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Push-Pull Phenomenon of a Dielectric Particle in a Rectangular Waveguide

Nayan K. Paul and Brandon A. Kemp*

Abstract—The electromagnetic force acting on a Rayleigh particle placed in a rectangular waveguide is studied. The particle is excited using the lowest order TE_{10} mode. It is determined that the particle is laterally trapped at the high intensity region of the electric field and either pushed away from or pulled toward the light source. This push-pull phenomenon depends on whether the frequency of the light wave is above or below the cutoff frequency (i.e., the particle can be pushed or pulled by tuning the frequency). While conventional optical tweezers rely on a balance of scattering and gradient force in the propagation direction, the phenomenon predicted here switches between the two forces near the lowest cutoff in a waveguide.

1. INTRODUCTION

Maxwell's theory predicts the radiation pressure of light [1]. Radiation pressure has held the attention of scientists for many years [2–5]. A challenge for electromagnetic theory is to model existing experiments and to predict new phenomena [6–9].

Application of optical trapping and manipulation of particles arise from the intricate forces due to the interaction of electromagnetic fields and matter [10, 11]. In optical trapping, electromagnetic force components give rise to a point of equilibrium where the gradient force dominates the scattering force of radiation pressure [11, 14]. The scattering force generally arises due to change of light momentum upon the scattering from the particle [15]. It is directly proportional to the intensity of light and acts towards the direction of wave propagation [11]. In the case of a counter propagating wave trap, the scattering force plays the dominant role in trapping the particle at the equilibrium position between the two incident waves [15]. The gradient of time averaged intensity of optical field can produce forces on a small dielectric particle, and trapping of the particle can be achieved by proper fashioned optical gradient [12, 13]. The gradient force is directly proportional to the gradient of the time-averaged intensity of optical field. It arises due to the interaction between the dipole and the gradient of the focused electromagnetic field, and the dipole tends to move towards the higher intensity region to minimize the interaction energy [15]. Optical binding is the stable spatial arrangement of several microparticles using light illumination where the position of a particle can be changed by light redistribution by another particle. Though optical trapping depends on the forces rising from incident field, optical binding forces depend on the modification of incident field in the presence of several illuminated objects [15]. Optical trapping can also be achieved as a balance between radiation pressure and optical binding forces [19, 20].

In biological applications of optical trapping, pulling, and pushing of a particle, controlled forces can be applied on internal parts of cells. Separation of living cells has been developed without any detectable optical damage, measurement of elastic properties of parts of the cell cytoplasm, and force generated by RNA polymerase as it moves along a DNA molecule are possible [10]. Moreover, optical trapping has been used to probe cell membranes, aggregate protein fibers, modify chromosomes in living cells, and transport and modify cells precisely which has led to clinical applications [16]. The ability

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* Corresponding author: Brandon A. Kemp (bkemp@astate.edu).
The authors are with the College of Engineering, Arkansas State University, P. O. Box 1740, State University, AR 72467, USA.

to trap, stretch, rotate, push, and pull particles or cells constitute an optical tool box for existing and emerging applications.

In this paper, we propose a method to create a pulling or pushing force on a small dielectric particle in a rectangular waveguide by varying the frequency of the exciting wave. We determine the cutoff frequency, vary the frequency, and calculate the gradient and scattering forces for the two cases (i.e., frequency below and above the cutoff). A theoretical study of pulling and pushing a small dielectric particle in a rectangular waveguide by varying the frequency is presented.

2. FIELD SOLUTION

We consider a PEC rectangular waveguide having dimensions $a = 300\,\mu\text{m}$ along x-axis and $b = 150\,\mu\text{m}$ along y-axis. Here, PEC stands for perfect electric conductor which is an ideal material that exhibits infinite electrical conductivity, or zero resistance. Though it does not exist in nature, we consider this concept since the electrical resistance is negligible in our model. The background medium in the waveguide is air, and a water drop of radius $r = 10\,\mu\text{m}$ is placed in the waveguide at a position $(x,\, y,\, z)$. All of the media are lossless. We solve the waveguide problem analytically and the scattering problem using Rayleigh approximation.

2.1. Waveguide

To represent the guided waves along the $\hat{z}$ direction, the z dependence of all field vectors can be represented as $e^{\pm ik_z z}$, where k_z is the propagation constant and '+' sign indicates the propagation along positive $\hat{z}$ direction [17]. For TE fields, the z-component of electric field is 0, and the magnetic field is [17]

$$H_z = \cos(k_x x)\cos(k_y y)\, e^{ik_z z}. \tag{1}$$

The transverse electric field components can be written as [17]

$$E_x = -\frac{i\omega\mu_b k_y}{\omega^2\mu_b\epsilon_b - k_z^2}\cos(k_x x)\sin(k_y y)\, e^{ik_z z}, \tag{2}$$

$$E_y = \frac{i\omega\mu_b k_x}{\omega^2\mu_b\epsilon_b - k_z^2}\sin(k_x x)\cos(k_y y)\, e^{ik_z z}. \tag{3}$$

Throughout this paper, the suffix b represents the parameters for the waveguide background medium and the suffix p represents the parameters for the particle. Here, ω, ϵ_b and μ_b represent the angular frequency, permittivity of the background medium, and permeability of the background medium, respectively. For the fundamental mode of propagation in a rectangular waveguide (i.e., TE_{10} mode) $k_y = 0$. From Equations (2) and (3), the transverse electric field components are

$$E_x = 0, \tag{4}$$

$$E_y = \frac{i\omega\mu_b}{k_x}\sin(k_x x)\, e^{ik_z z}. \tag{5}$$

The propagation constant is $k_z = \sqrt{\omega^2\mu_b\epsilon_b - k_x^2} = \frac{\omega}{v_b}$, where v_b is the phase velocity of wave in the background medium of the waveguide. To determine the cutoff frequency, the condition is

$$\omega_{cmn}^2\mu_b\epsilon_b = k_x^2. \tag{6}$$

If the frequency of the wave is less than the cutoff, the value of $k_z = k_z' + ik_z'' = ik_z''$ is imaginary. The y-field becomes

$$E_y = \frac{i\omega\mu_b}{k_x}\sin(k_x x)\, e^{-k_z'' z}. \tag{7}$$

The wave acts as an evanescent wave below cutoff frequency. On the other hand, if the frequency is above cutoff, then the value of k_z is real and there exists no imaginary part of k_z. Then the y-field becomes

$$E_y = \frac{i\omega\mu_b}{k_x}\sin(k_x x)e^{ik_z' z}. \tag{8}$$

The wave acts as a propagating wave above cutoff frequency.

2.2. Rayleigh Scattering

The Rayleigh model gives an approximation of the scattered field by a small particle, the radius of which is very small compared to the wavelength, but for larger or multiple particles this model has to used with caution [8]. The scattering by larger particle can be solved exactly using Mie theory [8, 21]. In the Rayleigh scattering regime, the particle acts as a simple dipole in the presence of electric field and its dipole moment can be represented by [7]

$$\mathbf{P}(\mathbf{r}, t) = 4\pi\epsilon_b r^3 \frac{\epsilon_p - \epsilon_b}{\epsilon_p + 2\epsilon_b} \mathbf{E}(\mathbf{r}, t). \tag{9}$$

The relative refractive index of the particle is $n_r = \frac{n_p}{n_b} = \sqrt{\frac{\epsilon_p}{\epsilon_b}}$, where n_p and n_b are the refractive indices of the particle and background media, respectively. Thus the dipole moment is

$$\mathbf{P}(\mathbf{r}, t) = \alpha\mathbf{E}(\mathbf{r}, t) = 4\pi\epsilon_b r^3 \frac{n_r^2 - 1}{n_r^2 + 2} \mathbf{E}(\mathbf{r}, t). \tag{10}$$

Here, α is the polarizability of the particle.

3. RADIATION FORCES ON A DIELECTRIC PARTICLE

For the calculation of radiation pressure on a dielectric particle, the Lorentz force can be applied to bound charges at the surface of the particle and bound currents distributed throughout the background medium of the particle [2, 8]. This radiation pressure induces two forces. One acts in the direction of wave propagation that is called the scattering force and another one acts toward the gradient of the incident field intensity that is called the gradient force.

3.1. Scattering Force

As the electric field oscillates with time, the induced dipole follows the electric field. The particle radiates scattered waves, and, as a result, changes of momentum take place. The resulting scattering force acting on the particle is [7]

$$\mathbf{F_{scat}} = \hat{z}\frac{n_b}{v_b} C_{scat} \langle \mathbf{S}(\mathbf{r}, t)\rangle, \tag{11}$$

where $\langle \mathbf{S}(\mathbf{r}, t)\rangle$ is the time-averaged Poynting power. It can be expressed in terms of the intensity

$$\langle \mathbf{S}(\mathbf{r}, t)\rangle = \hat{z}\frac{1}{2}\mathrm{Re}[\mathbf{E} \times \mathbf{H}^*] = \hat{z}\frac{\epsilon_b v_b}{2}|\mathbf{E}(\mathbf{r})|^2. \tag{12}$$

The scattering cross section relates the relative size of the particle to an electromagnetic wave [7]

$$C_{scat} = \frac{8}{3}\pi\left[\mathrm{Re}(k_z)\right]^4 r^6 \left(\frac{n_r^2 - 1}{n_r^2 + 2}\right)^2. \tag{13}$$

The scattering cross section depends on the real part of propagation constant k_z. If there exists no real part (i.e., for an evanescent wave), then the cross section is zero. For this reason, there exists no scattering force on the particle for an evanescent wave. On the other hand, propagating wave results in a scattering force on the particle.

3.2. Gradient Force

The gradient force always attracts the particle towards the higher intensity region of wave and the time-averaged force is given by [18]

$$\mathbf{F_{grad}}(\mathbf{r}) = \frac{1}{2}\mathrm{Re}\left[(\mathbf{p} \cdot \nabla)\mathbf{E}^*(\mathbf{r}) - i\omega\mu_b \mathbf{P}(\mathbf{r}) \times \mathbf{H}^*(\mathbf{r})\right]. \tag{14}$$

Since $\nabla \times \mathbf{E}(\mathbf{r}) = 0$ in the static limit, Equation (14) reduces to

$$\mathbf{F_{grad}}(\mathbf{r}) = \frac{1}{4}\mathrm{Re}\left\{\alpha\nabla|\mathbf{E}(\mathbf{r})|^2\right\}. \tag{15}$$

4. DISCUSSIONS AND RESULTS

In our study, the waveguide dimensions are $a = 300\,\mu\text{m}$ and $b = 150\,\mu\text{m}$ in x and y, respectively. The relative permittivity of the waveguide background medium is 1 (i.e., $\epsilon_b = \epsilon_0$), and the relative permeability is 1 (i.e., $\mu_b = \mu_0$). For TE_{10} mode, $m = 1$, $n = 0$, $k_x = \frac{m\pi}{a} = \frac{\pi}{300}$ rad/µm, and $k_y = \frac{n\pi}{b} = 0$. The cutoff frequency is $\omega_{c10} = \frac{kx}{\sqrt{\mu_b \epsilon_b}} = 1.77 \times 10^{12}$ rad/s or, $f_{c10} = 2.82 \times 10^{11}$ Hz.

4.1. Propagating Wave

We first consider that the wave propagates at a frequency of $f = 5 \times 10^{11}$ Hz (i.e., $\omega = 10\pi \times 10^{11}$ rad/s) which is above the cutoff frequency. The incident wavelength is $\lambda_0 = \frac{c}{f} = 600\,[\mu\text{m}]$. For Rayleigh scattering, the radius of the particle should must be less than $\frac{\lambda_0}{20}$ (i.e., $r \ll 30\,\mu\text{m}$) [7]. For TE_{10} mode, the scattering force does not depend on the b-dimension of the waveguide which is the smaller dimension, rather it changes with the a-dimension which is the longer dimension in xy plane. The electric field intensity is

$$|\mathbf{E}(\mathbf{r})|^2 = \mathbf{E_y}(r) \times \mathbf{E_y^*}(r) = \left(\frac{\omega\mu_b}{k_x}\right)^2 \sin^2(k_x x). \tag{16}$$

From Equations (11), (12), (13) and (16), it is clear that the scattering force on the particle is maximum when the particle is at a position of $\frac{a}{2}$ along the x-axis and gradually decreases as the position of the particle is changed towards the edge of the waveguide as shown in Figure 1. This is due to the fact that at $x = \frac{a}{2}$, the value of $\sin(k_x x)$ is maximum ($= 1$).

The y- and z-coordinates of the particle do not have any impact on the scattering force, the scattering force is same regardless the position of the particle for a propagating wave. At a certain position the scattering force increases as the radius of the particle increases as shown in Figure 2. The abrupt rise of scattering force with the increase of radius of the particle is due to the fact that the force rises at a rate of sixth power of the increment of the radius and it is noticeable in Figure 2. The gradient of the square of the magnitude of the wave from Equation (16) is

$$\triangledown|\mathbf{E}(\mathbf{r})|^2 = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\left[\left(\frac{\omega\mu_b}{k_x}\right)^2 \sin^2(k_x x)\right] = \hat{x}\frac{\omega^2\mu_b^2}{k_x}\sin(2k_x x). \tag{17}$$

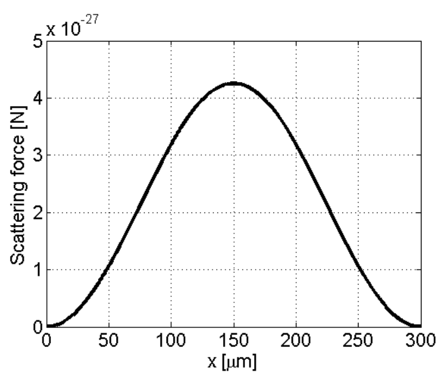


Figure 1. Variation of $\hat{z}$-directed scattering force on a water drop of $r = 10\,\mu\text{m}$ radius due to a propagating wave with the change of position along a-dimension of a dielectric rectangular waveguide with $a = 300\,\mu\text{m}$ and $b = 150\,\mu\text{m}$. The waveguide background medium is air, wave frequency is $f = 5 \times 10^{11}$ Hz, $\mu_p = \mu_b = \mu_0$, $E_0 = 1$, $\epsilon_p = 1.69\epsilon_0$, $\epsilon_b = \epsilon_0$, $n_p = 1.3$, and $n_b = 1$.

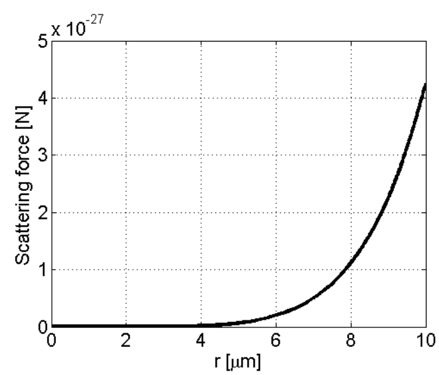


Figure 2. Variation of $\hat{z}$-directed scattering force on a water drop with the change of radius r of the particle located at a position (150, y, z) µm in a rectangular waveguide for a propagating wave. The waveguide background medium is air, wave frequency is $f = 5 \times 10^{11}$ Hz, $a = 300\,\mu\text{m}$, $b = 150\,\mu\text{m}$, $\mu_p = \mu_b = \mu_0$, $E_0 = 1$, $\epsilon_p = 1.69\epsilon_0$, $\epsilon_b = \epsilon_0$, $n_p = 1.3$, and $n_b = 1$.

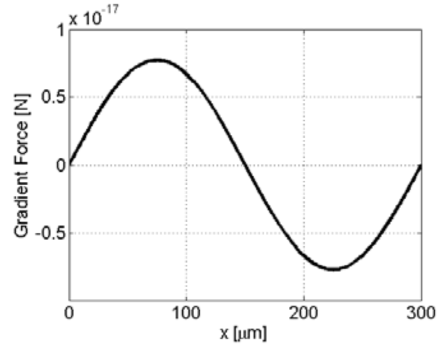


Figure 3. Variation of gradient force acting on a water drop of $r = 10\,\mu\text{m}$ radius within a rectangular waveguide for a propagating wave. The waveguide background medium is air, the wave frequency is $f = 5 \times 10^{11}$ Hz, $a = 300\,\mu\text{m}$, $b = 150\,\mu\text{m}$, $\mu_p = \mu_b = \mu_0$, $E_0 = 1$, $\epsilon_p = 1.69\epsilon_0$, $\epsilon_b = \epsilon_0$, $n_p = 1.3$ and $n_b = 1$.

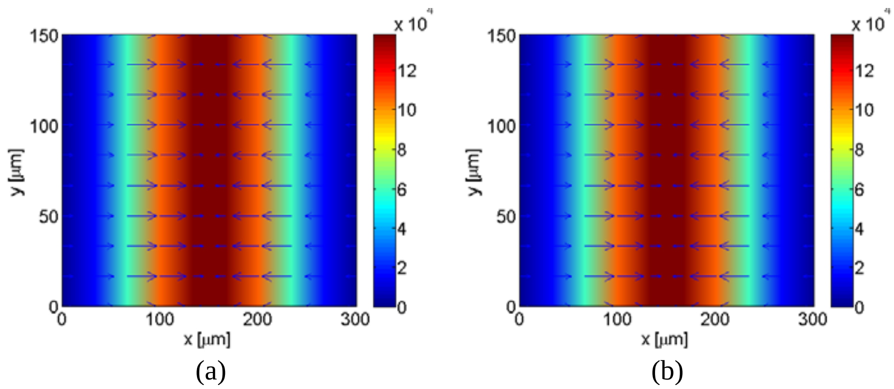


Figure 4. Plot of intensity and gradient force (arrows indicate the force direction) at the x-y plane of a rectangular waveguide acting on a water drop of $r = 10\,\mu\text{m}$ at two different positions along z-axis for a propagating wave. The waveguide background medium is air, the wave frequency is $f = 5 \times 10^{11}$ Hz, $a = 300\,\mu\text{m}$, $b = 150\,\mu\text{m}$, $\mu_p = \mu_b = \mu_0$, $E_0 = 1$, $\epsilon_p = 1.69\epsilon_0$, $\epsilon_b = \epsilon_0$, $n_p = 1.3$ and $n_b = 1$. (a) $z = 0\,\mu\text{m}$. (b) $z = 100\,\mu\text{m}$.

From Equation (15), the gradient force is

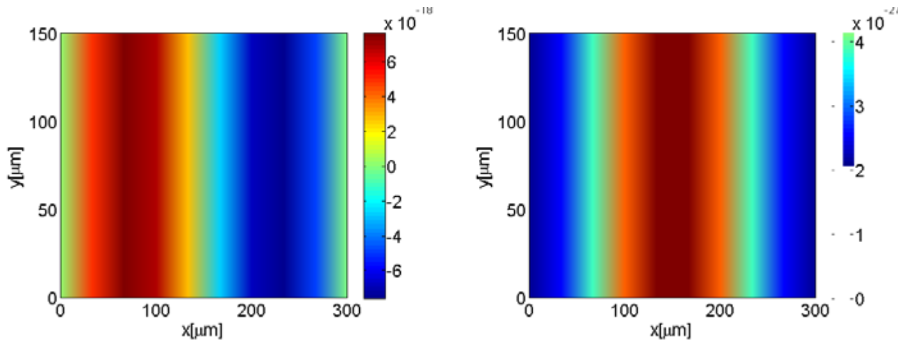
$$\mathbf{F_{grad}}(\mathbf{r}) = \hat{x}\frac{1}{4}\frac{\omega^2\mu_b^2}{k_x}\sin(2k_x x) \times \text{Re}[\alpha]. \tag{18}$$

From Equation (18) it is obvious that the magnitude of the gradient force is maximum and acts in the $+\hat{x}$ direction when the particle is at a position $(\frac{a}{4}, y, z)\,\mu\text{m}$. It reduces to zero when the particle is at a position $(\frac{a}{2}, y, z)\,\mu\text{m}$ and the magnitude becomes maximum, but acts along the $-\hat{x}$ direction when the particle is at a position $(\frac{3a}{4}, y, z)\,\mu\text{m}$ as shown in Figure 3. The particle always tends to move to the stable equilibrium point at $(\frac{a}{2}, y, z)\,\mu\text{m}$ as shown in Figure 4. The gradient force acting on the particle remains same as the wave propagates, since for a propagating wave it does not depend on the value of z.

The total force acting on the particle for a propagating wave is the sum of the gradient force which acts in the x-direction, and the scattering force which acts in the positive z-direction.

$$\mathbf{F_{total}} = \hat{x}\frac{1}{4}\frac{\omega^2\mu_b^2}{k_x}\sin(2k_x x) \times \text{Re}[\alpha] + \hat{z}\frac{n_b}{v_b}C_{scat}\left\langle \mathbf{S}(\mathbf{r}, t)\right\rangle. \tag{19}$$

Thus, for a propagating wave, the x-directed force always traps the particle at the midpoint of the a-edge (i.e., the higher intensity region, and the equilibrium point) and the z-directed force always pushes the particle towards the direction of propagation (i.e., the particle is forced to move towards $+z$ direction, or away from the source) as is shown in Figure 5.



(a) (b)

Figure 5. Plot of F_x and F_z components of total force acting on a water drop of $r = 10\,\mu\text{m}$ radius in a rectangular waveguide for a propagating wave. The waveguide background medium is air, the wave frequency is $f = 5 \times 10^{11}$ Hz, $a = 300\,\mu\text{m}$, $b = 150\,\mu\text{m}$, $\mu_p = \mu_b = \mu_0$, $E_0 = 1$, $\epsilon_p = 1.69\epsilon_0$, $\epsilon_b = \epsilon_0$, $n_p = 1.3$ and $n_b = 1$. The plot is shown at the x-y plane of the rectangular waveguide. (a) Plot of F_x component of total force given in Equation (19). The line plot is shown in Figure 3. (b) Plot of F_z component of total force given in Equation (19). The line plot is shown in Figure 1.

4.2. Evanescent Wave

Next we consider that the wave propagates at a frequency of $f = 10^{11}$ Hz which is less than the cutoff frequency. In this case, the wave acts like an evanescent wave. From Equation (7),

$$|\mathbf{E}(\mathbf{r})|^2 = \frac{\omega^2 \mu_b^2}{k_x^2} \sin^2(k_x x) e^{-2k_z'' z}. \tag{20}$$

In this case, the intensity gradient is

$$\begin{aligned}\bigtriangledown |\mathbf{E}(\mathbf{r})|^2 &= \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\left[\left(\frac{\omega \mu_b}{k_x}\right)^2 \sin^2(k_x x) e^{-2k_z'' z}\right] \\ &= \hat{x}\frac{2\omega^2\mu_b^2}{k_x}\sin(k_x x)\cos(k_x x) e^{-2k_z'' z} - \hat{z}k_z''\frac{2\omega^2\mu_b^2}{k_x^2}\sin^2(k_x x) e^{-2k_z'' z}.\end{aligned} \tag{21}$$

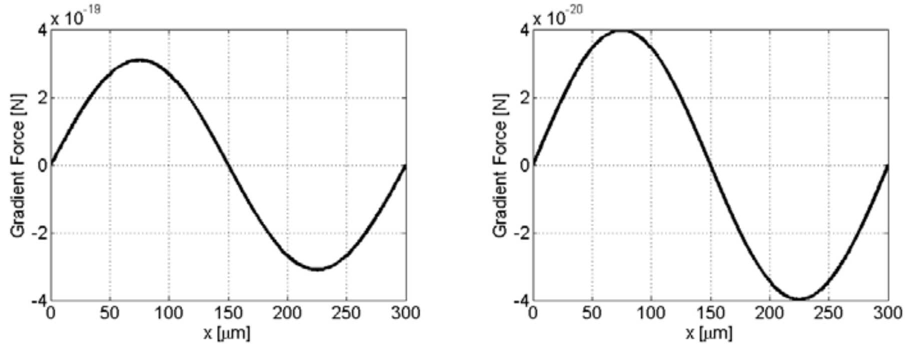
Therefore, the gradient force is

$$\mathbf{F_{grad}} = \left[\hat{x}\frac{1}{2}\frac{\omega^2\mu_b^2}{k_x}\sin(k_x x)\cos(k_x x) e^{-2k_z'' z} - \hat{z}\frac{k_z''}{2}\frac{\omega^2\mu_b^2}{k_x^2}\sin^2(k_x x) e^{-2k_z'' z}\right] \times \text{Re}[\alpha]. \tag{22}$$

The gradient force for an evanescent wave has two components. One component acts towards the x-direction and another component acts towards the z-direction. The magnitude of the x-component of the force is maximum when the particle is at a position $(\frac{a}{4}, y, z)$ µm and acts in the $+\hat{x}$ direction. It reduces to zero when the particle is at a position $(\frac{a}{2}, y, z)$ µm, and it becomes maximum and acts in the $-\hat{x}$ direction when the particle is at a position $(\frac{3a}{4}, y, z)$ µm. However, the force decays at a rate of $e^{-2k_z''}$ as shown in Figure 6.

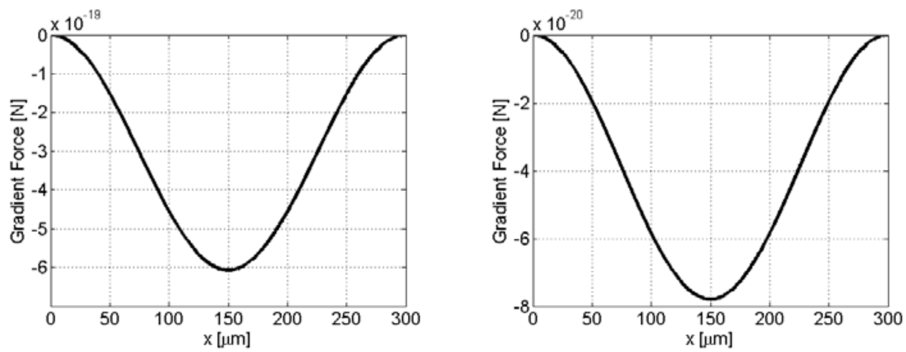
The z-component of gradient force is negative which means it acts along the negative z-axis. The magnitude of this force is maximum when the particle is at $(\frac{a}{2}, 0, 0)$ µm and gradually decreases as the particle is considered towards the edge of the waveguide as shown in Figure 7. It decays at a rate of $e^{-2k_z''}$ along the z-axis as shown in Figure 7.

From Figure 6 and Figure 7, it is obvious that for a wave which is evanescent along the positive z-axis, the particle always tends to move to the stable equilibrium at $x = a/2$. This force weakens as the wave propagates as shown in Figure 8. The z-component of the force which acts along the negative z-direction pulls the particle towards the light source as shown in Figure 7. Pulling the particle is only possible if it is placed within a certain distance from the light source. If it is located at a long distance away from the source where the x and z-component of the force diminish, then it is not possible to pull the particle towards the light source.



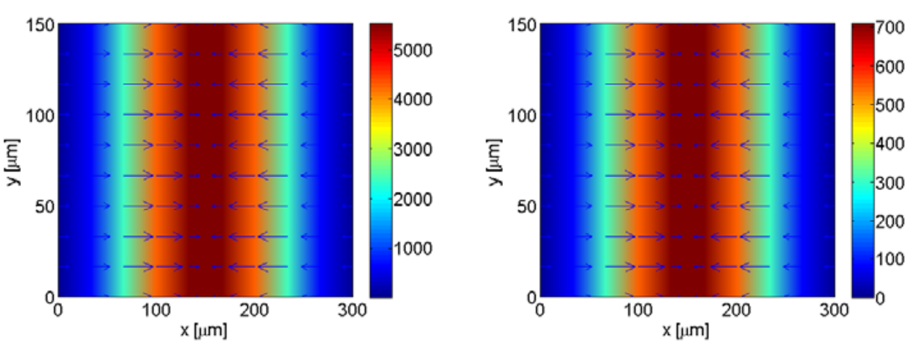
(a) (b)

Figure 6. x-component of gradient force acting on a water drop of $r = 10\,\mu\text{m}$ radius in a rectangular waveguide due to an evanescent wave. The waveguide background medium is air, the wave frequency is $f = 10^{11}$ Hz, $a = 300\,\mu\text{m}$, $b = 150\,\mu\text{m}$, $\mu_p = \mu_b = \mu_0$, $E_0 = 1$, $\epsilon_p = 1.69\epsilon_0$, $\epsilon_b = \epsilon_0$, $n_p = 1.3$ and $n_b = 1$. (a) $z = 0\,\mu\text{m}$. (b) $z = 100\,\mu\text{m}$.



(a) (b)

Figure 7. z-component of gradient force acting on a water drop of $r = 10\,\mu\text{m}$ radius in a rectangular waveguide due to an evanescent wave. The waveguide background medium is air, the wave frequency is $f = 10^{11}$ Hz, $a = 300\,\mu\text{m}$, $b = 150\,\mu\text{m}$, $\mu_p = \mu_b = \mu_0$, $E_0 = 1$, $\epsilon_p = 1.69\epsilon_0$, $\epsilon_b = \epsilon_0$, $n_p = 1.3$ and $n_b = 1$. (a) $z = 0\,\mu\text{m}$. (b) $z = 100\,\mu\text{m}$.



(a) (b)

Figure 8. Plot of intensity and gradient force (arrows indicate the force direction) acting on a water drop of $r = 10\,\mu\text{m}$ radius in a rectangular waveguide at the xy plane due to an evanescent wave. The waveguide background medium is air, the wave frequency is $f = 10^{11}$ Hz, $a = 300\,\mu\text{m}$, $b = 150\,\mu\text{m}$, $\mu_p = \mu_b = \mu_0$, $E_0 = 1$, $\epsilon_p = 1.69\epsilon_0$, $\epsilon_b = \epsilon_0$, $n_p = 1.3$ and $n_b = 1$. (a) $z = 0\,\mu\text{m}$. (b) $z = 100\,\mu\text{m}$.

5. CONCLUSION

We have calculated the radiation pressure acting on a particle placed in a rectangular waveguide for both propagating and evanescent waves using the Lorentz force. A particle in a rectangular waveguide can be pulled towards the light source or pushed away from the light source just by varying the frequency around the waveguide cutoff frequency. All of the graphical representations and calculations are accurate within the Rayleigh scattering regime, but may be inappropriate for a larger particle. All of the fields and forces are plotted using analytical calculations presented herein, and validated using COMSOL Multiphysics 4.3a.

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REFERENCES

1. Maxwell, J. C., *A Treatise on Electricity and Magnetism*, Volume II, Clarendon Press, Oxford, 1873.
2. Kemp, B. A., "Resolution of the Abraham-Minkowski debate: Implications for the electromagnetic wave theory of light in matter," *J. Appl. Phys.*, Vol. 109, No. 11, 111101, 2011.
3. Milonni, P. W. and R. W. Boyd, "Momentum of light in a dielectric medium," *Adv. Opt. Photon.*, Vol. 2, No. 4, 519–553, 2010.
4. Barnett, S. M. and R. Loudon, "The enigma of optical momentum in a medium," *Philos. Trans. R. Soc. A*, Vol. 368, No. 1914, 927–939, 2010.
5. Pfeifer, R. N., T. A. Nieminen, N. R. Heckenberg, and H. Rubinsztein-Dunlop, "Colloquium: Momentum of an electromagnetic wave in dielectric media," *Rev. Mod. Phys.*, Vol. 79, No. 4, 1197, 2007.
6. Gordon, J. P., "Radiation forces and momenta in dielectric media," *Phys. Rev. A*, Vol. 8, No. 1, 14–21, 1973.
7. Harada, Y. and T. Asakura, "Radiation forces on a dielectric sphere in the Rayleigh scattering regime," *Opt. Commun.*, Vol. 124, No. 5, 529–541, 1996.
8. Kemp, B. A., T. M. Grzegorczyk, and J. A. Kong, "Lorentz force on dielectric and magnetic particles," *Journal of Electromagnetic Waves and Applications*, Vol. 20, No. 6, 827–839, 2006.
9. Maslov, A. V., "Optomechanical properties of a particle-waveguide system," *Phys. Rev. A*, Vol. 90, No. 3, 033825, 2014.
10. Ashkin, A., "Optical trapping and manipulation of neutral particles using lasers," *Proc. Natl. Acad. Sci.*, Vol. 94, No. 10, 4853–4860, 1997.
11. Ashkin, A. and J. M. Dziedzic, "Optical trapping and manipulation of viruses and bacteria," *Science*, Vol. 235, No. 4795, 1517–1520, 1987.
12. Burns, M. M., J. M. Fournier, and J. A. Golovchenko, "Optical binding," *Phys. Rev. Lett.*, Vol. 63, No. 12, 1233, 1989.
13. Burns, M. M., J. M. Fournier, and J. A. Golovchenko, "Optical matter: Crystallization and binding in intense optical fields," *Science*, Vol. 249, No. 4970, 749–754, 1990.
14. Ashkin, A., "History of optical trapping and manipulation of small-neutral particle, atoms, and molecules," *IEEE J. Sel. Top. Quantum Electron.*, Vol. 6, No. 6, 841–856, 2000.
15. Dholakia, K. and P. Zemnek, "Colloquium: Gripped by light: Optical binding," *Rev. Mod. Phys.*, Vol. 82, No. 2, 1767, 2010.
16. Grier, D. G., "A revolution in optical manipulation," *Nature*, Vol. 424, No. 6950, 810–816, 2003.
17. Kong, J. A., *Electromagnetic Wave Theory*, Wiley-Interscience, New York, 1986.

18. Grier, D. G. and D. B. Ruffner, "Practical tractor beams," *10th International Conference on Laser-light and Interactions with Particles*, ME-5.1–ME-5.4, Marseille, France, Aug. 25–29, 2014.
19. Grzegorczyk, T. M., B. A. Kemp, and J. A. Kong, "Stable optical trapping based on optical binding forces," *Phys. Rev. Lett.*, Vol. 96, No. 11, 113903, 2006.
20. Ahlawat, S., R. Dasgupta, and P. K. Gupta, "Optical trapping near a colloidal cluster formed by a weakly focused laser beam," *J. Phy. D: Appl. Phys.*, Vol. 41, No. 10, 105107, 2008.
21. Kemp, B. A., T. M. Grzegorczyk, and J. A. Kong, "Optical momentum transfer to absorbing Mie particles," *Phys. Rev. Lett.*, Vol. 97, No. 13, 133902, 2006.