be defined once the angles of arrival of the D signals are known, but in the parameter estimation problem, these angles are unknown. So the received data from the sensors are utilized to estimate the signal subspace and, thereby, determine the angles of arrival. For this aim, the sample covariance matrix of the data R_x is expanded using the eigenvalues and eigenvectors,

$$R_x = \sum_{i=1}^{N} \lambda_i \Phi_i \Phi_i^H \tag{11}$$

where Φ_i are the eigenvectors and λ_i are the eigenvalues. In this way rewrite R_x as:

$$R_x = \Phi_i \Lambda \Phi_i^H \tag{12}$$

with $\Lambda = \text{diag}[\lambda_1, \lambda_2, \ldots, \lambda_N]$.

We refer to the first D eigenvalues, considered in order of decreasing size as the signal-subspace eigenvalues. Nevertheless, there is still a noise component in the signal subspace. The remaining eigenvectors define a noise subspace that does not contain any signal component.

So we define signal subspaces as:

$$U_S = [\Phi_1\ \Phi_2\ \ldots\ \Phi_D] \tag{13}$$

and noise subspace as:

$$U_N = [\Phi_{D+1}\ \Phi_{D+2}\ \ldots\ \Phi_N] \tag{14}$$

A way to determinate the previously subspaces is the Singular Value Decomposition (SVD) applied to the matrix R_x. After, the function $Q(\vartheta)$ is calculated, which is called null spectrum by projecting $v(\vartheta)$, defined for all possible values of ϑ in a fixed range $-\pi/2 \leq \vartheta \leq \pi/2$, on U_N. $Q(\vartheta)$ is expressed in terms of the eigenvectors of the noise subspace:

$$Q(\vartheta) = v^H(\vartheta)\left[U_N U_N^H\right] v(\vartheta) \tag{15}$$

The principle on which MUSIC is based is that the eigenvectors in the noise subspace are orthogonal to those of the signal subspace. Since the signal subspace contains information about the angles of arrival of each plane wave, the steering vectors of these angles are also orthogonal to the vectors in noise subspace. Thus if the product between a steering vector of the DOA of a wave and the vectors of the noise subspace is zero, the quantity defined in the Equation (15) is null too. The inverse of $Q(\vartheta)$ is called Music Spectrum:

$$P_{\text{MUSIC}}(\vartheta) = \frac{1}{v^H(\vartheta) U_N U_N^H v(\vartheta)} \tag{16}$$

Music spectrum will have peaks in correspondence of the angles where the steering vectors $v(\vartheta)$ are orthogonal to the noise eigenvectors of U_N.

We assess the estimate accuracy of MUSIC for different DOAs, SNR levels, and number of samples which, in this case, affects the performance of the algorithm. Let us consider the same tests used for the assessment of the Capon Beamforming technique. The results are shown in Figure 5 where the number of samples is 1024 as in the previous test.

We note an improvement in terms of resolution in solving the problem of DOA estimation using MUSIC method.

In Table 2 the respective estimates are reported too.

The limits of MUSIC in terms of spatial resolution are outlined in

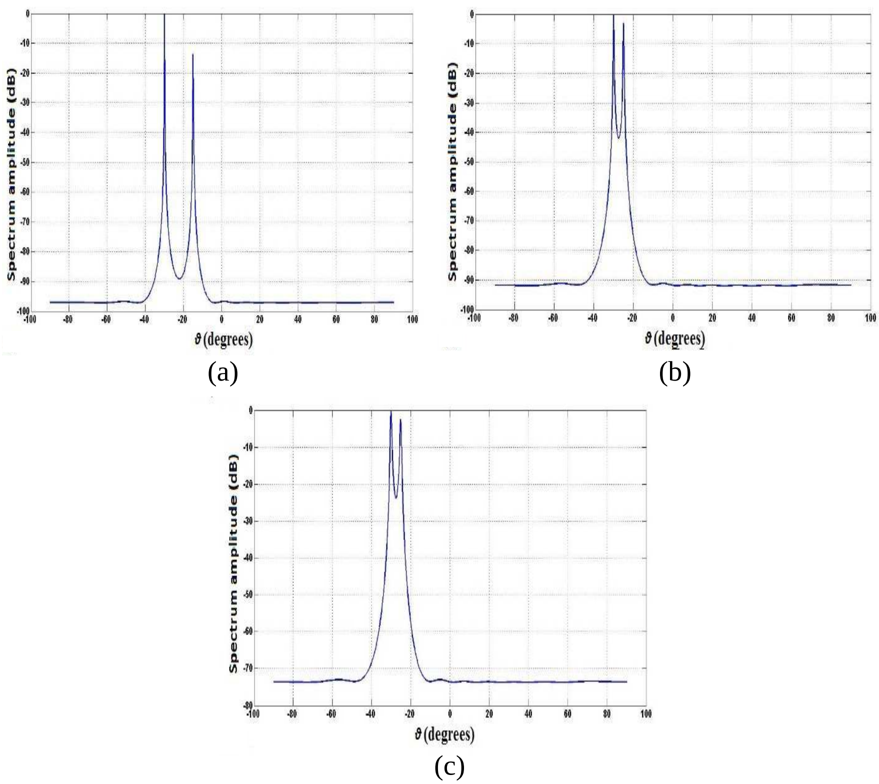


Figure 5. (a) $\mathrm{DOA}_1 = -30°$, $\mathrm{DOA}_2 = -15°$; SNR = 15 dB. (b) $\mathrm{DOA}_1 = -30°$, $\mathrm{DOA}_2 = -25°$; SNR = 15 dB. (c) $\mathrm{DOA}_1 = -30°$, $\mathrm{DOA}_2 = -25°$; SNR = 5 dB.

Figure 6(a). If the number of samples is small, a consistent degradation of the accuracy in DOA estimation can occur (see Table 3). Similarly, when the SNR level drops down, the MUSIC peaks can become not distinguishable even where a large number of samples is used (see Figure 6(b)).

3. PROPOSED METHOD

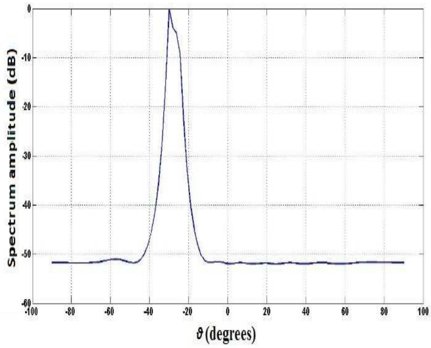
In the previous section, the limits, known in literature, of conventional DOA estimators such as the Capon Beamforming and MUSIC methods have been discussed. In order to achieve a finer spatial resolution in

Table 2. Estimate angles by MUSIC method (good spatial resolution).

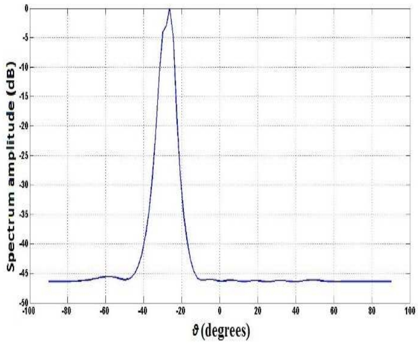
Case/Estimate	ϑ_1	ϑ_2	$\hat{\vartheta}_1$	$\hat{\vartheta}_2$
Case 1	$-30°$	$-15°$	$-30°$	$-15.04°$
Case 2	$-30°$	$-25°$	$-30°$	$-25.07°$
Case 3	$-30°$	$-25°$	$-30.18°$	$-25.25°$

Table 3. Estimate angles by MUSIC method (low spatial resolution).

Case/Estimate	ϑ_1	ϑ_2	$\hat{\vartheta}_1$	$\hat{\vartheta}_2$
Case 1	$-30°$	$-15°$	$-30°$	/
Case 2	$-30°$	$-25°$	/	$-26.36°$



(a)



(b)

Figure 6. (a) $\text{DOA}_1 = -30°$, $\text{DOA}_2 = -25°$; SNR $= 5$ dB; samples $= 100$. (b) $\text{DOA}_1 = -30°$, $\text{DOA}_2 = -25°$; SNR $= -5$ dB; samples $= 1024$.

DOA estimation, sparsity is exploited. In particular, a sparse solution for the unknown field signal is derived in order to obtain a sparse energy spectrum having peaks only along the directions of interest. The problem to solve is the following:

$$Y = C(\vartheta)s + w \tag{17}$$

where $Y \in \Re^{n\times k}$ is the vector containing the output of the sensor array due to the sampled signal, incident on it, with n the number of sensors in linear array and k the number of acquiring samples; $C(\vartheta) \in \Re^{n\times D}$ is the array manifold defined for the DOAs of the incident signals, D is the number of incident sources; $s \in \Re^{D\times k}$ is the vector of the field signals incoming on the sensor array from a certain spatial direction; $w \in \Re^{n\times k}$ is an additive Gaussian noise.

At the first step, the noise w is neglected. So, if the incident signals s on the sensor array and their DOAs are known, the matrix $C(\vartheta)$ is determined and the vector Y can be calculated in simple and unique way by Equation (17). The inverse problem consisting in the evaluation of the signal s starting from the Y measurements is not trivial because the matrix $C(\vartheta)$ is unknown. In order to circumvent this problem let us consider a great number of possible DOAs and redefine $C(\vartheta)$ for this set. So the problem (17) is reformulated as follows:

$$Y = C_{\tilde{\vartheta}}(\vartheta)s_{\tilde{\vartheta}} \tag{18}$$

where the noise is neglected for now as aforementioned. So $C_{\tilde{\vartheta}}(\vartheta) \in \Re^{n\times D_\vartheta}$ is the array manifold relative to set of possible DOAs with D_ϑ the number of possible DOAs, i.e., $\tilde{\vartheta} = [\vartheta_1, \vartheta_2, \ldots, \vartheta_{D_\vartheta}]$, $s_{\tilde{\vartheta}}$ is the vector of field signal defined for each DOAs of the set $\tilde{\vartheta}$. However this representation can lead to an ill-posed problem. In fact, $C_{\tilde{\vartheta}}(\vartheta)$ is, in the general case, an underdetermined matrix because there are more unknowns than independent linear equations. So the solution of the problem could be not uniquely defined. An easy and effective way for overcoming this obstacle is by Moore-Penrose pseudo-inverse [15] which calculates the least mean square solution. So the cost function,

$$J_1(s_{\tilde{\vartheta}}) = \left\|Y - Cs_{\tilde{\vartheta}}\right\|^2 \tag{19}$$

is minimized by the calculated solution which is among the all infinite solutions that a minimum norm (or least squares solution):

$$s_{\tilde{\vartheta}} = C_{\tilde{\theta}}^T \left(C_{\tilde{\theta}} C_{\tilde{\theta}}^T\right)^{-1} Y \tag{20}$$

In Equation (19) $C_{\tilde{\vartheta}}(\vartheta) = C$ and it defines a cost related to the goodness of the fitting of the data. However if r is the rank of matrix $C_{\tilde{\vartheta}}(\vartheta)$ and it is such that $r < n$, the problem is ill-posed, so the Singular

Value Decomposition is used for $C_{\tilde{\vartheta}}(\vartheta)$. The solution to problem (18) is that to least squares in Equation (20), but when in Equation (18) is added the noise,

$$Y = C_{\tilde{\vartheta}}(\vartheta) s_{\tilde{\vartheta}} + w \tag{21}$$

the noise components can be amplified in the directions of the singular vectors with small singular values. So we consider the measurements Y corrupted by the noise which defines the SNR level. In Figure 7 the least squares solution is depicted for three cases with different SNR. The DOAs to estimate are $\vartheta_1 = -30°$ and $\vartheta_2 = -15°$ in Figures 7(a)–(b) and so the angular distance between the two considered DOAs is such to be bigger of HPBW parameter. While in Figure 7(c) the DOAs are $\vartheta_1 = -30°$ and $\vartheta_2 = -25°$. The number of considered samples is equal to 100.

The least squares solution with respect to the considered set of

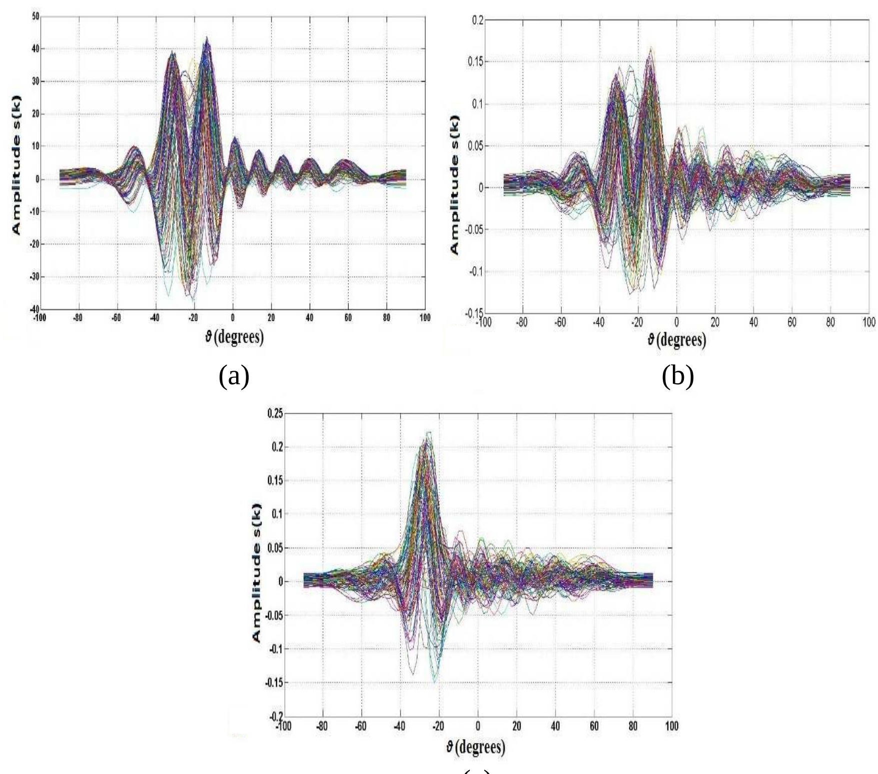


Figure 7. Least squares solution.

DOAs $\tilde{\vartheta}$ is shown, and each curve corresponds to a single sample k. It is possible to see in Figure 7(a) that with a SNR very high equal to 55 dB the two peaks are distinguishable, as also in Figure 7(b) with a minor SNR of 5 dB, but the energy of the signal in this case is lower than in the previous case. In Figure 7(c) the two peaks are not distinct due to the high noise in the signal (SNR = 5 dB) and a small angular distance of the sources.

So it is desirable to solve the problem (21) by a different approach, especially when the angular distance of the sources to be detected is small, and the solution calculated through the least squares would lead to a poor spatial resolution as seen in Figure 7(c).

In this case, regularization methods are used to solve ill-posed problems by including *a priori* knowledge about $s_{\tilde{\vartheta}}$ to stabilize the problem and to provide a reasonable and useful solution. So a second function in addition to that defined in Equation (19) is introduced to reinforce the desired reconstruction:

$$J_2(s_{\tilde{\vartheta}}) = \left\| s_{\tilde{\vartheta}} - s_0 \right\|^2 \tag{22}$$

which means that there is a preference for a reconstruction close to default solution s_0. We set this default solution in way to obtain a sparse spatial information for the field signal $s_{\tilde{\vartheta}}$ to be calculated.

Nevertheless $J_1(s_{\tilde{\vartheta}})$ and $J_2(s_{\tilde{\vartheta}})$ cannot generally be both minimized at the same time, so it is necessary to find a compromise, which can be simply obtained by taking a linear combination of the two:

$$J(s_{\tilde{\vartheta}}) = J_1(s_{\tilde{\vartheta}}) + \lambda J_2(s_{\tilde{\vartheta}}) \tag{23}$$

Scalar λ is the regularization parameter balancing the tradeoff between the two costs. A common method to resolve the problem (23) is Tikhonov Regularization:

$$s_{\tilde{\vartheta}_\lambda} = \arg\min \left\{ \left\| J_1(s_{\tilde{\vartheta}}) \right\|_2^2 + \lambda^2 \left\| J_2(s_{\tilde{\vartheta}}) \right\|_2^2 \right\} \tag{24}$$

There is a whole family of solutions indexed by λ. If this regularization parameter is very large, the solution favors the *a priori* information because the data and consequently the noise are ignored; while, if λ is small the solution is that non-regularized, i.e., that to least squares which is sensitive to noise. Simpler method to calculate this parameter is a graphical tool known as L-curve [16]. Tikhonov problem must be resolved for each k sample,

$$J(s_{\tilde{\theta}}(k)) = \left\| Y(k) - C_{\tilde{\theta}} s_{\tilde{\theta}}(k) \right\|_2^2 + \lambda^2 \left\| s_{\tilde{\theta}}(k) - s_0 \right\|_2^2 \tag{25}$$

The sparsity information will be maintained in s_0 defining it in this way:

$$s_0 = \sqrt{\sum_{i=1}^{k} (s_j(i))^2} \quad \forall \quad j = 1, \ldots, D_\vartheta \tag{26}$$

that is for each possible DOAs the $s_{\tilde{\vartheta}}(k)$ solutions calculated, by solving the k Tikhonov problem, are added together. This has the effect to reinforce the sparsity of the signal because the smallest terms in s_0 are set to zero, i.e., s_0 is such that it contains a great number of elements equal to zero and just few components are different from the null value. The steps to solve in iterative way the algorithm are shown in the block diagram of Figure 8.

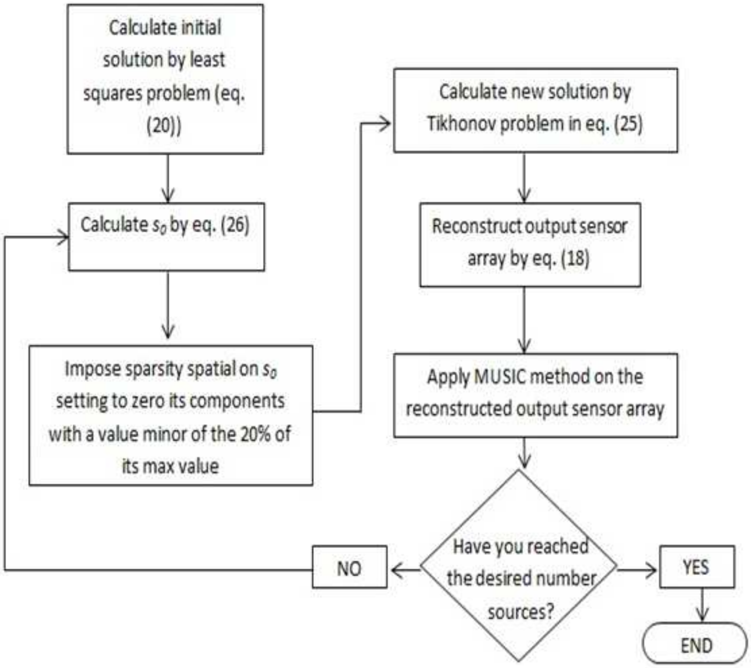


Figure 8. Steps of the proposed method.

With reference to the flow chart reported in Figure 8, a least square solution of the problem is derived, and, in this way, the vector s_0 is calculated. On this vector the spatial sparsity is imposed setting to zero the entries below the 20% of its maximum. This threshold value is established in such way to keep the information in the desirable range. In fact, with reference the least squares solution in Figure 7(c), s_0, calculated by Equation (26) is shown in Figure 9.

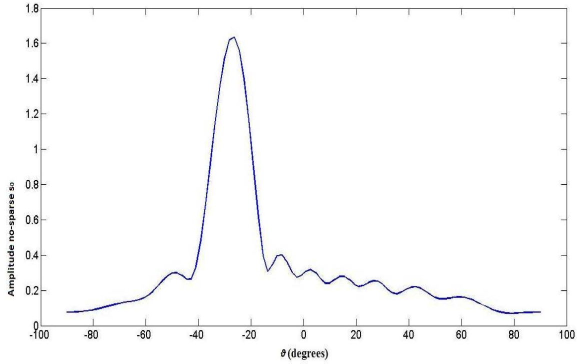


Figure 9. No-sparse initial s_0.

Table 4. Estimate angles by proposed method.

Case/Estimate	ϑ_1	ϑ_2	$\hat{\vartheta}_1$	$\hat{\vartheta}_2$
Case 1	$-30°$	$-25°$	$-30°$	$-24.55°$

So we are only interested in the range where there is a peak in order to be able to distinguish the two DOAs. $s_{\tilde{\vartheta}}(k)$ is calculated by solving Tikhonov problem and this solution is used to reconstruct the sensor array output because the sparsity of the signal has been reinforced and the energy of the field signal has been concentrated along DOAs of interest. So the imposed spatial sparsity has the effect to increase the SNR in the field signal. Consequently the new reconstructed Y vector results to be less corrupted by noise. MUSIC method is applied on this reconstructed output. The result is depicted in Figure 10 with reference to the case in Figures 7(c) and 9.

In Figure 10(a) the solution $s_{\tilde{\vartheta}}(k)$ at the second iteration of the algorithm is shown, while in Figure 10(b) the result by application of MUSIC method on the reconstructed data is depicted compared with the solution obtained by classical MUSIC method. The algorithm is stopped when the two desired DOAs are obtained. So the only information, required for this method are the DOAs to calculate, is the number of sources incident on the antenna sensor array. The final results are shown in Figures 10(c)–(d). It is clear that the sparse solution allows to obtain a better result compared to MUSIC, because the signal energy is concentrated along the DOAs of interest and so the two DOAs are detected. In Table 4 the estimate of the considered DOAs are reported.

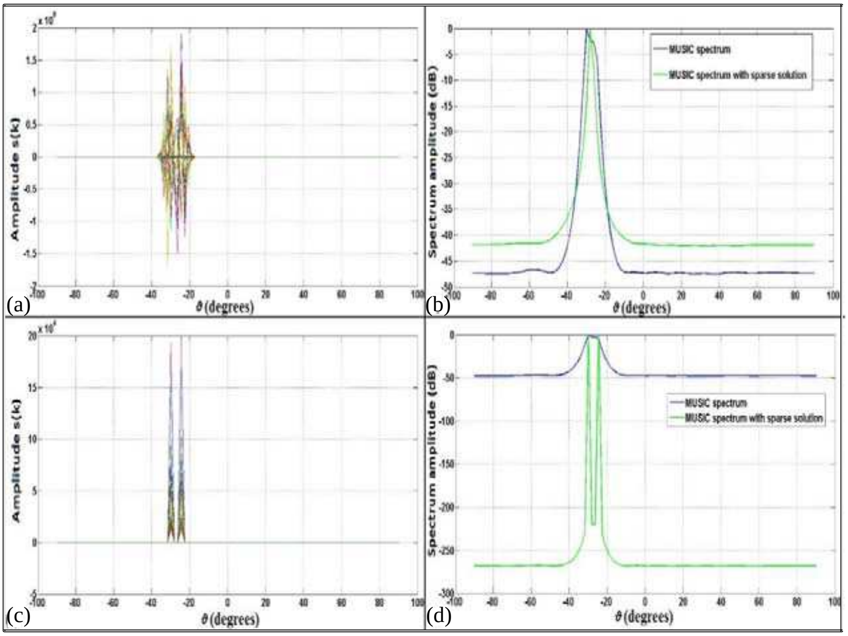


Figure 10. (a) Iteration #2: Tikhonov solution. (b) MUSIC spectrum without sparse solution vs. MUSIC spectrum with sparse solution. (c) Iteration #5: Tikhonov solution. (d) MUSIC spectrum without sparse solution vs. MUSIC spectrum with sparse solution.

In different problems related to search of a sparse solution, it has been proved that $l1$-norm is more indicated with respect to $l2$-norm, because a solution based on the least squares minimization as reported in Equation (25) tends to weight more residues larger than those small, and a few components in the solution near to zero cannot be completely cancelled [17, 18]. So, a sparse solution could not be obtained or the convergence of the problem can be more slow. However, the result in Figure 10 shows that through the use of $l2$-norm the energy of the signals is focused along the directions of interest because a sparse solution is calculated. That is due to the fact that computation of s by solving Equation (25) is based on the consideration that the non-zero coefficients in s_0 are relative to the columns in $C_{\tilde{\vartheta}}(\vartheta)$ necessary to concentrate the energy of the signal in certain directions. In Figure 9 is shown as to choose a possible location of the zeros in the solution, and consequently, the range where the solution must be searched, that is around the max of s_0. So only the columns of $C_{\tilde{\vartheta}}(\vartheta)$ related to this selected range are take in consideration during the minimization of the problem, the other ones reinforce the spatial sparsity because the corresponding steering vectors gives a null information for those directions out from the selected range.

Tikhonov problem defined in Equation (25) has a closed-form solution which can be calculated by setting:

$$\frac{\partial}{\partial s_{D_{\tilde{\vartheta}}}}\left\{\lambda^2(s-s_0)^T(s-s_0)+(y-C_{\tilde{\vartheta}}s)^T(y-C_{\tilde{\vartheta}}s)\right\}=0 \tag{27}$$

and so,

$$s_{\tilde{\theta}_i}(k)=\begin{cases} v_i\left(\frac{\lambda^2}{\lambda^2+\sigma_i^2}s_{0_i}+\frac{\sigma_i}{\sigma_i^2+\lambda^2}u_i^TY(k)\right) & \text{for } i=1,\ldots,r \\ v_i s_{0_i} & \text{for } i=r+1,\ldots,D_{\tilde{\vartheta}} \end{cases} \tag{28}$$

where r is the rank of matrix $C_{\tilde{\vartheta}}(\vartheta)$. Singular value decomposition is applied on $C_{\tilde{\vartheta}}(\vartheta)$, $C_{\tilde{\vartheta}}(\vartheta)=U\Sigma V=\sum_{i=1}^{r}u_i\sigma_i v_i^T$, and $s_{0_i}=v_i^Ts_0$, for $i=1,\ldots,D_{\tilde{\vartheta}}$. So the search of the minimum of the regularization residue for each algorithm iteration means that the best source estimate is used on each algorithm iteration. Figure 10 and Table 4 confirm the goodness of the proposed method.

However, since the method has been proposed as an improvement on MUSIC method, an analysis of its performance in terms of error behavior is reported. So, the limit of MUSIC is due to its low spatial resolution when two sources are separated by much less than the beamwidth of the array. In Figures 11 and 12 the location of the

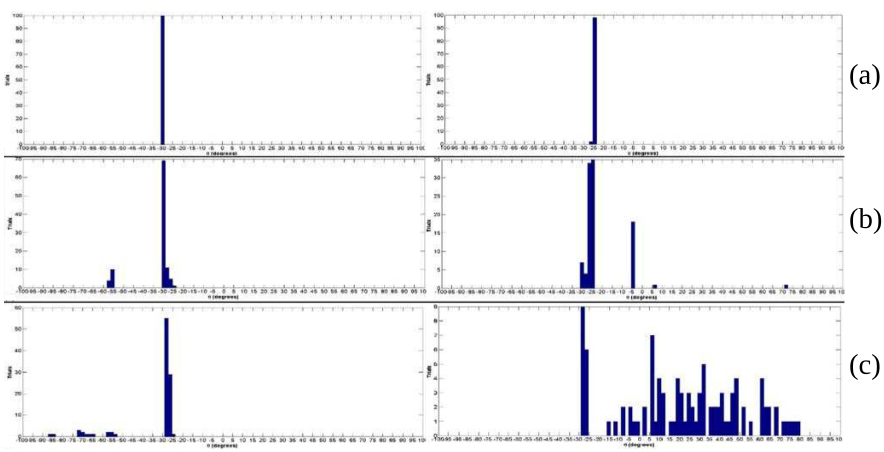


Figure 11. Error behavior for MUSIC method. (a) SNR = 15 dB. (b) SNR = 5 dB. (c) SNR = −8 dB.

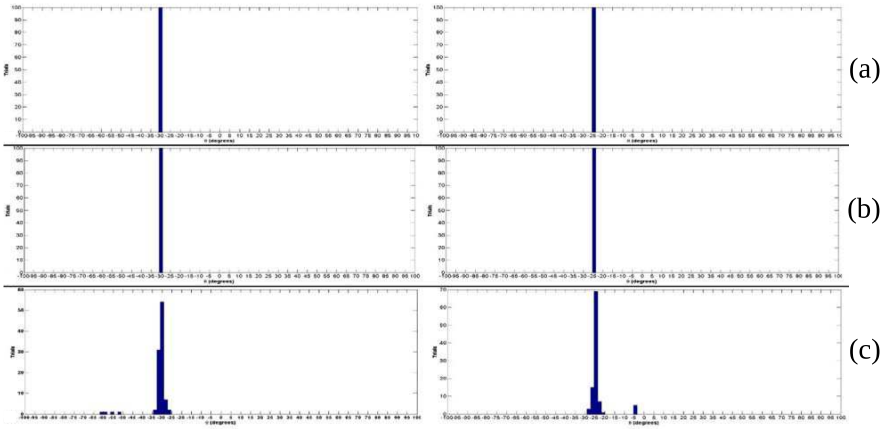


Figure 12. Error behavior for Sparse method. (a) SNR = 15 dB. (b) SNR = 5 dB. (c) SNR = −8 dB.

estimates obtained with the new method and MUSIC are depicted for 100 trials considering several SNR. The results are shown through a histogram representation. A linear array with 10 sensors is considered. The DOAs to estimate are: $\mathrm{DOA}_1 = -30°$, and $\mathrm{DOA}_2 = -25°$.

In Figure 11(a) the signals are resolved on all trials and the estimates are clustered around the correct value. In Figure 11(b) the signals are resolved on most trials, while on several trials the signals are not resolved. In Figure 11(c) more trials have unresolved signals.

In Figure 12 only in the case (c) a few of trials have unresolved signals. So the proposed method is more accurate than classical MUSIC method. MUSIC has estimation difficulties if impinging directions are closer to −90° and 90°. This problem is present in the proposed method also, and it is has not yet been resolved.

4. FINAL COMMENTS

The most important problem in antenna sensor array processing is the estimation of the position of sources emitting a signal (passive localization) or a point target illuminated by external signal (active localization).

A point in three dimensional space is defined by three parameters, namely, distance sensor-point, azimuth and elevation. The distance sensor-point is often measured by means of time of flight [12]. The azimuth and elevation angles are obtained from the measurements of direction of arrival (DOA). A source is assumed where there is a concentration of energy. This work has been focused on the development of an algorithm of DOA estimation to locate the sources of interest. Classical methods such as the Capon Beamformer and MUSIC have been analyzed in the context of DOA estimation problem. After having highlighted their limitations, an approach based on the sparsity of the signal incoming on the sensor array has been presented and discussed in detail. The sparsity is imposed *a priori* considering a representation of the problem in overcomplete form, due to the fact that the array manifold is constructed considering a great number of possible DOAs. The solution is achieved by the Tikhonov regularization method which provides a good trade-off in terms of fitting of the data and spatial sparsity of the field signal. The developed method allows concentrating the energy of the signal along the DOAs of interest. Cases of study are presented by different combinations of SNR, DOAs and number of samples. There is an improvement compared to the classical MUSIC method, and a smaller estimate error as it is possible to see from the analysis of the behavior error too. So spatial sparsity provides a clear advantage in DOA estimation. The only constraint is the knowledge of the number of sources to detect.

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