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DOMINANCE OF CREEPING WAVE MODES OF BACKSCATTERED FIELD FROM A CONDUCTING SPHERE WITH DIELECTRIC COATING

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1. INTRODUCTION

It is well known that the high frequency ray solution of the backscattered field from a coated conducting sphere which can be obtained from an asymptotic evaluation of the rigorous eigenfunction series solution consists of the reflected field and the diffracted field. The diffracted field from a convex surface such as the cylinder or the sphere is entirely associated with the creeping wave which propagates along the geodesic path on the curved surface. It is shown from the asymptotic evaluation of the creeping wave diffraction via the residue solution of the problem that there exists infinite number of creeping wave modes.

For a perfectly conducting convex surface, the first creeping wave mode is always dominant over all other modes and therefore only the first mode is adequate in practice. This is because the first mode has a lower attenuation constant and a larger diffraction coefficient than other higher modes. However this is not always true when the conducting surface is coated with a lossless dielectric material. For the coated cylinder, the dominance of the creeping wave mode changes as the thickness of the coating changes [1–3]. Thus, for some thickness of coating, the second or even higher mode is dominant over the first mode. The characteristics of the creeping wave on a conducting cylinder and a coated conducting cylinder were also investigated by Elliott [4], Paknys [5], Pearson [6], and Albertsen [7], Krasnojen [8].

For the case of a coated sphere, the diffracted field in the backscattering direction can not be cast in the format of the ordinary ray solution such as GTD (Geometrical Theory of Diffraction) since the backscattering is on the caustic line where infinite rays merge. Nevertheless the diffracted field is entirely associated with the creeping wave and therefore has a similar property as that of the diffracted field from a coated cylinder. The rigorous numerical calculations of RCS of a conducting sphere with a thin lossless coating were carried out by Rheinstein [9]. The high frequency ray solution of EM bistatic scattering from a coated conducting sphere was given by Kim [12]. It is observed from numerical results given by Rheinstein [9] that groups of resonance-like peaks occur as the size parameter of the coated sphere increases. Even though he obtained these resonance effects from the eigenfunction series solution, the effects can be explained by the changes of the attenuation coefficient of the surface wave. In this paper the resonance effects are analyzed in terms of the creeping wave diffraction obtained from the asymptotic solution of the backscattered field of a coated sphere and the groups of resonance peaks are explained as the change in the dominance of the creeping wave modes.

2. THE ASYMPTOTIC RAY SOLUTION OF THE CREEPING WAVE

Let us consider a coated conducting sphere illuminated by an incident plane wave. As illustrated in Fig. 1, the coated sphere with outer radius b and inner radius a is located at the coordinate origin. On the surface of the inner conducting sphere, the composite material with permittivity $(\epsilon_1 = \epsilon_r \epsilon_o)$ and permeability $(\mu_1 = \mu_r \mu_o)$ is coated with thickness $d(= b - a)$. The $\hat{x}$-polarized and $\hat{z}$-travelling incident plane

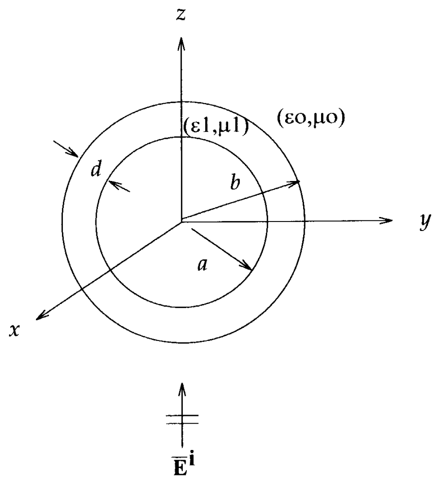

Figure 1. Geometry of the coated sphere.

wave can be expressed as

$$\bar{E}^i = \hat{x}E_o e^{-jk_o z} = \hat{x}E_o e^{-jk_o r\cos\theta} \tag{1}$$

An $e^{j\omega t}$ time dependence is assumed and suppressed in the expression of field. The eigenfunction series solution for the scattered field is given in [10]. Based on the Watson's transformation techniques [11, 12], the eigenfunction series solution for the scattered field can be transformed into a contour integral. This contour integral may then be separated into two integrals. The asymptotic evaluation of the first line integral results in the reflected field from the specular point on the coated sphere and the Cauchy residue evaluation of the second integral represents the diffracted field of the creeping wave. The asymptotic solution of the creeping wave diffraction for the far-zone backscattered field is given by [12, 13]

$$E_\theta^{c.w.} = E_\theta^{TE,\,c.w.} + E_\theta^{TM,\,c.w.} \tag{2}$$

$$E_\phi^{c.w.} = E_\phi^{TE,\,c.w.} + E_\phi^{TM,\,c.w.} \tag{3}$$

where

$$E_\theta^{TE,\,c.w.} = -E_o\cos\phi\frac{e^{-jk_o r}}{r}\sum_{m=1}^{\text{“}\infty\text{”}} D_m^s\frac{e^{-j\left(\nu_m^s+\frac{1}{2}\right)\pi}}{1+e^{-j2\pi\left(\nu_m^s+\frac{1}{2}\right)}} \tag{4}$$

$$E_\theta^{TM,\,c.w.} = E_o \cos\phi \frac{e^{-jk_or}}{r} \sum_{m=1}^{\text{“}\infty\text{”}} D_m^h \frac{e^{-j\left(\nu_m^h+\frac{1}{2}\right)\pi}}{1+e^{-j2\pi\left(\nu_m^h+\frac{1}{2}\right)}} \tag{5}$$

$$E_\phi^{TE,\,c.w.} = -E_o \sin\phi \frac{e^{-jk_or}}{r} \sum_{m=1}^{\text{“}\infty\text{”}} D_m^s \frac{e^{-j\left(\nu_m^s+\frac{1}{2}\right)\pi}}{1+e^{-j2\pi\left(\nu_m^s+\frac{1}{2}\right)}} \tag{6}$$

$$E_\phi^{TM,\,c.w.} = E_o \sin\phi \frac{e^{-jk_or}}{r} \sum_{m=1}^{\text{“}\infty\text{”}} D_m^h \frac{e^{-j\left(\nu_m^s+\frac{1}{2}\right)\pi}}{1+e^{-j2\pi\left(\nu_m^h+\frac{1}{2}\right)}} \tag{7}$$

$$D_m^s = \frac{\left(\nu_m^s + \frac{1}{2}\right)}{\hat{H}_{\nu_m^s}^{(2)}(k_ob)\frac{\partial}{\partial\nu}\left[\hat{H}_\nu^{(2)'} - A_\nu^s \hat{H}_\nu^{(2)}\right]_{\nu=\nu_m^s}} \tag{8}$$

$$D_m^h = \frac{\left(\nu_m^h + \frac{1}{2}\right)}{\hat{H}_{\nu_m^h}^{(2)}(k_ob)\frac{\partial}{\partial\nu}\left[\hat{H}_\nu^{(2)'} - A_\nu^h \hat{H}_\nu^{(2)}\right]_{\nu=\nu_m^h}} \tag{9}$$

$$A_\nu^s = \frac{Z_o}{Z_1} \frac{\hat{H}_\nu^{(1)'}(k_1b)\hat{H}_\nu^{(2)}(k_1a) - \hat{H}_\nu^{(2)'}(k_1b)\hat{H}_\nu^{(1)}(k_1a)}{\hat{H}_\nu^{(1)}(k_1b)\hat{H}_\nu^{(2)}(k_1a) - \hat{H}_\nu^{(2)}(k_1b)\hat{H}_\nu^{(1)}(k_1a)} \tag{10}$$

$$A_\nu^h = \frac{Z_1}{Z_o} \frac{\hat{H}_\nu^{(1)'}(k_1b)\hat{H}_\nu^{(2)'}(k_1a) - \hat{H}_\nu^{(2)'}(k_1b)\hat{H}_\nu^{(1)'}(k_1a)}{\hat{H}_\nu^{(1)}(k_1b)\hat{H}_\nu^{(2)}(k_1a) - \hat{H}_\nu^{(2)}(k_1b)\hat{H}_\nu^{(1)}(k_1a)} \tag{11}$$

The superscripts s and h in (4)–(11) denote the TE and TM case, respectively and Z_1 and k_1 represent the intrinsic impedance and the wave number of the coating material (i.e., $Z_1 = \sqrt{\mu_1/\epsilon_1}$, $k_1 = \omega\sqrt{\epsilon_1\mu_1}$). The subscript m denotes the mode of the creeping wave and the notation "∞" in (4)–(7) denotes that in actual calculation only one or two dominant modes are required to obtain an accurate result. $\hat{H}_\nu^{(1),(2)}$ and $\hat{H}_\nu^{(1)',(2)'}$ are the alternative spherical Hankel function and its derivative with respect to the argument, respectively [14].

Fig. 2 shows the comparison between RCS from the asymptotic ray solution and that from the eigenfunction series solution as the radius of the inner conducting sphere increases with the fixed thickness of coating. It is observed that two results show excellent agreement with each other when the inner radius of the sphere is larger than $1\lambda_o$. It should be noted that the asymptotic solution is valid for large scatterers. Thus, the investigation here is confined to a coated sphere whose radius is large in terms of the wavelength i.e., $a = 3\lambda_o$.

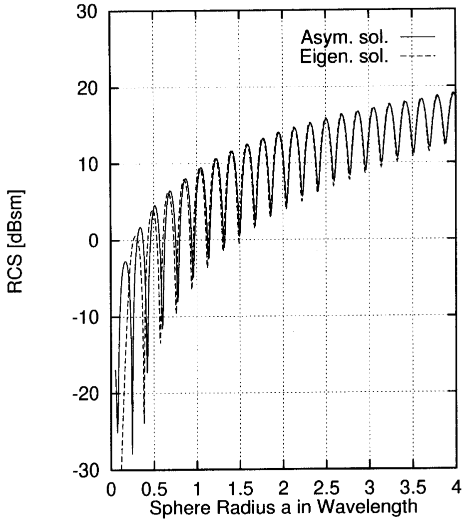

Figure 2. RCS from a coated sphere with the coating material $\epsilon_r = (2.5, -0.2)$ and $\mu_r = (1.6, -0.1)$ and the thickness of coating $d = 0.05\lambda_o$.

3. DOMINANCE OF CREEPING WAVE MODES

It is well known that for a planar dielectric slab on a ground plane there exists a cutoff thickness below which a surface wave mode can not exist and the strength of each surface mode changes with the thickness of the dielectric slab [15]. Meanwhile the coated sphere supports an infinite number of creeping wave modes regardless of the thickness of coating. For a bare conducting sphere the contribution from the first creeping wave mode to the total diffracted field is always dominant over other higher modes. However for the coated sphere the dominance of creeping wave mode changes with the thickness of coating. In the actual calculation of the diffracted field, one or two dominant modes are enough to obtain a numerically accurate result. It is important therefore to determine which mode is dominant over the other modes.

3.1 Roots of the Transcendental Equation

As discussed in [11–13], the eigenfunction series solution of a back-scattered field from a coated sphere can be transformed into a contour integral. The evaluation of the contour integral using Cauchy's residue theorem leads to another series expression which converges much more quickly than the eigenfunction series solution. Before the evaluation of the contour integral, the poles of the integrand must be found. The TE poles (ν_m^s) and the TM poles (ν_m^h) are determined from the roots of the transcendental equation given as

$$\left[\hat{H}_\nu^{(2)}(k_o b) - A_\nu^{s,h}\hat{H}_\nu^{(2)}(k_o b)\right]_{\nu=\nu_m^{s,h}} = 0 \tag{12}$$

where the subscript m in ν denotes the mode of the creeping wave and $A_\nu^{s,h}$ is in (10) and (11). The roots of the transcendental equation can be numerically searched via the Newton-Raphson method [12]. For a more details regarding the roots of the transcendental equation, the reader is referred to [1, 5–7, 12].

3.2 The Attenuation and the Propagation Constant

As the creeping wave travels along the geodesic path on the coated surface there exists the shift of its phase and the exponential decay of its strength. The propagation constant determines the phase velocity of the creeping wave and the attenuation constant represents the loss of the electromagnetic energy on the surface of the sphere. From (4)–(7) it is noted that

$$\begin{aligned}\frac{e^{-j\left(\nu_m+\frac{1}{2}\right)\pi}}{1+e^{-j2\pi\left(\nu_m+\frac{1}{2}\right)}} &= e^{-j\left(\nu_m+\frac{1}{2}\right)\pi}\left[\ 1+j^2 e^{-j2\pi\nu_m}+\ldots\ \right]\\ &\approx e^{-j\left(\nu_m+\frac{1}{2}\right)\pi}\end{aligned} \tag{13}$$

The physical significance of (13) is that the diffracted field is due to the creeping wave which travels the shortest geodesic path on the surface of the coated sphere whose angular range is π and the multiple encirclements of the creeping waves. However since its strength decays exponentially as the creeping wave propagates on the coated surface, it is seldom necessary to include the multiple encirclements terms of the creeping wave. Comparing the phase term of the asymptotic ray solution of the creeping wave in (4)–(7) with that of the ordinary GTD ray format of the diffracted field, it can be easily seen that

$$e^{-j(\nu_m+\frac{1}{2})\pi} = e^{-j\left[\frac{\nu_m+\frac{1}{2}}{b}\right]b\pi} = e^{-\alpha_m l_1}e^{-j\beta_m l_1} \tag{14}$$

where

$$\alpha_m = -\mathrm{Im}\left[\frac{\nu_\mathrm{m} + \frac{1}{2}}{\mathrm{b}}\right] \tag{15}$$

$$\beta_m = \mathrm{Re}\left[\frac{\nu_\mathrm{m} + \frac{1}{2}}{\mathrm{b}}\right] \tag{16}$$

$$l_1 = b\pi \tag{17}$$

b is the radius of the outer sphere and l_1 is the length of travel of the creeping wave on the surface of the coated sphere. α_m and β_m denote the attenuation constant and the propagation constant of the creeping wave mode, respectively.

3.3 The Diffraction Coefficient

As mentioned before the diffracted field in the direction of backscattering is on the caustic line where infinite rays merge. Thus the asymptotic solution of the creeping wave diffraction can not be given in a ray format as in an ordinary GTD ray format of creeping wave diffraction. Nevertheless, D_m^s and D_m^h in (8) and (9) can be defined as the diffraction coefficients which are related to the attachment and the launching effect of the creeping wave. As seen in (8) and (9), $D_m^{s,h}$ are the functions of the roots of the transcendental equation $\nu_m^{s,h}$, the inner radius of the sphere a, the thickness of coating d, and the coating material ϵ_1 and μ_1. It is clear from (4)–(7) and (13)–(14) that the magnitude of each mode of the creeping wave is determined by two factors: the diffraction coefficient and the attenuation coefficient. The diffraction coefficient is related to the amount of incident field attached to the surface of the coated sphere and the field shed to the field point after the creeping wave propagates along the great circle path on the coated sphere. The attenuation constant α_m determines the amount of field decayed exponentially as the creeping wave propagates on the geodesic path. Thus the contribution of each mode to the total diffracted field depends on both the diffraction coefficient and the attenuation constant of each mode for the given size parameters of the coated sphere and the given coating material.

4. NUMERICAL RESULTS

Fig. 3–6 show the attenuation constant and the diffraction coefficient which were defined in the previous section. The relative permittivity and permeability of the coating are $\epsilon_r = (2.56, 0.)$ and $\mu_r = (1., 0.)$, respectively and the radius of the inner conducting sphere is $3\lambda_o$. Fig. 3 and 4 are the results for the TM case and Fig. 5 and 6 are for the TE case.

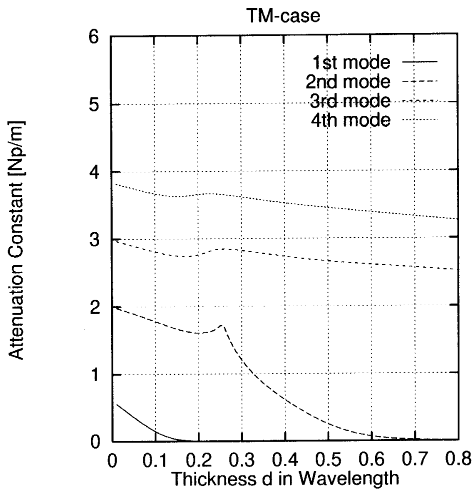

Figure 3. The attenuation constant of the creeping wave modes (TM-case) with $\epsilon_r = (2.56, 0.)$, $\mu_r = (1., 0.)$ and $a = 3\lambda_o$.

It is observed in Fig. 3 that the attenuation constant of the first mode is smaller than other modes and becomes negligibly small after the thickness of the coating exceeds $0.2\lambda_o$. As the thickness increases to $0.7\lambda_o$, the attenuation constant of the second mode also becomes negligibly small. It means that the first and the second mode of the creeping wave can propagate on the surface of the coated sphere without any attenuation practically when the thickness d is greater than $0.2\lambda_o$ and $0.7\lambda_o$, respectively. In Fig. 4, the diffraction coefficient of the first mode and the second mode are significant when the thickness is less than $0.2\lambda_o$ and $0.7\lambda_o$, respectively. As stated in the previous section, the magnitude of the creeping wave depends on two factors: the attenuation constant and the diffraction coefficient. Thus, it can be shown that the first mode of the creeping wave is dominant over others when the thickness is up to $0.2\lambda_o$ because the attenuation constant is

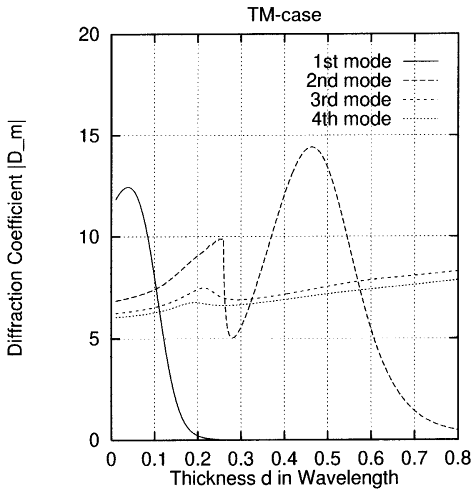

Figure 4. The diffraction coefficient of the creeping wave modes (TM-case) with $\epsilon_r = (2.56, 0.)$, $\mu_r = (1., 0.)$ and $a = 3\lambda_o$.

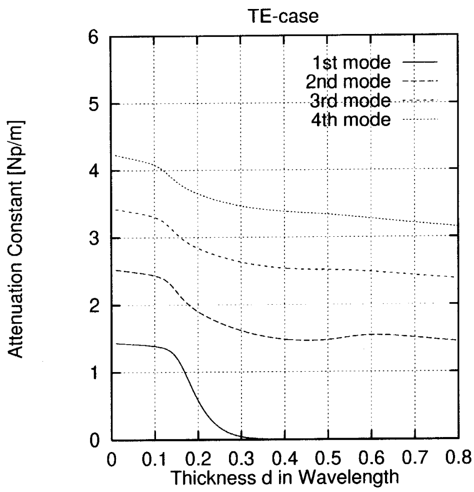

Figure 5. The attenuation constant of the creeping wave modes (TE-case) with the coating material $\epsilon_r = (2.56, 0.)$, $\mu_r = (1., 0.)$ and $a = 3\lambda_o$.

smaller and the diffraction coefficient is larger than other modes. For the range greater than $d = 0.2\lambda_o$, even the attenuation of the first mode is negligibly small, the diffraction coefficient of the first mode becomes smaller than that of the second mode and therefore, the con-

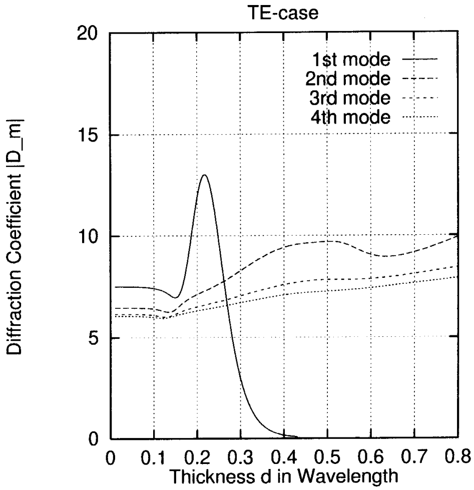

Figure 6. The diffraction coefficient of the creeping wave modes (TE-case) with $\epsilon_r = (2.56, 0.)$, $\mu_r = (1., 0.)$ and $a = 3\lambda_o$.

tribution of the first mode to the backscattered field is insignificant when compared to that of the second mode. Thus, the second mode of the creeping wave plays a more important role in the total backscattered field for coating thickness of $0.4\lambda_o \sim 0.8\lambda_o$. It is expected that if the coating thickness is increased further, the second mode of the creeping wave disappears and the third or even higher modes can be dominant. As shown in Fig. 5 and 6, the characteristics of the creeping wave modes for TE case are quite similar to those for TM case.

Fig. 7 and 8 show the magnitude of each mode of the creeping wave for the TM and TE case, respectively. For the TM case, the first mode and the second mode of the creeping wave are very significant when $d = 0.12\lambda_o$ and $d = 0.59\lambda_o$, respectively. The other modes can be ignored when $d = 0. \sim 0.8\lambda_o$. For the TE case, the first mode of the creeping wave is very significant when $d = 0.28\lambda_o$ and the other modes are negligible when $d = 0. \sim 0.8\lambda_o$.

Fig. 9 shows the RCS from a coated sphere with $\epsilon_r = (2.56, 0.)$, $\mu_r = (1., 0.)$ and $a = 3\lambda_o$. It is found that as the thickness of coating is increased, the groups of resonances occur. The groups of resonances are resulted from the interaction between the reflected field and the creeping wave whose dominance depends on the thickness of the coating.

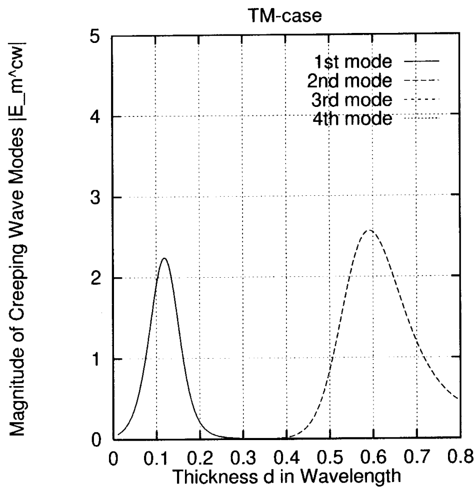

Figure 7. The magnitude of creeping wave modes from the attenuation constant (Fig. 3) and the diffraction coefficient (Fig. 4): TM-case. The first mode and the second mode of the creeping wave are very significant when $d = 0.12\lambda_o$ and $d = 0.59\lambda_o$, respectively. The other modes can be ignored when $d = 0. \sim 0.8\lambda_o$.

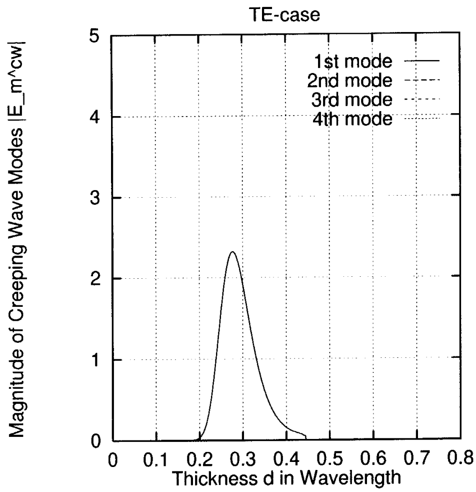

Figure 8. The magnitude of creeping wave modes from the attenuation constant (Fig. 5) and the diffraction coefficient (Fig. 6): TE-case. The first mode of the creeping wave is very significant when $d = 0.28\lambda_o$ and the other modes are negligible when $d = 0. \sim 0.8\lambda_o$.

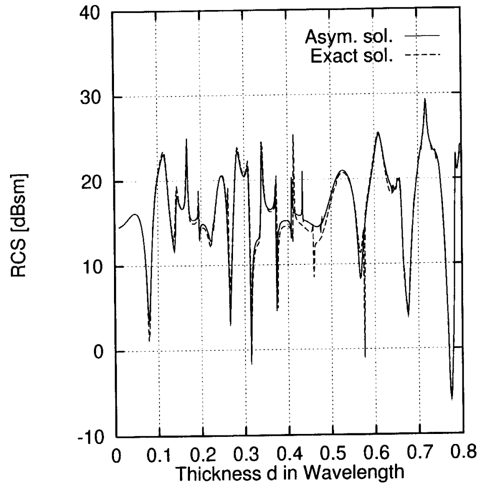

Figure 9. RCS from a coated sphere with $\epsilon_r = (2.56, 0.)$, $\mu_r = (1., 0.)$ and $a = 3\lambda_o$. It is found that as the thickness of coating is increased, the groups of resonances are occur.

5. CONCLUSIONS

In this study, the dominance of creeping wave modes of the backscattered field from a coated sphere is investigated. From the asymptotic evaluation of the eigenfunction series solution of the problem, the backscattered field is decomposed into the reflected field and the diffracted field. The diffracted field from the coated sphere is entirely associated with the creeping wave on the surface of the coated sphere. The magnitude of the each creeping wave depends on two factors; the attenuation constant and the diffraction coefficient. The attenuation constant determines the amount field decayed when the creeping wave travels along the geodesic path on the coated sphere and the diffraction coefficient are related to the amount of the incident field attached to the coated sphere and the field launched from the surface of the coated sphere to the field point. It is found that as the thickness of coating is increased, the groups of resonances occur. The groups of resonances are resulted from the interaction between the reflected field and the creeping wave whose dominance depends on the thickness of the coating.

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