Progress In Electromagnetics Research, PIER 15, 27–61, 1997

SINGLE SCATTERING AND DIFFUSION APPROXIMATIONS FOR MODIFIED RADIATIVE TRANSFER THEORY OF WAVE MULTIPLE SCATTERING IN DENSE MEDIA NEAR RESONANCE

Y. N. Barabanenkov, L. M. Zurk and M. Y. Barabanenkov

1. Introduction

Recently it was shown that the velocity of light in a strongly scattering medium can be an order of magnitude smaller than the velocity of light in vacuum because of the Mie resonance scattering [1]. It appears that the light resonates for awhile within the dielectric microspheres of which the medium is made so the light takes longer to travel through the sample. A velocity of light in the experiments [1] defined by the diffusion condtant D and the transport mean free path l_{tr} was neither phase nor group velocity, but the energy transport velocity $v_E = c_0^2/(1+a)c_{ph}$ where c_0 and c_{ph} are the phase velocity in a background and in the random medium, respectively and the quantity "a" characterizes an efficiency of the resonance. The time domain experiments were conducted to present additional confirmation of the slow sppeed of electromagnetic waves in a dense strongly scattering medium [2]. A wide-band microwave signal was propagated through randomly distributed glass spheres and the speed was measured directly, by the time at which 50% of the pulse energy arrives (so called "arrival time").

To consider the phenomenon of slowing down of a light or microwave pulse travel through a strongly scattering medium in detail a modified radiative transfer equation (MRTE) with weak time dispersion was derived for non-stationary electromagnetic wave multiple scattering in dense media near Mie resonance scattering at a heuristic level [3], semi-heuristic level [4], and using the Bethe-Salpeter equation in the two-frequency domain, i.e. "from the first principles" [5] (this paper is referred further as I). The tensor MRTE of I for the case of vector waves was derived on the basis of a generalized extinction theorem (two-frequency Ward-Takahashi indentity) [5–7] according to which the extinction of a pulse by propagation through a discrete random medium is conditioned by the incoherent scattering, the real absorption and by the change in time of the electromagnetic energy accumulated inside scatterers. This extinction theorem of principle was verified recently [8] in connection with the discussion [9,10] (especially p.92) and [11,12].

The MRTE of I enables one to describe not only the phenomenon of a pulse speed reduction but also the effects of the time delay and the electromagnetic energy accumulation inside scatterers on a pulse propagation through a strongly scattering medium. The tensor MRTE of I takes into account the dispersion in the time or frequency domain of both the mass and intensity operators. These two types of the dispersion give, respectively the coherent v_{coh}^{-1} and incoherent $v_{incoh}^{-1}(\overline{s}, \overline{s}')$ components of the inverse tensor-operator $v_E^{-1}(\overline{s}, \overline{s}')$ of the energy transport velocity with the matrix elements

$$\left[v_E^{-1}(\overline{s}, \overline{s}')\right]_{\alpha\beta,\alpha'\beta'} = \left(v_{coh}^{-1}\right)_{\alpha\beta,\alpha'\beta'} \cdot \delta(\overline{s} - \overline{s}') + \left[v_{incoh}^{-1}(\overline{s}, \overline{s}')\right]_{\alpha\beta,\alpha'\beta'} \quad (1)$$

in accordance with equation (96) of I (from here on the references on equations of I are denoted as (I.96) and so on). The coherent component of (1) being diagonal in the space of unit vectors $\overline{s}$ and $\overline{s}'$ is given by equation (I.96a) and defined roughly in the case of non-absorptive scatterers by the inverse value of the group velocity. The incoherent component of (1) being non-diagonal in the the space of unit vectors is given by equation (I.96b) and characterizes the quantity of the effect of the time delay on a pulse propagation. The same incoherent component makes also a principal contribution to the quantity "a" of the resonance efficiency in accordance with the sum rule (I.90) in the form

$$\int d\overline{s}' \left[v_E^{-1}(\overline{s}', \overline{s}) \right]_{\alpha\alpha,\beta\beta} = \frac{2}{v_E} \tag{2}$$

and therefore to the energy accumulation inside scatterers which is characterized by the ratio $a/(1+a)$ in the region of the diffusion regime. The last result is a particular consequence of the general equation

$$< w >=< w_{outside} > + < w_{inside} > \tag{3}$$

where $< w(\overline{R}, \omega) >$, $< w_{outside}(\overline{R}, \omega) >$ and $< w_{inside}(\overline{R}, \omega) >$ are the mean spectral components of the electromagnetic energy density in anpoint $\overline{R}$ of a random medium, in a point $\overline{R}$ outside scatterers and in a point $\overline{R}$ inside scatterers, respectively. It is of substantial importance that all three quantities of (3) can be calculated, actually with the aid of the same operator of the energy accumulation inside scatterers A^+ in accordance with equation (I.42a) for $< w >$ and also the following equation for the mean spectral component of the electromagnetic energy density inside scatterers

$$< w_{inside} >= -\frac{1}{2}\epsilon^{bac} \cdot \frac{\epsilon_1}{\epsilon'} \cdot \left(A^+ < \overline{E}_1 \otimes \overline{E}_2 > \right)_{\alpha\alpha} \tag{4}$$

As one can see from equation (1), the energy transport velocity in a dense random medium near resonance is in general an operator quantity in the space of unit vectors and indices of polarization but not a scalar quantity as in Reference 1. And only at large optical depth from field source in the random medium where the diffusion regime of a pulse propagation is established does the energy transport velocity become the scalar quantity v_E in accordance with the sum rule (2).

The aim of this paper is to demonstrate the above mentioned properties of the operator energy transport velocity of a pulse in a dense random medium near resonance scattering as well as the distribution of the mean electromagnetic energy density outside and inside scatterers by consideration of two simple solutions for the MRTE (I.87). First we find a solution for (I.87) in the single scattering approximation for the case of an optically thin random medium slab with the thickness L much less the transport mean free path, $L \ll l_{tr}$. This simple solution appears as physically meaningful by reveling an angular time delay of the pulse transmission through and reflection from the slab which is similar to the known angular time delay in the quantum mechanical scattering theory of particles [13]. The angular time delay being a microscopic quantity is defined in our case by the matrix elements of the incoherent component of the inverse tensor-operator of the energy transport velocity (1). It is worthwhile to note that this microscopic time delay is connected with such an energetic quantity as arrival time [2] and therefore can be measured experimentally.

Secondly we construct a solution for the MRTE (I.87) in the diffusion approximation for a pulse propagation in an unbounded random medium at large optical depth $R \gg l_{tr}$ from a field source. In this case the energy density of the pulse is propagated with the reduced diffusion constant D defined by the scalar energy transport velocity v_E and the transport mean free path. This solution in the diffusion approximation gives also simple formulas for the mean electromagnetic energy density distribution outside and inside scatterers during the pulse propagation.

The plane of the paper is as follows. In Section 2 the MRTE (I.87) is transformed to the integral form in the space and time domain. In Section 3 the iterative method is applied to the MRTE in the integral form that gives a solution in the single scattering approximation. The diffusion approximation is applied to find a solution for the MRTE in Section 4. It is helpful to mention that in these three sections only general propeties of the mass and intensity operators are used such as two trequency Ward-Takahashi identity (I.36). Accordingly, only general roperties of the coefficients in the MRTE (I.87) are used in the form of the sum rule (2) for the inverse tensor-operator of the energy transport velocity (1) and the optical theorem (I.82) connection the phase tensor $W_{\alpha\alpha,\alpha'\beta'}(\overline{s}, \overline{s}_0)$ and the extinction length l defined by equations (I.75) and (I.83), respectively. In Section 5 the obtained solutions for the MRTE are analyzed in the low-density limit and compared with results of the experiments [2]. Section 6 gives our conclusions and discussions. In particular, this section discusses briefly the problem of finding either experimentally or theoretically a concrete form for the integral kernels of the MRTE that should be considered in detail separately. Appendices contain the technical details. Some preliminary results of Sections 3 and 5 are presented in reports [14,15].

2. Modified Radiative Transfer Equation in an Integral Form

The MRTE (I.87) is written in the space-time Fourier transform representation with the wave vector $\overline{q}$ and frequency ω for the tensor-propagator $\mathcal{F}_{\alpha\beta,\alpha'\beta'}(\overline{s},\, \overline{s}';\, \overline{q},\, \omega)$ of the radiance where $\overline{s},\, \overline{s}'$ are unit vectors and $\alpha,\, \beta$ and $\alpha'\beta'$ are indices of polarization of the field and source, respectively. This equation was derived in I from the Bethe-Salpeter equation in kinetic form (I.53) by using the weak time-dispersion approximation along with the transeverse fields approximation ad quasi- far wave zone approximation. Let us introduce the space-time representation for the tensor-propagator of the radiance by the equation

$$\mathcal{F}(\overline{s},\, \overline{s}';\, \overline{q},\, \omega) = \int_0^\infty dt \int d\overline{R} e^{i(\omega t - \overline{q}\cdot\overline{R})} \cdot \mathcal{F}(\overline{s},\, \overline{s}';\, \overline{R},\, t) \tag{5}$$

and substitute

$$\mathcal{F}(\overline{s},\, \overline{s}';\, \overline{R},\, t) = \frac{1}{(4\pi)^2} \cdot \frac{k_{eff}}{k_0 n_{eff}} \Phi(\overline{s},\, \overline{s}';\, \overline{R},\, t) \tag{6}$$

Then the MRTE (I.87) takes the form

$$\begin{aligned} &\frac{\partial}{\partial t} \int d\overline{s}''\, v_E^{-1}(\overline{s},\, \overline{s}'')\, \Phi(\overline{s}'',\, \overline{s}';\, \overline{R},\, t) + \left(\overline{s} \cdot \overline{\nabla}_{\overline{R}} + \frac{1}{l} \right) \Phi(\overline{s},\, \overline{s}';\, \overline{R},\, t) \\ &\int d\overline{s}'' W(\overline{s},\, \overline{s}'') \Phi(\overline{s}'', \overline{s}';\, \overline{R},\, t) + \delta(\overline{s} - \overline{s}')\delta(\overline{R})\delta(t) P_{tr}(\overline{s}) \otimes P_{tr}(\overline{s}) \end{aligned} \tag{7}$$

In this equation the matrix notation is used according to which the product, e.g., of two tensors $W\Phi$ is equal to $W_{\alpha\beta,\alpha''\beta''}\, \Phi_{\alpha''\beta'',\alpha'\beta'}$ and repeated subscripts iimply the summation. The space-time representation of equation (I.88) gives

$$I(\overline{s};\,\overline{R},t) = \frac{1}{(4\pi)^2}\cdot\frac{k_{eff}}{k_0 n_{eff}} \cdot\int_0^t dt'\int d\overline{s}'\int d\overline{R}'\,\Phi(\overline{s},\overline{s}';\overline{R}-\overline{R}',t-t')\cdot J(\overline{s}';\overline{R}',t') \tag{8}$$

where the radiance tensor $I_{\alpha\beta}(\overline{s};\,\overline{R},t)$ and the source tensor $J_{\alpha\beta}(\overline{s};\,\overline{R},t)$ are expressed in terms of the tensor quantities $I_{\alpha\beta}(\overline{s};\,\overline{q},\omega)$ and $J_{\alpha\beta}(k_{eff}\overline{s};\,\overline{q},\omega)$ similar to equation (5). The application of the same transform (5) to equations (I.77), (I.78), and (I.79) gives the expressions of the mean electromagnetic energy density $< w(\overline{R},\,t) >$, Poynting's vector $< \overline{S}(\overline{R},\,t) >$, and field energy absorption because of conductivity $< Q^{con}(\overline{R},\,t) >$, respectively in terms of the radiance tensor (8). In particular, the mean Poynting's vector is given by

$$< \overline{S}(\overline{R},\,t) >= \frac{1}{2}\epsilon^{bac}\cdot\frac{c_0^2}{c_{ph}}\int d\overline{s}\cdot\overline{s}I_{\alpha\alpha}(\overline{s};\,\overline{R},t) \tag{9}$$

All these energetic quantities in the space-time representation $\overline{R},\,t$ characterize the electromagnetic field of a quasi-monochromatic pulse in a random medium with a middle frequency Ω being some functions of Ω as in the traditional radiative transfer theory [16]. Strictly speaking, for example the expression (9), becomes the true mean Poynting's vector after additional integration with respect to the middle frequency Ω in accordance with the complex convolution (I.7) resolved for $f(t)h(t)$.

The coherent component v_{coh}^{-1} of the inverse tensor-operator of the energy transport velocity (1) can be written on the base of equation (I.96a) in the case of isotropic (in average) and non-absorptive random medium as

$$v_{coh}^{-1}(\overline{s}) = v_{coh}^{-1}P_{tr}(\overline{s})\otimes P_{tr}(\overline{s}) \tag{10}$$

here a scalar velocity v_{coh} is defined by

$$\frac{c_0^2}{c_{ph}v_{coh}} = 1 - \frac{\partial\Re M_{tr}(k_{eff})}{\partial E} \tag{11}$$

where $M_{tr}(k) = M_{tr}(k,\,\Omega+i0)$ and the partial derivative with respect to the quantity $E = \Omega^2/c_0^2$ or frequency Ω is taken. The representation (10) enables one to transform equation (7) to an integral form.

This integral equation is derived in Appendix I and may be written as follws

$$\begin{aligned}\hat{\Phi}(\overline{s},\overline{s}';\overline{R},t) =& e^{-t}\delta(\overline{s}-\overline{s}')\delta(\overline{R}-\overline{s}t)P_{tr}(\overline{s})\otimes P_{tr}(\overline{s}) \\ & -\frac{\partial}{\partial t}\int_0^t dt'\int d\overline{R}'\int d\overline{s}''e^{-(t-t')} \\ & \cdot\delta\left(\overline{R}-\overline{R}'-\overline{s}(t-t')\right)\hat{v}^{-1}_{incoh}(\overline{s},\overline{s}'')\cdot\hat{\Phi}(\overline{s}'',\overline{s}';\overline{R}',t') \\ & +\int_0^t dt'\int d\overline{R}'\int d\overline{s}''\,e^{-(t-t')} \\ & \cdot\delta\left(\overline{R}-\overline{R}'-\overline{s}(t-t')\right)\hat{W}(\overline{s},\overline{s}'')\cdot\hat{\Phi}(\overline{s}'',\overline{s}';\overline{R}',t')\end{aligned} \tag{12}$$

Here the optical units are used for: (a) the space and time variables $\hat{\overline{R}}=\overline{R}/l$ and $\hat{t}=v_{coh}t/l$ which are denoted for simplicity again $\overline{R}$ and t, (b) the incoherent component of the ivnerse tensor-operator of the energy transport velocity $\hat{v}^{-1}_{incoh}(\overline{s},\overline{s}')=v_{coh}v^{-1}_{incoh}(\overline{s},\overline{s}'$, (c) the phase tensor $\hat{W}(\overline{s},\overline{s}')=lW(\overline{s},\overline{s}')$, and (d) the tensor propagator according to $\Phi=(v_{coh}/l^3)\hat{\Phi}$.

A solution for equation (12) has the form of the sum $\hat{\Phi}=\hat{\Phi}_{coh}+\hat{\Phi}_{incoh}$ of the coherent and incoherent components. The ;coherent component $\hat{\Phi}_{coh}$ is given by the first term of the right hand side (r.h.s.)of equation (12). The incoherent component $\hat{\Phi}_{incoh}$ is given by

$$\hat{\Phi}_{incoh}=\hat{\Phi}_{delay}+\hat{\Phi}_{scat} \tag{13}$$

where a "time delay" teimr $\hat{\Phi}_{delay}$ and "scattering term" $\hat{\Phi}_{scat}$ denote the second and third terms of the r.h.s. of (12), respectively. It is worthwhile to note here that equation (12) describes a delta-pulse propagation in a strongly scattering medium near resonance scattering in accordance with (8) and differs from the traditional radiative transfer equation [16] by the time delay term $\hat{\Phi}_{delay}$.

One can see that this term has equalitatively the form of the time derivative of the scattering term with a minus sign multiplied by a quantity t_{delay} of order of the microscopic time delay near resonance scattering defined below by equation (28), i.e. $\hat{\Phi}_{delay}(t)\sim(-1)t_{delay}\partial\hat{\Phi}_{scat}(t)/\partial t$. Let us suppose that the scattering term $\hat{\Phi}_{scat}(t)$ has a form similar to the typical detected time-dispersion curve when picosecond laser pulse is propagated through a highly scattering medium (see, e.g., Reference 17). In this case the time delay term will be negative and positive for the front and back parts of the scattering term, respectively with the same area between the curve $\hat{\Phi}_{delay}(t)$ adn t axis as it follows from equation

$$\int_0^\infty dt \hat{\Phi}_{delay}(t) = 0 \quad (14)$$

In Fig. 1 the quantity t_{delay} is given by $t_{delay} = 1$ nsec and it is shown qualitatively that the curve of the incoherent component $\hat{\Phi}_{incoh}(t)$ by multipl scattering near resonance goes lower and higher of the curve of the scattering term $\hat{\Phi}_{scat}(t)$ for the front and back parts of the scattering term, respectively. A non-physical negative part of the incoherent component in Fig. 1 related to the first time-interval from $t = 0.0$ nesc to $t = 0.8$ nesec appears because the MRTE (12) is unable to describe correctly this first small time interval of order of the microscopic time delay. In conclusion of this section one may say that the effect of resonance scattering on a pulse propagagted in a strongly scattering medium is similar to the effect of inductivity on a signal in an electrical circuit (see, e.g., Reference 18).

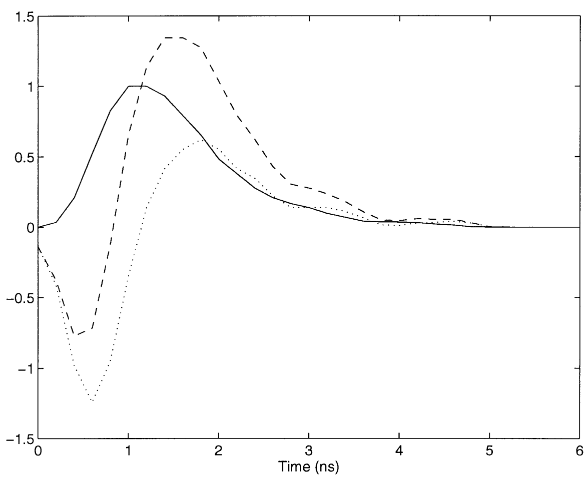

Figure 1. Qualitative illustration of equation (13) assuming $\hat{\Phi}_{scat}$ (solid line) has a form similar to the time dispersion curve in Ref. 17. The time delay term (dashed line) is proportional to the negative time derivatiove of $\hat{\Phi}_{scat}$ and the incoherent component (dotted line) does lower and higher than $\hat{\Phi}_{scat}$ for the front and back parts of the scattering term.

3. Solution for MRTE by Iterative Method

Equation (12) is suitable for application of the iterative method. In this method the first iteration is obtained by substituting the coherent component $\hat{\Phi}_{coh}$ into the second and third terms of the r.h.s. of (12) instead of the tensor popagtor $\hat{\Phi}$ that gives a solution for (12) in the single scattering approximation.

We will calculate further the mean transmitted and reflected energy flux of the field for an incident plane delta pulse radiated by an instantaneous source with tensor

$$J_{\alpha\beta}(\overline{s};\hat{\overline{R}},\hat{t}) = J_{\alpha\beta}\delta(\hat{z})\delta(\hat{t})\delta(\overline{s}-\overline{s}_0) \tag{15}$$

on the boundary $z = 0$ of the random medium slab $0 < z < l$ in the direction of the unit vector $\overline{s}_0$ with the projection $S_{0z} > 0$ in the principal coordinate system of the problem X, Y, Z. The polarization tensor $J_{\alpha\beta}$ is given by

$$J_{\alpha\beta} = (4\pi)^2\frac{2n_{eff}c_{ph}^2}{\epsilon^{bac}c_0^3 l}S^0 j^0_{\alpha\beta} \tag{16}$$

where S^0 is a magnitude of the mean Poynting's vector (9) for the coherent component of a solution for equation (12) in accordance with

$$< S_z(\tau,\hat{t}) >_{coh} = \mu_0 S^0 \cdot e^{-\hat{t}} \cdot \delta(\tau - \mu_0\hat{t}) \tag{17}$$

and a tensor $J^0_{\alpha\beta}$ has the unit trace, $J^0_{\alpha\alpha} = 1$. In equation (17) $\tau = z/l$ is the optical depth in the random medium and the denotation $\mu_0 = s_{0z}$ is used. We are interested in the quantity

$$\hat{\Phi}(\overline{s},\overline{s}_0;\,\tau,\hat{t}) = \int d\hat{\overline{R}}_\perp \hat{\Phi}(\overline{s},\overline{s}_0;\,\hat{\overline{R}},\hat{t}) \tag{18}$$

where integration is performed along a plane $\tau = const$. Straightforward calculations in the single scattering approximation give for the time delay and scattering terms of equation (13) the following expressions

$$\hat{\Phi}_{delay}(\overline{s},\overline{s}_0;\,\tau,\hat{t}) = -\hat{v}^{-1}_{incoh}(\overline{s},\overline{s}_0)\cdot\frac{\partial}{\partial\hat{t}}e^{-\hat{t}}\cdot f(\tau,\mu,\hat{t}) \tag{19}$$

and

$$\hat{\Phi}_{scat}(\overline{s}, \overline{s}_0;\ \tau, \hat{t}) = \hat{W}(\overline{s}, \overline{s}_0) \cdot e^{-\hat{t}} \cdot f(\tau, \mu, \hat{t}) \tag{20}$$

Here the scalar function $f(\tau, \mu, \hat{t})$ is defined by

$$f(\tau, \mu, \hat{t}) = \int_0^{\hat{t}} d\hat{t}' H(\tau - \mu(\hat{t} - \hat{t}');\ 0, \tau_0)\delta(\tau - \mu(\hat{t} - \hat{t}') - \mu_0\hat{t}') \tag{21}$$

where $H(\tau;\ a, b)$ is a step function equal to 1 for $a < \tau < b$ and equal to zero for all another τ. The quantity $\tau_0 = L/l$ denotes the optical thickness of the slab and $\mu = s_z$. Function (21) for the mean transmitted $f(\tau_0, \mu > 0, \hat{t})$ and reflected $f(0, \mu < 0, \hat{t})$ energy flux takes the form

$$f(\tau_0, \mu > 0, \hat{t}) = \frac{1}{|\mu - \mu_0|} H\left(\hat{t};\ \frac{\tau_0}{\mu_0}, \frac{\tau_0}{\mu}\right) \tag{22}$$

and

$$f(0, \mu < 0, \hat{t}) = \frac{1}{|\mu| + \mu_0} H\left(\hat{t};\ 0, \frac{|\mu| + \mu_0}{|\mu|\mu_0} \cdot \tau_0\right) \tag{23}$$

Here in the r.h.s. of (22) as well as further in the r.h.s. of (26) and (31) $\mu < \mu_0$, and μ and μ_0 are inverted if $\mu > \mu_0$.

Equations (19)-(23) give the incoherent component (13) of the solution for equation (12) in the single scattering approxmiation for the case of the delta pulse transmitted through or reflected from the random medium slab. To characterize this solution we will introduce, following Reference 2, an “arrival time” $\hat{t}_\Delta$ at which a Δ part, $0 < \Delta < 1$, of the pulse energy is transmitted through or reflected from the random medium slab. In accordance with equation (9) for the mean Poynting's vector the arrival time, e.g. of the pulse transmission, may be defined by equation

$$\int_0^{\hat{t}_\Delta} d\hat{t} \left(\hat{\Phi}_{incoh}\right)_{\alpha\alpha,\alpha'\beta'} (\overline{s}, \overline{s}_0;\ \tau_0, \hat{t}) J_{\alpha'\beta'} = \\ = \Delta \cdot \int_0^{\infty} d\hat{t} \left(\hat{\Phi}_{incoh}\right)_{\alpha\alpha,\alpha'\beta'} (\overline{s}, \overline{s}_0;\ \tau_0, \hat{t}) J_{\alpha'\beta'} \tag{24}$$

Substituting (22) into (19) and (20) and remembering (13), one can find for the arrival time of the pulse transmission through the slab the expression [14, 15]

$$\hat{t}_\Delta = \hat{t}^0_\Delta + \delta\hat{t} \tag{25}$$

where

$$\hat{t}^0_\Delta = Ln\frac{1}{(1-\Delta)\exp\left(-\frac{\tau_0}{\mu_0}\right) + \Delta\exp\left(-\frac{\tau_0}{\mu}\right)} \tag{26}$$

and

$$\delta\hat{t} = Ln(1 + \hat{t}_{delay}) \tag{27}$$

The meaningful quntity of angular time delay t_{delay} appearing in the r.h.s. of (27) is given by

$$\hat{t}_{delay}(\overline{s}, \overline{s}_0) = \frac{1}{\hat{W}_{\alpha\alpha,\alpha'\beta'}(\overline{s}, \overline{s}_0) J^0_{\alpha'\beta'}} \cdot \left[\hat{v}^{-1}_{incoh}(\overline{s}, \overline{s}_0)\right]_{\alpha\alpha,\alpha'\beta'} J^0_{\alpha'\beta'} \tag{28}$$

In the case of azimuthal symmetry of reception of the transmitted radiation the numerator and denominator of the r.h.s. of (28) are averaged additionally with respect to the azimuth of the $\overline{s}$ unit vector in the principal spherical coordinate system with polar axis Z along normal to the slab. In Reference 2 an "arrival velocity" v_A is considered which may be defined in our theory by $v_A/v_{coh} = \tau_0/\hat{t}_\Delta$ and obeys in the single scattering approximation the equation

$$\frac{v_{coh}}{v_A} - \frac{v_{coh}}{v^0_A} = \frac{\delta\hat{t}}{\tau_0} \tag{29}$$

where $v_{coh}/v^0_A = \hat{t}^0/\tau_0$. The arrival time of the pulse reflection is defined by equation (24) with τ_0 replaced by zero and has the form (25) where $\hat{t}^0_\Delta$ is given by

$$\hat{t}^0_\Delta = Ln\frac{1}{1-\Delta+\Delta\exp\left(-\frac{|\mu|+\mu_0}{|\mu|\mu_0}\cdot\tau_0\right)} \tag{30}$$

and $\delta\hat{t}$ is given by the same equations (27) and (28).

As one can see, in the equality (25) for the arrival time of the delta pulse transmitted through and reflected from a random medium slab the first term $\hat{t}^0_\Delta$ corresponds to the traditional radiative transfer theory [16]. The second term $\delta\hat{t}$ defined by equations (27) and (28) takes into account the effect of the time delay by resonance scattering and is connected directly because of equations (I.75) and (I.96b) with the scattering properties of the scatterers and their correlation functions. Similarly, the second term of the left hand side (l.h.s.) of equation (29) for the arrival velocity corresponds to the traditional radiative transfer theory and the term of the r.h.s. takes into account the effect of the time delay. An analysis of the expression (28) for the time delay is given in Section 5.

To characterize the physical meaning of the time delay (28) it is useful to write the solution (19) and (20) for a small time when the process of the pulse transmission through or reflection from the slab is not finished yet, respectively. One finds for the pulse transmission

$$\begin{aligned}\int_0^{\hat{t}} d\hat{t} (\hat{\Phi}_{incoh})_{\alpha\alpha,\alpha'\beta'}(\overline{s},\overline{s}_0;\,\tau_0,\hat{t}) J_{\alpha'\beta'} = \\ = \frac{1}{|\mu-\mu_0|} \hat{W}_{\alpha\alpha,\alpha'\beta'}(\overline{s},\overline{s}_0) J'_{\alpha}\beta' \cdot \left[\hat{t} - \frac{\tau_0}{\mu_0} - \hat{t}_{delay}(\overline{s},\overline{s}_0)\right]\end{aligned} \tag{31}$$

for

$$\frac{\tau_0}{\mu_0} < \hat{t} < \frac{\tau_0}{\mu} \ll 1$$

and for the pulse reflection

$$\begin{aligned}\int_0^{\hat{t}} d\hat{t} (\hat{\Phi}_{incoh})_{\alpha\alpha,\alpha'\beta'}(\overline{s},\overline{s}_0;\,0,\hat{t}) J_{\alpha'\beta'} = \\ = \frac{1}{|\mu|+\mu_0} \hat{W}_{\alpha\alpha,\alpha'\beta'}(\overline{s},\overline{s}_0) J_{\alpha'\beta'} \cdot \left[\hat{t} - \hat{t}_{delay}(\overline{s},\overline{s}_0)\right]\end{aligned} \tag{32}$$

for

$$0 < \hat{t} < \frac{|\mu|+\mu_0}{|\mu|\mu_0} \cdot \tau_0 \ll 1$$

The formulas (31) and (32) show that the quantity (28) defines really a time delay during the pulse transmission through or reflection from an optically thin slab. Comparsion shows the r.h.s. of (31) and (32) is similar to an expression for the average position at time t of the portion of the wave function for scattered particles found inside the given scattering cone (see equation (1.6) of Reference 13).

4. Diffusion Asymptotics of Solution for MRTE

Let us return to the MRTE (I.87) in the space-time Fourier transform representation and rewrite this one in the matrix notation as follows

$$(i\kappa - i\overline{s}\cdot\overline{q})\Phi(\overline{s},\overline{s}';\overline{q},\omega) - \int d\overline{s}''U(\overline{s},\overline{s}'';\,\omega)\Phi(\overline{s}'',\overline{s}';\,\overline{q},\omega) = -\mathcal{E}(\overline{s},\overline{s}') \tag{33}$$

Here the tensor-propagator $\Phi_{\alpha\beta,\alpha'\beta'}(\overline{s},\overline{s}';\,\overline{q},\omega)$ is defined by equation (6), a quantity $\kappa = \omega c_{ph}/c_0^2$, a tensor-operator $U_{\alpha\beta,\alpha'\beta'}(\overline{s},\overline{s}';\,\omega)$ is given by

$$U = \frac{1}{l}\mathcal{E} - W + i\kappa\mathcal{D} \tag{34}$$

with

$$\mathcal{E}_{\alpha\beta,\alpha'\beta'}(\overline{s},\overline{s}') = P^{tr}_{\alpha\alpha'}(\overline{s})P^{tr}_{\beta\beta'}(\overline{s})\delta(\overline{s}-\overline{s}') \tag{35}$$

and the tensor-operator of dispersion in frequency domain $\mathcal{D}_{\alpha\beta,\alpha'\beta'}(\overline{s},\overline{s}')$ is defined by equation (I.86) and connected with the inverse tensor-operator of the energy transport velocity (1) by equation (I.85). Construct a solution for equation (33) following the method of References 6,7 in terms of the tensor-eigenfunctions $\Phi^{(n)}_{\alpha\beta}(\overline{s};\,\overline{q},\omega)$ wich satisfy the homogeneous equation (33) for the eigenvalues $i\kappa = \lambda_n(q,\omega)$ and obey the orthogonality and completeness condition given by

$$\left(\Phi^{(n)},\overline{\Phi}^{(m)}\right) = \int d\overline{s}\Phi^{(n)}_{\alpha\beta}(\overline{s};\,\overline{q},\omega)\overline{\Phi}^{(m)}_{\alpha\beta}(\overline{s};\,\overline{q},\omega) = \delta_{nm} \tag{36}$$

and

$$\sum_n \Phi^{(n)}_{\alpha\beta}(\overline{s};\,\overline{q},\omega)\overline{\Phi}^{(n)}_{\alpha'\beta'}(\overline{s}';\,\overline{q},\omega) = \mathcal{E}_{\alpha\beta,\alpha'\beta'}(\overline{s},\overline{s}') \tag{37}$$

We denote $\overline{\Phi}^{(n)}_{\alpha\beta}$ the left tensor-eigenfunction related to $\Phi^{(n)}_{\alpha\beta}$ and satisfies the transposed homogeneous equation (33) for $i\kappa = \lambda_n$ and with the tensor-operator

$$\tilde{U}_{\alpha\beta,\alpha'\beta'}(\overline{s},\overline{s}';\,\omega) = U_{\alpha'\beta',\alpha\beta}(\overline{s}',\overline{s};\,\omega) \tag{38}$$

A general solution for equation (33) has the form of the series

$$\Phi_{\alpha\beta,\alpha'\beta'}(\overline{s},\overline{s}';\,\overline{q},\omega) = -\sum_n \frac{\Phi^{(n)}_{\alpha\beta}(\overline{s};\,\overline{q},\omega)\overline{\Phi}^{(n)}_{\alpha'\beta'}(\overline{s}';\,\overline{q},\omega)}{i\kappa - \lambda_n(q,\kappa)} \tag{39}$$

that is verified by direct substitution.

We seek further the singular part of the solution (39) as $q, \omega \to 0$ the existence of which can be established with the help of the optical theorem (I.82). Indeed, combining (33) and (I.82) at $q = 0$ one can show that the integral of $\Phi_{\alpha\beta,\alpha'\beta'}(\overline{s}, \overline{s}'; \mathbf{0}, \omega)$ over $\overline{s}$ behaves as $1/\omega$ as $\omega \to 0$ in the case of non-absorptive scatterers. On the other hand, from the optical theorem (I.82) it is seen that the tenso-eigenfunctions

$$\Phi^{(0)}_{\alpha\beta}(\overline{s};\, \mathbf{0}, 0) = \overline{\Phi}^{(0)}_{\alpha\beta}(\overline{s};\, \mathbf{0}, 0) = \frac{1}{\sqrt{8\pi}} P^{tr}_{\alpha\beta}(\overline{s}) \tag{40}$$

corresponding to the eigenvalue $\lambda_0(0,0) = 0$ responsible for the above singularity in the case of non-absorptive scatterers.

As it was shown, the $1/\omega$ singularity is closely connected to the energy conservation law which is unique in the problem considered. Therefore we shall assume that there is an unique tensor-eigenfunction $\Phi^{(0)}_{\alpha\beta}(\overline{s};\, \overline{q}, \omega)$ related to the eigenvalue which behaves as $\lambda_0(q,\omega) \to 0$ as $q, \omega \to 0$, and moreover that for sufficiently small q and ω there exists a finite gap between this and other eigenvallues, i.e. $|\lambda_0(q,\omega) - \lambda_n(q,\omega)| > 0$ for $n \neq 0$. It is worth noting that this is exactly the properties of the spectrum of the traditional non-stationary radiative transfer equation [16].

These assumptions allow one to separate the singular part of the spectral decomposition (39) of $\Phi_{\alpha\beta,\alpha'\beta'}$ and then to obtain $\lambda_0(q,\omega)$ and $\Phi^{(0)}_{\alpha\beta}(\overline{s};\, \overline{q}, \omega)$, $\overline{\Phi}^{(0)}_{\alpha\beta}(\overline{s};\, \overline{q}, \omega)$ with the aid of perturbation method with respect to q, ω. As a result the following diffusion asymptotics of (39) is derived (see details in Appendix II)

$$\Phi_{\alpha\beta,\alpha'\beta'}(\overline{s}, \overline{s}';\, \overline{q}, \omega) \simeq -\frac{c_0^2}{c_{ph}} \cdot \frac{\Phi_{\alpha\beta}(\overline{s};\, \overline{q}) \Phi_{\alpha'\beta'}(\overline{s}';\, \overline{q})}{i\omega - Dq^2 - \frac{1}{t_{abs}}} \tag{41}$$

Here the tensor-function $\Phi_{\alpha\beta}(\overline{s};\, \overline{q})$ is given by

$$\sqrt{8\pi(1+a)}\,\Phi_{\alpha\beta}(\overline{s};\, \overline{q}) = P^{tr}_{\alpha\beta}(\overline{s}) - i\tilde{\Phi}_{\alpha\beta}(\overline{s};\, \overline{q}) \tag{42}$$

with

$$\tilde{\Phi}_{\alpha\beta}(\overline{s};\, \overline{q}) = \lim_{\omega \to 0} \int d\overline{s}' \Phi_{\alpha\beta,\gamma\gamma}(\overline{s}, \overline{s}';\, 0, \omega)(\overline{s}' \cdot \overline{q}) \tag{43}$$

The diffusion constant D takes the form

$$D = \frac{1}{8\pi(1+a)} \cdot \frac{c_0^2}{c_{ph}} \cdot \frac{1}{q^2} \cdot \int d\overline{s}(\overline{s} \cdot \overline{q}) \tilde{\Phi}_{\alpha\alpha}(\overline{s};\, \overline{q}) = \frac{v_E l_{tr}}{3} \tag{44}$$

where the scalar energy transport velocity v_E was discussed in the Introduction. The transport mean free path is written as usual as $l_{tr} = l/(1- < \cos\theta >)$ with mean cosine defined by equation

$$\int d\overline{s} s_\alpha W_{\gamma\gamma,\alpha'\beta'}(\overline{s},\overline{s}') = \frac{1}{l} < \cos\theta > s'_\alpha P^{tr}_{\alpha'\beta'}(\overline{s}') \tag{45}$$

For the mean absorption time t_{abs} one finds

$$\frac{1}{t_{abs}} = \Omega \frac{\Im \hat{\epsilon}_1}{\epsilon'} \cdot \frac{a}{1+a} \tag{46}$$

where $\hat{\epsilon}_1$ is the complex dielectric permittivity of a conducting scatterer and $\epsilon' = \epsilon_1 - \epsilon^{bac}$ is deviation of the real part of one from a background constant.

Substitute the asymptotics (41) into equation (8) to obtain the radiance tensor and further the mean energetic quantities (I.77 - I.79) in the diffusion approximation. These straightforward calculations lead to the following physically transparent relations

$$< \overline{S}(\overline{R},\omega) >= -D\nabla < w(\overline{R},\omega) > \tag{47}$$

and

$$< Q^{con}(\overline{R},\omega) >= \frac{1}{t_{abs}} < w(\overline{R},\omega) > \tag{48}$$

Equation (47) is the Fick's law for the electromagnetic energy transport which relates the mean spectral component of the Poynting's vector $< \overline{S}(\overline{R},\omega) >$ with the mean spectral component of the electromagnetic energy density $< w(\overline{R},\omega) >$. Similarly, equation (48) relates the mean spectral component of the field energy absorption because of conductivity $< Q^{con}(\overline{R},\omega) >$ with the mean spectral component of the electromagnetic energy density. Direct verification shows that the mean energetic quantities in the diffusion approximation satisfy the mean Poynting's theorem (I.11) in the form

$$\left(-i\omega - D\Delta + \frac{1}{t_{abs}} \right) < w(\overline{R},\omega) > + < Q^{src}(\overline{R},\omega) >= 0 \tag{49}$$

Here the mean spectral component $< Q^{src}(\overline{R},\omega) >$ of the work of field source in an unit volume is given in accordance with equation (I.46) and (I.66) by

$$< Q^{src}(\overline{R},\omega) >= -\frac{1}{(4\pi)^2} \cdot \frac{k^2_{eff}}{2\Omega\mu k_0 n_{eff}} \cdot \int d\overline{s} \int_{\overline{q}} e^{i\overline{q}\cdot\overline{R}} J_{\alpha\alpha}(k_{eff}\overline{s};\overline{q},\omega) \tag{50}$$

To obtain further the mean spectral component of the electromagnetic energy density inside scatterers (4) it is more simple to use the following relationshi between this quantity and the mean spectral component of the field energy absorption because of conductivity

$$< w_{inside} >= \frac{\epsilon_1}{\Omega \Im \hat{\epsilon}_1} < Q^{con} > \tag{51}$$

Combining this relation with (48), one finds

$$< w_{inside}(\overline{R}, \omega) >= \frac{\epsilon_1}{\epsilon'} \cdot \frac{a}{1+a} < w(\overline{R}, \omega) > \tag{52}$$

and therefore the mean spectral component of the electromagnetic energy density outside scatterers is given in accordance with (3) by

$$< w_{outside}(\overline{R}, \omega) >= \left(1 - \frac{\epsilon_1}{\epsilon'} \cdot \frac{a}{1+a}\right) < w(\overline{R}, \omega) > \tag{53}$$

Equations (52) and (53) show that $(\epsilon_1/\epsilon')a/(1+a)$ part of the mean electromagnetic energy of a pulse during its propagation through a random medium is distributed inside scatterers and $(1-(\epsilon_1/\epsilon')a/(1+a)$ part of the mean energy distributed outside scatterers.

5. Low-Density Limit and Comparison with Experiments

In the lowest order of the number density of scatterers n the mass and intensity operators in the space Fourier transform representation are given by (see, e.g., Reference 19)

$$M_{\alpha\beta}(\overline{k}, \Omega) = nT_{\alpha\beta}(\overline{k}, \overline{k}; \Omega) \tag{54}$$

and

$$K_{\alpha\beta,\alpha'\beta'}(\overline{p}, \overline{p}'; 0, \omega) = nT_{\alpha\alpha'}(\overline{p}, \overline{p}'; \Omega + \frac{\omega}{2} + i0)T^*_{\beta\beta'}(\overline{p}, \overline{p}'; \Omega - \frac{\omega}{2} + i0) \tag{55}$$

Here $T_{\alpha\beta}(\overline{k}, \overline{k}'; \Omega)$ is the single scattering tensor-operator (T-matrix of one scatterer) in the jFourier transform representation

$$T_{\alpha\beta}(\overline{k}, \overline{k}'; \Omega) = \int d\overline{r} \int d\overline{r}' \exp\left[-i(\overline{k} \cdot \overline{r} - \overline{k}' \cdot \overline{r}')\right] T_{\alpha\beta}(\overline{r}, \overline{r}'; \Omega) \tag{56}$$

where the scattering tensor-operator in the space domain $T_{\alpha\beta}(\overline{r},\overline{r}';\,\Omega)$ obeys a tensor Lippman-Schwinger equation $T = V + V\mathcal{G}^0 T$ with the scattering potential V and the retarded Green tensor-function in a background medium $\mathcal{G}^0$ defined by equation (I. 22) and equation (I.24) at $V = 0$, respectively. The physical meaning of the single scattering tensor-operator (56) is characterized by that the scattering electric field $\overline{E}^S(\overline{r})$ in the Fraunhoffer zone of a scatterer has the form

$$\overline{E}^S(\overline{r}) = \overline{a}(\overline{s},\overline{s}_0)G_0(r) \tag{57}$$

where $\overline{s} = \overline{r}/r$ determines the scattering direction and the scattering vector-amplitude $\overline{a}(\overline{s},\overline{s}_0)$ is expressed through the scattering tensor-amplitude (I. 92) and the electric field $\overline{E}^0$ of an incident monochromatic plane wave propagated in the direction $\overline{s}_0$, $\overline{E}^0\overline{s}_0 = 0$, by equation

$$a_\alpha(\overline{s},\overline{s}_0) = a_{\alpha\alpha'}(\overline{s},\overline{s}_0)E^0_{\alpha'} \tag{58}$$

In the r.h.s. of (57) $G_0(r) = \exp(ik_0 r)/(-4\pi r)$ is the scalar Green function in a background medium.

Let us consider equation (57) in the coordinate system x, y, z natural to the scatterer as a dielectric sphere (see Reference 20, p.82, Fig. 3.3). In this coordinate system one can find the representation for the scattering tensor-amplitude (I. 92) in the form

$$a_{\alpha\beta}(\overline{s},\overline{s}_0) = \tilde{S}_1\hat{e}_{\phi\alpha}\hat{e}_{\phi\beta} + \tilde{S}_2\hat{e}_{\theta\alpha}\hat{e}_{\beta} \tag{59}$$

Here the quantities $\tilde{S}_{1,2} = (4\pi/ik_0)S^*_{1,2}$ where $S_{1,2} = S_{1,2}(\theta)$ are the scattering amplitude functions [21] of the copolarized and cross-polarized channels, θ is the scattering angle, $\hat{e}_\theta$ and $\hat{e}_\phi$ are the unit vectors of the natural spherical coordinate system with the polar axis z along the unit vector $\overline{s}_0$, and the unit vector $\hat{e}$, denoted by $\hat{e}_{\|\|i}$ in the Reference 20, is placed in an intersection of the x,y-plane and the scattering plane defined by the angle ϕ.

The representation (59) enables one to write all quantities of I which are needed to calculate the time delay (28) in terms of the scattering amplitude function $S_{1,2}$ and their partial derivatives with respect to the quantity E of frequency Ω. The transverse component $M_{tr}(k)$ of the mass operator at $k = k_0$ is given in the lowest order of the number density of scatterers by equality $M_{tr}(k_0) = n\tilde{S}(0^0)$ where $\tilde{S}(0^0) = \tilde{S}_1(0^0) = \tilde{S}_2(0^0)$ corresponds to the forward direction $\theta = 0^0$

of scattering. Therefore equation (I. 83) for the extinction length l takes the form $1/l \simeq -(n/k_0)\Im\tilde{S}(0^0) = n\sigma_{ex}$ where σ_{ex} is the extinction across section of single scattering which coincides in the case of non-absorptive scatterers with its scattering cross section σ_{sc}. For the phase velocity $c_{ph} = \Omega/k_{eff}$ in the random medium where the effective wave number k_{eff} is a root of equation (I.70) one finds

$$\frac{c_0}{c_{ph}} = \frac{k_{eff}}{k_0} \simeq \sqrt{1 - \frac{\Re M_{tr}(k_0)}{E}} \tag{60}$$

In the same low density limit one can replace k_{eff} by k_0 in the r.h.s. of equation (11) for the scalar coherent component v_{coh} of the energy transpport velocity in the case of non-absorptive spherical scatterers. Similarly one may replace $M_{tr}(k)$ by $M_{tr}(k_0)$ in the r.h.s. of equation (I. 65a) for the transverse component $G_{tr}(k)$ of the averaged Green-tensor function. The last approximation gives for the transverse component of the averaged Green-tensor function an expression which can be obtained from the transverse component of the Green-tensor function in a background medium by formal replacing the wave number k_0 in the background medium with the complex effective wave number k_1 in the random medium defined by equation $k_1^2 = k_0^2 - M_{tr}(k_0)$. In particular one finds

$$\int_P \frac{\partial \Re G_{tr}(p)}{\partial E} = \frac{1}{4\pi} \cdot \frac{\partial \Im k_1}{\partial E} \tag{61}$$

where the imaginary part of the complex effective wave number is given approximately by $\Im k_1 \simeq 1/2l$. The phase velocity $v_{ph} = \Omega/\Re k_1$ is connected with the phase velocity c_{ph}, defined by equation (60), as follows

$$\frac{c_{ph}}{v_{ph}} = \frac{\Re k_1}{k_{eff}} = \sqrt{1 + \left(\frac{\Im k_1}{k_{eff}}\right)^2} \tag{62}$$

In the lowest order of the density of scatterers the second term of the square root of (62) with the fraction $\Im k_1/k_{eff} \simeq 1/2k_0 l \ll 1$ may be ignored and the phase velocity v_{ph} coincides approximately with c_{ph}, $v_{ph} \simeq c_{ph}$. The group velocity $v_g = \partial\Omega/\partial\Re k_1$ obeys the equation

$$\frac{c_0^2}{v_{ph} v_g} = 1 - \frac{\partial \Re M_{tr}(k_0)}{\partial E} + 2(\Im k_1)\frac{\partial \Im k_1}{\partial E} \tag{63}$$

The third term in the r.h.s. of this equation is the second-order of the density of scatterers and therefore may be omitted. After that the group velocity v_g practically coincides with the scalar coherern commponent v_{coh} of the energy transport velocity defined by equation (11). The phase tensor (I. 75) in the lowest order of the density of scatterers takes the form

$$W_{\alpha\beta,\alpha'\beta'}(\overline{s},\overline{s}_0) \simeq \frac{1}{(4\pi)^2} n a_{\alpha\alpha'}(\overline{s},\overline{s}_0) \cdot a^*_{\beta\beta'}(\overline{s},\overline{s}_0) \tag{64}$$

and for the incoherent component (I. 96b) of the inverse tensor-operator of the energy transport velocity one finds

$$\begin{aligned} &\frac{c_0^2}{c_{ph}} \left[v^{-1}_{incoh}(\overline{s},\overline{s}_0\right]_{\alpha\beta,\alpha'\beta'} \simeq \\ &\simeq -\frac{1}{(4\pi)^2} k_0 n i \left[\frac{\partial a_{\alpha\alpha'}(\overline{s},\overline{s}_0)}{\partial E} a^*_{\beta\beta'}(\overline{s},\overline{s}_0) - a_{\alpha\alpha'}(\overline{s},\overline{s}_0)\frac{\partial a^*_{\beta\beta'}(\overline{s},\overline{s}_0)}{\partial E}\right] + \\ &\quad + \frac{\partial \Im k_1}{\partial E} W_{\alpha,\beta,\alpha'\beta'}(\overline{s},\overline{s}_0) \end{aligned} \tag{65}$$

Deriving the last equality, one replaced approximatedly in the integral term of the r.h.s. of (I. 96b) the tensor $K_{\alpha\beta,\alpha'\beta'}(p\overline{s}, k_{eff}\overline{s}_0;\, 0,0)$ by its value at $p = k_0$ and $k_{eff} \simeq k_0$ and used (61). The double trace of (65) on the base of the representation (59) is given by

$$\begin{aligned} \frac{c_0^2}{c_{ph}} \left[v^{-1}_{incoh}(\overline{s},\overline{s}_0)\right]_{\alpha\alpha\beta\beta} =& \frac{1}{(4\pi)^2} \cdot 2k_0 n \Im \left(\frac{\partial \tilde{S}_1}{\partial E}\tilde{S}_1^* + \frac{\partial \tilde{S}_2}{\partial E}\tilde{S}_2^*\right) + \\ & + \frac{1}{(4\pi)^2} \cdot \frac{\partial \Im k_1}{\partial E} \cdot n \left(|\tilde{S}_1|^2 + |\tilde{S}_2|^2\right) \end{aligned} \tag{66}$$

Substituting this euqality together with the double trace of the coherent component (10) of the inverse tensor-operator of the energy transport velocity into the sum rule (2) and using the optical theorem (I. 82) in the case of non-absorptive scatterers

$$\frac{1}{(4\pi)^2} \cdot n \int d\overline{s} \left(|\tilde{S}_1|^2 + |\tilde{S}_2|^2\right) = \frac{2}{l} \tag{67}$$

one can obtain the following expression for resonance efficiency "a" in the scalar energy transport velocity

$$a = -n\frac{\partial \Re \tilde{S}(0^0)}{\partial E} + \frac{k_0}{(4\pi)^2} \cdot n \int d\overline{s} \Im \left(\frac{\partial \tilde{S}_1}{\partial E}\tilde{S}_1^* + \frac{\partial \tilde{S}_2}{\partial E}\tilde{S}_2^*\right) + \frac{1}{l} \cdot \frac{\partial \Im k_1}{\partial E} \tag{68}$$

The sum a_{Lag} of the first two terms in the r.h.s. of eq. (68) is transformed to the form

$$a_{Lag} = a - \frac{1}{l}\partial\frac{\Im k_1}{\partial E} = \frac{3f}{4x^2}\sum_{n=1}^{\infty}(2n+1)\left(\frac{\partial\alpha_n}{\partial x} + \frac{\partial\beta_n}{\partial x}\right) - \frac{1}{2}fC(x) \quad (69)$$

Here $x = k_0 r_0$ and $f = n4\pi r_0^3/3$ are the size parameter and packing fraction of spherical scatterers with radius r_0, respectively. The phases $\alpha_n(x)$ and $\beta_n(x)$ of the different partial waves are defined in terms of Mie coefficients $a_n(x)$ and $b_n(x)$ by well known equations (see Reference 21, p.161). The parameter $C(x)$ defines the phase velocity c_{ph} in accordance with equation (60) by $c_0/c_{ph} = (1 + fC(x))^{1/2}$.

A version of formula (68) for scalar waves witout the "extra" third (last) term in the r.h.s. has been first obtained in the pioneering work [1]. There has been some confusion and contraversy in the literture about the origin and contribution of this "extra" term to the resonance efficiency "a". The existence of the "extra" term was connected directly with a corresponding term of the generalized Ward-Takahashi identity whcih was derived for multiple scattering of classical scalar waves in [6], confirmed in [8] and extended to the case of vector electromagnetic waves in [5,7]. Some later a version of formula (68) for scalar wave with "extra" term appeared in Reference 22. A contribution of the "extra" term to the resonance efficiency "a" is formally of the first order in density of scatterers according to [9] and as one can see also from the integral term in the r.h.s. of (I. 96b). The "extra" term of the second order in density is obtained in the r.h.s. of (68) within the "Wigner" approximation employd in Reference 22. By definition this approximation is obtained if p-dependence of the intensity operator in the integral term of the r.h.s. of (I. 96b) is ignored (compare with a sentence after equation (65)). The formula (69) has been first obtained in References 10 and 19 by a heuristic generalization of a result for the diffusion constant D of the scalar waves in a random medium near resonance scattering on the case of the vector waves and was used in Reference 2. The same problem of a diffusion constant D in a random medium near resonance scattering of vector waves was considered in Reference 3 by a similar to [10, 19] heuristic method but with a different from (69) result for resonance efficiency "a". The result of [3] for resonance efficiency which we denote a_{kav} coincides with the second term in the r.h.s. of equation (68) and has the form

$$a_{kav} = a_{Lag} + n\frac{\partial \Re \tilde{S}(0^0)}{\partial E} = \frac{3f}{2x^2}\sum_{n=1}^{\infty}(2n+1)\left(\frac{\partial \alpha_n}{\partial x}\sin^2\alpha_n + \frac{\partial \beta_n}{\partial x}\sin^2\beta_n\right) \tag{70}$$

Perhaps it would be helpful to note concluding this paragraph that exact formulas for resonance efficiency "a" were derived without using any perturbative expansion with respect to density of scatterers or their potential in References 9 and 7 for classical scalar and vector waves, respectively. From these exact formulas one may see in particular that in the case of quantum mechanical scattering (electron-impurity system) with energy-independent real scattering potential, when for example in equation (I. 22) a function $g(\Omega)$ is constant, the resonance efficiency "a" becomes equal to zero because the term in the Ward-Takahashi identity (I. 36) including a difference of values of this functions of two frequencies disappears.

Let us turn to the definition (28) of the time delay. in the case of completely polarized source the polarization tensor has the form $J^0_{\alpha\beta} = J^0_\alpha (J^0_\beta)^*$ where J^0_α is a complex unit vector. In the coordinate system natural to the scatterers this unit polarization vector can be characterized by its parallel $J^0_\| = \overline{J}^0 \cdot \hat{e}$ and perpendicular $J^0_\perp = -\overline{J}^0 \cdot \hat{e}_\phi$ components relatively to the scattering plane. In particular, for the linearly polarized source the parallel and perpendicular components of its unit polarization vector directed along X axis of the natural coordiante system are given by $J^0_\| = \cos\phi$ and $J^0_\perp = \sin\phi$ where ϕ is the azimuthal angle of the scattering plane in the natural coordinate system. In this case the deffinition (28) of the time delay can be transformed in the lowest order of the density of scatterers on the basis of equations (59), (64) and (65) to the following form

$$t_{delay}(\theta,\phi)/t_0 = -\frac{3f}{4\pi x^2}\cdot\frac{\sigma_{sc}}{\sigma_{sc}(\theta,\phi)}\Im\left(\frac{\partial S_1}{\partial x}S_1^*\sin^2\phi + \frac{\partial S_2}{\partial x}S_2^*\cos^2\phi\right) \tag{71}$$

Here t_0 is a scattering mean free time defined by $1/t_0 = c_0^2/lc_{ph}$ and $\sigma_{sc}(\theta,\phi) = (1/E)(|S_1|^2\sin^2\phi + |S_2|^2\cos^2\phi)$ is differential scattering cross section of spherical scatterer. A contribution of the last term in the r.h.s. of (65) to (71) is omitted. The time delay (71) satisfies the norm condition

$$\frac{1}{t_0}\int_0^{\pi}\sin\theta d\theta\int_0^{2\pi}d\phi\frac{\sigma_{sc}(\theta,\phi)}{\sigma_{sc}}\cdot t_{delay}(\theta,\phi)=\frac{1}{t_0}<t_{delay}(\theta,\phi)>=a_{kav} \tag{72}$$

where $< t_{delay}(\theta,\phi) >$ is the time delay averaged over all scattering directions θ,ϕ and the quantity a_{kav} is defined by equation (70). Comparison shows that averaged time delay $< t_{delay}(\theta,\phi) >$ may be thought of approximately as introduced in Reference 19 by heuristic approach a time Δt_w needed for incoming plane wave to "charge" the volume of a scatterer to a portion of electromagnetic energy. Actually, if one replace in equation (72) the scattering mean free time t_0 by l/c_0 and replace also in the r.h.s. of (70) the quantities $\sin^2\alpha_n$ and $\cos^2\alpha_n$ by $1/2$ then $< t_{delay}(\theta,\phi) >$ will become Δt_w in accordance with [19] (see equations (31) and (35) of this reference). We suppose further the incident plane delta pulse being radiated by the source is normal to the slab boundary direction, $s_{0z} = 1$, when the principal and natural coordinate systems are the same. This supposition enables one to obtain easily a time delay $t_{delay}(\theta)$ for the case of azimuthal symmetry of a reception of the transmitted or reflected radiation by averaging the numerator and denominator in the r.h.s. of (71) with respect to the angle ϕ that gives

$$t_{delay}(\theta)/t_0=-\frac{3f}{4\pi x^2}\cdot\frac{\sigma_{sc}}{\sigma_{sc}(\theta)}\Im\left(\frac{\partial S_1}{\partial x}S_1^*+\frac{\partial S_2}{\partial x}S_2^*\right) \tag{73}$$

where $\sigma_{sc}(\theta)=(1/E)(|S_1|^2+|S_2|^2)$.

Before going over to the actual computation of the time delay, let us stress that the r.h.s. of equations (71) and (73) involves the partial derivatives of the scattering amplitudes $S_{1,2}$ or because of the representation (59) and definition (I. 92) the partial derivatives of the single scattering tensor-operator (56) with respect to frequency Ω at constant wave number $\overline{k}=k_0\overline{s}$ and $\overline{k}'=k_0\overline{s}_0$ on the shel $k=k'=k_0$. As such it does not seem to be of any numerical use since text books provide the scattering amplitudes only. To resolve this problem approximately one could use for the first time the observation of Reference 10 (see p.92) that partial derivatives of (56) with respect to wve numbers $\overline{k}$ and $\overline{k}'$ are small in comparison with partial derivative with respect to frequency Ω near resonance of scattering where the Ω-dependence of (56) is expected to be large. This leads one to replace the partial derivatives with repsect to Ω by total derivatives in the r.h.s. of equations (71) and (73). For the exact resolution of the problem under consideration one can try to apply equations whcih connect the partial and total derivatives of the sacttering amplitudes with respect to Ω. Such equations can be derived by using the key indentity

$$2E\frac{\partial}{\partial E}\exp\left[-ik_0(\overline{s}\cdot\overline{r}-\overline{s}_0\cdot\overline{r}')\right]= \\ =\left(\overline{s}\frac{\partial}{\partial\overline{s}}+\overline{s}_0\frac{\partial}{\partial\overline{s}_0}\right)\exp\left[-ik_0(\overline{s}\cdot\overline{r}-\overline{s}_0\cdot\overline{r}')\right] \tag{74}$$

and for the scattering amplitude S_1 of the copolarized channel giveing (Appendix III)

$$\Im\left(\frac{d\tilde{S}_1}{dE}\tilde{S}_1^*\right)=\Im\left(\frac{\partial\tilde{S}_1}{\partial E}\tilde{S}_1^*\right)+\frac{\cos\theta}{E}\Im\left(\frac{d\tilde{S}_1}{d\cos\theta}\tilde{S}_1^*\right) \tag{75}$$

The same formula for the scattering amplitude S_2 of the cross polarized channel seems more complicated and is not discussed here. Fig 2. shows the numerical evaluation of equations (72) and (73) for the averaged time delay $< t_{delay}(\theta,\phi) >$ and the time delay $t_{delay}(\theta)$ in the case of azimuthal symmetry of a reception for Mie scatterers with index of refraction $m = 2.28 + i * .007$, pakcing fraction $f = 10.9\%$ angle $\theta = 10$ degree, and slab thickness $L = 62.5$ mm relevant for the time domain experiments [2]. The same figure displays the experimental time delay $t_{delay}(\theta)$ obtained from experiments [2] by help of equations (25) and (26). All time delays are given in Fig. 2 in units of t_0. In accordance wtih Introduction the single scattering approximation is applied under condition that parameter l_{tr}/L in Fig. 3 is more than one, $l_tr/L > 1$. Fig. 4 shows the numerical evaluation of equation (71) for the time delay $t_{delay}(\theta,\pi/2)$ in units of t_0 in the case of copolarized scattering $\phi=\pi/2$ when the scattering plane is perpendicular to the polarization plane of the incident pulse and using exact equation (75) for the partial derivative of the scattering amplitude S_1. This figure displays a small difference between exact and approximate evaluation of (71) by using equatiion (75) and by replacing of the partial derivative of S_1 with the total derivative of one, respectively.

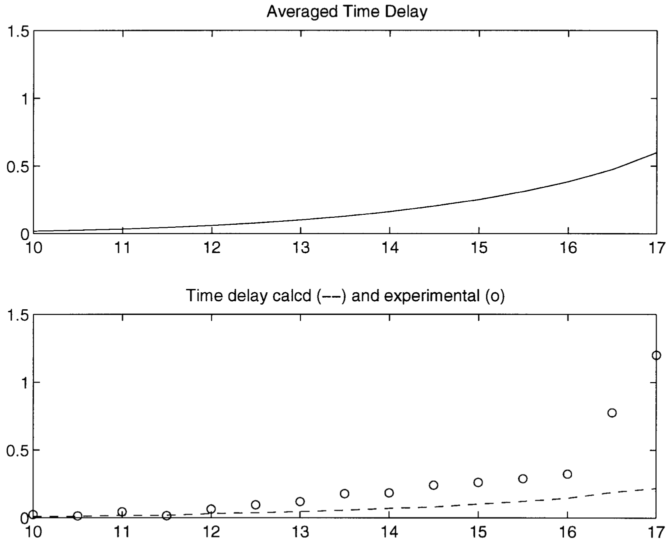

Figure 2. Top graph shows $< t_{delay}(\theta, \phi) >$ in units of t_0 for Mie scatterers with index of refraction $m = 2.28 + i * .007$, packing fraction $f = 10.9\%$ and slab thickness $L = 62.5$mm relevant for the time domain experiments. Bottom graph shows $t_{delay}(\theta)$ for the same medium sample, azimutal symmetry of a reception, and angle $\theta = 10$ degree with experimental data shown as o's. For both graphs the x-axis is frequency in GHz.

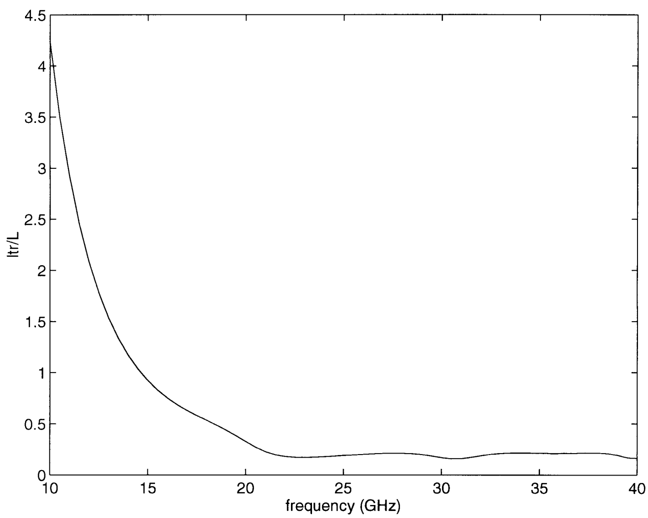

Figure 3. Ratio of transport mean free path l_{tr} to slab thickness $L = 62.5\,mm$ for experimental data with fractional volume 0.109.

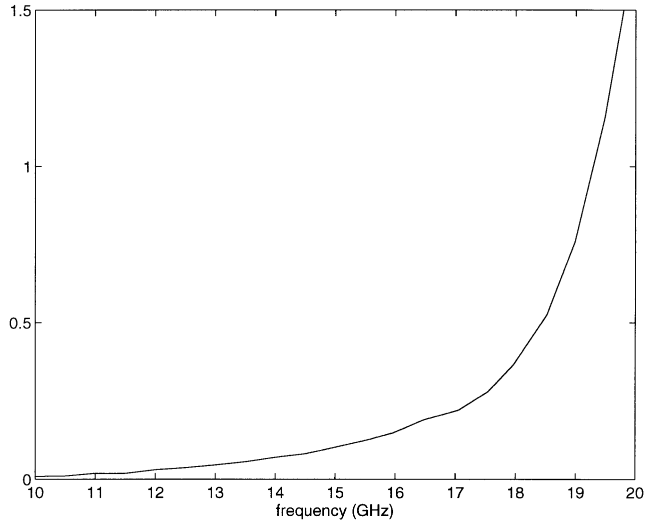

Figure 4. Numerical evaluation of equation (71) for the time delay $t_{delay}(\theta, \pi/2)$ in the units of t_0 in the case of copolarized scattering $\phi = \pi/2$ with $\theta = 10^0$ when the scattering plane is perpendicular to the polarization plane of the incident pulse. Evaluations using equation (75) for the partial derivative (solid line) and by replacing of the partial derivative with the total derivative (dashed line) are virtually indistinguishable for this case.

6. Conclusion and Discussion

In this paper two simple solutions for modified radiative transfer equation (MRTE) (7) are considered. The MRTE is more complicated than the traditional radiative transfer equation [16] due to the operator nature of the energy transport velocity (1) in the space of unit vectors and indices of polarization. The MRTE is transformed to the integral form (12) whcih enables one to conclude that effect of resonance scattering on a pulse propagation in a strongly scattering medium is similiar to the effect of inductivity on a signal in an electrical circuit in accordance with Fig. 1. The integral equation (12) is suitable for application of the iterative method. The first iteration gvies a solution for the problem of the delta pulse transmission through and reflection from an optically thin slab in the single scattering approximation. This solution shows that the "arrival time" (25) introduced according to Reference 2 which can be measured experimentally is simple connected with the angular time delay (28) of microscopic nature. The revealed possibility to measure the microscopic time delay (28) by incoherent reflection of a pulse from a random medium slab is especially worthwhile for the problems of remote sensing.

The time delay (28) is transformed in the low-density limit to the form (71) written in terms of well known scattering amplitude functions [21] and their partial derivatives with respect to frequency. The time delay(41) is an anisotropic quantity as a function of the scattering direction and only after averraging over all scattering directions in the form (72) conincides approximately with an isotropic time delay introduced earlier in Reference 19 by a heuristic approach. Fig 2 shows a qualitative agreement (in units of t_0) between the time delay obtained from experiment [2] on a pulse transmission through an optically thin random medium slab using equations (25-27) and the time delay obtained by evaluation of equation (73) in the interval from 10 GHz to 15 GHz of the middle frequency Ω of the pulse where the condition of application of the single scattering approximation is satisfied in accordance with Fig. 3. Nev ertheless, the accuracy of the single scattering approximation gets worse for the more frequencies Ω, in particular near Mie resonance frequency about 23 GHz in experiment [2]. A solution for MRTE (33) in diffusion approximation gives a result (41) consistent with general theory [6, 7] of wave diffusion in strongly scattering media. In the diffusion approximation simple formulas (52), (53) describe the distribution of the electromagnetic energy inside and outside scatterers, respectively which may be a subject of interest for photo-chemistry [23] and the medical laser tissue interaction [24].

There is an important problem concerning a way to obtain some idea of what the integral kernels in the MRTE (7) are like for concrete dense media cases. This problem may be resolved in principle either experimentally or theoretically.

One can see that the MRTE (7) being integrated with respect to time variable in the limits from zero to infinity coincides in its form with a traditional (conventional) radiative transfer equation for vector electromagnetic stationary radiation [25]. Therefore the problem to find a concrete form for the phase tensor and the extinction length in the MRTE (7) from experimental data or Monte Carlo simulations is the same with a conventional radiative transfer equation. This problem in part of Monte Carlo simulations for a conventional radiative transfer equation with a dense medium kernels was discussed recently in Reference 26. It is more difficult to determine from experimental data or Monte-Carlo simulations the inverse tensor - operator of the energy transport velocity (1). The formulas derived in Section 3 and including the angular time delay (28) may be a basis to resolve this problem.

The low-density limit (54), (55) is, of course not very interesting approximation to calculate the integral kernels of the MRTE (7). To calculate these kernels in some realistic form for dense media theoretical models one should use in accordance with definitions (I. 75) and (I. 96 a,b) the appropriate approximations for the mass and intensity operators which are energetically consistent with the two-frequance Ward-Tkahashi identity (I.36). In a next paper we intend to utilize a nonlinear (selfconsistent) generalization to the single-group approximation (NSGA) which was introduce in References 27 and 28 for a stationary problem The NSGA has got several advantages: it corresponds to a selfconsistent treatment of [29] for a generalized scattering operator or a dependent-scattering correction of [30] to this one, it includes as a consequence the quasi-crystalline approximation with coherent potential and a modified ladder approximation of [31], it gives a generalized Lorentz-Lorenz formula for the effective dielectric tensor to the cases of terers within the Lorentz cavity does not vanish. The NSGA could give an additional information to study the experimental results [34, 35] which have shown that as the packing fraction f of scatterers increase towrds clos packing $(f =\sim 0.60)$ there is no structure in the diffusion coefficient versus frequency and therefore according to [36] in the energy transport velocity too. This effect was considered recently in [36, 37] by applying some extentions of the well-known coherent-potential approximation. Having got advantages mentioned above the NSGA cannot nevetheless describe such effects as a space group resonance of [38] due to dipole-dipole interaction inside pairs of scatterers. For this purpose one may use a two-group approximation of [39] that was actually applied recently in [40] to study of the diffusion of scalar waves.

Acknowledgments

The research described in this publication was made possible in part by Grant N N71000 from the International Science Foundation. We are grateful to Professors A. Ishimaru, L. Tsang, Y. Kuga and D. Winebrenner for discussions which have stimulated this research.

Appendix I

Integrate equation (7) with respect to t in the limits from $t = 0$ to any $t > 0$ and take into account that $\Phi(\overline{s}, \overline{s}'; \overline{R}, t) = 0$ for $t < 0$. This gives an equation for the initial condition $\Phi(\overline{s}, \overline{s}'; \overline{R}, 0+0) = \Phi_0(\overline{s}, \overline{s}'; \overline{R})$ which has the form

$$\int d\overline{s}'' v_E^{-1}(\overline{s}, \overline{s}'')\Phi_0(\overline{s}'', \overline{s}'; \overline{R}) = \delta(\overline{R})\delta(\overline{s} - \overline{s}')P_{tr}(\overline{s}) \otimes P_{tr}(\overline{s}') \quad \text{(aI1)}$$

Now the tensor $\Phi(\overline{s}, \overline{s}'; \overline{R}, t)$ may be found as a soultion for the homogeneous equation (7) with the initial condition $\Phi_0(\overline{s}, \overline{s}'; \overline{R})$. This initial problem for the homogeneous equation (7) can be written in the tensor-operator denotations as follows

$$\left(v_{coh}^{-1}\frac{\partial}{\partial t} + \overline{s}\cdot\overline{\nabla} + \frac{1}{l}\right)\Phi(t) = \left(-v_{incoh}^{-1}\frac{\partial}{\partial t} + W\right)\Phi(t) \quad \text{(aI2)}$$

with $\Phi(0) = \Phi_0$.

Let us introduce the tensor $\Phi_{coh}(\overline{s}, \overline{s}'; \overline{R}, t)$ which satisfies the equation

$$\left(v_{coh}^{-1}\frac{\partial}{\partial t} + \overline{s}\cdot\overline{\nabla} + \frac{1}{l}\right)\Phi_{coh}(t) = 0 \quad \text{(aI3)}$$

with the initial condition

$$\Phi_{coh}(\overline{s}, \overline{s}'; \overline{R}, 0+0) = \delta(\overline{R})\delta(\overline{s} - \overline{s}')P_{tr}(\overline{s}) \otimes P_{tr}(\overline{s}') \quad \text{(aI4)}$$

A solution for the last initial problem is given on the base of (10) by

$$\Phi_{coh}(\overline{s}, \overline{s}'; \overline{R}, t) = \exp(-v_{coh}t/l)\cdot\delta(\overline{R} - v_{coh}\overline{s}t)\delta(\overline{s} - \overline{s}')P_{tr}(\overline{s}) \otimes P_{tr}(\overline{s}') \quad \text{(aI5)}$$

With the aid of the tensor (aI5) the initial problem (aI2) is transformed to equation which in the tensor-operator denotations has the form

$$\Phi(t) = \Phi_{coh}(t)\Phi_0 + v_{coh}\int_0^t dt'\Phi_{coh}(t - t')\left(-v_{incoh}^{-1}\frac{\partial}{\partial t'} + W\right)\Phi(t') \quad \text{(aI6)}$$

It is a subject of matter that initial tensor Φ_0 can be excluded from this equation by using the identity

$$\frac{\partial}{\partial t}\int_0^t dt'\Phi_{coh}(t - t')v_{incoh}^{-1}\Phi(t') = \\ = \Phi_{coh}(t)v_{incoh}^{-1}\Phi_0 + \int_o^t dt'\Phi_{coh}(t - t')v_{incoh}^{-1}\frac{\partial\Phi(t')}{\partial t'} \quad \text{(aI7)}$$

Applying this identity together with (aI1) to (aI6) gives equation (12).

Appendix II

We seek a solution for equation (33) in a space of transverse tensor-functions of unit vector $\overline{s}$ with scalar product defined by equation (36). The diffusion asymptotics (41) is obtained by successive approximations.

1. The case of $\overline{q} = 0$, $\kappa = 0$ and $B(\overline{s}) = 0$ where $B(\overline{s})$ is the absorption tensor defined by equation (I. 82) and (I. 84). In this case the tensor-operator (34) takes the form $U(0) = (1/l)\mathcal{E} - W$. Using the optical theorem (I. 82) for non-absorptive scatterers one finds

$$\int d\overline{s}' U_{\alpha\beta,\alpha'\beta'}(\overline{s},\overline{s}';0) P^{tr}_{\alpha'\beta'}(\overline{s}') = 0 \tag{aII1}$$

that gives (40).

2. The case of $\overline{q} = 0$, $\kappa = 0$ and $B(\overline{s}) \neq 0$. In the first order of the perturbation theory with respect to absorption the eigenvalue $\lambda_0(0,0)$ is given by

$$\lambda_0(0,0) = \left(\Phi^{(0)}, U(0)\Phi^{(0)}\right) = \frac{1}{2} B_{\alpha\alpha,\beta\beta}(\overline{s}) = \frac{k_0^2}{k_{eff}} \cdot \frac{\Im\hat{\epsilon}_1}{\epsilon'} \cdot a \tag{aII2}$$

where equation (I. 91) has been used

3. The case of $\overline{q} = 0$, $\kappa \neq 0$ and $B(\overline{s}) = 0$. In the first order of the perturbation theory with respect to κ a correction $\delta\lambda_0(0,\kappa)$ to the eigenvalue $\lambda_0(0,0)$ is given by

$$\begin{aligned}\delta\lambda_0(0,\kappa) &= \left(\Phi^{(0)}, i\kappa\mathcal{D}\Phi^{(0)}\right) = i\kappa\frac{1}{8\pi}\int d\overline{s}\int d\overline{s}'\mathcal{D}_{\alpha\alpha,\beta\beta}(\overline{s},\overline{s}') \\ &= i\kappa\frac{1}{8\pi}\int d\overline{s}' A^{+}_{\alpha\alpha,\beta\beta}(\overline{s}') = -i\kappa a\end{aligned} \tag{aII3}$$

where the sum rule (I. 81) has been used.

4. The case of $\overline{q} \neq 0$, $\kappa = 0$ and $B(\overline{s}) = 0$. In the lowest order of the perturbation theory with respect to $\overline{q}$ a correction $\delta\lambda_0(q,0)$ to the eigenvalue $\lambda_0(0,0)$ is given by

$$\delta\lambda_0(q,0) = \left(\Phi^{(0)}, i(\overline{s}\cdot\overline{q})\delta\Phi^{(0)}\right) \tag{aII4}$$

Here $\delta\Phi^{(0)}$ is a correction to the eigenfunction (40). This correction satisfies equation

$$U(0)\delta\Phi^{(0)} = -i(\overline{s}\cdot\overline{q})\Phi^{(0)} \qquad \text{(aII5)}$$

and the orthogonality condition

$$\left(\delta\Phi^{(0)},\ \Phi^{(0)}\right) = 0 \qquad \text{(aII6)}$$

A solution for equation (aII5) can be written in terms of solution for equation (33) as follows

$$\delta\Phi^{(0)}_{\alpha\beta}(\overline{s};\ \overline{q},0) = -\frac{i}{\sqrt{8\pi}}\tilde{\Phi}_{\alpha\beta}(\overline{s};\ \overline{q}) \qquad \text{(aII7)}$$

where tensor $\tilde{\Phi}_{\alpha\beta}(\overline{s};\ q)$ is defined by (43). Substituting (aII7) in the r.h.s. of (aII4), one finds

$$\delta\lambda_0(\overline{q},0) = (1+a)\frac{c_{ph}}{c_0^2}\cdot D\overline{q}^2 \qquad \text{(aII8)}$$

where D is the diffusion constant (44).

After the successive approximation 1-4 the diffusion asymptotics (41) is obtained by substituting the eigenvalue $\lambda_0(0,0)+\delta\lambda_0(0,\kappa)+\delta\lambda_0(q,0)$ and the eigenfunction $\Phi^{(0)}+\delta\Phi^{(0)}$ into the term $n=0$ of the r.h.s. of equation (39).

Appendix III

The scattering amplitudes $\tilde{S}_1$ and $\tilde{S}_2$ are written in terms of the single scattering tensor-operator (56) on the base of (59) and (I. 92) in accordance with

$$\tilde{S}_a = \hat{e}_{\phi\alpha}T_{\alpha\beta}(k_0\overline{s},k_0\overline{s}_0)\hat{e}_{\phi\beta} \qquad \text{(aIII1)}$$

and

$$\tilde{S}_2 = \hat{e}_{\theta\alpha}T_{\alpha\beta}(k_0\overline{s},k_0\overline{s}_0)\hat{e}_{\beta} \qquad \text{(aIII2)}$$

where $T_{\alpha\beta}(\overline{k},\overline{k}') = T_{\alpha\beta}(\overline{k},\overline{k}';\ \Omega+i0)$. The total derivative for example of $\tilde{S}_1$ with respect to E is defined by

$$\frac{d\tilde{S}_1}{dE} = \frac{\partial\tilde{S}_1}{\partial E} + \left[\frac{\partial\tilde{S}_1}{\partial E}\right]|_{space} \qquad \text{(aIII3)}$$

Here the second term in the r.h.s. takes into account the effect of finite size of the scatterer (space dispersion) and is given according to the key identity (74) by

$$\left[\frac{\partial \tilde{S}_1}{\partial E}\right]|_{space} = \frac{1}{2E}\hat{e}_{\phi\alpha}\left[\hat{N}T_{\alpha\beta}(k_0\overline{s}, k_0\overline{s}_0)\right]\hat{e}_{\phi\beta} \qquad \text{(aIII4)}$$

where $\hat{N}$ denotes the operator

$$\hat{N} = \overline{s}\frac{\partial}{\partial\overline{s}} + \overline{s}_0\frac{\partial}{\partial\overline{s}_0} \qquad \text{(aIII5)}$$

Apply further this operator to (aIII1) to obtain the equation of the form

$$\hat{N}\tilde{S}_1 = \hat{e}_{\phi\alpha}(\hat{N}T_{\alpha\beta})\hat{e}_{\phi\alpha} + (\hat{N}\hat{e}_{\phi\alpha})T_{\alpha\beta}\hat{e}_{\phi\beta} + \hat{e}_{\phi\alpha}T_{\alpha\beta}(\hat{N}\hat{e}_{\phi\beta}) \qquad \text{(aIII6)}$$

Because the scattering amplitude S_1 is a function of $\cos\theta$ one can write

$$\hat{N}\tilde{S}_1 = 2\cos\theta\frac{d\tilde{S}_1}{d\cos\theta} \qquad \text{(aIII7)}$$

Now one needs to calculate $N\hat{e}_\phi$. There is the following equation

$$\hat{N}\hat{e}_\phi = \frac{2}{\sin^2\theta}\hat{e}_\phi \qquad \text{(aIII8)}$$

On the base of (aIII4) and (aIII6)-(aIII8) one finds

$$\left[E\frac{\partial \tilde{S}_1}{\partial E}\right]|_{space} = \cos\theta\frac{d\tilde{S}_1}{d\cos\theta} - \frac{2}{\sin^2\theta}\cdot\tilde{S}_1 \qquad \text{(aIII9)}$$

that after substituting in the r.h.s. of (aIII3) gives (75).

References

1. Van Albada, M.(P)., B. A. van Tiggelen, A. Lagendijk, and A. Tip, "Speed of propagation of classical waves in strongly scattering media," *Phys. Rev. Lett.,* Vol. 66, 3132–3134, 1991.
2. Kuga, Y., A. Ishimaru, and D. Rice, "The velocity of coherent and incoherent electromagnetic waves in a dense strongly scattering medium," *Phys. Rev. B,* Vol. 48, 13155–13158, 1993.
3. Kogan, E., and M. Kaveh, "Diffusion constant in a random system near resonance," *Phys. Rev. B.,* Vol. 46, 10636–10640, 1992.
4. Barabanenkov, Yu. N., L. M. Zurk, "Radiative transfer with time delay near resonance scattering: wave energy transport velocity approximation," *Turkish J. of Physics,* Vol. 18, 978–981, 1994.
5. Barabanenkov, Yu. N., L. M. Zurk, and M. Yu. Barabanenkov, "Poynting's theorem and electromagnetic wave multiple scattering in dense media near resonance: modified radiative transfer equation," *J. Elect. Waves and Applications,* Vol. 9, 1393–1420, 1995.
6. Barabanenkov, Yu. N., and V. D. Ozrin, "Asymptotic solution of the Bethe-Salpeter equation and the Green-Kubo formula for the diffusion constant for wave propagation in random media," *Phys. Lett. A,* Vol. 154, 38–42, 1991.
7. Barabanenkov, Yu, N., and V. D. Ozrin, "Diffusion asymptotics of the Bethe-Salpeter equation for electromagnetic waves in discrete random media," *Phys. Lett. A,* Vol. 206, 116–122, 1995.
8. Kroha, J., C. M. Soukoulis, and P. Wolfle, "Localization of classical waves in a random medium: A self consistent theory," *Phys. Rev. B,* Vol. 47, 11093–11096, 1993.
9. Barabanenkov, Yu. N., and V. D. Ozrin, "Problem of light diffusion in strongly scattering media," *Phys. Rev. Lett.,* Vol 69, 1364–1366, 1992.
10. Ban Tiggelen, B. A., "Multiple Scattering and Localization of LIght," (PhD thesis). University of Amsterdam, 1992.
11. Van Tiggelen, B. A., A. Lagendijk, A. Tip, "Comment on 'The problem of light diffusion in strongly scattering media'," *Phys. Rev. Lett.,* Vol. 71, 1284–1285, 1993.
12. Barabanenkov, Yu. N., and V. D. Ozrin, "Problem of light diffusion in strongly scattering media - Reply," *Phys. Rev. Lett.,* Vol. 71, 1285, 1993.

13. Bolle, D., and T. A. Osborn, "Concepts of multiparticle time delay," *Phys. Rev. D,* Vol. 13, 299–311, 1976.
14. Barabanenkov, Yu. N., L. M. Zurk, and M. Yu. Barabanenkov, "Solution for modified radiative transfer equation near resonance scattering," submitted to 1995 Progress in Electromagnetic Research Symposium, July 24–28, 1995, Seattle, USA.
15. Zurk, L. M., "Modified radiative transfer theory for wave multiple scattering near resonance and experiments on pulse transmission through dense strongly scattering medium," Proceeding s of the 1995 URSI International Symposium on Electromagnetic Theory, May 23–26, 1995, St. Petersburg, Russia, 640–642.
16. Case, K. M., and P. F. Zweifel, *Linear Transport Theory,* Addision–Wesley, Massachusetts, 1967.
17. Andersson-Engles, S., R. Berg, S. Svanberg, and O. Jarlman, "Time-resolved transillumination for medical diagnostics," *Optics Leters,* Vol. 15, 1179–1181, 1990.
18. Stratton, J. A., Electromagnetic Theory, McGraw-Hill, New York, 1941.
19. Van Tiggelen, V. A., A. Lagendijk, M. P. van Albada, and A. Tip, "Speed of light in random media," *Phys. Rev. B.,* Vol. 45, 12233–12243, 1992.
20. Bohren, C. F., and D. R. Huffman, *Absorption and Scattering of Light by Small Particles,* J. Willey, New York, 1983.
21. Van de Hulst, H. C., *Light Scattering by Small Prticles,* Dover, New York, 1981.
22. Van Tiggelen, and A. Lagendijk, "Rigorous treatment of the speed of diffusion classical waves," *Europhys. Lett.,* Vol. 23, 311–316, 1993.
23. Bott, A. and W. Zdunkowski, "Electromagnetic energy within dielectric spheres," *J. Opt. Soc. Am. A.,* Vol. 4, 1361–1365, 1987.
24. Katzir, A., *Lasers and Optical Fibres in Medicine,* Acad. Press, New York, 1994.
25. Chandrasekhar, S., *Radiative Transfer,* Dover, New York, 1960.
26. Zurk, L. M., L. Tsang, and D. P. Winebrenner, "Scattering properties of dense media from Monte Carlo simulation with application of active remote sensing of snow," accepted for publication in *Radio Science,* July 1995.

27. Barabanenkov, Yu. N., "Energy equivalence of application of Dyson and Bethe-Salpeter equations in the theory of multiple wave scattering," *Izv. Vyssch. Uchebn. Zaved. Radiofiz.,* Vol. 12, 113–120, 1974.
28. Barabanenkov, Yu. N., and M. I. Kalinin, "Weakly nonuncoupled relations in wave multiple scattering theory for dense discrete random media," *Phys. Lett. A.,* Vol. 163, 214–218, 1992.
29. Mahan, G. D., *Many Particle Physics,* Plenum Press, New York, 1981.
30. B. A. Van Tiggelen, B. A., A. Lagendijk, and A. Tip, "Multiple Scattering effects for propagation of light in 3D slabs," *J. Phys.: Condens. Matter,* Vol. 2, 7653–7677, 1990.
31. Tsang, L., and A. Ishimary, "Radiative wave equations for vector electromagnetic propogation in dense nontenuous media," *J. Elect. Waves and Applications,* Vol. 1, 59–79, 1987.
32. Tsang, L., I. A. Kong, and R. T. Shin, *Theory of Microwave Remote Sensing,* Wiley-Interscience, New York, 1985.
33. Leibsh, A., and B. N. J. Persson, "Optical properties of small metallic particles in a continuous dielectric medium," *J. Phys. C.: Solid State Phys.,* Vol. 16, 5376–5391, 1983.
34. Garcia, N., A. Z. Genack, and A. A. Lisyansky, "Measurement of the transport mean free path of diffusion photons," *Phys. Rev. B.,* Vol. 46, 14475–14479, 1992.
35. Genack, A. Z., J. H. Li, N. Garsia, and A. A. Lisyansky, "Photon diffusion, correlation and localization," in *Photonic Band Gaps and Localization,* edited by C. M. Soukoulis, Plenum, New York, 1993, p. 23–55.
36. Bush, K., and C. M. Soukoulis, "Transport properties of random media: A new effective medium theory," *Phys. Rev. Lett.,* Vol. 75, 3442–3445, 1995.
37. Soukoulis, C. M., S. Datta, and E. N. Economou, "Propagation of classical waves in random media," *Phys. Rev. B.,* Vol. 49, 3800–3810, 1994.
38. Barabanenkov, Yu. N., and V. V. Shlyapin, "Space group resonance in electromagnetic wave multiple scattering," *Phys. Lett. A.,* 170, 239–244, 1992.
39. Barabanenkov, Yu. N., and V. M. Finkelberg, "Optical theorem in the theory of multiple wave scattering," *Izv. Vyssch. Uchebu. Zaved. Radiofiz.,* Vol. 11, 719–725, 1968.

40. Van Tiggelen, B. A., and A. Lagendijk, "Resonantly induced dipole-dipole interactions in the diffusion of scalar waves," *Phys. Rev. B.,* Vol. 50, 16729–16732, 1994.