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ASYMPTOTIC SOLUTIONS FOR THE SCATTERED FIELD OF PLANE WAVE BY A CYLINDRICAL OBSTACLE BURIED IN A GROUNDED DIELECTRIC LAYER

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1. INTRODUCTION

Scattering of electromagnetic waves from a cylindrical obstacle which is buried in a grounded dielectric layer is considered. This geometry is commonly used while studying the microwave circuits. Grounded dielectric layer geometry can also be a good model for various scattering problems such as scattering by a cylindrical object buried in a layer of ice over sea water where sea water may be considered as a perfect conductor. Another example of this geometry could be a cylindrical object above the earth surface camouflaged by dry grass in which earth is modeled as a perfect conductor.

The exact analytical solution of this problem is not available. To obtain an approximate solution for the scattered field numerical techniques such as method of moments [1–2] or analytical techniques as discussed in [3–4] may be employed. In this paper, it is desired to solve this problem by an analytical technique. The problem is complicated due to the presence of interfaces on both sides of the buried object. Therefore, it is quite difficult to incorporate all the multiple reflections among the buried object and interfaces surrounding the object.

Hongo and Hamamura [3] have obtained an asymptotic solution for a cylindrical object buried in a dielectric half-space. They considered a perfectly conducting strip of finite width buried in a dielectric half-space. They obtained an asymptotic solution for the far-zone scattered field from the buried strip which contains a pattern function. The pattern function is the only factor in the scattered field expression which is dependent on the shape of the buried object. They replaced the pattern function of the strip by the pattern function of a circular cylinder. The solution resulting from their work has been verified using method of moments by Naqvi et al. [5]. This technique of applying the boundary conditions on a strip and then replacing the pattern function of the strip by the pattern function of the desired object in the final result is very convenient and gives satisfactory results. Therefore this technique will be adopted to obtain far-zone field scattered by cylindrical object buried in grounded dielectric layer.

In this paper plane wave spectrum representation of the scattered fields is considered in different regions in terms of unknown spectrum functions. Application of boundary conditions on these spectrum functions yields a dual integral equation which is reduced to a matrix equation. The elements of matrix are definite integrals which are solved asymptotically for large separation of buried object from both the interfaces. Only the dominant part of reflected scattered fields is considered. It is assumed that only those reflected scattered fields interact with the buried object which suffer only one reflection from any of the interfaces.

2. FORMULATION

Geometry for the scattering problem is illustrated in Fig. 1, along with the coordinates and notations to be used. A perfectly conducting sheet lies at $y = -d$. Space $y > 0$ has propagation constant k_0 and is called medium 1, while space $-d < y < 0$ has propagation constant k and is called medium 2. It is assumed that $k > k_0$. Both media are assumed to be homogeneous and lossless. A perfectly conducting strip of width $2a$ resides at $y = -h$ in the medium 2. The strip is of infinite length in the z-direction. It is assumed that a perpendicularly polarized plane

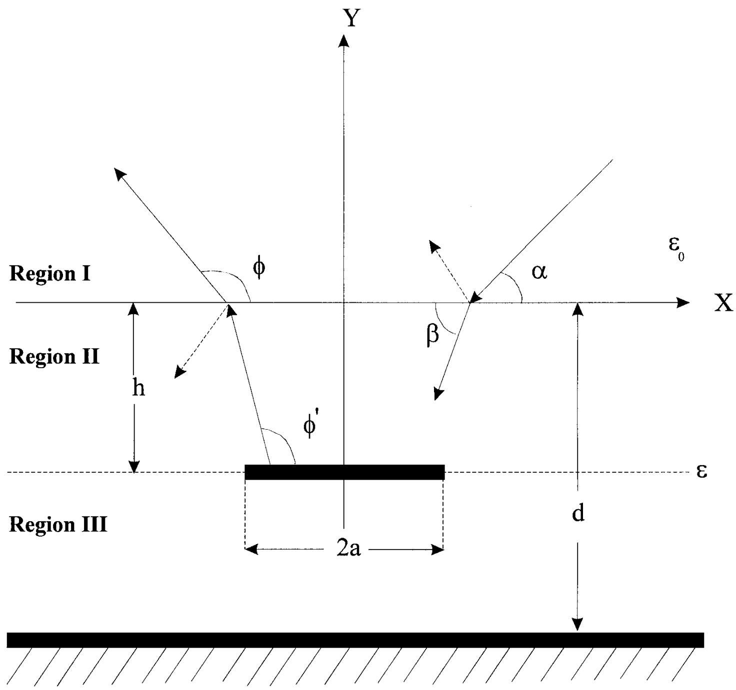

Figure 1. Perfectly conducting strip buried in a grounded dielectric layer.

wave is incident with angle α from medium 1, i.e.,

$$E_z^i = \exp\{jk_0(x\cos\alpha + y\sin\alpha)\}. \tag{1}$$

The time factor is taken to be $\exp(j\omega t)$ throughout the present analysis. At the interface $y = 0$ the incident plane wave generates two waves: a reflected wave in medium 1 propagating in direction $\pi - \alpha$ and a transmitted wave in the grounded dielectric layer propagating in direction β. For angle α and β given in Fig. 1, Snell's law takes the form

$$k_0 \cos\alpha = k\cos\beta. \tag{2}$$

When no inhomogeneity is present total field inside the dielectric layer can be written as

$$E_{2z} = \frac{2jk_0 \sin\alpha \sin\{k(y+d)\sin\beta\}}{k\sin\beta\cos(kd\sin\beta) + jk_0\sin\alpha\sin(kd\sin\beta)} \exp(jkx\cos\beta). \tag{3}$$

This field in the dielectric layer shows a standing wave pattern in the y-direction with zero at the conducting sheet. Now scattered field from the strip, buried in a geometry shown in Fig. 1, will be calculated taking E_{2z} as a field incident on the buried strip.

In order to calculate the scattered field from the buried strip, whole space is subdivided into three subregions for convenience of imposing the required boundary conditions. Region I is $y > 0$ and the scattered field in region I is denoted by $E_z^{(I)}$, region II is $-h < y < 0$ and the scattered field is denoted by $E_z^{(II)}$ while region III is $-d < y < -h$ and the scattered field is denoted by $E_z^{(III)}$. The fields $E_z^{(I)}$, $E_z^{(II)}$ and $E_z^{(III)}$ satisfy homogeneous Helmholtz's equation and they can be expressed in terms of spectrum of plane waves. The explicit expression for the scattered field in each region is given by [3]

$$E_z^{(I)} = \sqrt{\frac{\pi x}{2a}} \int_0^\infty f_{e1}(\xi) J_{-1/2}\left(\frac{x\xi}{a}\right) \exp\left(\frac{-\eta_0 y}{a}\right) \sqrt{\xi} d\xi + CO \tag{4a}$$

$$\begin{aligned} E_z^{(II)} = \sqrt{\frac{\pi x}{2a}} \int_0^\infty J_{-1/2}\left(\frac{x\xi}{a}\right) \left\{ f_{e2}(\xi) \exp\left(\frac{\eta y}{a}\right) \right. \\ \left. + f_{e3}(\xi) \exp\left(\frac{-\eta y}{a}\right) \right\} \sqrt{\xi} d\xi + CO \end{aligned} \tag{4b}$$

$$\begin{aligned} E_z^{(III)} = \sqrt{\frac{\pi x}{2a}} \int_0^\infty J_{-1/2}\left(\frac{x\xi}{a}\right) \left[f_{e4} \exp\left\{(y+h)\frac{\eta}{a}\right\} \right. \\ \left. + f_{e5}(\xi) \exp\left\{-(y+h)\frac{\eta}{a}\right\} \right] \sqrt{\xi} d\xi + CO \end{aligned} \tag{4c}$$

where

$$\begin{aligned} \eta_0 &= \sqrt{\xi^2 - \kappa_0^2}, \quad \mathcal{R}(\eta_0) > 0 \\ \eta &= \sqrt{\xi^2 - \kappa^2}, \quad \mathcal{R}(\eta) > 0 \\ \kappa_0 &= k_0 a, \qquad \kappa = ka. \end{aligned} \tag{5}$$

$\mathcal{R}$ stands for real part while $J_{-1/2}(x)$ is the Bessel function of order $-1/2$ and is given as

$$\sqrt{\frac{\pi t}{2}} J_{-1/2}(t) = \cos t.$$

CO in (4) means corresponding odd function terms which are given by replacing $\sqrt{\pi x\xi/2a}J_{-1/2}(x\xi/a) = \cos(x\xi/a)$ and $f_{ei}(\xi)$, with $\sqrt{\pi x\xi/2a}J_{1/2}(x\xi/a) = \sin(x\xi/a)$ and $f_{oi}(\xi)$ respectively, where $i = 1, 2, 3, 4$ and 5. Functions $f_{ei}(\xi)$ and $f_{oi}(\xi)$ are unknown and will be determined from the boundary conditions.

3. IMPOSITION OF BOUNDARY CONDITIONS

Using the boundary conditions that E_z and H_x are continuous at $y = 0$, and E_z is continuous at $y = -h$ the following relations among the spectrum functions are obtained

$$f_{\substack{e1\\o1}}(\xi) = f_{\substack{e2\\o2}}(\xi) + f_{\substack{e3\\o3}}(\xi) \tag{6a}$$

$$-\eta_o f_{\substack{e1\\o1}}(\xi) = \eta\left\{f_{\substack{e2\\o2}}(\xi) - f_{\substack{e3\\o3}}(\xi)\right\} \tag{6b}$$

$$f_{\substack{e2\\o2}}(\xi)\exp\left(-\eta\frac{h}{a}\right) + f_{\substack{e3\\o3}}(\xi)\exp\left(\eta\frac{h}{a}\right) = f_{\substack{e4\\o4}}(\xi) + f_{\substack{e5\\o5}}(\xi). \tag{6c}$$

Condition $E_z^{(III)} + E_{2z} = 0$ for $|x| \le a$ in the plane $y = -h$, yields the following relation

$$\begin{aligned}\sqrt{\frac{\pi x}{2a}}\int_0^\infty &\left\{\Big[f_{e4}(\xi) + f_{e5}(\xi)\Big]J_{-1/2}\left(\frac{x\xi}{a}\right)\right.\\ &\left.+\Big[f_{o4}(\xi) + f_{05}(\xi)\Big]J_{1/2}\left(\frac{x\xi}{a}\right)\right\}\sqrt{\xi}d\xi\\ = E_0 &\exp(jkx\cos\beta)\end{aligned} \tag{7a}$$

where

$$E_0 = \frac{-2jk_0\sin\alpha\sin\{k(d-h)\sin\beta\}}{k\sin\beta\cos(kd\sin\beta) + jk_0\sin\alpha\sin(kd\sin\beta)}.$$

At $y = -d$, tangential component of the total field $E^{(III)} + E_{2z} = 0$. This boundary condition yields the following

$$f_{\substack{e4\\o4}}(\xi) = -f_{\substack{e5\\o5}}(\xi)\exp\left\{2\frac{\eta}{a}(d-h)\right\}. \tag{7b}$$

Equations (6) and (7) are dual integral equations. In order to reduce these integral equations into matrix equations consider the following expansion

$$f_{\substack{e2\\o2}} \exp\left(-\eta\frac{h}{a}\right) - f_{\substack{e4\\o4}}(\xi) = \sum_{m(\text{even})} \frac{1}{\eta} A_{em} J_{em}(\xi) + \sum_{m(\text{odd})} \frac{1}{\eta} A_{om} J_{om}(\xi)$$
$$= \sum_{m=0}^{m=\infty} \frac{1}{\eta} A_{\substack{em\\om}} J_{\substack{em\\om}}(\xi) \tag{8}$$

where $J_{em}(\xi) = J_{2m}(\xi)$ and $J_{om}(\xi) = J_{2m+1}(\xi)$ and A_{em} and A_{om} are expansion coefficients. The unknown functions $f_{ei}(\xi)$ and $f_{oi}(\xi)$ can be expressed in terms of expansion coefficients A_{em} and A_{om}. It is convenient to express $f_{\substack{e3\\o3}}(\xi)$ directly in terms expansion coefficients and the other spectrum functions can be expressed in terms of $f_{\substack{e3\\o3}}(\xi)$. This results in the following expressions for $f_{\substack{ei\\oi}}(\xi)$.

$$f_{\substack{e1\\o1}}(\xi) = \frac{2\eta}{\eta+\eta_0} f_{\substack{e3\\o3}}$$
$$f_{\substack{e2\\o2}}(\xi) = R(\xi) f_{\substack{e3\\o3}}$$
$$f_{\substack{e3\\o3}}(\xi) = \frac{1}{L(\xi)}\left[\exp\left(\frac{\eta}{a}h\right) - \exp\left\{\frac{\eta}{a}(2d-h)\right\}\right] \sum_{m=0}^{\infty} \frac{1}{\eta} A_{\substack{em\\om}} J_{\substack{em\\om}}(\xi)$$
$$f_{\substack{e4\\o4}}(\xi) = f_{\substack{e3\\o3}} R(\xi) \exp\left(-\eta\frac{h}{a}\right) - \sum_{m=0}^{m=\infty} \frac{1}{\eta} A_{\substack{em\\om}} J_{\substack{em\\om}}(\xi)$$
$$f_{\substack{e5\\o5}}(\xi) = f_{\substack{e3\\o3}} \exp\left(\eta\frac{h}{a}\right) + \sum_{m=0}^{m=\infty} \frac{1}{\eta} A_{\substack{em\\om}} J_{\substack{em\\om}}(\xi)$$

where

$$R(\xi) = \frac{\eta-\eta_0}{\eta+\eta_0}, \qquad L(\xi) = \frac{\eta-\eta_0}{\eta+\eta_0} + \exp\left(2\frac{d}{a}\eta\right).$$

Trigonometric factors in $(7a)$ which are function of x are expanded in Jacobi series [6]. Substituting the value of functions $f_{\substack{e4\\o4}}(\xi)$ and $f_{\substack{e5\\o5}}(\xi)$ in equation $(7a)$ after some manipulation yields the following

$$\int_0^\infty \left[1 - \exp\left\{2\frac{\eta}{a}(d-h)\right\} + \frac{1}{L(\xi)}\left\{\exp\left(\frac{2\eta}{a}h\right) + \exp\left\{\frac{2\eta}{a}(2d-h)\right\}\right.\right.$$
$$\left.\left. - 2\exp\left(\frac{2\eta}{a}d\right)\right\}\right] \sum_{m=0}^{\infty} A_{\substack{em\\om}} \frac{1}{\sqrt{\xi^2-\kappa^2}} J_{\substack{en\\on}}(\xi) J_{\substack{em\\om}}(\xi) d\xi$$
$$= \delta_{\substack{en\\on}} E_0 J_{\substack{en\\on}}(\kappa\cos\beta) \tag{9}$$

where $\delta_{en} = 1,\ \delta_{on} = j$.

It is assumed that thickness d of dielectric layer is very large and strip is far away from both interfaces. To utilized this assumption a few words about (9) are in order. The R.H.S. is the incident field on the strip and hence the L.H.S. is negative of field scattered by the strip and evaluated at the strip. The expression in square brackets may expanded in powers of R. The resulting series of integrals represents the scattered waves which hit the strip after multiple reflections from the surrounding interfaces. Since the scattered waves diverge after a reflection, therefore each reflection contribute less and less to the total scattered field. Only those integrals are retained which contributes to the scattered field after suffering at most one reflection from any interface. After this approximation (9) reduces to the following

$$\begin{aligned}\int_0^\infty &\left[1 - \exp\{2\frac{\eta}{a}(d-h)\} + R(\xi)\exp\left(2\frac{\eta}{a}h\right)\right] \\ &\cdot \sum_{m=0}^{\infty} A_{\substack{em\\om}} \frac{1}{\sqrt{\xi^2-\kappa^2}} J_{\substack{en\\on}}(\xi) J_{\substack{em\\om}}(\xi) d\xi \\ =&\delta_{\substack{en\\on}} E_0 J_{\substack{en\\on}}(\kappa\cos\beta).\end{aligned} \tag{10}$$

Above expression will now be used to calculate the unknown expansion coefficients $A_{\substack{em\\om}}$. If needed higher order multiple reflections may also be considered to calculate the unknown expansion coefficients.

4. APPROXIMATE SOLUTION FOR THE EXPANSION COEFFICIENTS

It is difficult to get the exact solutions for A_{em} and A_{om} except for the case in which reflected scattered field has no interaction with the buried strip. This situation has been discussed by Naqvi [2] for the case of a circular cylinder. An iterative method can be used to obtained approximate solutions for A_{em} and A_{om} when the conditions $k(d-h),\ kh \gg 1$ are satisfied. Equation (10) is written in matrix form as

$$\begin{aligned}[G_e][A_{em}] &= E_0[J_{en}] + [M_e][A_{em}] \\ [G_o][A_{om}] &= jE_0[J_{on}] + [M_o][A_{om}]\end{aligned} \tag{11}$$

where $[A_{em}]$, $[A_{om}]$, $[J_{en}]$ and $[J_{on}]$ are the column matrices with elements A_{em}, A_{om}, $J_{2n}(\kappa\cos\beta)$ and $J_{2n+1}(\kappa\cos\beta)$, respectively.

Matrices G_e, G_o, M_e and M_o have order (n, m) with elements $G(2n, 2m)$, $G(2n+1, 2m+1)$, $M(2n, 2m)$ and $M(2n+1, 2m+1)$ respectively. In general, matrix elements $G(n, m)$ and $M(n, m)$ are given by

$$G(n,m) = \int_0^\infty \frac{J_n(\xi)J_m(\xi)}{\sqrt{\xi^2 - \kappa^2}} d\xi \tag{12a}$$

$$M(n,m) = \int_0^\infty \frac{J_n(\xi)J_m(\xi)}{\sqrt{\xi^2 - \kappa^2}} \cdot \left[\exp\left(\frac{2\eta}{a}(d-h)\right) - R(\xi)\exp\left(\frac{2\eta}{a}h\right)\right] d\xi. \tag{12b}$$

It is obvious from (12) that $[G_{\substack{e\\o}}]$ and $[M_{\substack{e\\o}}]$ are symmetric matrices. $[M_{\substack{e\\o}}]$ considers the interaction effects of the reflected scattered field with the buried strip. Each element of the matrix $M_{\substack{e\\o}}$ has two terms. The asymptotic expression of first term for $k(d-h) \gg 1$ and second term for $kh \gg 1$ is now derived. The series expression for Bessel function of the first kind, of order n is

$$J_n(\xi) = \sum_{l=0}^{\infty} \frac{(-1)^l \xi^{n+2l}}{2^{n+2l} l!(l+n)!}.$$

It may be noted that in expression (12) both m and n can either be even or odd. Using the fact that the integrand of $(12b)$ is even function of ξ, since $m+n$ is always even number, the limit of integration is extended to $(-\infty, \infty)$. Making the transformation of variable, $\xi = \kappa\cos\theta$ and using the steepest descent method of integration [7] yields the following

$$M(n,m) = C\{2k(d-h)\}P_d(D)Q_1\left(\frac{\pi}{2}\right) - C(2kh)P_d(D)Q_2\left(\frac{\pi}{2}\right) \tag{13a}$$

where

$$Q_1(\theta) = J_m(\kappa\cos\theta)J_n(\kappa\cos\theta)$$
$$Q_2(\theta) = J_m(\kappa\cos\theta)J_n(\kappa\cos\theta)R(\theta) \tag{13b}$$

$$P_d(D) = \sum_{m=0}^{m=\infty} \frac{(1+4D^2)(9+4D^2)\cdots([2m-1]^2+4D^2)}{(-j8kd)^m m!} \tag{13c}$$

$$C(x) = -j\sqrt{\frac{\pi}{2x}} \exp\left[j\left(x - \frac{\pi}{4}\right)\right]. \tag{13d}$$

In the above equation $D = \partial/\partial\theta$, and $P_d(D)Q(\pi/2)$ means $P_d(D) \cdot Q(\theta)|_{\theta=(\pi/2)}$. It is important to note that the matrix $M(n,m)$ may be written as

$$\begin{aligned}[M] =& C\{2k(d-h)\}P_d(D)[J_n(\kappa\cos\theta)][J_m(\kappa\cos\theta)]^T \\ &- C(2kh)P_d(D)R(\theta)[J_n(\kappa\cos\theta)][J_m(\kappa\cos\theta)]^T. \end{aligned} \tag{14}$$

Expression (11) may be solved in an iterative manner. The zeroth order approximation for $A^{(0)}_{em}$ and $A^{(0)}_{om}$, and the first-order corrections $A^{(1)}_{em}$ and $A^{(1)}_{om}$ are given by

$$\begin{aligned} [A^{(0)}_{\substack{em\\om}}] &= E_o\delta_{\substack{en\\on}}[G_{\substack{e\\o}}]^{-1}[J_{\substack{en\\on}}] \\ [A^{(1)}_{\substack{em\\om}}] &= [G_{\substack{e\\o}}]^{-1}[M_{\substack{e\\o}}][A^{(0)}_{\substack{em\\om}}]\Big|_{\theta=(\pi/2)}. \end{aligned} \tag{15a}$$

It may be noted that zeroth order solution $[A^{(0)}_{\substack{em\\om}}]$ corresponds to solution for the case in which the scattered fields after reflection from the interfaces have no interaction with the buried strip. First order correction $[A^{(1)}_{\substack{em\\om}}]$ considers the first order reflection of scattered field from both the interfaces towards the buried strip. In this way higher order multiple reflections between the buried strip and the interfaces may be considered, but in this work only the first reflection, of scattered field by each interface, towards the buried strip is considered. Substituting the value of $[A^{(0)}_{\substack{em\\om}}]$ from $(15a)$ in the above expression yields the following

$$[A_{\substack{em\\om}}]^{(1)} = E_0\delta_{\substack{en\\on}}[G_{\substack{e\\o}}]^{-1}[M_{\substack{e\\o}}][G_{\substack{e\\o}}]^{-1}[J_{\substack{en\\on}}]\Big|_{\theta=(\pi/2)}$$

Substituting the value of $[M_{\substack{e\\o}}]$ in the above expression and taking the transpose of both sides of resulting expression, yields the following

$$\begin{aligned}[A^{(1)}_{\substack{em\\om}}]^T =& E_0\delta_{\substack{en\\on}}P_d(D)\Big[C\{2k(d-h)\} - C(2kh)R(\theta)\Big] \\ &\cdot \Gamma_{\substack{e\\o}}(\beta,\theta)[J_{\substack{en\\on}}(\kappa\cos\theta)]^T[G_{\substack{e\\o}}]^{-1}\Big|_{\theta=(\pi/2)} \end{aligned} \tag{15b}$$

where

$$\Gamma_{\substack{e\\o}}(\theta,\phi) = [J_{\substack{e\\o}}(\kappa\cos\theta)]^T[G_{\substack{e\\o}}]^{-1}[J_{\substack{e\\o}}(\kappa\cos\phi)]. \tag{15c}$$

Zeroth order solution $[A^{(0)}_{\substack{em\\om}}]$ and first order correction $[A^{(1)}_{\substack{em\\om}}]$ will now be used to calculate the far-zone scattered field from the buried strip, i.e., $[A_{\substack{em\\om}}] \sim [A^{(0)}_{\substack{em\\om}}] + [A^{(1)}_{\substack{em\\om}}]$.

5. FAR-FIELD EXPRESSION FOR THE SCATTERED FIELD

It is desired to calculate the far-zone scattered field from the buried obstacle. Substituting the functions $f_{\substack{e1\\o1}}$ in terms of expansion coefficients $A_{\substack{em\\om}}$ in $(4a)$ and using the saddle point method of integration [7] yields the following expression for the far-zone scattered field

$$E_z^{(I)} = \frac{2k_0 \sin\phi}{L_2(\phi)} C_1(k_0\rho)[f(\phi', \beta) + \zeta(\beta, \phi')] \tag{16}$$

where

$$\begin{aligned}
L_2(\phi) &= 2j\left[\beta_1 \cos\beta_1 d + jk_0 \sin\phi \sin\beta_1 d\right]\\
\beta_1 &= \sqrt{k^2 - k_0^2 \cos^2\phi}\\
C_1(x) &= \sqrt{\frac{\pi}{2x}} \exp\left[-j\left(x - \frac{\pi}{4}\right)\right]\\
f(\phi', \beta) &= P(\phi', \beta)\exp\{-j\beta_1(d-h)\} - P(-\phi', \beta)\exp\{j\beta_1(d-h)\}\\
P(\phi', \beta) &= E_1\Gamma(\phi', \beta)\exp\{jk(d-h)\sin\beta\}\\
&\quad - E_1\Gamma(\phi', -\beta)\exp\{-jk(d-h)\sin\beta\}\\
\Gamma(\phi', \beta) &= \Gamma_e(\phi', \beta) - \Gamma_o(\phi', \beta)\\
E_1 &= \frac{k_0 \sin\alpha}{k\sin\beta\cos(kd\sin\beta) + jk_0\sin\alpha\sin(kd\sin\beta)}\\
\phi' &= \tan^{-1}\sqrt{(k/k_0\cos\phi)^2 - 1}\\
\zeta(\beta, \phi') &= P_d(D)\Big\{C\{2k(d-h)\} - C(2kh)R(\theta)\Big\}\Gamma(\beta, \theta)f(\theta, \phi')\Big|_{\theta=(\pi/2)}
\end{aligned}$$

Far-zone scattered field from the buried cylindrical obstacle is written such that only dominant contribution in the asymptotic series of first order correction is considered, i.e.,

$$\begin{aligned}
E_z^{(I)} = \frac{2k_0 \sin\phi}{L_2(\phi)} C_1(k_0\rho)\Big[& f(\phi', \beta)\\
&+ \Big\{C\{2k(d-h)\} - C(2kh)R(\theta)\Big\} f(\theta, \phi')\Gamma(\beta, \theta)\Big|_{\theta=(\pi/2)}\Big].
\end{aligned} \tag{17}$$

In the above equation, $\Gamma(\phi, \beta)$ is the plane wave pattern function of the strip in a homogeneous medium (see Appendix). β and ϕ are the angles of incidence and observation respectively. It is important to note that all the terms in above expression except pattern function Γ are independent of the shape of buried cylindrical obstacle. This observation leads to the conclusion that far-zone scattered field expression (17) may be utilized to calculate far-zone scattered field for other cylindrical objects using the technique of Hongo and Hamamura [3] as discussed previously. This can be done by substituting the plane wave pattern function of the corresponding object in the far-zone scattered field expression (17).

It is desired to calculate the far-zone scattered field from a perfectly conducting circular cylinder buried in a grounded dielectric layer using the scattered field expression (17). For this purpose, the pattern function of perfectly conducting cylinder of radius a is required. The far-zone scattered field from a perfectly conducting circular cylinder of radius a when it is excited by a plane wave may be written as

$$E_z = C_1(x)\Gamma(\theta_1, \theta_2)$$

where θ_1 is the incidence angle of field incident on the cylinder while θ_2 is the observation angle of the scattered field. The plane wave pattern function of a circular cylinder is given by the following equation

$$\Gamma(\theta_1, \theta_2) = \frac{-2j}{\pi} \sum_{n=-\infty}^{n=\infty} \frac{J_n(ka)}{H_n^{(2)}(ka)} \exp\{jn(\theta_2 - \theta_1 + \pi)\}$$

Substituting above pattern function one can calculate the corresponding far-zone field expression for the case of a circular cylinder of radius a and buried in a grounded dielectric layer. Scattered field pattern is presented in Fig. 2 for a perfectly conducting, infinite circular cylinder, which is buried in a grounded dielectric layer. Comparison of the scattered field pattern is presented in Fig. 3 with the case where reflected scattered field has no interaction with the buried cylinder.

APPENDIX

Consider a perfectly conducting strip of width a placed in a homogeneous medium. The propagation constant of the medium is k. The strip is excited by a plane wave with field expression given in (1). The scattered field expression in regions above and below the strip in terms of spectrum of plane waves is written as

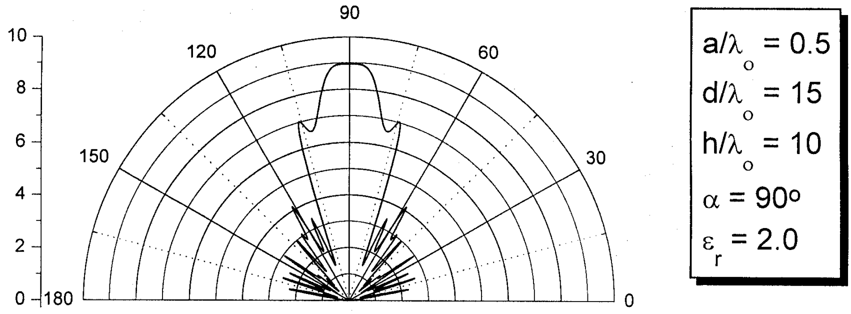

Figure 2. Scattered field pattern of a perfectly conducting circular cylinder buried in a grounded dielectric layer.

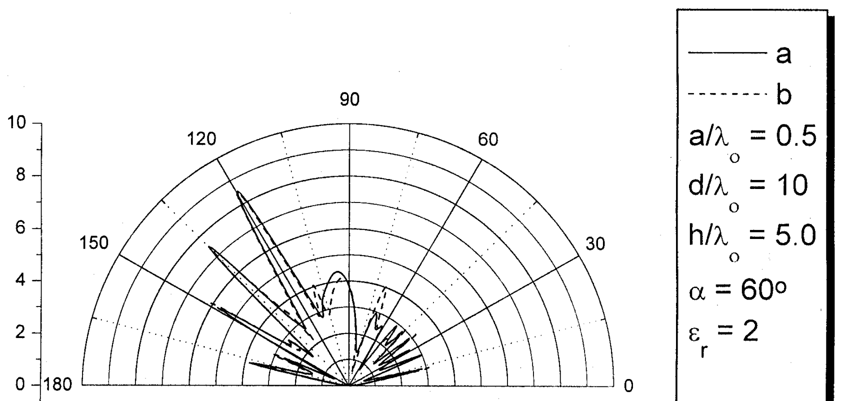

Figure 3. Comparison of the scattered field patterns of a perfectly conducting circular cylinder. (a) corresponds to the field without interaction, (b) corresponds to the field with interaction.

$$E_z^{(I)} = \sqrt{\frac{\pi x}{2a}} \int_0^\infty f_{e1}(\xi) J_{-1/2}\left(\frac{x\xi}{a}\right) \exp\left(\frac{-\eta y}{a}\right) \sqrt{\xi} d\xi + CO, \;\; y > 0 \qquad (A1)$$

$$E_z^{(II)} = \sqrt{\frac{\pi x}{2a}} \int_0^\infty f_{e2}(\xi) J_{-1/2}\left(\frac{x\xi}{a}\right) \exp\left(\frac{\eta y}{a}\right) \sqrt{\xi} d\xi + CO, \ y < 0. \tag{A2}$$

The application of boundary conditions at the interface $y = 0$ yields the following relations between the spectrum functions

$$f_{\substack{e1\\o1}}(\xi) = f_{\substack{e2\\o2}}(\xi) \tag{A3}$$

$$\sqrt{\frac{\pi x}{2a}} \int_0^\infty \left\{ f_{e1}(\xi) J_{-1/2}\left(\frac{x\xi}{a}\right) + f_{o1}(\xi) J_{1/2}\left(\frac{x\xi}{a}\right) \right\} \sqrt{\xi} d\xi$$
$$= \exp(jkx\cos\alpha). \tag{A4}$$

The representation of the spectrum function in terms of unknown expansion coefficients is assumed as

$$f_{\substack{e1\\o1}}(\xi) = \sum_{m=0}^{m=\infty} \frac{1}{\eta} A_{\substack{em\\om}} J_{\substack{em\\om}}(\xi). \tag{A5}$$

Substitution of $(A5)$ in $(A1)$ yields the following resulting expression for the expansion coefficients

$$\int_0^\infty \sum_{m=0}^{\infty} A_{\substack{em\\om}} \frac{1}{\sqrt{\xi^2 - \kappa^2}} J_{\substack{en\\on}}(\xi) J_{\substack{em\\om}}(\xi) d\xi = \delta_{\substack{en\\on}} J_{\substack{en\\on}}(\kappa \cos\alpha). \tag{A6}$$

The above expression in a matrix form may be written as

$$[A_{\substack{em\\om}}] = \delta_{\substack{en\\on}} [G_{\substack{e\\o}}]^{-1} [J_{\substack{en\\on}}]. \tag{A7}$$

It is obvious from expressions $(A5)$ and $(A7)$ that the spectrum functions $f_{\substack{ei\\oi}}(\alpha, \theta)$, $i = 1, 2$ may be written as

$$f_{\substack{ei\\oi}}(\alpha, \theta) = \delta_{\substack{en\\on}} \frac{1}{\eta} \Gamma_{\substack{e\\o}}(\alpha, \theta), \quad i = 1, 2 \tag{A8}$$

where

$$\xi = \kappa \cos\theta$$
$$\Gamma_{\substack{e\\o}}(\alpha, \theta) = [J_{\substack{e\\o}}(\kappa\cos\alpha)]^T [G_{\substack{e\\o}}]^{-1} [J_{\substack{e\\o}}(\kappa\cos\theta)].$$

The dominant contribution of the far-zone field scattered from the strip is given by

$$E_z = C_1(k\rho)\Gamma(\alpha, \phi) \tag{A9}$$

where

$$\Gamma(\alpha, \phi) = \Gamma_e(\alpha, \phi) - \Gamma_o(\alpha, \phi).$$

In the expression $(A9)$ factor $\Gamma(\alpha, \theta)$ contains the information about the shape of the object and is termed as the pattern function of the object.

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