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MUTUAL COUPLING BETWEEN BIANISOTROPIC PARTICLES: A THEORETICAL STUDY

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1. INTRODUCTION

So far research on the electromagnetic modelling of chiral and bianisotropic composite media has mainly focused on dilute mixtures, for which the mutual coupling between particles is small. The macroscopic constitutive parameters for such composites can be predicted through the use of a variety of mixing formulas. Among these mixing rules, the Maxwell-Garnett (MG) model for bianisotropics [1–3] has proven to be simple and reasonably accurate for isotropic chiral media [4,5]. If the particles are sufficiently small as compared to the ambient wavelength, then a dipole model can be employed. Based upon the computed dipolarisability dyadics for a single particle [6], the macroscopic medium parameters can then be obtained from the Lorenz-Lorentz formulae in the MG model.

Qualitative interaction phenomena that are a result of the mixing are apparent from the MG equations, such as the coupling of the electric, magnetic and magneto-electric parameters of the individual inclusions through mixing, as well as the non-linear dependence of the macroscopic parameters on the particle concentration. However, the applicability of the MG equations is severely restricted to dilute particle concentrations: the extent of mutual coupling that is taken into account is limited because the depolarisation fields are computed from the dipole moments of isolated (*ie* non-interacting) particles. The interaction which results from multiple scattering between particles accounts for quantitative discrepancies between MG estimates and measured [4] or numerically predicted [7] values. High-density chiral and bianisotropic materials are of considerable interest, in view of the second-order effect of chirality on the wave numbers and wave impedance. Hence a more detailed study of coupling phenomena is of interest, which is the subject of the present paper.

To tackle the problem of mutual coupling, explicit [7] and implicit [8] full-wave numerical techniques as well as experimental analyses [4] have been initiated in the recent past. These studies are here complemented by an analytical solution to the problem of coupling between two general bianisotropic particles using the concept of interaction (reaction). A compact interaction matrix formalism is formulated based on dyadic and tryadic algebra. The analytical results are illustrated with numerical computations for some canonical examples of two anisotropic chiral particles with various relative orientations. In an alternative approach, modified dipolarisability dyadics are derived analytically for a system of two bianisotropic particles. These enable the perturbations that are a result of mutual coupling to be quantified for use with existing mixing rules. The results are illustrated for two identical, closely-spaced helices with parallel or perpendicular relative orientation.

2. INTERACTION BETWEEN PAIRED DIPOLES

Figure 1a shows the configuration under investigation. Two paired electric-magnetic dipoles are spaced by a vector $\underline{d}$ and located in an isotropic host medium with permittivity ϵ_o and permeability μ_o. The respective dipole moments $\underline{p}_{e1}$, $\underline{p}_{m1}$ and $\underline{p}_{e2}$, $\underline{p}_{m2}$ take arbitrary orientations and magnitude. At this stage, magneto-electric coupling is not a prerequisite: the precise relation of the electric and magnetic dipole moments to the source fields is left unspecified. Hence the result applies equally well to two dipoles (or pairs thereof) which do not originate from magneto-electric coupling.

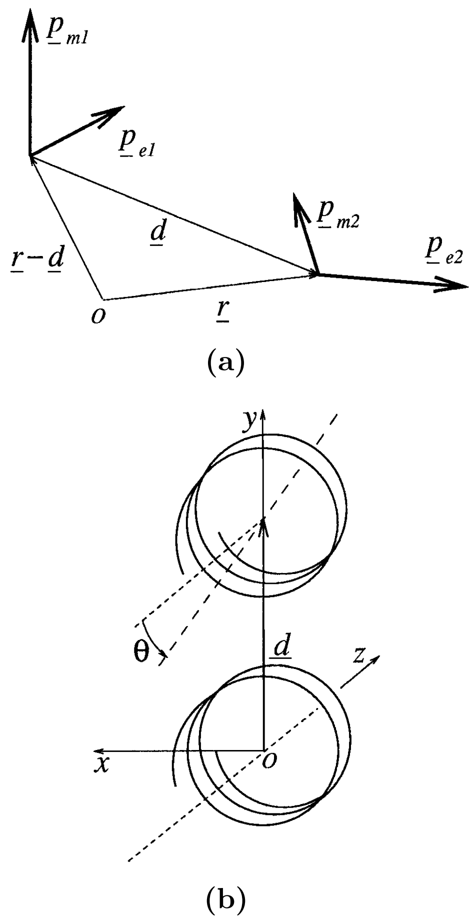

Figure 1. (a) Pair of paired dipoles and (b) Two strictly planar helices.

The interaction W_{12} of the fields $\underline{E}_2$, $\underline{H}_2$ radiated by the pair $\underline{p}_{e2}$, $\underline{p}_{m2}$ on the dipole sources $\underline{p}_{e1}$, $\underline{p}_{m1}$ follows from a generalisation of its definition for a pair of electric dipoles, as given in [13]:-

$$W_{12} \stackrel{\Delta}{=} -\left[\underline{p}_{e1} \cdot \underline{E}_2\left(\underline{r}-\underline{d}\right)+\underline{p}_{m1} \cdot \underline{H}_2\left(\underline{r}-\underline{d}\right)\right] \equiv W_e+W_m \tag{1}$$

W_{12} has the dimension of energy. This generalisation for W_{12} is consistent with the definition of reaction $<1,2>$ [12, 14] for the point sources $\underline{J}_{e1}$, $\underline{J}_{m1}$ at $\underline{r}=\underline{d}$ onto the fields $\underline{E}_2$, $\underline{H}_2$:-

$$\begin{aligned} <1,2> &\stackrel{\Delta}{=} (j\omega)^{-1} \iiint_V \left(\underline{J}_{e1} \cdot \underline{E}_2 - \underline{J}_{m1} \cdot \underline{H}_2\right) \mathrm{d}V \\ &\equiv <1,2>_e + <1,2>_m \end{aligned} \tag{2}$$

where integration is performed over all space. Similar expressions are obtained for W_{21} and $<2,1>$, through interchange of indices and replacing $\underline{d}$ by $-\underline{d}$. The analogy between W_e and $<1,2>_e$ follows directly from the definition of the electric dipole moment:-

$$\underline{p}_{e1} = (j\omega)^{-1} \iiint_V \underline{J}_{e1}\, \mathrm{d}V \tag{3}$$

The equivalence of W_m and $<1,2>_m$ follows by noting the source equivalence between an axial magnetic current $\underline{J}_{m1} = J_{m1}\underline{1}_z$ and an equivalent rotational (azimuthal) electric current $\underline{J}^{eq}_{e\,1} = \underline{\nabla} \times \underline{J}_{m1}/(j\omega\mu_o)$:-

$$\begin{aligned} -(j\omega)^{-1} \iiint_V \underline{J}_{m1}\, \mathrm{d}V &= -(j2\omega)^{-1} \iiint_V \underline{\nabla} \times \left(\underline{J}_{m1} \times \underline{\rho}\right)\, \mathrm{d}V \\ &= \frac{\mu_o}{2} \iiint_V \underline{\rho} \times \underline{J}^{eq}_{e\,1}\, \mathrm{d}V = \underline{p}_{m1} \end{aligned} \tag{4}$$

using the identities $\underline{1}_z = \frac{1}{2}\left[\underline{\nabla} \times \left(\underline{1}_z \times \underline{\rho}\right)\right]$ and $\underline{\nabla} \times \left(\underline{J}_{m1} \times \underline{\rho}\right) = -\underline{\rho} \times \left(\underline{\nabla} \times \underline{J}_{m1}\right)$. Thus the reaction Eq. (2) for a pair of dipole sources is the integral representation of the interaction Eq. (1), regardless of the nature of the sources. It is to be remembered that for induced dipole moments, W_{12} as defined by Eq. (1) quantifies coupling only to first order: the radiated fields $\underline{E}_2(\underline{r}-\underline{d})$ and $\underline{H}_2(\underline{r}-\underline{d})$ affect $\underline{p}_{e1}$ and $\underline{p}_{m1}$, whose radiated fields $\underline{E}_1(\underline{r})$ and $\underline{H}_1(\underline{r})$ in turn affect $\underline{p}_{e2}$ and $\underline{p}_{m2}$. This process of multiple coupling is discussed in more detail later. Finally, it is noted that in this treatment particles are considered as point sources (dipoles), which is an idealisation of any realistic case of particles with nonzero dimensions. In particular, particles exhibit a nonzero thickness in the direction $\underline{1}_d$. Because of proximity effects, the mutual coupling at points on opposite sides of a particle in the direction $\underline{1}_d$ is not equal: the nearest opposite points show a lower charge density as compared to the distant opposite points. Thus the coupling is spatially dispersive within the volume of each particle. Hence deviations may occur when comparing the analytically obtained values with those obtained from numerical simulation or from experimental investigation.

The radiated fields at $\underline{r} = \underline{d}$ in Eq. (1) can be expressed as a multipole expansion [15], which is here limited to dipole moment terms and rewritten using dyadic notation:-

$$\begin{aligned}\underline{E}_2(\underline{d}) =& \frac{\exp\left(-j\underline{k}\cdot\underline{d}\right)}{4\pi\epsilon_o d^3}\Big\{(1+jkd)\left(3\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{p}_{e2} \\ &+(jkd)^2\left(\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{p}_{e2}+j\omega\epsilon_o d\,(1+jkd)\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{p}_{m2}\Big\} \quad (5)\\ \underline{H}_2(\underline{d}) =& \frac{\exp\left(-j\underline{k}\cdot\underline{d}\right)}{4\pi\mu_o d^3}\Big\{(1+jkd)\left(3\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{p}_{m2} \\ &+(jkd)^2\left(\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{p}_{m2}-j\omega\mu_o d\,(1+jkd)\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{p}_{e2}\Big\} \quad (6)\end{aligned}$$

where $\underline{\underline{I}}$ represents the unit dyadic. Substitution of Eqs. (5–6) into Eq. (1) yields W_{12} as a function of the dipole moments:-

$$\begin{aligned}W_{12} =& -\frac{\exp\left(-j\underline{k}\cdot\underline{d}\right)}{4\pi d^3}\Big\{\epsilon_o{}^{-1}\Big[(1+jkd)\,\underline{p}_{e1}\cdot\left(3\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{p}_{e2} \\ &\qquad+(jkd)^2\underline{p}_{e1}\cdot\left(\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{p}_{e2}\Big] \\ &+\mu_o{}^{-1}\Big[(1+jkd)\,\underline{p}_{m1}\cdot\left(3\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{p}_{m2} \\ &\qquad+(jkd)^2\underline{p}_{m1}\cdot\left(\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{p}_{m2}\Big] \\ &+j\omega d\,(1+jkd)\Big[\underline{p}_{e1}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{p}_{m2}-\underline{p}_{m1}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{p}_{e2}\Big]\Big\} \\ \triangleq& W_{ee}+W_{mm}+W_{em}+W_{me} \quad (7)\end{aligned}$$

In high-density composite media which are excited at sufficiently low frequencies, the near field terms ($kd \ll 1$) dominate the expressions.

As the frequency or separation increases, terms of higher order in (kd) become more significant. Provided the medium is reciprocal, which in the case of bianisotropics implies self-complementarity [14], it follows from simple calculation that the interaction satisfies the reciprocity condition: $W_{12} = W_{21} \stackrel{\Delta}{=} W \Longleftrightarrow <1,2> = <2,1>$. Since $\underline{\underline{I}} \times \underline{1}_d$ is the general anti-symmetric dyadic, $W_{em} + W_{me}$ can be written as:-

$$\begin{aligned} W_{em} + W_{me} = &- \frac{\exp(-j\underline{k}\cdot\underline{d})}{4\pi d^3}\Big\{ j\omega d\,(1 + jkd) \\ &\cdot \Big[\underline{p}_{e1} \cdot \big(\underline{\underline{I}} \times \underline{1}_d\big) \cdot \underline{p}_{m2} + \underline{p}_{e2} \cdot \big(\underline{\underline{I}} \times \underline{1}_d\big) \cdot \underline{p}_{m1} \Big] \Big\} \\ = &\frac{j\omega\,(1 + jkd)\exp(-j\underline{k}\cdot\underline{d})}{4\pi d^2}\underline{1}_d \cdot \Big[\underline{p}_{e1} \times \underline{p}_{m2} + \underline{p}_{e2} \times \underline{p}_{m1} \Big] \end{aligned} \tag{8}$$

If both $(\underline{d}, \underline{p}_{e1}, \underline{p}_{m2})$ and $(\underline{d}, \underline{p}_{e2}, \underline{p}_{m1})$ are coplanar triplets or if $\underline{p}_{e1} \times \underline{p}_{m2} = \underline{p}_{m1} \times \underline{p}_{e2}$ (in particular, if both $(\underline{p}_{e1}, \underline{p}_{m2})$ and $(\underline{p}_{e2}, \underline{p}_{m1})$ are parallel pairs), then $W_{em} + W_{me} = 0$.

3. INTERACTION BETWEEN PAIRED COUPLED DIPOLES

Substitution of the microscopic constitutive equations for general bianisotropic media:-

$$\begin{bmatrix} \underline{p}_e \\ \underline{p}_m \end{bmatrix} = \begin{bmatrix} \epsilon_o \underline{\underline{p}}_{ee} & \sqrt{\mu_o \epsilon_o}\,\underline{\underline{p}}_{em} \\ \sqrt{\mu_o \epsilon_o}\,\underline{\underline{p}}_{me} & \mu_o \underline{\underline{p}}_{mm} \end{bmatrix} \cdot \begin{bmatrix} \underline{E} \\ \underline{H} \end{bmatrix} \tag{9}$$

into Eq. (7) yields W_{12} in terms of the constitutive dyadics and the separation distance. $\underline{E}$ and $\underline{H}$ denote the external fields, incident onto a single particle. $\underline{\underline{p}}_{kl}$ are the dipolarisability [1] dyadics of the particle, all being expressed in m^3. Then, Eq. (7) can be written as a six-vector bilinear form:-

$$W_{12} = [\,\underline{E}_1^T \quad \underline{H}_1^T\,] \cdot \underline{\underline{M}} \cdot \begin{bmatrix} \underline{E}_2 \\ \underline{H}_2 \end{bmatrix} \tag{10}$$

Six-vectors and six-dyadics have recently been reviewed in the context of electromagnetics [10]. The explicit form of the interaction six-dyadic

[1] The term dipolarisability dyadics is preferred over the more common term polarisability dyadics, to distinguish them from multi-polarisabilities.

$\underline{\underline{M}}$ in Eq. (10) will now be derived building on the results for several special cases.

3.1 Near Field Interaction ($kd \ll 1$)

Retaining zeroth-order terms in (kd) in Eq. (7) only, $\underline{\underline{M}}_{near}$ is computed as:-

$$\underline{\underline{M}}_{near} = -\frac{\exp(-j\underline{k}\cdot\underline{d})}{4\pi d^3}\begin{bmatrix}\underline{\underline{M}}_{11} & \underline{\underline{M}}_{12}\\ \underline{\underline{M}}_{21} & \underline{\underline{M}}_{22}\end{bmatrix} \tag{11}$$

with

$$\underline{\underline{M}}_{11} = \epsilon_o\left[\underline{\underline{p}}^T_{ee_1}\cdot\left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{ee2} + \underline{\underline{p}}^T_{me_1}\cdot\left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{me2}\right] \tag{12}$$

$$\underline{\underline{M}}_{12} = \sqrt{\mu_o\epsilon_o}\left[\underline{\underline{p}}^T_{ee_1}\cdot\left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{em2} + \underline{\underline{p}}^T_{me_1}\cdot\left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{mm2}\right] \tag{13}$$

$$\underline{\underline{M}}_{21} = \sqrt{\mu_o\epsilon_o}\left[\underline{\underline{p}}^T_{em_1}\cdot\left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{ee2} + \underline{\underline{p}}^T_{mm_1}\cdot\left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{me2}\right] \tag{14}$$

$$\underline{\underline{M}}_{22} = \mu_o\left[\underline{\underline{p}}^T_{em_1}\cdot\left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{em2} + \underline{\underline{p}}^T_{mm_1}\cdot\left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{mm2}\right] \tag{15}$$

in which $\underline{\underline{p}}_{kli}$ are the dipolarisability dyadics for particle i ($i = 1,\, 2$), and the superscript T denotes transposition.

3.2 Far Field Interaction ($kd \gg 1$)

Retaining second-order terms in (kd) in Eq. (7) only, the interaction dyadic $\underline{\underline{M}}_{far}$ can be written as:-

$$\underline{\underline{M}}_{far} = \frac{k^2\exp(-j\underline{k}\cdot\underline{d})}{4\pi d}\begin{bmatrix}\underline{\underline{M}}_{11} & \underline{\underline{M}}_{12}\\ \underline{\underline{M}}_{21} & \underline{\underline{M}}_{22}\end{bmatrix} \tag{16}$$

with

$$\begin{aligned}\underline{\underline{M}}_{11} = &\epsilon_o\Big[\underline{\underline{p}}^T_{ee_1}\cdot\left(\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{ee2} + \underline{\underline{p}}^T_{me_1}\cdot\left(\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{me2}\\ &+\underline{\underline{p}}^T_{ee_1}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{\underline{p}}_{me2} - \underline{\underline{p}}^T_{me_1}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{\underline{p}}_{ee2}\Big]\end{aligned} \tag{17}$$

$$\begin{aligned}\underline{\underline{M}}_{12} = &\sqrt{\mu_o\epsilon_o}\Big[\underline{\underline{p}}^T_{ee_1}\cdot\left(\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{em2} + \underline{\underline{p}}^T_{me_1}\cdot\left(\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{mm2}\\ &+\underline{\underline{p}}^T_{ee_1}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{\underline{p}}_{mm2} - \underline{\underline{p}}^T_{me_1}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{\underline{p}}_{em2}\Big]\end{aligned} \tag{18}$$

$$\begin{aligned}\underline{\underline{M}}_{21} =&\sqrt{\mu_o\epsilon_o}\left[\underline{\underline{p}}^T_{em_1}\cdot\left(\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{ee2}+\underline{\underline{p}}^T_{mm_1}\cdot\left(\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{me2}\right.\\&\left.+\underline{\underline{p}}^T_{em_1}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{\underline{p}}_{me2}-\underline{\underline{p}}^T_{mm_1}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{\underline{p}}_{ee2}\right] \qquad (19)\\ \underline{\underline{M}}_{22} =&\mu_o\left[\underline{\underline{p}}^T_{em_1}\cdot\left(\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{em2}+\underline{\underline{p}}^T_{mm_1}\cdot\left(\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{mm2}\right.\\&\left.+\underline{\underline{p}}^T_{em_1}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{\underline{p}}_{mm2}-\underline{\underline{p}}^T_{mm_1}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{\underline{p}}_{em2}\right] \qquad (20)\end{aligned}$$

The product terms containing the skew-symmetric dyadic can be written more compactly when using tryadics:-

$$\begin{aligned}&\underline{\underline{p}}^T_{kli}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{\underline{p}}_{rsj}=\underline{1}_d\cdot\left(\underline{\underline{p}}^T_{kli}\times\underline{\underline{p}}_{rsj}\right)\\&(k,l,r,s=e,m;\quad i,j=1,2)\end{aligned} \qquad (21)$$

in which $\underline{\underline{p}}^T_{kli}\times\underline{\underline{p}}_{rsj}$ is a tryadic. Tryadics are tensors of rank three, *ie* having three indices, and map a vector onto a dyadic.

3.3 Boundary Zone Interaction Terms ($kd\sim 1$)

In the boundary zone, additional first-order terms in (kd) become significant. In this case, the dyadic $\underline{\underline{M}}_{boundary}$:-

$$\underline{\underline{M}}_{boundary}=\frac{-jk\ \exp\left(-j\underline{k}\cdot\underline{d}\right)}{4\pi d^2}\begin{bmatrix}\underline{\underline{M}}_{11} & \underline{\underline{M}}_{12}\\ \underline{\underline{M}}_{21} & \underline{\underline{M}}_{22}\end{bmatrix} \qquad (22)$$

with elements:-

$$\begin{aligned}\underline{\underline{M}}_{11}=&\quad\epsilon_o\left[\underline{\underline{p}}^T_{ee_1}\cdot\left(3\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{ee2}+\underline{\underline{p}}^T_{me_1}\cdot\left(3\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{me2}\right.\\&\left.+\underline{\underline{p}}^T_{ee_1}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{\underline{p}}_{me2}-\underline{\underline{p}}^T_{me_1}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{\underline{p}}_{ee2}\right] \qquad (23)\end{aligned}$$

$$\begin{aligned}\underline{\underline{M}}_{12}=&\quad\sqrt{\mu_o\epsilon_o}\left[\underline{\underline{p}}^T_{ee_1}\cdot\left(3\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{em2}+\underline{\underline{p}}^T_{me_1}\cdot\left(3\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{mm2}\right.\\&\left.+\underline{\underline{p}}^T_{ee_1}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{\underline{p}}_{mm2}-\underline{\underline{p}}^T_{me_1}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{\underline{p}}_{em2}\right] \qquad (24)\\ \underline{\underline{M}}_{21}=&\quad\sqrt{\mu_o\epsilon_o}\left[\underline{\underline{p}}^T_{em_1}\cdot\left(3\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{ee2}+\underline{\underline{p}}^T_{mm_1}\cdot\left(3\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{me2}\right.\\&\left.+\underline{\underline{p}}^T_{em_1}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{\underline{p}}_{me2}-\underline{\underline{p}}^T_{mm_1}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{\underline{p}}_{ee2}\right] \qquad (25)\\ \underline{\underline{M}}_{22}=&\quad\mu_o\left[\underline{\underline{p}}^T_{em_1}\cdot\left(3\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{em2}+\underline{\underline{p}}^T_{mm_1}\cdot\left(3\underline{1}_d\underline{1}_d-\underline{\underline{I}}\right)\cdot\underline{\underline{p}}_{mm2}\right.\\&\left.+\underline{\underline{p}}^T_{em_1}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{\underline{p}}_{mm2}-\underline{\underline{p}}^T_{mm_1}\cdot\left(\underline{\underline{I}}\times\underline{1}_d\right)\cdot\underline{\underline{p}}_{em2}\right] \qquad (26)\end{aligned}$$

is to be added to $\underline{\underline{M}}_{near}$ and $\underline{\underline{M}}_{far}$ in order to obtain the complete interaction dyadic.

3.4 General Interaction

The interaction for the general case of arbitrary particle separation is simply constructed from the previous results as:-

$$\underline{\underline{M}} = \underline{\underline{M}}_{near} + \underline{\underline{M}}_{boundary} + \underline{\underline{M}}_{far} \tag{27}$$

3.5 Examples

Figures 2–3 show $-W = \mathrm{Re}\,[-W] - j\mathrm{Im}\,[-W]$ as a function of p/L_{turn} and the relative orientation angle θ between the axes of two identical 3-turn helices with pitch $p = 0.5\,\mathrm{mm}$, free length 1.5mm, wire length $L = 3L_{turn} = 9.54\,\mathrm{mm}$ and gauge 0.15mm. The operation frequency is $0.25\,\mathrm{GHz}$ ($kL = 0.05 \ll 1$). The helices are separated by a distance $\underline{d} = (10\mathrm{mm})\,\underline{1}_y$ and remain strictly planar at all relative orientations, *ie* one helix is rotated around the ox-axis by θ in this plane with respect to the other helix (Figure 1b). oz defines the direction of the axis of the fixed helix and ox contains its starting point. The incident wave is linearly polarized along the oz-direction. The computations are based on the previously calculated dipolarisability dyadics of this particular isolated helix by using the method of counterpropagating waves with averaged direction of propagation [6]. This reference also contains a detailed discussion of the dependence of all 36 complex dipolarisabilities on p/L_{turn} for a single helix at quasi-static and quasi-resonance conditions.

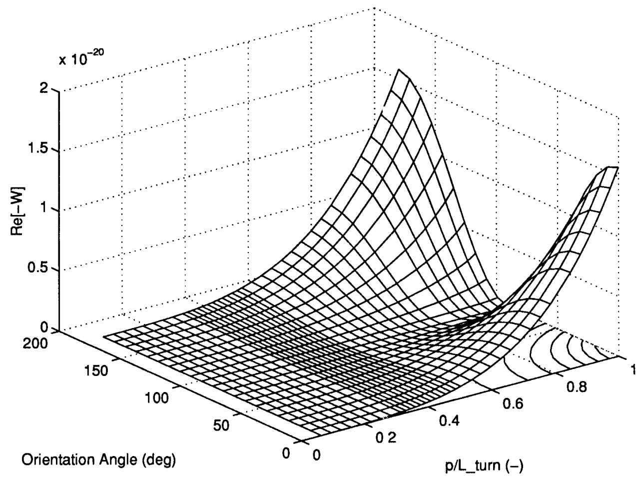

Figure 2. Real part of $-W$ as a function of relative orientation angle θ and helix pitch-to-length ratio p/L_{turn} for two identical, strictly planar helices. $kd = 0.05$.

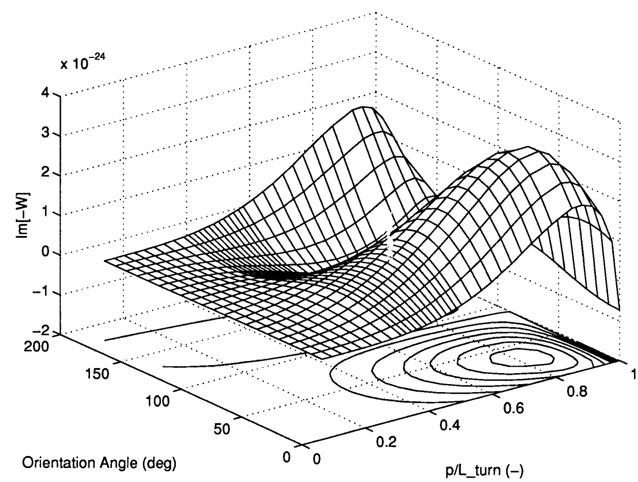

Figure 3. Imaginary part of $-W$ as a function of relative orientation angle θ and helix pitch-to-length ratio p/L_{turn} for two identical, strictly planar helices. $kd = 0.05$.

For arbitrary θ, $\mathrm{Re}\,[-W]$ increases as the pitch increases, because of the dominance of the elements of $\underline{\underline{p}}_{ee}$ over the other dipolarisabilities [6]. For arbitrary p/L_{turn}, $\mathrm{Re}\,[-W]$ reaches a maximum near parallel ($\theta \approx 0$ or π) and a minimum near perpendicular ($\theta \approx \pm\pi/2$) relative orientation. The absolute maxima are obtained for $p = L_{turn}$. For $\mathrm{Im}\,[-W]$, absolute maxima are reached for $p/L_{turn} < 1$, as a result of the $\mathrm{Im}\left[\underline{\underline{p}}_{kl}\right]$ reaching maxima at different p/L_{turn} ratios than $\mathrm{Re}\left[\underline{\underline{p}}_{kl}\right]$ at non-zero frequencies [6]. The slight asymmetry of the θ-dependence with respect to $\theta = \pi/2$ (which also exists for $\mathrm{Re}\,[-W]$) is a result of the induced dipole moments being not exactly parallel to the helix axis [9], hence perpendicular helices do not result in orthogonal electric or magnetic dipole moments. Except for this asymmetry, the θ-dependence can be easily understood by considering the overlap between the radiation patterns of two electric or magnetic dipole antennas: the overlap (interaction) is maximal for co-polarised dipoles and minimal for cross-polarised dipoles. For a fixed orientation and p/L_{turn} ratio, $\mathrm{Im}\,[-W]$ is approximately four orders of magnitude smaller than $\mathrm{Re}\,[-W]$, as a result of the real and imaginary parts of $\underline{\underline{p}}_{kl}$ showing a very large difference for quasi-static operation [6, 9].

Figures 4–5 show $\mathrm{Re}\,[-W]$ and $\mathrm{Im}\,[-W]$ as a function of p/L_{turn} and the logarithm of the relative separation, $\log_{10}(kd)$, between two parallel, strictly planar helices ($\theta = 0$ or π). As expected, the interaction is seen to increase with decreasing distance, first at higher p/L_{turn}-ratios, then at smaller ratios. Notice the disappearance of the maximum for $\mathrm{Im}\,[-W]$ in the boundary zone and in the far zone.

Figures 6–7 show $\mathrm{Re}\,[-W]$ and $\mathrm{Im}\,[-W]$ as a function of p/L_{turn} and the logarithmic relative distance between two perpendicular, strictly planar helices ($\theta = \pi/2$). It now emerges that, unlike the parallel configuration, $\mathrm{Re}\,[-W]$ reaches a maximum at $p/L_{turn} \approx 0.69 < 1$. This is understood by comparing $\left[\underline{\underline{p}}_{ee}\right]_{zy}$ with $\left[\underline{\underline{p}}_{ee}\right]_{zz}$ as a function of p/L_{turn} in [6, Figure 2]: the former shows a maximum at $p/L_{turn} \approx 0.69$, whereas the latter increases monotonically, hence their product also shows a maximum. In the far field $\mathrm{Im}\,[-W]$ reaches a minimum, whereas in the near field $\mathrm{Im}\,[-W]$ reaches a maximum. As expected, $\mathrm{Im}\,[-W] \to 0$ if $p/L_{turn} \to 0$ or $p/L_{turn} \to 1$.

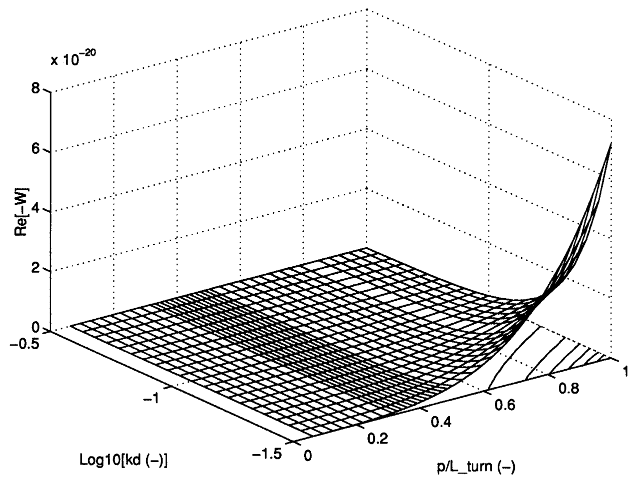

Figure 4. Real part of $-W$ as a function of helix separation kd and helix pitch-to-length ratio p/L_{turn} for two identical, strictly planar parallel helices.

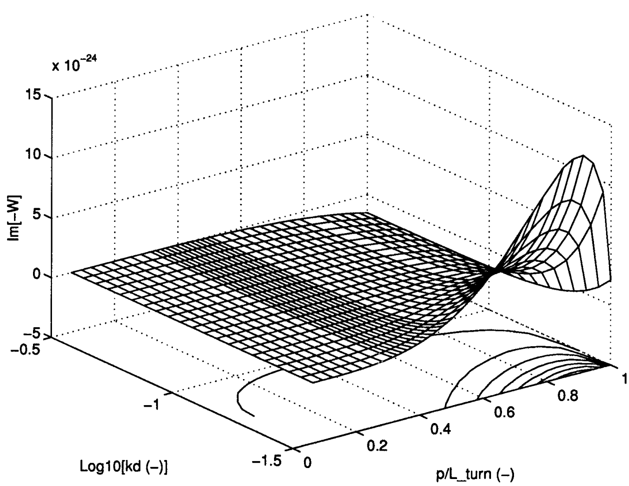

Figure 5. Imaginary part of $-W$ as a function of helix separation kd and helix pitch-to-length ratio p/L_{turn} for two identical, strictly planar parallel helices.

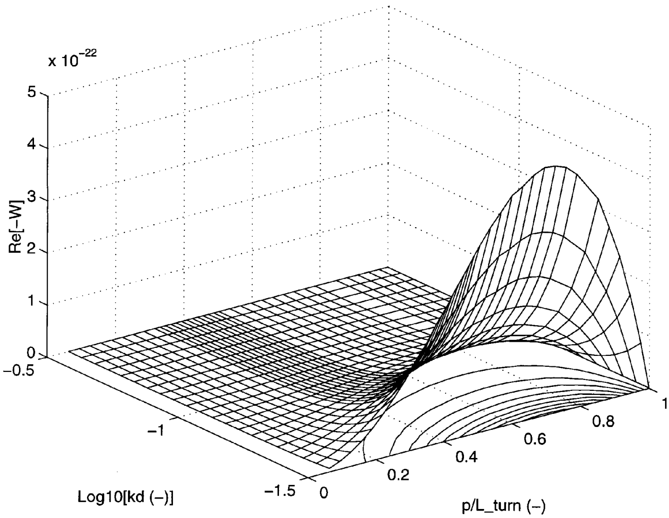

Figure 6. Real part of $-W$ as a function of helix separation kd and helix pitch-to-length ratio p/L_{turn} for two identical, strictly planar perpendicular helices.

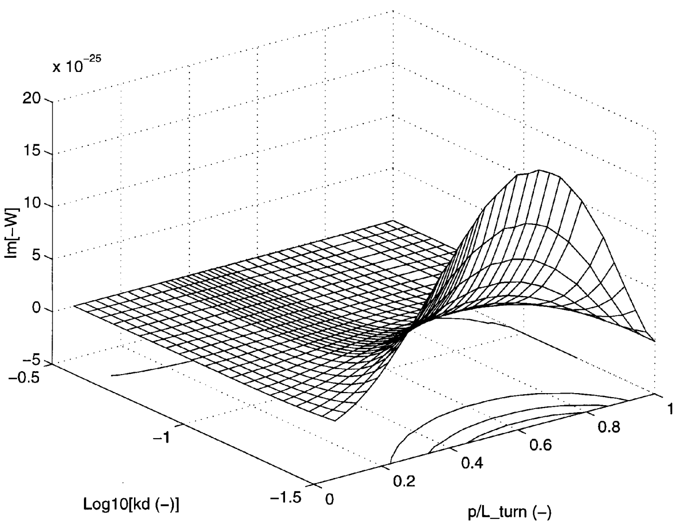

Figure 7. Imaginary part of $-W$ as a function of helix separation kd and helix pitch-to-length ratio p/L_{turn} for two identical, strictly planar perpendicular helices.

Comparing Figures 4 and 6 shows that, for a fixed separation and p/L_{turn} ratio, $\mathrm{Re}\,[-W]$ and $\mathrm{Im}\,[-W]$ are about two orders of magnitude smaller for a perpendicular helix pair as compared to a parallel pair. This follows from a similar result for the real and imaginary parts of $\underline{\underline{p}}_{kl}$ for the quasi-static operation [6] and has been confirmed by experimental observations [4].

A similar analysis has been performed for one of the helices being rotated around the oy-axis. In this case, the helices are in general not coplanar, except for parallel and anti-parallel orientations. The results are qualitatively the same, although small numerical differences have been noticed. Further analysis has also shown that, for a fixed $0 < p/L_{turn} < 1$ and $kd \ll 1$, $\mathrm{Im}\,[-W] > 0$ for $(\theta_o - \pi/2) < \theta < \theta_o$ and $\mathrm{Im}\,[-W] < 0$ for $\theta_o < \theta < (\theta_o + \pi/2)$ with $\theta_o \approx \pi/2$.

Figures 8 and 9 show $\mathrm{Re}\,[-W]$ and $\mathrm{Im}\,[-W]$ as a function of p/L_{turn} and the logarithmic relative distance between two parallel, co-linear helices ($\theta = 0$ or π, $\underline{d} = d\underline{1}_z$) The real part $\mathrm{Re}\,[-W]$ shows a similar dependence as that for parallel, strictly planar helices and is of the same order of magnitude. The inversion of its sign as compared to Figure 4 is due to the inversion of the sign of the zz-component of $(3\underline{1}_d\underline{1}_d - \underline{\underline{I}})$, in Eqs. (11–15).

It can be concluded that, apart from the separation distance of the helices, the value of W depends on the product $\underline{1}_{a_1} \cdot (3\underline{1}_d\underline{1}_d - \underline{\underline{I}}) \cdot \underline{1}_{a_2}$, *ie.* on the direction of $\underline{1}_d$ relative to that of the axes a_1 and a_2 of the two helices, and the relative orientation of the latter with respect to the polarisation direction of the incident wave.

Finally, it is noted that the interaction was defined and computed for the dipole moments associated with the helices, not for the helices as such. The latter requires the additional assessment of quadrupole, octopole, ...moments, along with an extended definition for W that incorporates such higher-order moments. This is beyond the scope of the dipole approximation that has been used throughout this paper, and requires separate investigation.

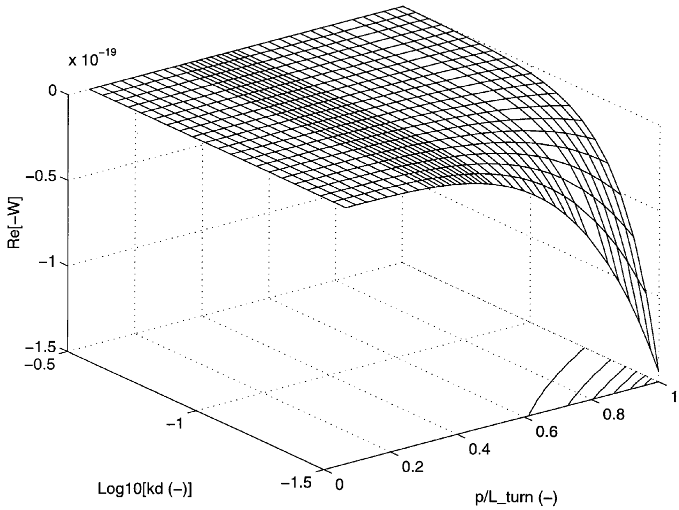

Figure 8. Real part of $-W$ as a function of helix separation and helix pitch-to-length ratio p/L_{turn} for two identical, co-linear helices.

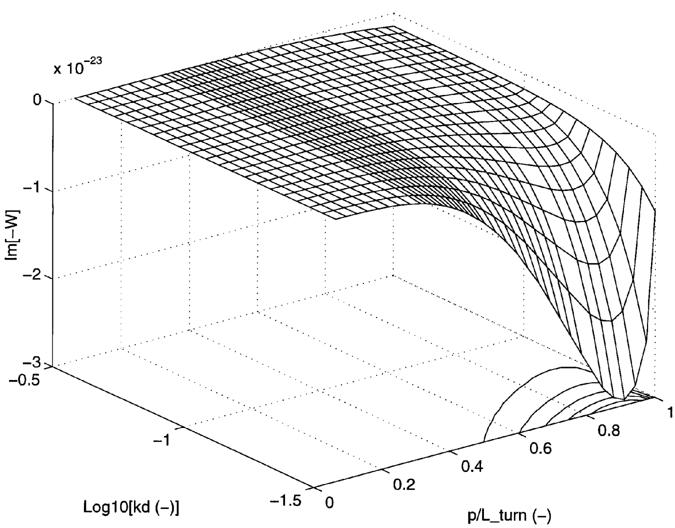

Figure 9. Imaginary part of $-W$ as a function of helix separation kd and helix pitch-to-length ratio p/L_{turn} for two identical, co-linear helices.

4. DE-EMBEDDING OF PARTICLES USING MODIFIED DIPOLARISABILITY DYADICS

4.1 De-embedding *vs* Homogenisation

Although the concept of interaction is valuable as a measure for mutual coupling, its usefulness in the estimation of the macroscopic medium parameters of high-density composites is limited. Therefore a different approach is pursued here that incorporates mutual coupling between particles into the dipolarisability dyadics of each individual particle. This procedure will be referred to as de-embedding, for it enables the embedded particle to be characterised as if no other (embedding) particles were present.

Induced dipole moments are related via the dipolarisability dyadics to the local fields $\underline{E}'$, $\underline{H}'$ that induce them. Local fields are usually taken as the vector sum of the macroscopic (external incident) fields, $\underline{E}$ and $\underline{H}$, and the homogenised depolarisation fields. Depolarisation fields represent the contribution of the electric and magnetic polarisation in the ambient medium, $\underline{P}_e(\underline{r})$ and $\underline{P}_m(\underline{r})$, for the particle under consideration. Hence depolarisation fields carry implicitly some of the interaction between particles. The homogenised depolarisation fields are usually assumed to satisfy the Lorentzian estimates, so that the local (internal incident) fields are expressed as:-

$$\underline{E}' = \underline{E} + \underline{\underline{L}} \cdot \frac{\underline{P}_e}{\epsilon}, \qquad \underline{H}' = \underline{H} + \underline{\underline{L}} \cdot \frac{\underline{P}_m}{\mu} \tag{28}$$

where $\underline{\underline{L}}$ is the depolarisation dyadic which depends on the geometry of the particle under consideration and ϵ, μ are the unknown effective medium parameters of the (homogenised) background medium. In the MG model, ϵ and μ are approximated by the parameters of the host medium, ϵ_o and μ_o. The validity of the homogenisation procedure is severely restricted by the condition that the particle concentration N be very low. In this case, the interaction between particles is negligible and $\underline{P}_e = \sum_{i=1}^{N} \underline{p}_{ei}$, $\underline{P}_m = \sum_{i=1}^{N} \underline{p}_{mi}$ hold to good approximation with no spatial dispersion, where $\underline{p}_{ei}$, $\underline{p}_{mi}$ are the dipole moments of the isolated particle i. In the following, the homogenisation procedure is replaced by an explicit multiple scattering scheme, *ie* the depolarisation fields for each bianisotropic particle are computed directly from the fields $\underline{E}' - \underline{E}$, $\underline{H}' - \underline{H}$ scattered by the surrounding particles. Hence the incident fields for the particles are the macroscopic fields in the (homogeneous) host medium, $\underline{E}$ and $\underline{H}$, augmented by the scattered fields.

As will be shown, the use of modified dipolarisability dyadics enables coupling between particles to be quantified explicitly and to be included into $\underline{P}_e$ and $\underline{P}_m$. This leads to increased accuracy when used in mixing formulae at higher particle concentrations. Computations are restricted here to a system of two particles, although the method and concepts developed are applicable to any number of particles.

4.2 First-Order Coupling

A system of two arbitrary particles is considered here. This comprises the particular cases of two identical particles with aligned anisotropies, two identical particles with non-aligned anisotropies ('rotated particles'), two different particles with aligned anisotropies ('parallel particles'), and the general case of two different particles with non-aligned anisotropies. Here the case of interaction of the neighbouring particle 2 onto particle 1 is considered.

For particle 1, the fields scattered by particle 2 add to the external incident fields $\underline{E}$ and $\underline{H}$ and give rise to first-order modified (perturbed) dipole moments $\underline{p}_e^{(1)}$, $\underline{p}_m^{(1)}$. The perturbations $\underline{E}^{(1)} - \underline{E}$ and $\underline{H}^{(1)} - \underline{H}$ that lead to the first-order local fields $\underline{E}^{(1)}$, $\underline{H}^{(1)}$ can be incorporated into the dipolarisability dyadics, so that the incident fields $\underline{E}$ and $\underline{H}$ can still be used in the two-particle configuration:-

$$\begin{aligned}\begin{bmatrix} \underline{p}_{e_1}^{(1)} \\ \underline{p}_{m_1}^{(1)} \end{bmatrix} &= \begin{bmatrix} \epsilon_o \underline{\underline{p}}_{ee_1} & \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{em_1} \\ \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{me_1} & \mu_o \underline{\underline{p}}_{mm_1} \end{bmatrix} \cdot \begin{bmatrix} \underline{E}^{(1)} \\ \underline{H}^{(1)} \end{bmatrix} \\ &\triangleq \begin{bmatrix} \epsilon_o \underline{\underline{p}}_{ee_1}^{(1)} & \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{em_1}^{(1)} \\ \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{me_1}^{(1)} & \mu_o \underline{\underline{p}}_{mm_1}^{(1)} \end{bmatrix} \cdot \begin{bmatrix} \underline{E} \\ \underline{H} \end{bmatrix}\end{aligned} \tag{29}$$

$\underline{\underline{p}}_{kl}^{(1)}$ $(k, l = e, m)$ are denoted as the first-order modified dipolarisability dyadics. For first-order scattering (perturbation), the sources of the fields scattered by particle 2 are its unperturbed dipole moments, hence:-

$$\begin{aligned}\underline{E}^{(1)} =& \underline{E} + \underline{E}_2(\underline{d}) \\ =& \underline{E} + \frac{\exp\left(-j\underline{k}\cdot\underline{d}\right)}{4\pi\epsilon_o d^3}\Big\{\left[(1+jkd)\left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\right.\end{aligned}$$

$$+ (jkd)^2 \left(\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\Big] \cdot \underline{p}_{e2} + j\omega\epsilon_o d\,(1+jkd)\left(\underline{\underline{I}} \times \underline{1}_d\right) \cdot \underline{p}_{m2}\Big\} \tag{30}$$

$$\begin{aligned}\underline{H}^{(1)} =& \underline{H} + \underline{H}_2(\underline{d})\\ =& \underline{H} + \frac{\exp\left(-j\underline{k}\cdot\underline{d}\right)}{4\pi\mu_o r^3}\Big\{\Big[(1+jkd)\left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\\ &+ (jkd)^2\left(\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\Big] \cdot \underline{p}_{m2} - j\omega\mu_o d\,(1+jkd)\left(\underline{\underline{I}} \times \underline{1}_d\right) \cdot \underline{p}_{e2}\Big\}\end{aligned} \tag{31}$$

If Eqs. (30–31) are substituted into Eq. (29) the first-order local (perturbed incident) fields are obtained in terms of the unperturbed fields:-

$$\begin{bmatrix} \underline{E}^{(1)} \\ \underline{H}^{(1)} \end{bmatrix} = \begin{bmatrix} \underline{\underline{I}} + \underline{\underline{P}}_2^{(1)} & \underline{\underline{Q}}_2^{(1)} \\ \underline{\underline{U}}_2^{(1)} & \underline{\underline{I}} + \underline{\underline{V}}_2^{(1)} \end{bmatrix} \cdot \begin{bmatrix} \underline{E} \\ \underline{H} \end{bmatrix} \tag{32}$$

with

$$\begin{aligned}\underline{\underline{P}}_2^{(1)} =& \frac{\exp\left(-j\underline{k}\cdot\underline{d}\right)}{4\pi d^3}\Big\{\Big[(1+jkd)\left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\\ &+ (jkd)^2\left(\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\Big] \cdot \underline{\underline{p}}_{ee_2} + \Big[jkd\,(1+jkd)\left(\underline{\underline{I}} \times \underline{1}_d\right)\Big] \cdot \underline{\underline{p}}_{me_2}\Big\}\end{aligned} \tag{33}$$

$$\begin{aligned}\underline{\underline{Q}}_2^{(1)} =& \sqrt{\frac{\mu_o}{\epsilon_o}}\frac{\exp\left(-j\underline{k}\cdot\underline{d}\right)}{4\pi d^3}\Big\{\Big[(1+jkd)\left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\\ &+ (jkd)^2\left(\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\Big] \cdot \underline{\underline{p}}_{em_2} + \Big[jkd\,(1+jkd)\left(\underline{\underline{I}} \times \underline{1}_d\right)\Big] \cdot \underline{\underline{p}}_{mm_2}\Big\}\end{aligned} \tag{34}$$

$$\begin{aligned}\underline{\underline{U}}_2^{(1)} =& \sqrt{\frac{\epsilon_o}{\mu_o}}\frac{\exp\left(-j\underline{k}\cdot\underline{d}\right)}{4\pi d^3}\Big\{\Big[(1+jkd)\left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\\ &+ (jkd)^2\left(\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\Big] \cdot \underline{\underline{p}}_{me_2} - \Big[jkd\,(1+jkd)\left(\underline{\underline{I}} \times \underline{1}_d\right)\Big] \cdot \underline{\underline{p}}_{ee_2}\Big\}\end{aligned} \tag{35}$$

$$\begin{aligned}\underline{\underline{V}}_2^{(1)} =& \frac{\exp\left(-j\underline{k}\cdot\underline{d}\right)}{4\pi d^3}\Big\{\Big[(1+jkd)\left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\\ &+ (jkd)^2\left(\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right)\Big] \cdot \underline{\underline{p}}_{mm_2} - \Big[jkd\,(1+jkd)\left(\underline{\underline{I}} \times \underline{1}_d\right)\Big] \cdot \underline{\underline{p}}_{em_2}\Big\}\end{aligned} \tag{36}$$

This yields the first-order modified dipolarisability dyadics $\underline{\underline{p}}_{kl}^{(1)}$ for the de-embedded particle in terms of the dipolarisability dyadics $\underline{\underline{p}}_{kl}$ for the same, but isolated particle:-

$$\begin{bmatrix} \epsilon_o \underline{\underline{p}}_{ee_1}^{(1)} & \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{em_1}^{(1)} \\ \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{me_1}^{(1)} & \mu_o \underline{\underline{p}}_{mm_1}^{(1)} \end{bmatrix} = \begin{bmatrix} \epsilon_o \underline{\underline{p}}_{ee_1} & \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{em_1} \\ \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{me_1} & \mu_o \underline{\underline{p}}_{mm_1} \end{bmatrix} \cdot \begin{bmatrix} \underline{\underline{I}} + \underline{\underline{P}}_2^{(1)} & \underline{\underline{Q}}_2^{(1)} \\ \underline{\underline{U}}_2^{(1)} & \underline{\underline{I}} + \underline{\underline{V}}_2^{(1)} \end{bmatrix} \tag{37}$$

The dyadic perturbations, defined by $\Delta\underline{\underline{p}}_{kl}^{(1)} \stackrel{\Delta}{=} \underline{\underline{p}}_{kl}^{(1)} - \underline{\underline{p}}_{kl}$, follow as:-

$$\begin{bmatrix} \epsilon_o \Delta\underline{\underline{p}}_{ee_1}^{(1)} & \sqrt{\mu_o\epsilon_o}\,\Delta\underline{\underline{p}}_{em_1}^{(1)} \\ \sqrt{\mu_o\epsilon_o}\,\Delta\underline{\underline{p}}_{me_1}^{(1)} & \mu_o \Delta\underline{\underline{p}}_{mm_1}^{(1)} \end{bmatrix} = \begin{bmatrix} \epsilon_o \underline{\underline{p}}_{ee_1} \cdot \underline{\underline{P}}_2^{(1)} + \sqrt{\mu_o\epsilon_o}\underline{\underline{p}}_{em_1} \cdot \underline{\underline{U}}_2^{(1)} & \epsilon_o \underline{\underline{p}}_{ee_1} \cdot \underline{\underline{Q}}_2^{(1)} + \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{em_1} \cdot \underline{\underline{V}}_2^{(1)} \\ \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{me_1} \cdot \underline{\underline{P}}_2^{(1)} + \mu_o \underline{\underline{p}}_{mm_1} \cdot \underline{\underline{U}}_2^{(1)} & \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{me_1} \cdot \underline{\underline{Q}}_2^{(1)} + \mu_o \underline{\underline{p}}_{mm_1} \cdot \underline{\underline{V}}_2^{(1)} \end{bmatrix} \tag{38}$$

It is verified that in the dilute limit the unperturbed solution is retrieved: $d \to \infty$ results in $\underline{\underline{P}}_2^{(1)}, \underline{\underline{V}}_2^{(1)}, \underline{\underline{Q}}_2^{(1)}, \underline{\underline{U}}_2^{(1)} \to \underline{\underline{0}}$, yielding $\underline{\underline{p}}_{kl}^{(1)} \to \underline{\underline{p}}_{kl}$. Furthermore, $\Delta\underline{\underline{p}}_{ee_1}^{(1)}$ depends on $\underline{\underline{p}}_{ee2}$, $\underline{\underline{p}}_{em2}$ and $\underline{\underline{p}}_{me2}$; $\Delta\underline{\underline{p}}_{mm_1}^{(1)}$ depends on $\underline{\underline{p}}_{mm2}$, $\underline{\underline{p}}_{em2}$ and $\underline{\underline{p}}_{me2}$, and $\Delta\underline{\underline{p}}_{em_1}^{(1)}$ and $\Delta\underline{\underline{p}}_{me_1}^{(1)}$ depend on all four $\underline{\underline{p}}_{ee2}$, $\underline{\underline{p}}_{mm2}$, $\underline{\underline{p}}_{em2}$ and $\underline{\underline{p}}_{me2}$.

It is a simple matter to verify that the interaction six-dyadic for $\underline{p}_{e1}$, $\underline{p}_{m1}$ and $\underline{E}_2(\underline{d})$, $\underline{H}_2(\underline{d})$, as defined by Eq. (10):

$$\begin{aligned} W_{12} &= -\left[\underline{p}_{e1} \cdot \underline{E}_2(\underline{d}) + \underline{p}_{m1} \cdot \underline{H}_2(\underline{d})\right] \\ &= -\left[\, \underline{E}^T \quad \underline{H}^T \,\right] \cdot \begin{bmatrix} \epsilon_o \underline{\underline{p}}_{ee_1}^T \cdot \underline{\underline{P}}_2^{(1)} + \sqrt{\mu_o\epsilon_o}\underline{\underline{p}}_{me_1}^T \cdot \underline{\underline{U}}_2^{(1)} & \epsilon_o \underline{\underline{p}}_{ee_1}^T \cdot \underline{\underline{Q}}_2^{(1)} + \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{me_1}^T \cdot \underline{\underline{V}}_2^{(1)} \\ \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{em_1}^T \cdot \underline{\underline{P}}_2^{(1)} + \mu_o \underline{\underline{p}}_{mm_1}^T \cdot \underline{\underline{U}}_2^{(1)} & \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{em_1}^T \cdot \underline{\underline{Q}}_2^{(1)} + \mu_o \underline{\underline{p}}_{mm_1}^T \cdot \underline{\underline{V}}_2^{(1)} \end{bmatrix} \cdot \begin{bmatrix} \underline{E} \\ \underline{H} \end{bmatrix} \end{aligned} \tag{39}$$

is identical to Eq. (27). Note the differences between the interaction six-dyadic $\underline{\underline{M}}$ in Eq. (39) and the perturbation six-dyadic $\left[\Delta \underline{\underline{p}}^{(1)}_{kl_1}\right]$ given by Eq. (38), even for reciprocal particles, for which $\underline{\underline{p}}_{ee_i} = \underline{\underline{p}}^T_{ee_i}$, $\underline{\underline{p}}_{mm_i} = \underline{\underline{p}}^T_{mm_i}$, $\underline{\underline{p}}_{em_i} = -\underline{\underline{p}}^T_{me_i}$. Thus, although $\left[\Delta \underline{\underline{p}}^{(1)}_{kl_1}\right]$ contains the effect of mutual coupling between particles, it does not represent their interaction six-dyadic in the strict sense.

The following generalised reciprocity relations hold between the perturbations for two reciprocal particles embedded in a reciprocal host medium:-

$$\Delta \underline{\underline{p}}^{(1)}_{ee_1} = \left(\Delta \underline{\underline{p}}^{(1)}_{ee_2}\right)^T, \qquad \Delta \underline{\underline{p}}^{(1)}_{em_1} = -\left(\Delta \underline{\underline{p}}^{(1)}_{me_2}\right)^T \tag{40}$$

$$\Delta \underline{\underline{p}}^{(1)}_{me_1} = -\left(\Delta \underline{\underline{p}}^{(1)}_{em_2}\right)^T, \qquad \Delta \underline{\underline{p}}^{(1)}_{mm_1} = \left(\Delta \underline{\underline{p}}^{(1)}_{mm_2}\right)^T \tag{41}$$

which follows from equating W_{12} and W_{21}. Hence if the particles are identical with aligned anisotropies ($\underline{\underline{p}}_{kli} = \underline{\underline{p}}_{klj} \stackrel{\Delta}{=} \underline{\underline{p}}_{kl}$), then the perturbed dipolarisabilities satisfy the same reciprocity conditions as the unperturbed dyadics:-

$$\underline{\underline{p}}^{(1)}_{ee_i} = \left(\underline{\underline{p}}^{(1)}_{ee_i}\right)^T, \quad \underline{\underline{p}}^{(1)}_{mm_i} = \left(\underline{\underline{p}}^{(1)}_{mm_i}\right)^T, \quad \underline{\underline{p}}^{(1)}_{em_i} = -\left(\underline{\underline{p}}^{(1)}_{me_i}\right)^T \quad (i = 1,\, 2) \tag{42}$$

It is instructive to consider the special case of near-field interaction ($kd \to 0$). Only in this case the perturbations become significant, since they are of second order in $\underline{\underline{p}}_{kli}$ and in practice[2] $\left\|\underline{\underline{p}}_{kli}\right\| \ll 1$. In this case the terms in $\underline{\underline{I}} \times \underline{d}$, representing the cross-products between electric and magnetic dipole moments, do not contribute:-

$$\begin{aligned}
\epsilon_o \Delta \underline{\underline{p}}^{(1)}_{ee_1} \to \quad & \epsilon_o \frac{\exp(-j\underline{k}\cdot\underline{d})}{4\pi d^3}\left[\underline{\underline{p}}_{ee_1} \cdot \left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right) \cdot \underline{\underline{p}}_{ee2}\right. \\
& \left. + \underline{\underline{p}}_{em_1} \cdot \left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right) \cdot \underline{\underline{p}}_{me2}\right]
\end{aligned} \tag{43}$$

$$\sqrt{\mu_o\epsilon_o}\, \Delta \underline{\underline{p}}^{(1)}_{em_1} \to \sqrt{\mu_o\epsilon_o} \frac{\exp(-j\underline{k}\cdot\underline{d})}{4\pi d^3}\left[\underline{\underline{p}}_{em_1} \cdot \left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right) \cdot \underline{\underline{p}}_{mm2}\right.$$

[2] The choice of definition for the norm of a dyadic is not relevant here.

$$+\underline{\underline{p}}_{ee_1} \cdot \left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right) \cdot \underline{\underline{p}}_{em2}\Big] \quad (44)$$

$$\sqrt{\mu_o\epsilon_o}\,\Delta\underline{\underline{p}}^{(1)}_{me_1} \to \sqrt{\mu_o\epsilon_o}\frac{\exp\left(-j\underline{k}\cdot\underline{d}\right)}{4\pi d^3}\Big[\underline{\underline{p}}_{me_1} \cdot \left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right) \cdot \underline{\underline{p}}_{ee2}$$
$$+\underline{\underline{p}}_{mm_1} \cdot \left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right) \cdot \underline{\underline{p}}_{me2}\Big] \quad (45)$$

$$\mu_o\Delta\underline{\underline{p}}^{(1)}_{mm_1} \to \mu_o\frac{\exp\left(-j\underline{k}\cdot\underline{d}\right)}{4\pi d^3}\Big[\underline{\underline{p}}_{mm_1} \cdot \left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right) \cdot \underline{\underline{p}}_{mm2}$$
$$+\underline{\underline{p}}_{me_1} \cdot \left(3\underline{1}_d\underline{1}_d - \underline{\underline{I}}\right) \cdot \underline{\underline{p}}_{em2}\Big] \quad (46)$$

For example, for two identical helices embedded in a dielectric host medium, for which $\left\|\underline{\underline{p}}_{ee_i}\right\| \gg \left(\left\|\underline{\underline{p}}_{emi}\right\|, \left\|\underline{\underline{p}}_{mei}\right\|\right) \gg \left\|\underline{\underline{p}}_{mmi}\right\|$ ($i = 1, 2$), the ratios:-

$$\left\|\underline{\underline{p}}^{(1)}_{mm_i}\right\| / \left\|\underline{\underline{p}}^{(1)}_{ee_i}\right\|, \; \left\|\underline{\underline{p}}^{(1)}_{em_i}\right\| / \left\|\underline{\underline{p}}^{(1)}_{ee_i}\right\| \text{ and } \left\|\underline{\underline{p}}^{(1)}_{me_i}\right\| / \left\|\underline{\underline{p}}^{(1)}_{ee_i}\right\| \quad (47)$$

decrease as d decreases. As a result, mutual coupling between helices results in a smaller increase of the magnetic and chiral effects as compared to the effect on the dielectric properties. It is emphasized that the near field is defined as the region where the particle separation is small relative to the ambient wavelength ($kd \ll 1$). This does not necessarily imply that the particle separation is of the same order of magnitude as that of the characteristic size of the individual particles.

The previous results can be readily generalised to a system of N ($N \geq 2$) particles. The coupling for each particle i is obtained by replacing all indices '2' by $j = 2, \ldots, N$; by replacing $\underline{d}$ by $\underline{d}_{ij}$ which measures the distance between particles i and j , and summing over all $N-1$ individual perturbations. The first-order coupling for the complete system of N particles is then obtained by summation over all N particles i and averaging:-

$$\begin{bmatrix} \epsilon_o\Delta\underline{\underline{p}}^{(1)}_{ee} & \sqrt{\mu_o\epsilon_o}\,\Delta\underline{\underline{p}}^{(1)}_{em} \\ \sqrt{\mu_o\epsilon_o}\,\Delta\underline{\underline{p}}^{(1)}_{me} & \mu_o\Delta\underline{\underline{p}}^{(1)}_{mm} \end{bmatrix} = \frac{1}{N}\sum_{i=1}^{N}\sum_{j\neq i}^{N}$$
$$\begin{bmatrix} \epsilon_o\underline{\underline{p}}_{ee_i} \cdot \underline{\underline{P}}^{(1)}_j + \sqrt{\mu_o\epsilon_o}\underline{\underline{p}}_{emi} \cdot \underline{\underline{U}}^{(1)}_j & \epsilon_o\underline{\underline{p}}_{ee_i} \cdot \underline{\underline{Q}}^{(1)}_j + \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{emi} \cdot \underline{\underline{V}}^{(1)}_j \\ \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{mei} \cdot \underline{\underline{P}}^{(1)}_j + \mu_o\underline{\underline{p}}_{mmi} \cdot \underline{\underline{U}}^{(1)}_j & \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{mei} \cdot \underline{\underline{Q}}^{(1)}_j + \mu_o\underline{\underline{p}}_{mmi} \cdot \underline{\underline{V}}^{(1)}_j \end{bmatrix} \quad (48)$$

Since first-order scattering only involves pairs of particles, this is also the total first-order coupling for the system.

4.3 Higher-Order Coupling

Higher-order multiple scattering can equally be incorporated into the dipolarisability dyadics. For the q th-order multiple scattering onto particle 1, the incident wave is the $(q-1)$ th-order multiply scattered wave from any other particle. Thus, generalising Eq. (37), the total interaction is:-

$$\begin{bmatrix} \epsilon_o \underline{\underline{p}}^{(\infty)}_{ee_1} & \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}^{(\infty)}_{em_1} \\ \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}^{(\infty)}_{me_1} & \mu_o \underline{\underline{p}}^{(\infty)}_{mm_1} \end{bmatrix} \cdot \begin{bmatrix} \underline{E} \\ \underline{H} \end{bmatrix} =$$

$$\begin{bmatrix} \epsilon_o \underline{\underline{p}}_{ee_1} & \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{em_1} \\ \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}_{me_1} & \mu_o \underline{\underline{p}}_{mm_1} \end{bmatrix} \cdot \begin{bmatrix} \underline{E} + \sum_{q=1}^{\infty} \left[\underline{E}^{(q)} - \underline{E}^{(q-1)}\right] \\ \underline{H} + \sum_{q=1}^{\infty} \left[\underline{H}^{(q)} - \underline{H}^{(q-1)}\right] \end{bmatrix} \quad (49)$$

where ${}^{(\infty)}$ signifies modification for all higher-order scattering and with $\underline{E}^{(0)} \equiv \underline{E}$, $\underline{H}^{(0)} \equiv \underline{H}$. q th-order scattering involves a series of q first-order scattering processes in a string of maximal $q+1$ scatterers. The waves incident on the first and last particle of the string are related by:-

$$\begin{bmatrix} \underline{E}^{(q)} - \underline{E}^{(q-1)} \\ \underline{H}^{(q)} - \underline{H}^{(q-1)} \end{bmatrix} = \prod_{r}^{q} \begin{bmatrix} \underline{\underline{P}}^{(1)}_{r} & \underline{\underline{Q}}^{(1)}_{r} \\ \underline{\underline{U}}^{(1)}_{r} & \underline{\underline{V}}^{(1)}_{r} \end{bmatrix} \cdot \begin{bmatrix} \underline{E} \\ \underline{H} \end{bmatrix} \quad (50)$$

where r runs over the indices of the particles contained in a particular string of length q (not necessarily enumerated in ascending order and with the possibility of repetition of indices). Hence the explicit form of the modified dipolarisability dyadics is strongly dependent on the specific configuration. Its complexity increases rapidly as the number of particles increases and if the symmetry of the arrangement decreases. For the two-particle system, the total scattering is the sum of all even- and odd-order scattering:-

$$\begin{bmatrix} \epsilon_o \underline{\underline{p}}^{(\infty)}_{ee_i} & \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}^{(\infty)}_{em_i} \\ \sqrt{\mu_o\epsilon_o}\,\underline{\underline{p}}^{(\infty)}_{me_i} & \mu_o \underline{\underline{p}}^{(\infty)}_{mm_i} \end{bmatrix}$$

$$
\begin{aligned}
&= \begin{bmatrix} \epsilon_o \underline{\underline{p}}_{ee_i} & \sqrt{\mu_o \epsilon_o}\, \underline{\underline{p}}_{em_i} \\ \sqrt{\mu_o \epsilon_o}\, \underline{\underline{p}}_{me_i} & \mu_o \underline{\underline{p}}_{mm_i} \end{bmatrix} \left\{ \sum_{q=1}^{\infty} \left[\begin{bmatrix} \underline{\underline{P}}_{\bar{i}}^{(1)} & \underline{\underline{Q}}_{\bar{i}}^{(1)} \\ \underline{\underline{U}}_{\bar{i}}^{(1)} & \underline{\underline{V}}_{\bar{i}}^{(1)} \end{bmatrix} \right.\right. \\
&\quad \left.\left. \cdot \begin{bmatrix} \underline{\underline{P}}_{i}^{(1)} & \underline{\underline{Q}}_{i}^{(1)} \\ \underline{\underline{U}}_{i}^{(1)} & \underline{\underline{V}}_{i}^{(1)} \end{bmatrix} \right]^{q-1} \cdot \left[\underline{\underline{I}} + \begin{bmatrix} \underline{\underline{P}}_{\bar{i}}^{(1)} & \underline{\underline{Q}}_{\bar{i}}^{(1)} \\ \underline{\underline{U}}_{\bar{i}}^{(1)} & \underline{\underline{V}}_{\bar{i}}^{(1)} \end{bmatrix} \right] \right\} \\
&= \begin{bmatrix} \epsilon_o \underline{\underline{p}}_{ee_i} & \sqrt{\mu_o \epsilon_o}\, \underline{\underline{p}}_{emi} \\ \sqrt{\mu_o \epsilon_o}\, \underline{\underline{p}}_{mei} & \mu_o \underline{\underline{p}}_{mmi} \end{bmatrix} \left\{ \left[\underline{\underline{I}} - \begin{bmatrix} \underline{\underline{P}}_{\bar{i}}^{(1)} & \underline{\underline{Q}}_{\bar{i}}^{(1)} \\ \underline{\underline{U}}_{\bar{i}}^{(1)} & \underline{\underline{V}}_{\bar{i}}^{(1)} \end{bmatrix} \right.\right. \\
&\quad \left.\left. \cdot \begin{bmatrix} \underline{\underline{P}}_{i}^{(1)} & \underline{\underline{Q}}_{i}^{(1)} \\ \underline{\underline{U}}_{i}^{(1)} & \underline{\underline{V}}_{i}^{(1)} \end{bmatrix} \right]^{-1} \cdot \left[\underline{\underline{I}} + \begin{bmatrix} \underline{\underline{P}}_{\bar{i}}^{(1)} & \underline{\underline{Q}}_{\bar{i}}^{(1)} \\ \underline{\underline{U}}_{\bar{i}}^{(1)} & \underline{\underline{V}}_{\bar{i}}^{(1)} \end{bmatrix} \right] \right\} \qquad (51)
\end{aligned}
$$

where $i = 1, 2$ and $\bar{1} = 2$, $\bar{2} = 1$. Writing Eq. (37) as:-

$$
\begin{aligned}
&\begin{bmatrix} \underline{\underline{P}}_{i}^{(1)} & \underline{\underline{Q}}_{i}^{(1)} \\ \underline{\underline{U}}_{i}^{(1)} & \underline{\underline{V}}_{i}^{(1)} \end{bmatrix} = \\
&\begin{bmatrix} \epsilon_o \underline{\underline{p}}_{ee_{\bar{i}}} & \sqrt{\mu_o \epsilon_o}\, \underline{\underline{p}}_{em_{\bar{i}}} \\ \sqrt{\mu_o \epsilon_o}\, \underline{\underline{p}}_{me_{\bar{i}}} & \mu_o \underline{\underline{p}}_{mm_{\bar{i}}} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \epsilon_o \Delta \underline{\underline{p}}_{ee_{\bar{i}}}^{(1)} & \sqrt{\mu_o \epsilon_o}\, \Delta \underline{\underline{p}}_{em_{\bar{i}}}^{(1)} \\ \sqrt{\mu_o \epsilon_o}\, \Delta \underline{\underline{p}}_{me_{\bar{i}}}^{(1)} & \mu_o \Delta \underline{\underline{p}}_{mm_{\bar{i}}}^{(1)} \end{bmatrix} \qquad (52)
\end{aligned}
$$

and denoting all six-dyadics containing the four $\underline{\underline{p}}_{kl\,i}$ by $\left[\underline{\underline{p}}_i\right]$, the total modified dipolarisability dyadics can be expressed in terms of the unperturbed dyadics and the first-order dyadic perturbations:-

$$
\begin{aligned}
\left[\underline{\underline{p}}_i^{(\infty)}\right] &= \left[\underline{\underline{p}}_i\right] \cdot \left\{ \left[\underline{\underline{I}} - \left[\underline{\underline{p}}_i\right]^{-1} \cdot \left[\Delta \underline{\underline{p}}_i^{(1)}\right] \cdot \left[\underline{\underline{p}}_{\bar{i}}\right]^{-1} \cdot \left[\Delta \underline{\underline{p}}_{\bar{i}}^{(1)}\right] \right]^{-1} \right. \\
&\quad \left. \cdot \left[\underline{\underline{I}} + \left[\underline{\underline{p}}_i\right]^{-1} \cdot \left[\Delta \underline{\underline{p}}_i^{(1)}\right] \right] \right\} \qquad (53)
\end{aligned}
$$

It is verified that if the perturbations are small, the total perturbation $\left[\Delta \underline{\underline{p}}_i^{(\infty)}\right] \triangleq \left[\underline{\underline{p}}_i^{(\infty)}\right] - \left[\underline{\underline{p}}_i\right]$ reduces to the first-order perturbation:-

$$
\left[\Delta \underline{\underline{p}}_i^{(\infty)}\right] \approx \left[\Delta \underline{\underline{p}}_i^{(1)}\right] \qquad (54)
$$

The computation of the total mutual coupling for multi-particle systems is of considerable complexity. The part which involves all pairs of particles serves as a first approximation and is obtained by replacing $\bar{i}$ in Eq. (53) by $j \neq i$ and summation:-

$$\left[\underline{\underline{p}}_i^{(\infty)}\right] \approx \sum_{j \neq i}^{N} \left[\underline{\underline{p}}_i\right] \cdot \left\{ \left[\underline{\underline{I}} - \left[\underline{\underline{p}}_i\right]^{-1} \cdot \left[\Delta \underline{\underline{p}}_i^{(1)}\right] \cdot \left[\underline{\underline{p}}_j\right]^{-1} \cdot \left[\Delta \underline{\underline{p}}_j^{(1)}\right] \right]^{-1} \cdot \left[\underline{\underline{I}} + \left[\underline{\underline{p}}_i\right]^{-1} \cdot \left[\Delta \underline{\underline{p}}_i^{(1)}\right] \right] \right\} \tag{55}$$

Further refinements involving all contributions of three-, four-, ..., N-particle strings may be added.

Assuming the total coupling has been obtained for each particle i in a particular configuration, the individual $\left[\underline{\underline{p}}_i^{(\infty)}\right]$ enable the computation of the perturbed dipole moments, $\underline{p}_{e,m_i}^{(\infty)}$, to be made:-

$$\begin{bmatrix} \underline{p}_{e_i}^{(\infty)} \\ \underline{p}_{m_i}^{(\infty)} \end{bmatrix} = \begin{bmatrix} \epsilon_o \underline{\underline{p}}_{ee_i}^{(\infty)} & \sqrt{\mu_o \epsilon_o}\, \underline{\underline{p}}_{em_i}^{(\infty)} \\ \sqrt{\mu_o \epsilon_o}\, \underline{\underline{p}}_{me_i}^{(\infty)} & \mu_o \underline{\underline{p}}_{mm_i}^{(\infty)} \end{bmatrix} \cdot \begin{bmatrix} \underline{E} \\ \underline{H} \end{bmatrix} \tag{56}$$

from which the perturbed depolarisation fields are obtained as:-

$$\underline{\underline{L}}_i \cdot \sum_{j \neq i}^{N} \frac{\underline{p}_{e_j}^{(\infty)}}{\epsilon_o}, \qquad \underline{\underline{L}}_i \cdot \sum_{j \neq i}^{N} \frac{\underline{p}_{m_j}^{(\infty)}}{\mu_o} \tag{57}$$

in which ϵ_o and μ_o, the parameters of the host medium, are now the *exact* values to be used, by virtue of the de-embedding (although they are obviously not the parameters of the effective medium). Hence estimates in the form of apparent medium parameters [11] are no longer required. The depolarisation fields can be homogenised by averaging Eq. (57) with respect to all N particles i.

4.4. Examples

The modified dipolarisabilities have been computed numerically for two identical parallel and two identical orthogonal helices. The helices form a strictly planar pair in the plane of the incident wave front and are identical to those used in section 3.5.

Figures 10–11 show the real and imaginary parts of the 36 modified dipolarisabilities $\left[\underline{\underline{p}}_{kl}^{(\infty)}\right]_{ij}$ as a function of p/L_{turn} for the system of two parallel helices. field operation ($kd \leq 0.01$). In Figure 10, the distance between the helix centres is $\underline{d} = (10\text{mm})\,\underline{1}_y$. The dipolarisabilities are almost identical to those of the isolated helix [6]. In Figure 11, the distance is $\underline{d} = (3\text{mm})\,\underline{1}_y$. The dominant $\left[\underline{\underline{p}}^{(\infty)}_{ee}\right]_{zz}$ is overall lowered, predominantly at larger p/L_{turn} -ratios, demonstrating the depolarisation. The maxima of several off-diagonal elements are seen to have been shifted, usually towards lower values of p/L_{turn} , as compared to the corresponding elements for an isolated helix. In particular the dominant chirality term, $\left[j\underline{\underline{p}}_{em}\right]_{zz}$, has its maximum shifted towards lower values of p/L_{turn} . If d is further decreased, the maxima shift further down and $\left[\underline{\underline{p}}^{(\infty)}_{ee}\right]_{zz}$ starts to increase less than proportionally with p/L_{turn} .

Figure 12 shows $\underline{\underline{p}}^{(\infty)}_{kl}$ for each one of two perpendicular[3] helices for the same distance[4] $\underline{d} = (3\text{mm})\,\underline{1}_y$. The coupling is clearly much smaller than for parallel helices, and mainly affects the off-diagonal elements only.

Figure 13 shows $\underline{\underline{p}}^{(\infty)}_{kl}$, for each one of two parallel, co-linear helices with $\underline{d} = (3\,\text{mm})\underline{1}_z$. The coupling leads to a maximum for $\text{Re}\left[j\underline{\underline{p}}_{em}\right]_{zz}$ that is about twice the value as that for an isolated helix. This maximum shifts towards higher p/L_{turn} as the separation distance is decreased. The value of $\text{Re}\left[\underline{\underline{p}}_{ee}\right]_{zz}$ has increased by one order of magnitude for large p/L_{turn} as compared to an isolated helix. Unlike strictly planar parallel helices, the co-linear configuration gives rise to an increase of the elements of $\text{Re}\left[\underline{\underline{p}}_{mm}\right]_{zz}$, as a result of the magnetic flux lines that couple to the other helix being parallel rather than anti-parallel with the screw sense of the latter helix.

---

[3] For the 90deg rotated helix, whose axis is along $\underline{1}_y$, the dyadics have been transformed accordingly, through multiplication by the 90deg-rotation dyadic.

[4] This small value of d would cause the helices to start making contact for large p/L_{turn} -ratios. The situation is therefore idealised here by considering both helices as point-polarisable particles.

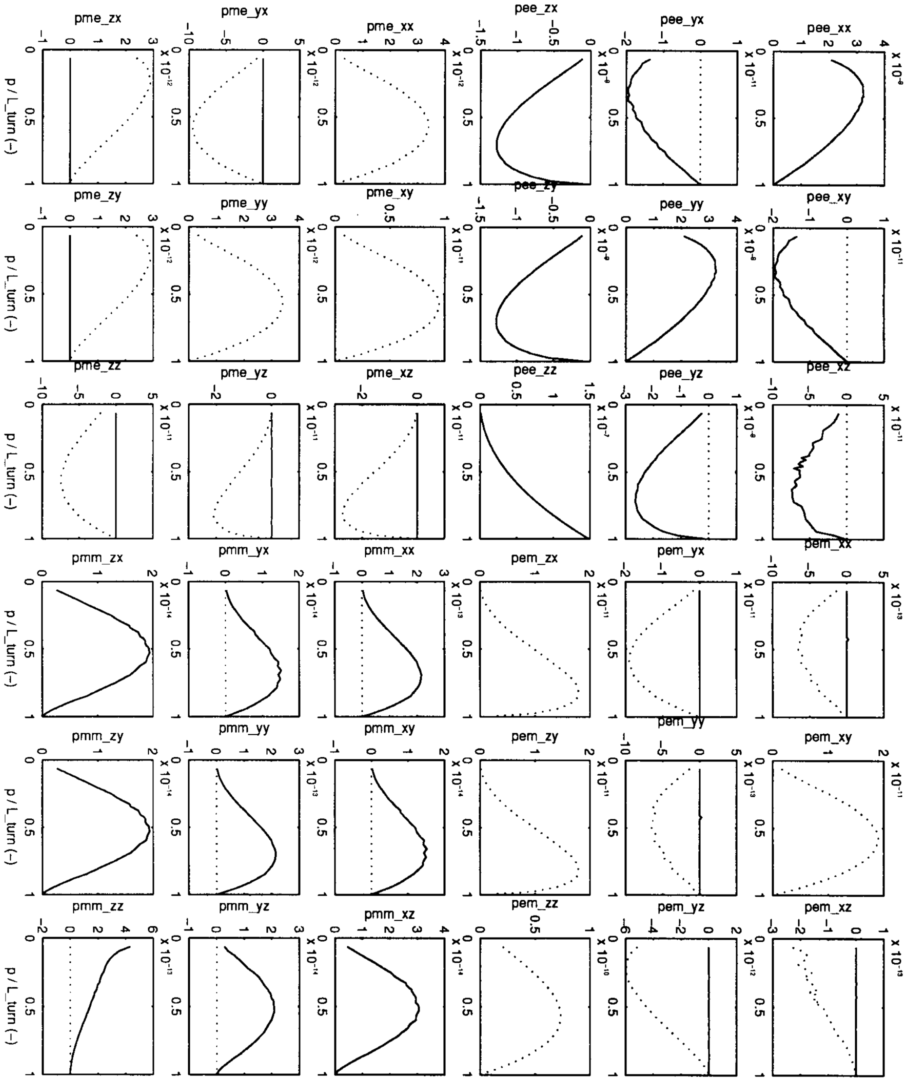

Figure 10. Complex modified dipolarisabilities for two strictly planar, parallel helices. $\underline{d} = (10\,\mathrm{mm})\ \underline{1}_y$. —: real part, $\cdots$: imaginary part.

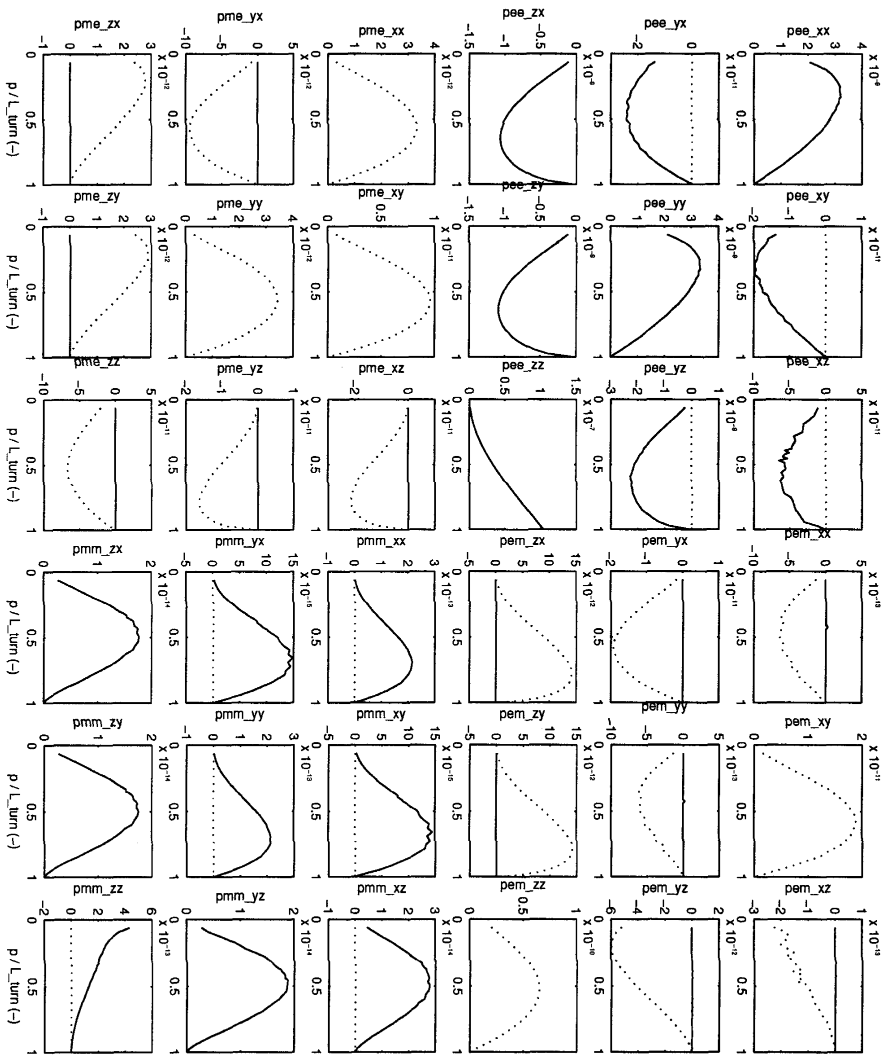

Figure 11. Complex modified dipolarisabilities for two strictly planar, parallel helices. $\underline{d} = (3\,\mathrm{mm})\ \underline{1}_y$. —: real part, $\cdots$: imaginary part.

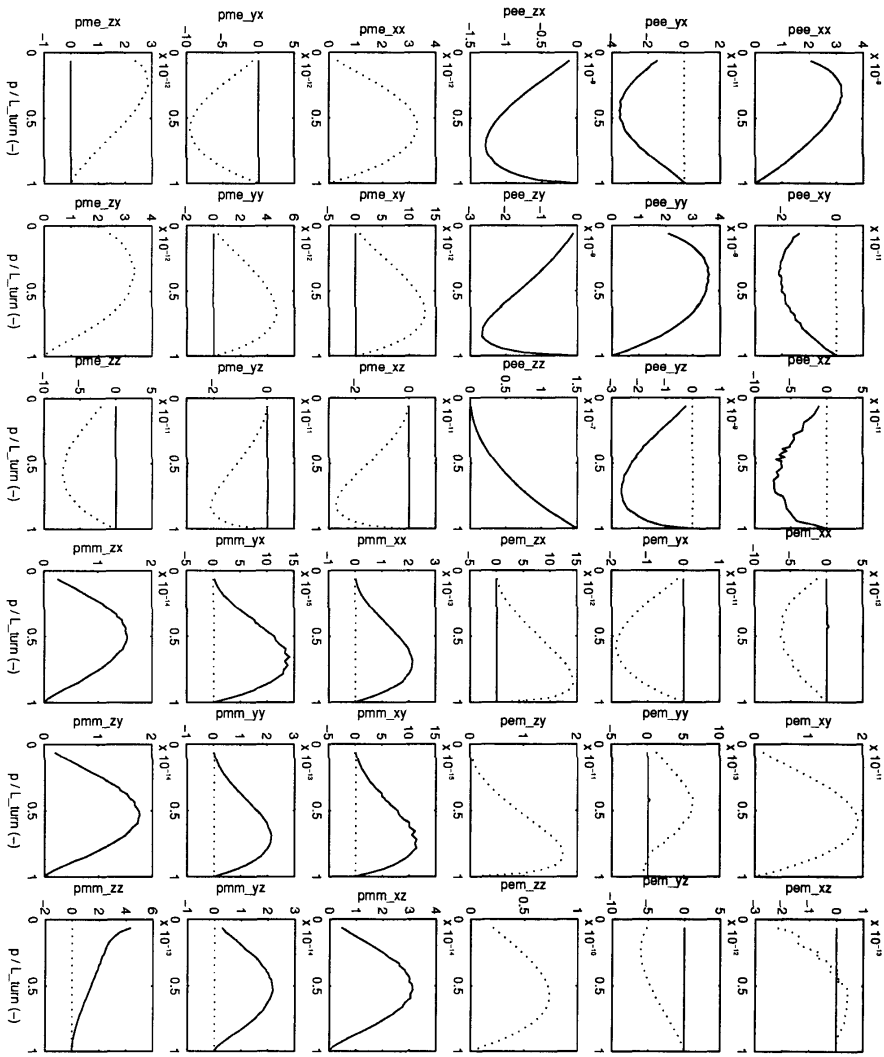

Figure 12. Complex modified dipolarisabilities for two strictly planar, perpendicular helices. $\underline{d} = (3\,\mathrm{mm})\ \underline{1}_y$. —: real part, $\cdots$: imaginary part.

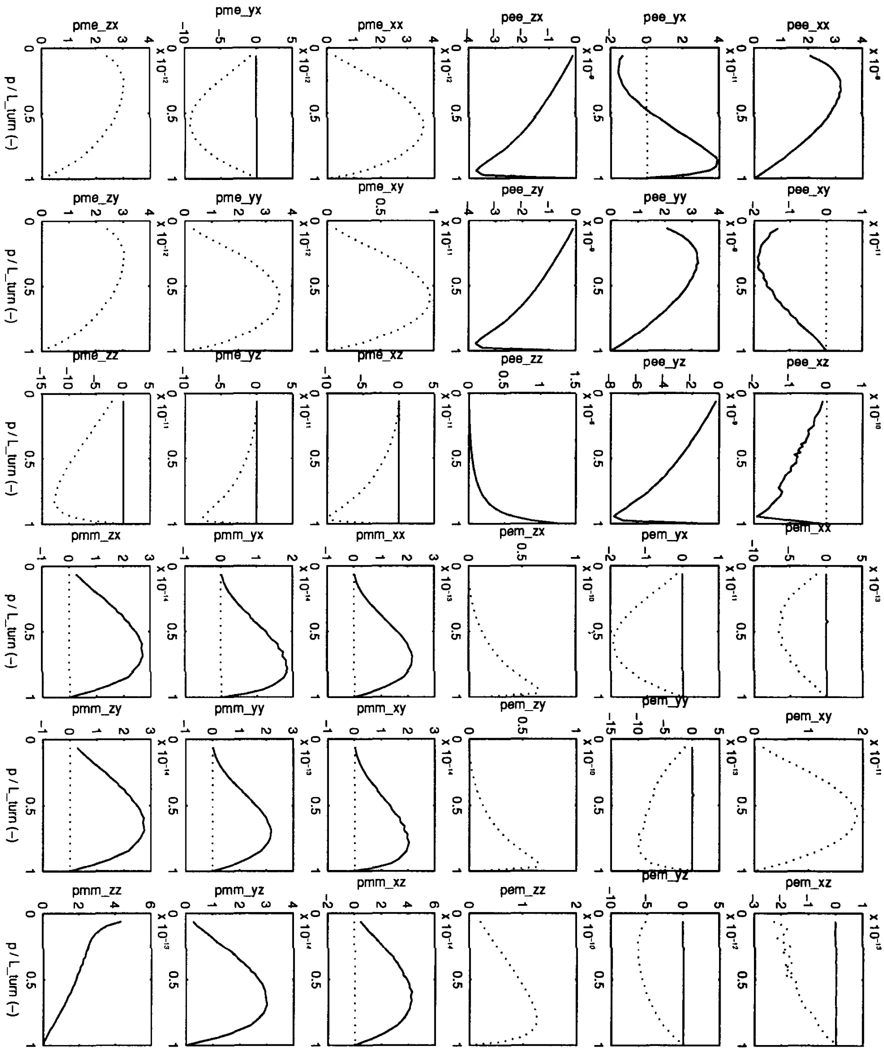

Figure 13. Complex modified dipolarisabilities for two strictly co-linear helices. $\underline{d} = (3\,\mathrm{mm})\ \underline{1}_z$. —: real part, $\cdots$: imaginary part.

5. CONCLUSIONS

A six-dyadic $\underline{\underline{M}}$ has been determined for quantifying the interaction between two general electrically small bianisotropic particles, modelled as a pair of paired dipoles, for known inducing fields. Near field, far field and boundary field contributions have been identified in the general expression for $\underline{\underline{M}}$. Numerical results for three canonical configurations, parallel, perpendicular and co-linear helix pairs, have been given and explained by considering the characteristics of the dipolarisabilities of an isolated helix in combination with radiation patterns of dipole antennas. In particular, co-linear helices show higher interaction than parallel helices which in turn show higher interaction than perpendicular helices.

De-embedding of bianisotropic particles has been introduced to account for mutual coupling in the Maxwell-Garnett mixing formulas, by replacing dipolarisabilities of single isolated particles by modified dipolarisabilities of individual embedded particles. The total coupling Eq. (53) between particles has been shown to be computable from the first-order coupling perturbations Eq. (37), involving only the unperturbed dyadics and the separation distance. Mutual coupling between a strictly planar pair of parallel identical helices has been found to influence the optimisation of the helix geometry in the sense of shifting the maximum of the chiral dipolarisability. The observed differences are small because a system of only two particles has been considered here. Also, mutual coupling was shown to give rise to a smaller increase of the chiral and magnetic properties as compared to the dielectric (magnetic) properties of a helix-loaded dielectric (magnetic) medium. Therefore, this coupling causes the *relative* chirality (that is the ratio of chirality factor to the average refractive index of the material, when expressed in the $\underline{E}$, $\underline{H}$ formalism) of such a medium to decrease.

Although only pairs of particles have here been focused upon, it is expected that this should lead to sufficient accuracy for small particles, in view of their small scattering cross-section, which makes the contribution of higher-order scattering to the total coupling comparatively small. Finally it is noticed that coupling in isotropic high-density helix-based composites is less spatially dispersive than in isotropic composites containing straight wire inclusions: parallel helices cannot be brought as close together without entanglement as straight wires can, whereas perpendicular helices can be brought closer together than straight wire sections, for a given wire length.

REFERENCES

1. Weiglhofer, W. S., A. Lakhtakia, and J. C. Monzon, "Maxwell-Garnett model for composites of electrically small uniaxial objects," *Microw Opt Tech Lett.*, Vol. 6, No. 112, 681–684, 1993.
2. Sihvola, A. H., "On polarizability properties of bianisotropic spheres with noncomplete magnetoelectric dyadics," *Microw Opt Tech Lett.*, Vol. 7, No. 14, 658–661, 1994.
3. Tretyakov, S. A., and F. Mariotte, "Maxwell Garnett modeling of uniaxial chiral composites with bianisotropic inclusions," *J. Electromagn Waves Applic.*, Vol. 9, No. 7/8, 1011–1025, 1995.
4. Guérin, F., "Contribution à l'étude théorique et expérimentale des matériaux composites chiraux et bianisotropes dans le domain microonde," Doctoral Thesis, Université de Limoges, 154–158, July 1995.
5. Mariotte, F., S. A. Tretyakov, and B. Sauviac, "Isotropic chiral composite modeling: comparison between analytical, numerical and experimental results," *Microw Opt Tech Lett.*, Vol. 7, No. 18, 861–864, 1994.
6. Arnaut, L. R., "Computation of dipolarisability tensors of helix-based bianisotropic media," Proc 2nd Journées Maxwell – International Conference on Electromagnetics (6–9 Jun 1995, Bordeaux, France), 319–327.
7. Arnaut, L. R., and L. E. Davis, "Effect of mutual coupling between aligned helices on the constitutive dyadics of high-density bianisotropic composites," Proc "Chiral'95" International Conference (11–14 Oct 1995, State College, PA), 80–86. Note that the sign of W_m used in this paper does not comply with source equivalence for the magnetic current in the reaction integral, and has therefore been changed here.
8. Whites, K. W., "Full-wave computation of constitutive parameters for lossless composite materials", *IEEE Trans Antennas Propag.*, Vol. 43, No. 4, 376–384, 1995.
9. Arnaut, L. R., "Axial representation of the dipolarisability dyadics of chiral structures," Proc "Chiral'95" International Conference (11–14 Oct 1995, State College, PA), 95–100.
10. Lindell, I. V., A. H. Sihvola, and K. Suchy, " Six-vector formalism in electromagnetics of bi-anisotropic media," *J. Electromag. Waves Applic.*, Vol. 9, No. 7/8, 887–903, 1995.

11. Sihvola, A. H., and J. A. Kong, "Effective permittivity of dielectric mixtures," *IEEE Trans Geoscience and Remote Sensing*, Vol. 26, nr. 4, 420–429, 1988; Corrections, *ibid*, Vol. 27, No. 1, 101–102, 1989.
12. Rumsey, V. H., "Reaction concept in electromagnetic theory," *Phys Rev*, Vol. 94, 1483–1491, 1954.
13. Jackson, J. D., *Classical Electrodynamics*, Wiley, NY, 2nd ed., Ch 4, 1975.
14. Kong, J. A., *Electromagnetic Wave Theory*, Wiley Interscience, NY, 2nd ed., 1990.
15. Van Bladel, J., *Electromagnetic Fields*, Hemisphere, NY, 2nd ed., 213–218. 1985.
16. Lindell, I. V., *Methods for Electromagnetic Field Analysis*, Clarendon, Oxford, 1992.