Progress In Electromagnetics Research, PIER 21, 137–151, 1999

A NEW TECHNIQUE FOR THE ANALYSIS OF DISCONTINUITIES IN MICROWAVE PLANAR CIRCUITS

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1. INTRODUCTION

The characterization of discontinuities in microwave planar circuits has become an important research topic for developing accurate and fast

CAD tools. In the literature many techniques have been suggested to characterize these discontinuities [1–5]. The Finite Difference Time Domain method (FDTD) [1,2] and the Finite Element Method (FEM) [3,4] have been used to provide accurate modeling of the discontinuities. Even though these methods are accurate, they are highly expensive in computational time. In an attempt to reduce the computational labor, other techniques based on the Galerkin Method of Moments [5], such as the Transverse Resonance Technique (TRT), have been proposed [6]. While the TRT preserves the accuracy and presents a faster approach with a simple formulation, this method requires repeated analysis at each frequency point.

In the proposed approach, an excitation term is added to the formulation. The computational effort is therefore reduced to the solving of an inhomogeneous linear system of equations at each frequency point. The admittance matrix is derived from the solution of the matrix equation $AX = B$, where A is a matrix representing the boundary conditions of the electromagnetic fields (EM), and B accounting for the excitation term.

In the literature, several definitions of the excitation source have been suggested. Some authors [7,8] have defined the sources on the circuit plane. With this consideration, there is a discontinuity in the transition source-line, so additional mathematical tools are introduced to correct the numerical results. The mathematical tools used to solve the problem of the discontinuity are frequency dependent. For example a two-port circuit or "coupling quadripole" used in [7], involves additional computation since its parameters are computed at each frequency point.

With the present approach, the excitation source is the fundamental mode of the circuit feed line defined in a vertical section of the circuit. Accordingly, there is no need for additional mathematical tools to adapt the source to the circuit. This source can be derived from a coaxial excitation mechanism having the same characteristic impedance as the feeding line.

2. THEORETICAL FORMULATION

In this section, an integral equation is derived by applying the boundary conditions of the electromagnetic fields of the source and of the circuit. The integral equation will be solved by using the Galerkin MoM to determine the normalized admittance matrix of the circuit.

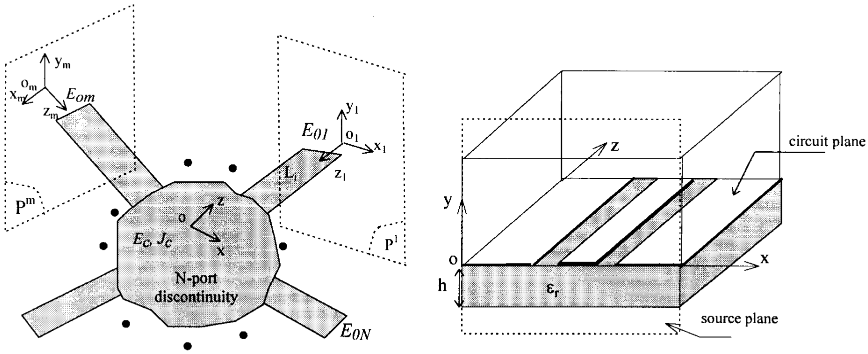


Figure 1. (a) Schematic of a multiport discontinuity. (b) transition source-line (case of a CPW)

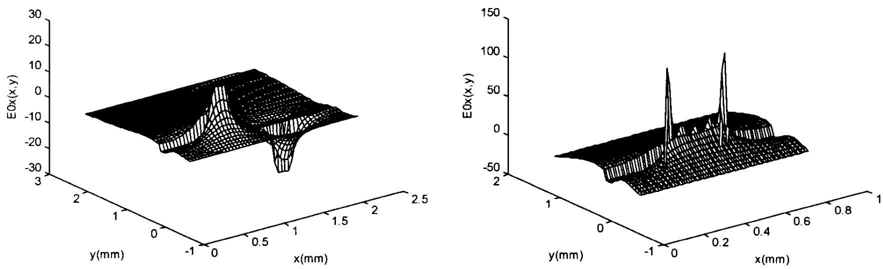
We consider linear equations of the inhomogeneous type $L(f) = g$, where L is an operator, g is the excitation source and f is the unknown function to be determined. The analysis problem involves the determination of f when L and g are given.

We will consider the case of computing the normalized admittance matrix of an N-port discontinuity in a planar circuit, where the N ports are connected to N sources. Figure 1 shows the structure under investigation excited by voltage sources E_{01}, $E_{02}, \ldots, E_{0N}$.

2.1 Excitation Sources

For many uniform guiding structures, the electromagnetic field distribution is known and can therefore be used to excite the fields in a port of the structure. The fundamental mode of coplanar wave guides (CPW) is computed by many numerical methods. In this work, we use the generalized transverse resonance method to compute the source. The main steps are to:

- compute the propagation constant β of the fundamental mode;
- derive the electric field and the current in a cross section of the line considered as infinite;
- normalize the obtained field.



(a) (b)

Figure 2. x-component of voltage sources. (a) Coplanar wave guide ($w = 0.9\,\mathrm{mm}$, $s = 0.5\,\mathrm{mm}$, substrate thickness $h = 0.635\,\mathrm{mm}$, $\varepsilon_r = 9.9$, housing WR28, F = 15 GHz) (b) Finline (gap $w = 0.3\,\mathrm{mm}$, substrate thickness $h = 0.11\,\mathrm{mm}$, $\varepsilon_r = 3.75$, F = 80 GHz).

Let $E_0 = V_0 e_0$ be the voltage source used to excite the structure under investigation, and H_0 the magnetic field. The current density is derived from H_0 as: $j_0 = \vec{u} \wedge H_0$, where $\vec{u}$ is the direction of the line. The source is unitary, so E_0 and j_0 verify the condition: $\langle E_0 | j_0 \rangle = 1$, where the scalar product of E_0 and j_0 is $\langle E_0 | j_0 \rangle = \int_S E_0^* \cdot j_0 ds$.

Plots of the excitation source E_0 in the case of a CPW and a finline are shown in Figure 2.

2.2 Computation of the Input Admittance Matrix

To compute the input admittance matrix, the concept of extended operator [9] is used allowing generalized field expansion functions. The characterization is made by modeling the electric field. The expansion or trial functions $\phi_k(x, z)$ used to describe the electric field on the circuit plane $(y = 0)$, reduce the full wave analysis to a two-dimensional problem. These functions verify the electric field boundary conditions on the plane $(y = 0)$:

$$H_I J = 0, \tag{1a}$$

$$H_M E = 0, \quad H_M \phi_k = 0, \tag{1b}$$

where,

$H_I = 1,\ H_M = 0$ on the dielectric and $H_I = 0,\ H_M = 1$ on the metal;
$E:$ the electric field;
$J:$ the current density.

The circuit current is related to the tangential electric fields of both the sources and the circuit by the admittance operators $\hat{Y}_{cm}$ ($m = 1, \ldots, N,$ or $m = c$) as:

$$|J_c\rangle = \hat{Y}_{c1}|E_{01}\rangle + \cdots + \hat{Y}_{cN}|E_{0N}\rangle \hat{Y}_{cc}|E_c\rangle \tag{2}$$

where,

$|E_c\rangle:$ the unknown tangential electric field on the circuit plane (P_c) to be determined;

$|J_c\rangle:$ the current density on (P_c);

$\hat{Y}_{cm}$ $(m = 1, \ldots, N):$ the admittance operator which defines the relationship between the TE and TM mode fields on the m'th plane (P^m) and the current generated by these modes on the circuit plane. $\hat{Y}_{cm}$ is defined in the next section;

$\hat{Y}_{cc}:$ the admittance operator which defines the current generated on the circuit plane by the TE and TM modes of the empty wave guide, with respect to a propagation perpendicular to the circuit plane.

Similarly, the currents of the sources are related to the fields by:

$$\begin{cases} |J_{01}\rangle = \hat{Y}_{11}|E_{01}\rangle + \cdots + \hat{Y}_{1N}|E_{0N}\rangle + \hat{Y}_{1c}|E_c\rangle \\ \vdots \\ |J_{0N}\rangle = \hat{Y}_{N1}|E_{01}\rangle + \cdots + \hat{Y}_{NN}|E_{0N}\rangle + \hat{Y}_{Nc}|E_c\rangle \end{cases} \tag{3}$$

To model the tangential electric field E_c on the circuit plane, we choose a basis of two-dimensional wavelet functions ϕ_k, thus E_c can be written as:

$$E_c = \sum_{k=1}^{K} x_k \phi_k(x, z) \tag{4a}$$

where, K is the number of trial functions used to expand the field.

The functions $\phi_k (k = 1, \ldots, K)$ verify the boundary condition (1b):

$$\langle \phi_k | J_c \rangle = 0,\ k = 1, \ldots, K \tag{4b}$$

In order to solve for E_c, we apply the Galerkin Method of Moments to equation (2), under the boundary condition (1a). The trial functions ϕ_k ($\phi_k = 0$ on the metal) verify the boundary condition (1b). Multiplying (2) by each trial function ϕ_k, we get the following K-dimensional system:

$$\begin{cases} \langle\phi_1|J_c\rangle = \langle\phi_1|\hat{Y}_{c1}E_{01}\rangle + \cdots + \langle\phi_1|\hat{Y}_{cN}E_{0N}\rangle + \sum_i x_i\langle\phi_1|\hat{Y}_{cc}\phi_i\rangle = 0 \\ \vdots \\ \langle\phi_K|J_c\rangle = \langle\phi_K|\hat{Y}_{c1}E_{01}\rangle + \cdots + \langle\phi_K|\hat{Y}_{cN}E_{0N}\rangle + \sum_i x_i\langle\phi_K|\hat{Y}_{cc}\phi_i\rangle = 0 \end{cases} \tag{5a}$$

This system can be rewritten in the matrix form:

$$AX = B, \tag{5b}$$

$$A(k,k') = \langle\phi_k|Y_{cc}\phi_{k'}\rangle, \qquad k,k' = 1,\ldots,K \tag{6a}$$

$$B(k) = -\sum_{n=1}^{N}\langle\phi_k|Y_{cn}E_{0n}\rangle = -\sum_{n=1}^{N} V_{0n}B_n(k), \quad k = 1,\ldots,K \tag{6b}$$

where V_{0n} is the amplitude of E_{0n}, $n = 1,\ldots,N$.

The column vector $X = (x_1,\ldots,x_k,\ldots,x_K)^t$ is composed of the unknown weights x_k of the trial functions used to model E_c. The vector X is determined by solving (5b)

$$X = -\sum_n V_{0n}A^{-1}B_n \tag{7}$$

Next, taking into account (7) and multiplying each equation (m'th equation) of the system (3) by its corresponding source (E_{0m}), we obtain the system:

$$\begin{cases} I_{01} = V_{01}\langle e_{01}|\hat{Y}_{11}e_{01}\rangle + \cdots + V_{0N}\langle e_{01}|\hat{Y}_{1N}e_{0N}\rangle - \sum_{n=1}^{N} V_{0n}C_1A^{-1}B_n \\ \vdots \\ I_{0m} = V_{0m}\langle e_{0m}|\hat{Y}_{m1}e_{01}\rangle + \cdots + V_{0N}\langle e_{0m}|\hat{Y}_{mN}e_{0N}\rangle - \sum_{n=1}^{N} V_{0n}C_mA^{-1}B_n \\ \vdots \\ I_{N} = V_{01}\langle e_{0N}|\hat{Y}_{N1}e_{01}\rangle + \cdots + V_{0N}\langle e_{0N}|\hat{Y}_{NN}e_{0N}\rangle - \sum_{n=1}^{N} V_{0n}C_NA^{-1}B_n \end{cases} \tag{8a}$$

where,

$$\begin{aligned} &I_{0n}: \ \text{the amplitude of } J_{0n}, \ n = 1\ldots N, \ J_{0n} = I_{0n}j_{0n}; \\ &\langle E_{0n}|j_{0n}\rangle = 1, \ \text{the sources are unitary;} \\ &C_m: \ C_m(k) = \langle \phi_k|\hat{Y}_{mc}e_{0m}\rangle, \ k = 1, \ldots, K. \end{aligned} \tag{8b}$$

The elements of the input admittance matrix Y_{in} are determined by rewriting the system (8a) in the form:

$$\begin{cases} I_{01} = y_{11}V_{01} + \cdots + y_{1N}V_{0N} \\ \vdots \\ I_{0N} = y_{N1}V_{01} + \cdots + y_{NN}V_{0N} \end{cases} \tag{9}$$

where

$$y_{mn} = Y_{mn} - C_mA^{-1}B_n \tag{10a}$$

$$Y_{mn} = \langle e_{0m}|\hat{Y}_{mn}e_{0n}\rangle \tag{10b}$$

Note that the y_{mn} $(m, n = 1, \ldots, N)$ are the elements of the input admittance matrix Y_{in} characterizing the discontinuity.

Once the normalized input admittance matrix Y_{in} is determined the scattering matrix S characterizing the discontinuity is derived.

2.3 Determination of the Admittance Operators

The admittance operator $\hat{Y}_{mn}$ $(m, n = 1, \ldots, N)$ defines the currents $j_{pq}^{(m)}$ on the plane P_m generated by each mode $(f_{pq}^{(n)})^{\mathrm{TE}}$ and $(f_{pq}^{(n)})^{\mathrm{TM}}$ $(n = 1, \ldots, N,\ p, q = 0, \ldots, \infty)$ of the plane of the source E_{0n}, where all sources $E_{0s},\ s \neq n$ are off. So we can write: $j_{pq}^{(m)} = \hat{Y}_{mn} f_{pq}^{(n)}|_{\text{ports } p \neq n \text{ are short-circuited}}$.

The admittance operator is then: $\hat{Y}_{mn} = \sum_{pq} |j_{pq}^{(m)}\rangle\langle f_{pq}^{(n)}|^{\mathrm{TE}} + |j_{pq}^{(m)}\rangle\langle f_{pq}^{(n)}|^{\mathrm{TM}}$.

Similarly, the operator $\hat{Y}_{cm}$ $(m = 1, \ldots, N)$ defines the current generated on the circuit plane by each mode TE and TM on the plane of the source E_{0m}. The admittance operator $\hat{Y}_{mc}$ defines the current on the plane of the source E_{0m} generated by each mode TE or TM on the circuit plane.

3. IMPLEMENTATION OF THE METHOD

In order to demonstrate the use of the proposed technique, the theoretical formulation is implemented in a C-coded program to compute the input admittances and scattering matrices characterizing discontinuities in coplanar wave guides and finlines.

Two simplifying assumptions are made to reduce the complexity of the formulation without any noticeable loss in accuracy:

1. the strips of the circuit are perfect conductors with an infinitesimal thickness;
2. the substrates are isotropic and lossless.

The program has been used to characterize a number of discontinuities in planar circuits:

- A short-circuited finline considered as an one-port circuit;
- Step discontinuities in coplanar wave guides considered as two-port circuits.

3.1 Case of One-Port Circuit: a Short-Circuited Finline

The circuit under study has a single port excited by the voltage source $|e_0\rangle$. The source and the source current $|J_0\rangle$ are defined on the source plane (xoy). Similarly, both the tangential electric field $|E_c\rangle$ and the current $|J_c\rangle$ of the circuit are defined in the circuit plane (xoz). The input admittance of the line is obtained from equation (10a), as follows:

$$y_{in} = Y_{11} - C_1 A^{-1} B_1, \tag{11a}$$

where,
$$Y_{11} = \langle e_0 | \hat{Y}_{11} e_0 \rangle; \tag{11b}$$

$$B_1[k] = \langle e_0 | \hat{Y}_{12} \phi_k \rangle; \qquad k = 1, \ldots, K \tag{11c}$$

$$C_1[k] = \langle \phi_k | \hat{Y}_{21} e_0 \rangle; \qquad k = 1, \ldots, K \tag{11d}$$

$$A[k, k'] = \langle \phi_k | \hat{Y}_{22} \phi_{k'} \rangle; \qquad k, k' = 1, \ldots, K \tag{11e}$$

The proposed technique is applied to compute the input admittance of the short-circuited finline with dimensions: gap $w = 0.3\,\text{mm}$, substrate thickness $h = 0.11\,\text{mm}$, substrate $\varepsilon_r = 3.75$, and at the frequency 80 GHz. The short-circuit is placed far enough from the source so that the reflected higher order modes are vanished. The admittance y_{in} has been computed and compared with the theoretical values obtained from the known analytical formula y_{in}^{th}:

$$y_{in}^{th} = -j\cot(\beta l), \tag{12}$$

where l is the distance between the source and the short circuit;
β is the propagation constant of the line.

The input admittances y_{in} and y_{in}^{th} are plotted as a function of the length l and shown in Figure 3. We notice a good agreement between y_{in} and y_{in}^{th}.

Note that the numerical accuracy of the solution depends strongly on the choice of the trial functions. The use of wavelet functions defined in the entire domain of the circuit led to an acceptable accuracy in the computation of y_{in}. The convergence of the algorithm has been obtained using only ten (10) trial functions and one hundred (100) basis functions (TE and TM modes).

3.2 Case of Two Port Discontinuities in Coplanar Wave Guides

The proposed technique is used to analyze more complicated discontinuities in coplanar wave guides. This involves the computation of scattering matrices of a number of step discontinuities.

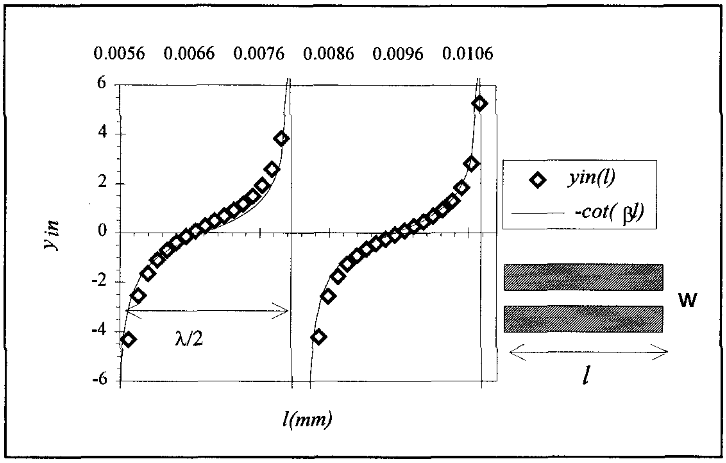


Figure 3. The input admittance y_{in} of a short circuited fin line as function of its length l; Dimensions: gap $w = 0.3\,\text{mm}$, substrate thickness $h = 0.11\,\text{mm}$, $\varepsilon_r = 3.75$, Frequency = 80 GHz.

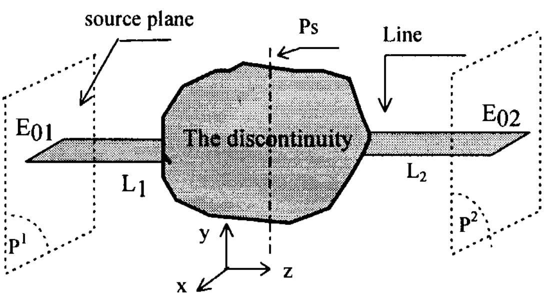


Figure 4. Schematic of 2-port discontinuity.

Figure 4 shows the general schematic geometry of the structure under study. The structure is excited by two sources E_{01} and E_{02} connected to the structure ports via two lines L_1 and L_2 respectively. The discontinuity has an arbitrary shape. Three different structures are considered: a symmetric double step discontinuity, a simple step discontinuity and a cascaded step discontinuity in coplanar wave guides.

a. Symmetric Double Step Discontinuities

We apply the method to characterize a double step discontinuity in a coplanar wave guide, the same structure used by Alessandri et al [10] to implement a 3-D mode matching technique. The inner line length is approximately half a wave length $(\lambda/2)$ at 15 GHz. The two symmetry planes P_s (the symmetry plane in the propagation direction) and the x-direction symmetry plane are taken into account in this implementation. Therefore, the analysis is reduced to the computation of the input admittance for the even mode and odd mode excitations.

To compute the admittance matrix of the equivalent circuit, the above analysis (computation of the input admittance) is applied twice. First, the symmetry plane P_s is replaced by a magnetic wall, then the input admittance relative to the even mode excitation y_{in}^e is computed. In the second application, P_s is replaced by an electric wall, the odd mode component y_{in}^o is computed. As a result, the elements of the admittance matrix Y_{in} are :

$$y_{11} = \frac{y_{in}^e + y_{in}^o}{2} \tag{13a}$$

$$y_{12} = \frac{y_{in}^e - y_{in}^o}{2} \tag{13b}$$

The algorithm convergence is reached using 15 test functions, leading to a matrix dimension 30 by 30. The elements of the scattering matrix of the structure are deduced from the computed normalized Y-parameters. Figure 5 shows the frequency dependent S-parameters of the discontinuity. Comparison of the obtained results with those of Alessandri et al [10] shows slight differences in some points. Alessandri et al [10] took into account the influence of the metallization thickness on the behavior of the strips. However, in our study, the strips are assumed to have infinitesimal thickness. This observation explains the differences between our numerical results and those of [10] in some frequency points.

b. Simple Step Discontinuity

This discontinuity is excited by two voltage sources E_{01} and E_{02}. According to the general formulation (10a) derived in section (II), the elements of the admittance matrix are:

$$y_{11} = Y_{11} - C_1 A^{-1} B_1 \tag{14a}$$

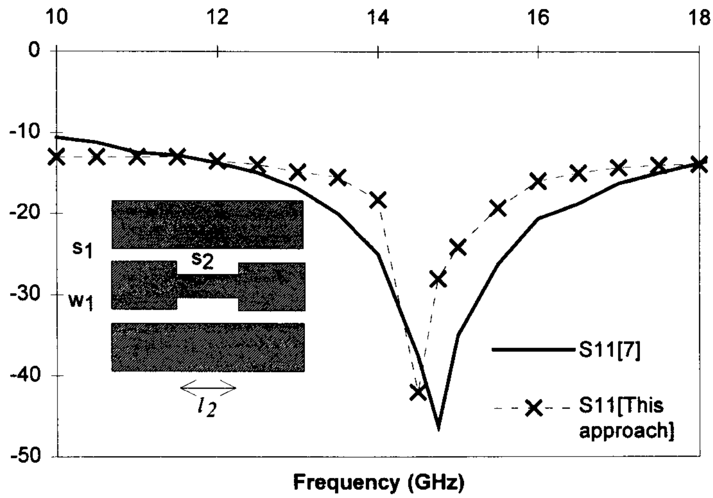


Figure 5. $S11$-parameter of a double step discontinuity in a coplanar circuit; Dimensions: $s_1 = 0.2\,\text{mm}$, $w_1 = 0.5\,\text{mm}$, $s_2 = 0.35\,\text{mm}$, $l_2 = 4.36\,\text{mm}$, $h = 0.635\,\text{mm}$, $\varepsilon_r = 9.9$, housing WR28.

$$y_{12} = Y_{12} - C_1 A^{-1} B_2 \tag{14b}$$

$$y_{21} = Y_{21} - C_2 A^{-1} B_1 \tag{14c}$$

$$y_{22} = Y_{22} - C_2 A^{-1} B_2 \tag{14d}$$

The scattering parameters S_{mn} of the discontinuity are derived from the admittance matrix. The numerical results computed by the proposed technique are compared to published data using other methods [11-13].

Figure 6 compares the computed S-parameters for the step discontinuity in a coplanar wave guide computed by the proposed technique with the results of [11] and [12]. From the figure, we can see that the computed results are in good agreement with those of Jin and Vahldieck [11] using the Frequency Domain Transmission Line Matrix method (TLM) and the results of Kuo and Kitazawa [12] using the Mode Matching technique. The convergence of our algorithm has been reached with a minimum of 20 trial functions.

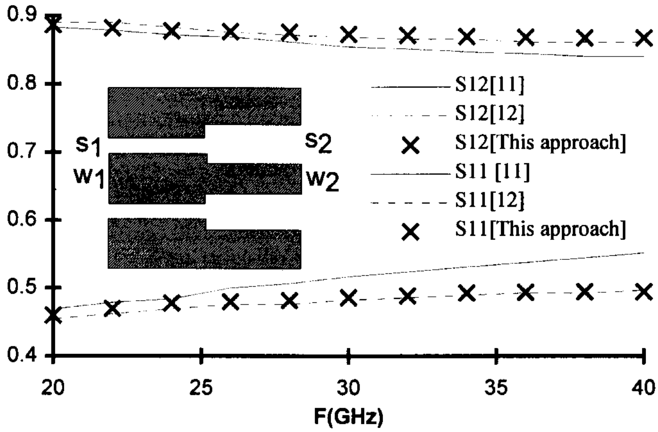


Figure 6. S-parameters of a simple step discontinuity in a coplanar wave guide; Dimensions: $s_1 = 0.1\,\text{mm}$, $s_2 = 0.4\,\text{mm}$, $w_1 = 0.4\,\text{mm}$, $w_2 = 0.1\,\text{mm}$, dielectric constant $\varepsilon = 9.6$ and thickness $h = 0.254\,\text{mm}$.

c. <u>Cascaded Step Discontinuity</u>

In this case, the discontinuity is a quarter wave $\lambda/4$ impedance transformer. Similar to the simple step discontinuity, the structure is excited by the two voltage sources E_{01} and E_{02}. The dimensions of this transformer are the same used by Schmidt and Russer [13]. The structure dimensions are: $\varepsilon_r = 12.9$, thickness $= 200\mu\text{m}$, wave guide 35 ohm ($w_1 = 20\mu\text{m}$, $s_1 = 5\mu\text{m}$), wave guide 50 ohm ($w_2 = 15\mu\text{m}$, $s_2 = 10\mu\text{m}$, $l_2 = 3.104\,\text{mm}$), wave guide 70 ohm ($w_3 = 8\mu\text{m}$, $s_3 = 17\mu\text{m}$). The convergence is reached with a minimum of 20 trial functions. The obtained results for the reflection coefficient $S11$ and the transmission coefficient $S21$ are slightly different from those obtained using the mode matching technique and the generalized scattering matrix method [13] (Figure 7). Schmidt and Russer [13] took into account the strip thickness, whereas in our study the metallization thickness is assumed to be zero. The results obtained using the proposed method show a satisfactory agreement with the published data [13].

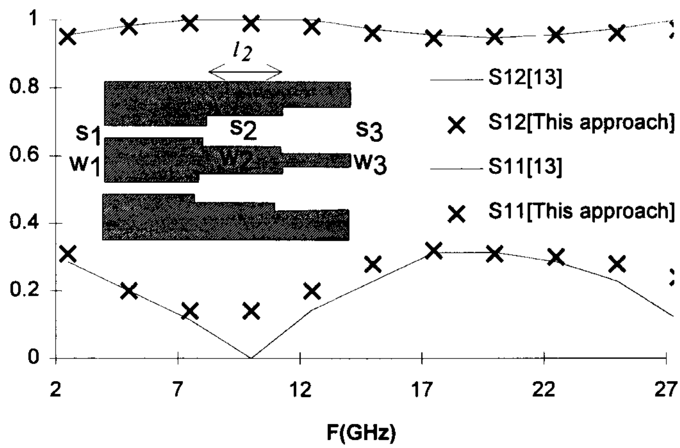


Figure 7. S-parameters of a cascaded step discontinuity in a coplanar wave guide: $\lambda/4$ transformer; Dimensions: $\varepsilon_r = 12.9$, thickness $= 200\mu$m, wave guide 35 ohm ($w = 20\mu$m, $s = 5\mu$m), wave guide 50 ohm ($w = 15\mu$m, $s = 10\mu$m, $l = 3.104$ mm), wave guide 70 ohm ($w = 8\mu$m, $s = 17\mu$m).

4. CONCLUSION

This paper presented a new technique allowing an accurate analysis of discontinuities in planar circuits, with a new consideration of the sources. With the new consideration of the sources the computational effort of the input admittance is reduced to the numerical solution of an inhomogeneous linear system of equations. The technique is applied first to compute the input admittance of a short-circuited line, then to compute the S-parameters of a number of step discontinuities in coplanar circuits. The obtained results are in good agreement with the analytic formula and the published data. This study can be further extended to analyze discontinuities taking into consideration the strip thickness.

REFERENCES

1. Maeda, S., T. Kashiwa, and I. Fukai, "Full wave analysis of propagation characteristics of a through hole using the finite difference time domain method," *IEEE Trans. Microwave Theory Tech.*, Vol. 39, 2154–2159, December 1991.

2. Cherry, P. C., and M. F. Iscander, "FDTD analysis of high frequency electronic interconnection effects," *IEEE Trans. Microwave Theory Tech.*, Vol. 43, 2445–2451, October 1995.
3. Ferrari, R. L., and R. L. Naidu, "Finite-element modeling of high-frequency electromagnetic problems with material discontinuities," *IEE Proc.*, Vol. 137, 313–320, November 1990.
4. Jianming, J., *The Finite Element Method in Electromagnetics*, John Wiley & Sons, 1993.
5. Harrington, R. F., *Field Computation by Moment Methods*, New York: Macmillan, 1968.
6. Sorrentino, R., and T. Itoh, "Transverse resonance analysis of finline step discontinuities," *IEEE Trans. Microwave Theory Tech.*, Vol. 32, 1633–1638, December 1984.
7. Pujol, S., H. Baudrand, and F. Hanna, "A complete description of a source-type method for modeling planar structures," *Annales Télécommunications*, Vol. 48, No. 9-10, 459–470,1993.
8. Nadarassin, M., H. Aubert, and H. Baudrand, "Analysis of planar structures by an integral approach using entire domain trial functions," *IEEE Trans. Microwave Theory Tech.*, Vol. 43, 2492–2495, October 1995.
9. Souny, B., H. Aubert, and H. Baudrand, "Elimination of spurious solutions in the calculation of eigenmodes by moment method," *IEEE Trans. Microwave Theory Tech.*, Vol. 44, 154–157, January 1996.
10. Alessandri, F., G. Baini, M. Mongiardo, and R. Sorrentino, "A 3-D mode matching technique for the efficient analysis of coplanar MMIC discontinuities with finite metallization thickness," *IEEE Trans. Microwave Theory Tech.*, Vol. 41, 1625–1629, September 1993.
11. Jin, H., and R. Vahldieck, "The frequency-domain transmission line matrix method –A new concept," *IEEE Trans. Microwave Theory and Tech.*, Vol. 40, 2207–2218, December 1992.
12. Kuo. C. K., T. Kitazawa, and T. Itoh, "Analysis of shield coplanar waveguide step discontinuity considering the finite metallization thickness effect," *IEEE-S International Symposium Digest*, Boston, 473–475, 1991
13. Schmid, R., and P. Russer, "Modeling of cascaded coplanar waveguide discontinuities by the mode-matching approach," *IEEE Trans. Microwave Theory and Tech.*, Vol. 43, 2910–2917, December 1992.