Progress In Electromagnetics Research, *PIER 19, 301–318, 1998*

SINGULARITIES AND DISCONTINUITIES IN THE EIGENFUNCTION EXPANSIONS OF THE DYADIC GREEN'S FUNCTIONS FOR BIISOTROPIC MEDIA

E. L. Tan and S. Y. Tan

School of Electrical and Electronic Engineering,
Nanyang Technological University,
Singapore 639798, Republic of Singapore.

1. INTRODUCTION

The dyadic Green's function technique [1–4] is a very powerful and elegant method in the study of electromagnetic wave propagation and scattering, and has been extensively developed in recent years to solve many (reciprocal) chiral- and biisotropic-related problems [5–11]. In many applications such as multi-layered media [2, 12–14] and waveguides [1], [15], the dyadic Green's function is expanded in terms of vector wave functions or the eigenfunctions of a particular geometry. For the last two decades, there have been active discussions regarding the singularity and the completeness of the eigenfunction representations of the dyadic Green's function for isotropic media [16, 17]. There has also been discussion on the discontinuous behavior of these eigenfunction expansions across the source point in a isotropic medium [18, 19]. For the case of chiral, biisotropic and even more complex media, the literature on the singularities of dyadic Green's functions can be found in [12, 20–23]. These singularities have been treated mostly based on rigorous extraction from integrals or principal-volume method [4, 24].

In this paper, the singularities and discontinuities present in the eigenfunction expansions of the dyadic Green's functions for biisotropic media are revisited and discussed based on the theory of distributions [25]. These singularities and discontinuities are derived directly from Maxwell equations cast into dyadic forms prior to explicit determination of the dyadic Green's function formula. For both electric and magnetic dyadic Green's functions, the source point singular terms are obtained without any integration and the discontinuities in their tangential and normal components across the source point are determined in compact forms directly from Maxwell dyadic equations. From the discontinuity relations, boundary conditions involving the tangential and normal components of electric and magnetic fields are derived. Due to the availability of different sets of constitutive relations characterizing a biisotropic medium [26, 11], the source point singular terms and the discontinuity relations are discussed based on the commonly used Post-Jaggard (Section 2) and Drude-Born-Federov (Section 3) relations. In view of the possible distinctions between these relations, particularly when the constitutive parameters and sources do not conform to certain transformations [26, 27], the permittivity, permeability, vectors and dyadics will be augmented with P (for Post-Jaggard) and F (for Drude-Born-Federov) in the subscripts to avoid any confusion. Throughout the following analysis, $e^{-i\omega t}$ time dependence is assumed and suppressed.

2. POST-JAGGARD RELATIONS

A biisotropic medium can be characterized by Post-Jaggard constitutive relations [26]:

$$\overline{D}_P = \epsilon_P \overline{E}_P + (\psi + i\xi)\overline{B}_P \tag{1}$$

$$\overline{H}_P = \frac{1}{\mu_P}\overline{B}_P - (\psi - i\xi)\overline{E}_P \tag{2}$$

where chirality is introduced through ξ and nonreciprocity through ψ. Incorporating equations (1) and (2) into Maxwell equations and defining the electric and magnetic dyadic Green's functions as

$$\overline{E}_P(\overline{r}) = \iiint\limits_{V'} dv' \,\overline{\overline{G}}_{eP}(\overline{r}, \overline{r}') \cdot \overline{J}_P(\overline{r}') \tag{3}$$

$$\overline{H}_P(\overline{r}) = \iiint\limits_{V'} dv' \,\overline{\overline{G}}_{mP}(\overline{r},\overline{r}') \cdot \overline{J}_P(\overline{r}'), \tag{4}$$

we obtain the Maxwell dyadic equations

$$\nabla \times \overline{\overline{G}}_{eP} = i\omega\mu_P \overline{\overline{G}}_{mP} + i\omega\mu_P(\psi - i\xi)\overline{\overline{G}}_{eP} \tag{5}$$

$$\begin{aligned}\nabla \times \overline{\overline{G}}_{mP} = & -i\omega[\epsilon_P + \mu_P(\psi^2 + \xi^2)]\overline{\overline{G}}_{eP} \\ & - i\omega\mu_P(\psi + i\xi)\overline{\overline{G}}_{mP} + \overline{\overline{I}}\delta(\overline{r}' - \overline{r})\end{aligned} \tag{6}$$

$$\nabla \cdot \overline{\overline{G}}_{eP} = -\frac{1}{i\omega\epsilon_P}\nabla'\delta(\overline{r}' - \overline{r}) \tag{7}$$

$$\nabla \cdot \overline{\overline{G}}_{mP} = \frac{\psi - i\xi}{i\omega\epsilon_P}\nabla'\delta(\overline{r}' - \overline{r}) \tag{8}$$

where $\overline{\overline{I}}$ is the idemfactor and $\delta(\overline{r}' - \overline{r})$ is the three-dimensional Dirac delta function defined as

$$\overline{J}_P(\overline{r}) = \iiint\limits_{V'} dv' \,\overline{\overline{I}}\delta(\overline{r}' - \overline{r}) \cdot \overline{J}_P(\overline{r}'). \tag{9}$$

As it should be, noting that

$$\nabla'\delta(\overline{r}' - \overline{r}) = -\nabla\delta(\overline{r} - \overline{r}'), \tag{10}$$

the above dyadic equations reduce to those given by Tai [1] for isotropic media where $\psi = \xi = 0$.

Following the notations in [19], let the electric and magnetic dyadic Green's functions be written in general forms as

$$\overline{\overline{G}}_{eP} = \overline{\overline{G}}^0_{eP}\delta(p' - p) + \overline{\overline{G}}^>_{eP}U(p - p') + \overline{\overline{G}}^<_{eP}U(p' - p) \tag{11}$$

$$\overline{\overline{G}}_{mP} = \overline{\overline{G}}^0_{mP}\delta(p' - p) + \overline{\overline{G}}^>_{mP}U(p - p') + \overline{\overline{G}}^<_{mP}U(p' - p). \tag{12}$$

Here, we have expanded each dyadic Green's function into three parts weighted by different distributions: $\delta(p' - p)$ is the one-dimensional Dirac delta function and $U(\pm p \mp p')$ are the Heaviside unit step functions. The '0' part together with $\delta(p' - p)$ gives the singular dyadic term required in biisotropic source region, while the '$>$' and '$<$' parts correspond to $\hat{p}$-propagating solenoidal eigenfunction expansion for

$p > p'$ and $p < p'$ respectively [19]. Substituting (11) and (12) into the Maxwell dyadic equations (5)–(8) and taking the derivatives in the sense of distributions using [30]

$$\nabla \times [\overline{\overline{G}}^{>}_{<} U(\pm p \mp p')] = (\nabla \times \overline{\overline{G}}^{>}_{<}) U(\pm p \mp p') \pm (\hat{p} \times \overline{\overline{G}}^{>}_{<}) \delta(p - p') \quad (13)$$

$$\nabla \cdot [\overline{\overline{G}}^{>}_{<} U(\pm p \mp p')] = (\nabla \cdot \overline{\overline{G}}^{>}_{<}) U(\pm p \mp p') \pm (\hat{p} \cdot \overline{\overline{G}}^{>}_{<}) \delta(p - p') \quad (14)$$

$$\nabla \times [\overline{\overline{G}}^{0} \delta(p' - p)] = (\nabla \times \overline{\overline{G}}^{0}) \delta(p' - p) + \nabla \delta(p' - p) \times \overline{\overline{G}}^{0} \quad (15)$$

$$\nabla \cdot [\overline{\overline{G}}^{0} \delta(p' - p)] = (\nabla \cdot \overline{\overline{G}}^{0}) \delta(p' - p) + \nabla \delta(p' - p) \cdot \overline{\overline{G}}^{0}, \quad (16)$$

we are able to deduce directly the singularities and discontinuities present in the eigenfunction expansions of $\overline{\overline{G}}_{eP}$ and $\overline{\overline{G}}_{mP}$.

For $p = p'$, we have the following equations corresponding to $\frac{\partial}{\partial p'} \delta(p' - p)$:

$$\hat{p} \times \overline{\overline{G}}^{0}_{eP} = \overline{\overline{0}} \quad (17)$$

$$\hat{p} \times \overline{\overline{G}}^{0}_{mP} = \overline{\overline{0}} \quad (18)$$

$$\hat{p} \cdot \overline{\overline{G}}^{0}_{eP} = \frac{1}{i\omega\epsilon_P} \delta_t \hat{p}' \quad (19)$$

$$\hat{p} \cdot \overline{\overline{G}}^{0}_{mP} = -\frac{\psi - i\xi}{i\omega\epsilon_P} \delta_t \hat{p}' \quad (20)$$

where δ_t is the transverse delta function which together with $\delta(p' - p)$ forms the three-dimensional Dirac delta function, i.e. $\delta_t \delta(p' - p) = \delta(\overline{r}' - \overline{r})$. From the above, it follows immediately that

$$\overline{\overline{G}}^{0}_{eP} \delta(p' - p) = \frac{1}{i\omega\epsilon_P} \delta(\overline{r}' - \overline{r}) \hat{p}\hat{p}' \quad (21)$$

$$\overline{\overline{G}}^{0}_{mP} \delta(p' - p) = -\frac{\psi - i\xi}{i\omega\epsilon_P} \delta(\overline{r}' - \overline{r}) \hat{p}\hat{p}'. \quad (22)$$

Hence, we have obtained in a simple manner the explicit expressions of the source point dyadic delta function terms for $\overline{\overline{G}}_{eP}$ and $\overline{\overline{G}}_{mP}$ as given by (21) and (22) respectively. These singular terms are obtained directly from Maxwell equations, prior to explicit determination of the dyadic Green's function formula — which for the unbounded case can be expressed in terms of two isotropic ones with parameters k_+ and

k_- respectively [20], and also prior to thorough investigation of the eigenfunction properties in biisotropic media — where the fields are known to be decomposable in terms of left- and right-circularly polarized components [7, 9]. Moreover, the delta function singularities have been surfaced without an apparent need to specify the shape of an exclusion volume and this approach thus avoids the (sometimes tedious) task of integration over the surface of the exclusion volume as required by the principal-volume method [29, 31]. From these expressions, it is noteworthy that in contrast to the isotropic case, the magnetic dyadic Green's function features an extra delta function term in addition to its solenoidal eigenfunction expansion, in accordance with [21] using Tellegen constitutive relations. The presence of Post-Jaggard chirality and/or nonreciprocity does not affect the source point term for electric dyadic Green's function but it introduces one for magnetic dyadic Green's function.

Employing (21) and (22), we obtain the following equations for $p = p'$ corresponding to $\delta(p' - p)$ in the Maxwell dyadic equations:

$$\hat{p} \times (\overline{\overline{G}}^{>}_{eP} - \overline{\overline{G}}^{<}_{eP}) = -\frac{1}{i\omega\epsilon_P}\nabla_t\delta_t \times \hat{p}\hat{p}' \tag{23}$$

$$\hat{p} \times (\overline{\overline{G}}^{>}_{mP} - \overline{\overline{G}}^{<}_{mP}) = \overline{\overline{I}}_t\delta_t + \frac{\psi - i\xi}{i\omega\epsilon_P}\nabla_t\delta_t \times \hat{p}\hat{p}' \tag{24}$$

$$\hat{p} \cdot (\overline{\overline{G}}^{>}_{eP} - \overline{\overline{G}}^{<}_{eP}) = -\frac{1}{i\omega\epsilon_P}\nabla'_t\delta_t \tag{25}$$

$$\hat{p} \cdot (\overline{\overline{G}}^{>}_{mP} - \overline{\overline{G}}^{<}_{mP}) = \frac{\psi - i\xi}{i\omega\epsilon_P}\nabla'_t\delta_t. \tag{26}$$

In the above, $\overline{\overline{I}}_t$ is the transverse (to $\hat{p}$) part of idemfactor and ∇_t is the gradient operator taken with respect to the transverse coordinates. Equations (23)–(26) describe the discontinuities in the eigenfunction expansions of the dyadic Green's functions for Post-Jaggard biisotropic media. These discontinuities have been expressed in compact forms (not involving summations or integrations of eigenfunctions) for both tangential $(\hat{p}\times)$ and normal $(\hat{p}\cdot)$ components of the dyadics. Note the additional term weighted by $(\psi - i\xi)$ in (24) and (26) compare to isotropic case. Also, notice that the tangential component of the electric dyadic Green's function is not continuous for biisotropic as well as isotropic (eq. (4.33) in [1]) media. The above discontinuities have emerged naturally due to the representations (eigenfunction expansions) of $\overline{\overline{G}}_{eP}$ and $\overline{\overline{G}}_{mP}$ in the forms of (11) and (12) where a point singularity at $\overline{r} = \overline{r}'$ has been modeled by an equivalent layer of surface singularity at $p = p'$ [18]. These discontinuity relations can be verified readily as demonstrated in Appendix A, using the explicit expressions of $\overline{\overline{G}}^{>}_{<}$ which depend on the boundary and/or radiation conditions to be satisfied in a particular interior or exterior boundary value problem.

3. DRUDE-BORN-FEDEROV RELATIONS

Another set of biisotropic constitutive relations which has been in much use is that of Drude-Born-Federov (DBF) [26]:

$$\overline{D}_F = \epsilon_F[\overline{E}_F + (\beta - i\alpha)\nabla \times \overline{E}_F] \tag{27}$$

$$\overline{B}_F = \mu_F[\overline{H}_F + (\beta + i\alpha)\nabla \times \overline{H}_F] \tag{28}$$

where now β is the chirality parameter and α is the nonreciprocity parameter. Following the same token as in the previous section, the electric and magnetic dyadic Green's functions are defined as

$$\overline{E}_F(\overline{r}) = \iiint\limits_{V'} dv'\,\overline{\overline{G}}_{eF}(\overline{r}, \overline{r}') \cdot \overline{J}_F(\overline{r}') \tag{29}$$

$$\overline{H}_F(\overline{r}) = \iiint\limits_{V'} dv'\,\overline{\overline{G}}_{mF}(\overline{r}, \overline{r}') \cdot \overline{J}_F(\overline{r}') \tag{30}$$

with

$$\overline{\overline{G}}_{eF} = \overline{\overline{G}}^0_{eF}\delta(p' - p) + \overline{\overline{G}}^{>}_{eF}U(p - p') + \overline{\overline{G}}^{<}_{eF}U(p' - p) \tag{31}$$

$$\overline{\overline{G}}_{mF} = \overline{\overline{G}}^0_{mF}\delta(p' - p) + \overline{\overline{G}}^{>}_{mF}U(p - p') + \overline{\overline{G}}^{<}_{mF}U(p' - p). \tag{32}$$

Then, the source point dyadic delta function terms are found to be

$$\overline{\overline{G}}^0_{eF}\delta(p' - p) = \frac{1}{i\omega\epsilon_F}\delta(\overline{r}' - \overline{r})\hat{p}\hat{p}' \tag{33}$$

$$\overline{\overline{G}}^0_{mF}\delta(p' - p) = \overline{\overline{0}}. \tag{34}$$

The discontinuity relations can also be determined as

$$\hat{p} \times (\overline{\overline{G}}^{>}_{eF} - \overline{\overline{G}}^{<}_{eF}) = -\frac{1}{i\omega\epsilon_F}\nabla_t\delta_t \times \hat{p}\hat{p}' - \frac{\omega\mu_F k_0^2(\alpha - i\beta)}{k_F^2}\overline{\overline{I}}_t\delta_t \tag{35}$$

$$\hat{p} \times (\overline{\overline{G}}^{>}_{mF} - \overline{\overline{G}}^{<}_{mF}) = \frac{k_0^2}{k_F^2} \overline{\overline{I}}_t \delta_t \tag{36}$$

$$\hat{p} \cdot (\overline{\overline{G}}^{>}_{eF} - \overline{\overline{G}}^{<}_{eF}) = -\frac{1}{i\omega\epsilon_F} \nabla'_t \delta_t \tag{37}$$

$$\hat{p} \cdot (\overline{\overline{G}}^{>}_{mF} - \overline{\overline{G}}^{<}_{mF}) = \overline{0} \tag{38}$$

where

$$k_F^2 = \omega^2 \mu_F \epsilon_F \tag{39}$$

$$k_0^2 = \frac{k_F^2}{1 - k_F^2(\alpha^2 + \beta^2)}. \tag{40}$$

Equations (33)–(34) and (35)–(38) give respectively the singularities and discontinuities in the eigenfunction expansions of the dyadic Green's functions for DBF biisotropic media. Note their differences compare to those of Post-Jaggard (21)–(26). Due to the solenoidal property of DBF magnetic field, the source point dyadic delta function term (34), and the discontinuity relations (36) and (38) for the magnetic dyadic Green's function $\overline{\overline{G}}_{mF}$ are analogous to those of isotropic $\overline{\overline{G}}_m$. For the electric dyadic Green's function $\overline{\overline{G}}_{eF}$, the source point term (33) and the discontinuity in the normal component of the dyadic (37) have the same forms as those of isotropic and Post-Jaggard biisotropic media, while there is an additional term weighted by $(\alpha - i\beta)$ in (35) for the tangential component of the dyadic.

Although the above DBF singularities and discontinuities are seen to be distinct from those of Post-Jaggard, the two sets are intimately related to each other. In fact, it can be shown that they both satisfy the following relations for their singularities

$$\overline{\overline{G}}^0_{eF} = [1 - k_0^2(\alpha - i\beta)^2] \overline{\overline{G}}^0_{eP} - \frac{\omega\mu_F k_0^2(\alpha - i\beta)}{k_F^2} \overline{\overline{G}}^0_{mP} \tag{41}$$

$$\overline{\overline{G}}^0_{mF} = \frac{k_0^2}{k_F^2} [\overline{\overline{G}}^0_{mP} + \omega\epsilon_F(\alpha - i\beta)\overline{\overline{G}}^0_{eP}], \tag{42}$$

provided the constitutive parameters are mapped as in [26]. Similarly, the discontinuities and even the dyadic Green's functions themselves will conform to these transformations, i.e. $\overline{\overline{G}}^0$ replaced by $\hat{p} \times (\overline{\overline{G}}^{>} - \overline{\overline{G}}^{<})$, $\hat{p} \cdot (\overline{\overline{G}}^{>} - \overline{\overline{G}}^{<})$ or $\overline{\overline{G}}$ in (41) and (42). With these transformations, the Green's functions derived in one set of constitutive relations can be translated readily to the other set if desired.

4. APPLICATIONS

While the source point dyadic delta function terms (21)–(22) and (33)–(34) certainly play an important role in numerical integral equation approaches, particularly when computing fields inside the source region [28], the discontinuity relations obtained above can be used to derive the boundary conditions for the electric and magnetic fields across a current sheet $\overline{J}_s$. This current sheet could be fictitious and might be introduced only to facilitate analysis in dealing with certain (e.g., diffraction) problems. In general, the current may consist of tangentially and/or normally (which constitutes a double layer of charge) directed components. Denoting these tangential and normal parts as $\overline{J}_{ts} = \overline{\overline{I}}_t \cdot \overline{J}_s$ and $\overline{J}_{ps} = \hat{p}\hat{p} \cdot \overline{J}_s$ respectively, we shall obtain the boundary conditions involving field components across $\overline{J}_{ts}$ and/or $\overline{J}_{ps}$ directly from the discontinuity relations. In usual practice, one actually elevates the (field) boundary conditions into the (dyadic) discontinuity relations [1, 19].

For Post-Jaggard biisotropic media, we have from (23)–(26),

$$\hat{p} \times (\overline{E}_P^{>} - \overline{E}_P^{<}) = -\frac{1}{i\omega\epsilon_P} \nabla \times \overline{J}_{psP} \tag{43}$$

$$\hat{p} \times (\overline{H}_P^{>} - \overline{H}_P^{<}) = \overline{J}_{tsP} + \frac{\psi - i\xi}{i\omega\epsilon_P} \nabla \times \overline{J}_{psP} \tag{44}$$

$$\hat{p} \cdot (\overline{E}_P^{>} - \overline{E}_P^{<}) = \frac{1}{i\omega\epsilon_P} \nabla \cdot \overline{J}_{tsP} \tag{45}$$

$$\hat{p} \cdot (\overline{H}_P^{>} - \overline{H}_P^{<}) = -\frac{\psi - i\xi}{i\omega\epsilon_P} \nabla \cdot \overline{J}_{tsP}. \tag{46}$$

Similarly, for DBF biisotropic media, (35)–(38) yield

$$\hat{p} \times (\overline{E}_F^{>} - \overline{E}_F^{<}) = -\frac{1}{i\omega\epsilon_F} \nabla \times \overline{J}_{psF} - \frac{\omega\mu_F k_0^2(\alpha - i\beta)}{k_F^2} \overline{J}_{tsF} \tag{47}$$

$$\hat{p} \times (\overline{H}_F^{>} - \overline{H}_F^{<}) = \frac{k_0^2}{k_F^2} \overline{J}_{tsF} \tag{48}$$

$$\hat{p} \cdot (\overline{E}_F^{>} - \overline{E}_F^{<}) = \frac{1}{i\omega\epsilon_F} \nabla \cdot \overline{J}_{tsF} \tag{49}$$

$$\hat{p} \cdot (\overline{H}_F^> - \overline{H}_F^<) = 0 \tag{50}$$

Note the manner $\overline{J}_{ts}$ and $\overline{J}_{ps}$ contribute to the discontinuous change in the tangential and normal components of $\overline{E}$ and $\overline{H}$. For the case of Post-Jaggard media, $\overline{J}_{psP}$ can produce discontinuities in the tangential components of $\overline{E}_P$ and $\overline{H}_P$, while $\overline{J}_{tsP}$ gives rise to discontinuities in their normal components as well as in the tangential $\overline{H}_P$ component. For the case of DBF media, $\overline{J}_{psF}$ produces discontinuity in the tangential component of $\overline{E}_F$ only, while $\overline{J}_{tsF}$ leads to discontinuities in the tangential components of $\overline{E}_F$ and $\overline{H}_F$ in addition to the normal component of $\overline{E}_F$. When the medium becomes isotropic, i.e. $\psi = \xi = \alpha = \beta = 0$ and $\epsilon_P = \epsilon_F = \epsilon$, $\mu_P = \mu_F = \mu$, the above relations coincide with those (particularly the tangential electric field component) derived in [3] using the conventional integral-and-limiting approach.

Apart from straightforward derivation of boundary conditions, the discontinuity relations also find applications in the method of $\overline{\overline{G}}_m$ [1] or method of $\nabla \times \overline{\overline{G}}_e$ [16], where one often encounters the terms such as $\hat{p} \times (\overline{\overline{G}}_m^> - \overline{\overline{G}}_m^<)\delta(p-p')$ or $\hat{p} \times (\overline{\overline{G}}_e^> - \overline{\overline{G}}_e^<)\delta(p-p')$. These terms can be determined directly from (23)–(24) and (35)–(36) for Post-Jaggard and DBF media respectively.

5. CONCLUSION

Based on the theory of distributions, this paper has presented a simple derivation of the singularities and discontinuities associated with the eigenfunction expansions of the dyadic Green's functions for biisotropic media. The approach deals directly with Maxwell dyadic equations prior to explicit determination of the dyadic Green's function formula and also prior to thorough investigation of the eigenfunction properties in biisotropic media. In obtaining the source point singularities for electric and magnetic dyadic Green's functions, there is no need to specify the shape of an exclusion volume and this avoids any surface integration as required by the principal-volume method. The discontinuity relations describing the changes in the tangential and normal components of the dyadics across a source point have been obtained in compact forms directly from Maxwell dyadic equations. With the aid of explicit eigenfunction expansions of dyadic Green's functions, these discontinuity relations can be verified readily as demonstrated in Appendix A, where we provide the expressions for the unbounded dyadic Green's functions expanded in terms of rectangular, cylindrical and spherical vector wave functions. As an application for the discontinuity relations, boundary conditions involving various field components are derived directly from them. Although these boundary conditions are meant for general biisotropic media, the special case for isotropic media has helped to reassert the fact that the tangential components of both electric dyadic Green's function and electric field will undergo discontinuous changes across a surface containing source point or normally directed current. Due to the availability of different sets of constitutive relations characterizing a biisotropic medium, the singularities and discontinuities have been discussed using Post-Jaggard and Drude-Born-Federov relations. Between these two sets of relations, the transformations for their singularities, discontinuities and the dyadic Green's functions themselves have been given. With these transformations, the Green's functions derived in one set of constitutive relations can be translated readily to the other set if desired. As a final note, the approach described above is seen to be very versatile and its applications to more complex media are currently under investigation.

APPENDIX A

To verify the discontinuity relations in (23)–(26) and (35)–(38), let us choose $\hat{p} = \hat{z}$ and consider the eigenfunction expansion of unbounded (for simplicity) dyadic Green's functions in terms of rectangular vector wave functions, which are defined as [4]

$$\overline{M}(\overline{r}; k_x, k_y, k_z) = [\hat{x}ik_y - \hat{y}ik_x]e^{ik_x x + ik_y y + ik_z z} \tag{A1}$$

$$\overline{N}(\overline{r}; k_x, k_y, k_z) = \frac{1}{k}[-\hat{x}k_z k_x - \hat{y}k_z k_y + \hat{z}(k_x^2 + k_y^2)]e^{ik_x x + ik_y y + ik_z z}, \quad k^2 = k_x^2 + k_y^2 + k_z^2. \tag{A2}$$

The Post-Jaggard and DBF dyadic Green's functions can be expanded as

$$\overline{\overline{G}}_{eP} = \frac{1}{i\omega\epsilon_P}\delta(\overline{r}' - \overline{r})\hat{z}\hat{z} - \frac{\omega\mu_P}{8\pi^2(k_+ + k_-)}\int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \frac{1}{k_x^2 + k_y^2}$$
$$\left\{\left[\frac{k_+}{k_{z+}}\overline{V}^{>}\overline{V}'^{>} + \frac{k_-}{k_{z-}}\overline{W}^{>}\overline{W}'^{>}\right]U(z - z')\right.$$

$$+\left[\frac{k_+}{k_{z+}}\overline{V}^{<}\overline{V}'^{<}+\frac{k_-}{k_{z-}}\overline{W}^{<}\overline{W}'^{<}\right]U(z'-z)\Bigg\} \tag{A3}$$

$$\overline{\overline{G}}_{mP}=-\frac{\psi-i\xi}{i\omega\epsilon_P}\delta(\overline{r}'-\overline{r})\hat{z}\hat{z}+\frac{i}{8\pi^2}\int_{-\infty}^{\infty}dk_x\int_{-\infty}^{\infty}dk_y\frac{1}{k_x^2+k_y^2}$$
$$\Bigg\{\Bigg[\left(\frac{1}{2}-\frac{i\omega\mu_P\psi}{k_++k_-}\right)\frac{k_+}{k_{z+}}\overline{V}^{>}\overline{V}'^{>}$$
$$-\left(\frac{1}{2}+\frac{i\omega\mu_P\psi}{k_++k_-}\right)\frac{k_-}{k_{z-}}\overline{W}^{>}\overline{W}'^{>}\Bigg]U(z-z')$$
$$+\Bigg[\left(\frac{1}{2}-\frac{i\omega\mu_P\psi}{k_++k_-}\right)\frac{k_+}{k_{z+}}\overline{V}^{<}\overline{V}'^{<}$$
$$-\left(\frac{1}{2}+\frac{i\omega\mu_P\psi}{k_++k_-}\right)\frac{k_-}{k_{z-}}\overline{W}^{<}\overline{W}'^{<}\Bigg]U(z'-z)\Bigg\} \tag{A4}$$

$$\overline{\overline{G}}_{eF}=\frac{1}{i\omega\epsilon_F}\delta(\overline{r}'-\overline{r})\hat{z}\hat{z}-\frac{\omega\mu_F k_0^2}{8\pi^2(k_++k_-)k_F^2}\int_{-\infty}^{\infty}dk_x\int_{-\infty}^{\infty}dk_y\frac{1}{k_x^2+k_y^2}$$
$$\Bigg\{\Bigg[\Big(1+ik_+(\alpha-i\beta)\Big)\frac{k_+}{k_{z+}}\overline{V}^{>}\overline{V}'^{>}$$
$$+\Big(1-ik_-(\alpha-i\beta)\Big)\frac{k_-}{k_{z-}}\overline{W}^{>}\overline{W}'^{>}\Bigg]U(z-z')$$
$$+\Bigg[\Big(1+ik_+(\alpha-i\beta)\Big)\frac{k_+}{k_{z+}}\overline{V}^{<}\overline{V}'^{<}$$
$$+\Big(1-ik_-(\alpha-i\beta)\Big)\frac{k_-}{k_{z-}}\overline{W}^{<}\overline{W}'^{<}\Bigg]U(z'-z)\Bigg\} \tag{A5}$$

$$\overline{\overline{G}}_{mF}=\frac{ik_0^2}{8\pi^2(k_++k_-)k_F^2}\int_{-\infty}^{\infty}dk_x\int_{-\infty}^{\infty}dk_y\frac{1}{k_x^2+k_y^2}$$
$$\Bigg\{\left[\frac{k_+^2}{k_{z+}}\overline{V}^{>}\overline{V}'^{>}-\frac{k_-^2}{k_{z-}}\overline{W}^{>}\overline{W}'^{>}\right]U(z-z')$$
$$+\left[\frac{k_+^2}{k_{z+}}\overline{V}^{<}\overline{V}'^{<}-\frac{k_-^2}{k_{z-}}\overline{W}^{<}\overline{W}'^{<}\right]U(z'-z)\Bigg\} \tag{A6}$$

where

$$\overline{V}^{\stackrel{>}{<}}=\overline{M}(\overline{r};k_x,k_y,\pm k_{z+})+\overline{N}(\overline{r};k_x,k_y,\pm k_{z+}) \tag{A7}$$

$$\overline{V}'^{>}_{<} = \overline{M}(\overline{r}'; -k_x, -k_y, \mp k_{z+}) + \overline{N}(\overline{r}'; -k_x, -k_y, \mp k_{z+}) \tag{A8}$$

$$\overline{W}^{>}_{<} = \overline{M}(\overline{r}; k_x, k_y, \pm k_{z-}) - \overline{N}(\overline{r}; k_x, k_y, \pm k_{z-}) \tag{A9}$$

$$\overline{W}'^{>}_{<} = \overline{M}(\overline{r}'; -k_x, -k_y, \mp k_{z-}) - \overline{N}(\overline{r}'; -k_x, -k_y, \mp k_{z-}) \tag{A10}$$

$$k_{z\pm}^2 = k_\pm^2 - k_x^2 - k_y^2 \tag{A11}$$

$$k_\pm = \pm\omega\mu_P\xi + \sqrt{\omega^2\mu_P\epsilon_P + (\omega\mu_P\xi)^2} = \pm k_0^2\beta + k_0\sqrt{1 + k_0^2\beta^2} \tag{A12}$$

From these expressions, the $\overline{\overline{G}}^{>}_{<}$ parts in (11)–(12) and (31)–(32) can be identified easily. Furthermore, the transverse delta function can be expanded as

$$\delta_t = \delta(x - x')\delta(y - y') = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y e^{ik_x(x-x')+ik_y(y-y')} \tag{A13}$$

$$\overline{\overline{I}}_t\delta_t = \frac{1}{4\pi^2}\int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y e^{ik_x(x-x')+ik_y(y-y')} \left[\hat{k}_t\hat{k}_t + (\hat{k}_t \times \hat{z})(\hat{k}_t \times \hat{z})\right] \tag{A14}$$

where $\overline{k}_t = k_x\hat{x} + k_y\hat{y}$ and $\hat{k}_t$ is the unit vector of $\overline{k}_t$. Employing (A3)–(A6) and recognizing (A13)–(A14), we can prove readily the discontinuity relations (23)–(26) and (35)–(38) for rectangular coordinate system. Similarly, one can verify these relations for other Dupin coordinate systems [1]. For convenience, we will provide the explicit expressions for the unbounded dyadic Green's functions expanded in cylindrical and spherical coordinate systems.

In terms of cylindrical vector wave functions defined as [4]

$$\overline{M}_n^{\overset{H}{J}}(\overline{r}; \lambda, h) = \left[\hat{\rho}\frac{in}{\rho}Z_n^{\overset{H}{J}}(\lambda\rho) - \hat{\phi}\frac{\partial}{\partial\rho}Z_n^{\overset{H}{J}}(\lambda\rho)\right]e^{in\phi+ihz} \tag{A15}$$

$$\overline{N}_n^{\overset{H}{J}}(\overline{r}; \lambda, h) = \frac{1}{k}\left[\hat{\rho}ih\frac{\partial}{\partial\rho}Z_n^{\overset{H}{J}}(\lambda\rho) - \hat{\phi}\frac{hn}{\rho}Z_n^{\overset{H}{J}}(\lambda\rho) + \hat{z}\lambda^2 Z_n^{\overset{H}{J}}(\lambda\rho)\right]e^{in\phi+ihz}, \qquad k^2 = \lambda^2 + h^2 \tag{A16}$$

$$Z_n^{\overset{H}{J}}(\lambda\rho) = \begin{cases} H_n^{(1)}(\lambda\rho) \\ J_n(\lambda\rho) \end{cases} \tag{A17}$$

the Post-Jaggard and DBF dyadic Green's functions expanded using $\hat{p} = \hat{\rho}$ are

$$\begin{aligned}
\overline{\overline{G}}_{eP} &= \frac{1}{i\omega\epsilon_P}\delta(\overline{r}' - \overline{r})\hat{\rho}\hat{\rho}' - \frac{\omega\mu_P}{8\pi(k_+ + k_-)}\int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty} \\
&\quad \left\{ \left[\frac{k_+}{\lambda_+^2}\overline{V}^{>}\overline{V}'^{>} + \frac{k_-}{\lambda_-^2}\overline{W}^{>}\overline{W}'^{>}\right] U(\rho - \rho') \right. \\
&\quad \left. + \left[\frac{k_+}{\lambda_+^2}\overline{V}^{<}\overline{V}'^{<} + \frac{k_-}{\lambda_-^2}\overline{W}^{<}\overline{W}'^{<}\right] U(\rho' - \rho) \right\}
\end{aligned} \tag{A18}$$

$$\begin{aligned}
\overline{\overline{G}}_{mP} &= -\frac{\psi - i\xi}{i\omega\epsilon_P}\delta(\overline{r}' - \overline{r})\hat{\rho}\hat{\rho}' + \frac{i}{8\pi}\int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty} \\
&\quad \left\{ \left[\left(\frac{1}{2} - \frac{i\omega\mu_P\psi}{k_+ + k_-}\right)\frac{k_+}{\lambda_+^2}\overline{V}^{>}\overline{V}'^{>} \right.\right. \\
&\quad \left. - \left(\frac{1}{2} + \frac{i\omega\mu_P\psi}{k_+ + k_-}\right)\frac{k_-}{\lambda_-^2}\overline{W}^{>}\overline{W}'^{>}\right] U(\rho - \rho') \\
&\quad + \left[\left(\frac{1}{2} - \frac{i\omega\mu_P\psi}{k_+ + k_-}\right)\frac{k_+}{\lambda_+^2}\overline{V}^{<}\overline{V}'^{<} \right. \\
&\quad \left.\left. - \left(\frac{1}{2} + \frac{i\omega\mu_P\psi}{k_+ + k_-}\right)\frac{k_-}{\lambda_-^2}\overline{W}^{<}\overline{W}'^{<}\right] U(\rho' - \rho) \right\}
\end{aligned} \tag{A19}$$

$$\begin{aligned}
\overline{\overline{G}}_{eF} &= \frac{1}{i\omega\epsilon_F}\delta(\overline{r}' - \overline{r})\hat{\rho}\hat{\rho}' - \frac{\omega\mu_F k_0^2}{8\pi(k_+ + k_-)k_F^2}\int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty} \\
&\quad \left\{ \left[\Big(1 + ik_+(\alpha - i\beta)\Big)\frac{k_+}{\lambda_+^2}\overline{V}^{>}\overline{V}'^{>} \right.\right. \\
&\quad \left. + \Big(1 - ik_-(\alpha - i\beta)\Big)\frac{k_-}{\lambda_-^2}\overline{W}^{>}\overline{W}'^{>}\right] U(\rho - \rho') \\
&\quad + \left[\Big(1 + ik_+(\alpha - i\beta)\Big)\frac{k_+}{\lambda_+^2}\overline{V}^{<}\overline{V}'^{<} \right. \\
&\quad \left.\left. + \Big(1 - ik_-(\alpha - i\beta)\Big)\frac{k_-}{\lambda_-^2}\overline{W}^{<}\overline{W}'^{<}\right] U(\rho' - \rho) \right\}
\end{aligned} \tag{A20}$$

$$\overline{\overline{G}}_{mF} = \frac{ik_0^2}{8\pi(k_+ + k_-)k_F^2} \int_{-\infty}^{\infty} dh \sum_{n=-\infty}^{\infty}$$
$$\left\{ \left[\frac{k_+^2}{\lambda_+^2} \overline{V}^> \overline{V}'^> - \frac{k_-^2}{\lambda_-^2} \overline{W}^> \overline{W}'^> \right] U(\rho - \rho') \right.$$
$$\left. + \left[\frac{k_+^2}{\lambda_+^2} \overline{V}^< \overline{V}'^< - \frac{k_-^2}{\lambda_-^2} \overline{W}^< \overline{W}'^< \right] U(\rho' - \rho) \right\} \tag{A21}$$

where

$$\overline{V}^{>}_{<} = \overline{M}^{H}_{n}{}^{J}(\overline{r}; \lambda_+, h) + \overline{N}^{H}_{n}{}^{J}(\overline{r}; \lambda_+, h) \tag{A22}$$
$$\overline{V}'^{>}_{<} = \overline{M}^{J}_{-n}{}^{H}(\overline{r}'; -\lambda_+, -h) + \overline{N}^{J}_{-n}{}^{H}(\overline{r}'; -\lambda_+, -h) \tag{A23}$$
$$\overline{W}^{>}_{<} = \overline{M}^{H}_{n}{}^{J}(\overline{r}; \lambda_-, h) - \overline{N}^{H}_{n}{}^{J}(\overline{r}; \lambda_-, h) \tag{A24}$$
$$\overline{W}'^{>}_{<} = \overline{M}^{J}_{-n}{}^{H}(\overline{r}'; -\lambda_-, -h) - \overline{N}^{J}_{-n}{}^{H}(\overline{r}'; -\lambda_-, -h) \tag{A25}$$
$$\lambda_\pm^2 = k_\pm^2 - h^2 \tag{A26}$$

In terms of spherical vector wave functions defined as [4]

$$\overline{M}^{h}_{nm}{}^{j}(\overline{r}; k) = \sqrt{\frac{(2n+1)(n-m)!}{4\pi(n+m)!}} z^{h}_{n}{}^{j}(kr) [\hat{\theta} \frac{im}{\sin\theta} P_n^m(\cos\theta)$$
$$- \hat{\phi} \frac{\partial}{\partial\theta} P_n^m(\cos\theta)] e^{im\phi} \tag{A27}$$

$$\overline{N}^{h}_{nm}{}^{j}(\overline{r}; k) = \sqrt{\frac{(2n+1)(n-m)!}{4\pi(n+m)!}} \Big[\hat{r} \frac{n(n+1)}{kr} z^{h}_{n}{}^{j}(kr) P_n^m(\cos\theta)$$
$$+ \frac{1}{kr} \frac{\partial}{\partial r} (r z^{h}_{n}{}^{j}(kr)) (\hat{\theta} \frac{\partial}{\partial\theta} P_n^m(\cos\theta)$$
$$+ \hat{\phi} \frac{im}{\sin\theta} P_n^m(\cos\theta)) \Big] e^{im\phi} \tag{A28}$$

$$z^{h}_{n}{}^{j}(kr) = \begin{cases} h_n^{(1)}(kr) \\ j_n(kr) \end{cases} \tag{A29}$$

the Post-Jaggard and DBF dyadic Green's functions expanded using $\hat{p} = \hat{r}$ are

$$\overline{\overline{G}}_{eP} = \frac{1}{i\omega\epsilon_P}\delta(\overline{r}' - \overline{r})\hat{r}\hat{r}' - \frac{\omega\mu_P}{k_+ + k_-}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\frac{1}{n(n+1)}$$

$$\left\{\left[k_+^2\overline{V}^{>}\overline{V}'^{>} + k_-^2\overline{W}^{>}\overline{W}'^{>}\right]U(r-r')\right.$$

$$\left. + \left[k_+^2\overline{V}^{<}\overline{V}'^{<} + k_-^2\overline{W}^{<}\overline{W}'^{<}\right]U(r'-r)\right\} \tag{A30}$$

$$\overline{\overline{G}}_{mP} = -\frac{\psi - i\xi}{i\omega\epsilon_P}\delta(\overline{r}' - \overline{r})\hat{r}\hat{r}' + \sum_{n=1}^{\infty}\sum_{m=-n}^{n}\frac{i}{n(n+1)}$$

$$\left\{\left[\left(\frac{1}{2} - \frac{i\omega\mu_P\psi}{k_+ + k_-}\right)k_+^2\overline{V}^{>}\overline{V}'^{>}\right.\right.$$

$$\left. - \left(\frac{1}{2} + \frac{i\omega\mu_P\psi}{k_+ + k_-}\right)k_-^2\overline{W}^{>}\overline{W}'^{>}\right]U(r-r')$$

$$ + \left[\left(\frac{1}{2} - \frac{i\omega\mu_P\psi}{k_+ + k_-}\right)k_+^2\overline{V}^{<}\overline{V}'^{<}\right.$$

$$\left.\left. - \left(\frac{1}{2} + \frac{i\omega\mu_P\psi}{k_+ + k_-}\right)k_-^2\overline{W}^{<}\overline{W}'^{<}\right]U(r'-r)\right\} \tag{A31}$$

$$\overline{\overline{G}}_{eF} = \frac{1}{i\omega\epsilon_F}\delta(\overline{r}' - \overline{r})\hat{r}\hat{r}' - \frac{\omega\mu_F k_0^2}{(k_+ + k_-)k_F^2}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\frac{1}{n(n+1)}$$

$$\left\{\left[\Big(1 + ik_+(\alpha - i\beta)\Big)k_+^2\overline{V}^{>}\overline{V}'^{>}\right.\right.$$

$$\left. + \Big(1 - ik_-(\alpha - i\beta)\Big)k_-^2\overline{W}^{>}\overline{W}'^{>}\right]U(r-r')$$

$$ + \left[\Big(1 + ik_+(\alpha - i\beta)\Big)k_+^2\overline{V}^{<}\overline{V}'^{<}\right.$$

$$\left.\left. + \Big(1 - ik_-(\alpha - i\beta)\Big)k_-^2\overline{W}^{<}\overline{W}'^{<}\right]U(r'-r)\right\} \tag{A32}$$

$$\overline{\overline{G}}_{mF} = \frac{ik_0^2}{(k_+ + k_-)k_F^2}\sum_{n=1}^{\infty}\sum_{m=-n}^{n}\frac{1}{n(n+1)}$$

$$\left\{\left[k_+^3\overline{V}^{>}\overline{V}'^{>} - k_-^3\overline{W}^{>}\overline{W}'^{>}\right]U(r-r')\right.$$
$$\left. + \left[k_+^3\overline{V}^{<}\overline{V}'^{<} - k_-^3\overline{W}^{<}\overline{W}'^{<}\right]U(r'-r)\right\} \tag{A33}$$

where

$$\overline{V}^{>}_{<} = \overline{M}^{h}_{\substack{j\\nm}}(\overline{r};k_+) + \overline{N}^{h}_{\substack{j\\nm}}(\overline{r};k_+) \tag{A34}$$
$$\overline{V}'^{>}_{<} = \overline{M}^{j}_{\substack{h\\n,-m}}(\overline{r}';k_+) + \overline{N}^{j}_{\substack{h\\n,-m}}(\overline{r}';k_+) \tag{A35}$$
$$\overline{W}^{>}_{<} = \overline{M}^{h}_{\substack{j\\nm}}(\overline{r};k_-) - \overline{N}^{h}_{\substack{j\\nm}}(\overline{r};k_-) \tag{A36}$$
$$\overline{W}'^{>}_{<} = \overline{M}^{j}_{\substack{h\\n,-m}}(\overline{r}';k_-) - \overline{N}^{j}_{\substack{h\\n,-m}}(\overline{r}';k_-) \tag{A37}$$

REFERENCES

1. Tai, C. T., *Dyadic Green's Functions in Electromagnetic Theory*, New York: IEEE Press, 1993.
2. Kong, J. A., *Electromagnetic Wave Theory*, New York: Wiley, 1990.
3. Collin, R. E., *Field Theory of Guided Waves*, New York: IEEE Press, 1991.
4. Chew, W. C., *Waves and Fields in Inhomogeneous Media*, New York: Van Nostrand, 1990.
5. Jaggard, D. L., A. R. Mickelson, and C. H. Papas, "On electromagnetic waves in chiral media," *Appl. Phys.*, Vol. 18, 211–216, 1979.
6. Bassiri, S., N. Engheta, and C. H. Papas, "Dyadic Green's function and dipole radiation in chiral media," *Alta Freq.*, Vol. 55, 83–88, 1986.
7. Jaggard, D. L., X. Sun, and N. Engheta, "Canonical sources and duality in chiral media," *IEEE Trans. Antennas Propagat.*, Vol. 36, No. 7, 1007–1013, 1988.
8. Engheta, N., and M. W. Kowarz, "Antenna radiation in the presence of a chiral sphere," *J. Appl. Phys.*, Vol. 67, No. 2, 639–647, 1990.
9. Monzon, J. C., "Radiation and scattering in homogeneous general biisotropic regions," *IEEE Trans. Antennas Propagat.*, Vol. 38, No. 2, 227–235, 1990.

10. Lindell, I. V., A. H. Sihvola, S. A. Tretyakov, and A. J. Viitanen, *Electromagnetic Waves in Chiral and Bi-Isotropic Media*, London: Artech House, 1994.
11. Lakhtakia, A., *Beltrami Fields in Chiral Media*, Singapore: World Scientific, 1994.
12. Ali, S. M., T. M. Habashy, and J. A. Kong, "Spectral-domain dyadic Green's function in layered chiral media," *J. Opt. Soc. Am. A*, Vol. 9, No. 3, 413–423, 1992.
13. Yin, W., and W. Wang, "Dyadic Green's function of cylindrical multilayered chiral media and its applications," *J. Electrom. Waves Applicat.*, Vol. 7, No. 7, 1005–1027, 1993.
14. Li, L. W., P. S. Kooi, M. S. Leong, and T. S. Yeo, "A general expression of dyadic Green's functions in radially multilayered chiral media," *IEEE Trans. Antennas Propagat.*, Vol. 43, No. 3, 232–238, 1995.
15. Hui, H. T., and E. K. N. Yung, "The eigenfunction expansion of dyadic Green's functions for chirowaveguides," *IEEE Trans. Microwave Theory Tech.*, Vol. 44, No. 9, 1575–1583, 1996.
16. Tai, C. T., "On the eigenfunction expansion of dyadic Green's functions," *Proc. IEEE*, Vol. 61, 480–481, 1973.
17. Collin, R. E., "On the incompleteness of E and H modes in waveguides," *Can. J. Phys.*, Vol. 51, 1135–1140, 1973.
18. Tai, C. T., "Equivalent layers of surface charge, current sheet, and polarization in the eigenfunction expansions of Green's functions in electromagnetic theory," *IEEE Trans. Antennas Propagat.*, Vol. 29, No. 5, 733–739, 1981.
19. Pathak, P. H., "On the eigenfunction expansion of electromagnetic dyadic Green's functions," *IEEE Trans. Antennas Propagat.*, Vol. 31, No. 6, 837–846, 1983.
20. Lindell, I. V., and A. H. Sihvola, "Singularity of Green dyadic for bi-isotropic media," *Electron. Lett.*, Vol. 30, No. 11, 842–843, 1994 (and erata: *Electron. Lett.*, Vol. 30, No. 20, 1722, 1994).
21. Jacoby, B., and F. Olyslager, "Singularity in Green dyadics for uniaxial bianisotropic media," *Electron. Lett.*, Vol. 31, No. 10, 779–781, 1995 (and erata: *Electron. Lett.*, Vol. 31, No. 16, 1396, 1995).
22. Lakhtakia, A., and B. Shanker, "Beltrami fields within continuous source regions, volume integral equations, scattering algorithms and the extended Maxwell-Garnett Model," *Int. J. Appl. Electromagn. Mater.*, Vol. 4, 65–82, 1993.
23. Shanker, B., "Green's dyadic and its associated singularities for a chiral-biaxial Omega medium," *Microwave Opt. Technol. Lett.*, Vol. 13, No. 3, 1996.

24. Van Bladel, J., *Singular Electromagnetic Fields and Sources*, Oxford: Clarendon, 1991.
25. Gel'fand, I. M., and G. E. Shilov, *Generalized Functions*, Vol. I. New York: Academic, 1964.
26. Sihvola, A. H., and I. V. Lindell, "Bi-isotropic constitutive relations," *Microwave Opt. Technol. Lett.*, Vol. 4, No. 8, 295–297, 1991.
27. Lakhtakia, A., "A case for magnetic sources," *Physics Essays*, Vol. 4, 105–108, 1991.
28. Wang, J. J. H., *Generalized Moment Methods in Electromagnetics*, New York: Wiley, 1991.
29. Chew, W. C., "Some observations on spatial and eigenfunction representations of dyadic green's functions," *IEEE Trans. Antennas Propagat.*, Vol. 37, No. 10, 1322–1327, 1989.
30. Van Bladel, J., *Electromagnetic Fields*, Washington:Hemisphere, 1985.
31. Yaghjian, A., "Electric dyadic Green's functions in the source region," *Proc. IEEE,* Vol. 68, 248–263, 1980.