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EXTENSION OF THE MEI METHOD AND ITS APPLICATIONS ON THIN WIRE ANTENNAS

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1. INTRODUCTION

Since the proposal of the MEI concept [1, 2], it has been widely applied in many computational electromagnetic areas, such as scattering by conducting objects or complex media [2–4], discontinuities of microstrips [5], and parameter extraction of microwave circuits [6], etc. The method itself, has also been studied, such as error analysis [7, 8], choice of metrons [9], extrapolation and interpolation techniques [10], and on surface method [11, 12], etc. The advantage of the MEI method is that it can truncate the computation domain very closely to the object. Only components of electric fields or magnetic fields are included in the original MEI equations. If components of both electric fields and magnetic fields are coupled in the MEI equations, the MEI equations can operate on the object surface. This method is called as the on surface MEI method [11].

In the MEI method, a series of possibly existing currents (metrons) on the object surface is selected at first, then all the components in the MEI equations are computed and substituted into the MEI equations, the MEI coefficients can be determined. When the real exciting source is imposed, the system of the MEI equations gives the real solution [2]. It is clear that the key of the MEI method is to *measure out the MEI coefficients*. The basic postulate of the MEI is that all the coefficients are independent on the exciting source, hence the metrons excited by a series of assumed exciting sources can be applied to find the MEI coefficients.

It is known that any electromagnetic problem can be depicted by one or more than one differential equations with determinative conditions, which include boundary conditions and source conditions. Generally speaking, this system can not be solved directly except numerical methods, even the differential equations themselves can not be given explicitly. In this paper, the system of the MEI equations are summarized as an approximation of the differential equations, both for the truncated boundary problems or the on surface MEI method. For the same problem, the solutions under different source conditions or different boundary conditions should satisfy the same system of MEI equations, the MEI coefficients are thus determined by a series of measuring functions. When both the source conditions and the boundary conditions are given, the real solution can be obtained by solving the system of MEI equations. Hence we can measure the MEI coefficients either by adjusting the source conditions or by adjusting the boundary conditions. This idea gives us more degrees of freedom to find the MEI coefficients, and it is possible for us to select the easiest way for the measurement in a certain problem.

As an elementary study, in this paper, we have focused the applications on thin wire antennas where the source conditions and boundary conditions can be defined easily. In the section 3, a new kind of MEI equations is given, which proved that the surface current density and scattered field satisfy a linear relationship on the object surface. To determine the MEI coefficients, both the source-conditions-measuring strategy (SCMS) and boundary-conditions-measuring strategy (BCMS) are discussed in section 4 and 5 respectively. The original

MEI method and the on surface MEI method are categorized into SCMS. For the BCMS, the MEI coefficients are measured out though loading the ends of the wire with different impedance.

2. EXTENSION OF THE MEI CONCEPT

The electromagnetic boundary-value field problems as shown in figure 1, can be depicted by

$$\begin{cases} D(\Phi) = 0 \\ B(\Phi) = 0 \end{cases} \tag{1}$$

where Φ is the unknowns to be determined, D is the differential operator, and B is the boundary condition operator. The exciting source condition has been included in these equations. Here we express the source condition explicitly, and the problem to be solved becomes

$$\begin{cases} D(\Phi) = 0 \\ B(\Phi) = 0 \\ S(\Phi) = 0 \end{cases} \tag{2}$$

where S is the source condition operator.

In this paper, eq. (2) is not only used to characterize the boundary-value problems, but also used to characterize the artificial truncated boundary problems. Figure 1 shows an electromagnetic boundary-value problem, and its exciting source conditions have been included in the boundary conditions as shown in eq. (1). In figure 2, the solid arc refers to one segment of the artificial truncated boundary, which is used to truncate the computation domain in many computational methods, both in time domain [13] or frequency domain [7]. Suppose point A on the truncated boundary is to be considered, its vicinal points are all closed in the dashed circlet. Based on the Huygens' equivalence principle, the field value of the point A can be determined by the fields along the closed dashed circlet. Since no incidence wave comes from the outside, the closed circlet is equal to the dashed arc $\overline{BCD}$ and the solid arc $\overline{BAD}$, and the truncated boundary problem can be depicted as

$$\begin{cases} D(\Phi) = 0 \\ S(\Phi) = 0 \end{cases} \tag{3}$$

where the operator D represents the relationship among fields on the truncated point A and points in its vicinity. D is the operator that all kinds of truncated boundary conditions try to find.

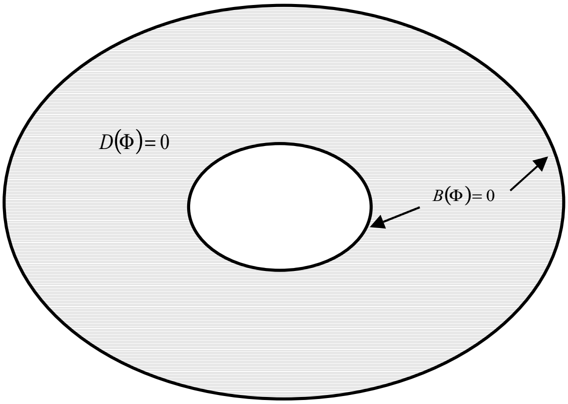


Figure 1. Boundary-value problems (Fields in the shadow domain are to be found).

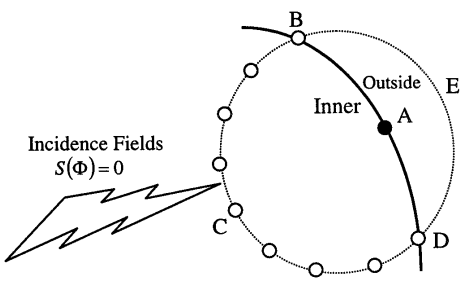


Figure 2. Artificial truncated boundary conditions (Solid line is the truncated boundary).

In the MEI method, the MEI equations are used to approximate the operator D in finite-difference form,

$$D(\Phi) \approx M(\Phi) \tag{4}$$

where M is called as the MEI operator, then eq. (2) becomes

$$\begin{cases} M(\Phi) = 0 \\ B(\Phi) = 0 \\ S(\Phi) = 0 \end{cases} \tag{5}$$

When the MEI method works as a truncated boundary condition which is away from the object surface [1–7] as shown in fig. 2, it can be depicted as

$$\begin{cases} M(\Phi) = 0 \\ SB(\Phi) = 0 \end{cases} \tag{6}$$

in which the operator SB means the boundary conditions of the truncated boundary problem are hinted in the source condition. For the on surface MEI method [11, 12], the problem can be depicted as

$$\begin{cases} M\left(\Phi, \dfrac{\partial}{\partial n}\Phi\right) = 0 \\ BS(\Phi) = 0, \text{or } BS\left(\dfrac{\partial}{\partial n}\Phi\right) = 0 \end{cases} \tag{7}$$

where the operator BS means the source condition have been hinted in the boundary condition.

In fact, M is the finite-difference approximation of the differential operator D. The dominant factor is how to find this operator as precise as possible using reasonable time and storage. In the original MEI method and the On-surface MEI method, the strategy to find the operator can be summarized as

$$\begin{cases} M(\Phi) = 0 \\ B\ (\Phi) = 0 \\ \{S_i\ (\Phi) = 0, i = 1, 2, 3, \ldots\} \end{cases} \tag{8}$$

where S_i refers to one of the assumed source conditions. From the second and the third equation in (8), a series of fields $\{\Phi_i\}$ can be obtained directly, and they are substituted into the first equation to get the MEI operator by the least square algorithm. After the determination of the MEI operator, the real solution can be obtained by solving a linear system of equations as shown in eq. (5), when the real source condition is imposed.

It should be pointed out that the source conditions in eq. (8) are the values of the measuring functions $\{\Phi_i\}$ on the dashed closed circlet in fig. 2. In the original MEI method, to produce these functions, a series of currents on the object surface (metrons) is selected and integrated along the surface. Since the information of the object geometry can be included, the truncated boundary can be posited very closely to the object surface [2].

We call the method to determine the MEI operator indicated by eq. (8) as source-conditions-measuring strategy (SCMS), because when we find the MEI operator, we have assumed a series of exciting sources to induce the series of metrons [11]. Our question is, can we produce the measuring functions by a series of boundary conditions? We call this kind of measuring way as boundary-conditions-measuring strategy (BCMS). The strategy is depicted as

$$\begin{cases} M(\Phi) = 0 \\ \{B_i(\Phi) = 0, i = 1, 2, 3, \ldots\} \\ S(\Phi) = 0 \end{cases} \tag{9}$$

Since the MEI operator is independent on the source conditions and boundary conditions, the strategy depicted in eq. (9) or eq. (8) should give the same results. Hence either selecting reasonable source conditions or boundary conditions as adjusters to produce measuring functions, can we obtain the MEI operator.

We found that the realization of these strategies depends on the correct division of the source conditions and boundary conditions in practical problems. For the artificial truncated boundary condition problems as shown in figure 2, the application of the SCMS is direct as detailed in many previous works [1–10]. As an elementary study, we will study the linear thin wire structure as depicted in figure 4, where $a \ll \lambda$, so that the surface current has only z-direction component and $I = 2\pi a J_z$. In section 3, a new kind of MEI equations, which couples the surface current density with scattered field on the object surface, is derived. In section 4, the SCMS is used to measure out the MEI operator. In section 5, the BCMS is used to measure out the MEI operator though loading the ends of the wire with different impedance. For wire structures, the boundary conditions include the boundary condition on the wire surface and the zero-current conditions at the two ends of the wire. It is found that it is practical to blend the surface boundary condition into the source condition, while the ends condition is still kept as boundary condition, as

$$\begin{cases} M(\Phi) = 0 \\ B_e(\Phi) = 0 \\ SB_s(\Phi) = 0 \end{cases} \tag{10}$$

where B_e is the ends-condition operator and SB_s hints that the source condition has included the surface boundary condition.

3. MEI EQUATIONS FOR THE SURFACE CURRENT DENSITY

The reaction integral equation is [16],

$$\int_S [\vec{J}_S \cdot \vec{E}^m - \vec{M}_S \cdot \vec{H}^m] ds + \int_V [\vec{J}^i \cdot \vec{E}^m - \vec{M}^i \cdot \vec{H}^m] d\nu = 0 \tag{11}$$

where V is the volume closed by the enclosing surface S, $\vec{J}^i$ and $\vec{M}^i$ are the electric source current density and magnetic source current density respectively, $\vec{J}_S$ and $\vec{M}_S$ are the equivalent electric surface current density and magnetic surface current density on S due to $\vec{J}^i$ and $\vec{M}^i$, $\vec{E}^m$ and $\vec{H}^m$ is the electric field and magnetic field excited by the test source $\vec{J}^m$ and $\vec{M}^m$ in the free space.

From the reciprocity theorem, we have

$$\int_V \vec{J}^i \cdot \vec{E}^m d\nu = \int_V \vec{J}^m \cdot \vec{E}^i d\nu \tag{12}$$

$$\int_V \vec{M}^i \cdot \vec{H}^m d\nu = \int_V \vec{M}^m \cdot \vec{H}^i d\nu \tag{13}$$

where $\vec{E}^i$ and $\vec{H}^i$ is the incidence field due to the source $\vec{J}^i$ and $\vec{M}^i$, respectively. And let's consider the case in which the magnetic test current $\vec{M}^m = 0$, and the volume is occupied by a perfect conducting object, $\vec{M}_S = 0$. Substitute eq. (3) and (2) into (1), we have

$$\int_S \vec{J}_S \cdot \vec{E}^m ds + \int_S \vec{J}^m \cdot \vec{E}^i ds = 0 \tag{14}$$

Since the test source $\vec{J}^m$ can be selected arbitrarily, we can set the test source on a small part of the surface as in [11],

$$\vec{J}_n^m(\vec{r}) = \begin{cases} \vec{J}^m(\vec{r}), & \vec{r} \in S_n \\ 0, & \vec{r} \in \overline{S}_n \end{cases} \tag{15}$$

From eq. (4) and (5), we have

$$\int_{S_n} \vec{J}_S \cdot \vec{E}_n^m ds + \int_{S_n} \vec{J}_n^m \cdot \vec{E}^i ds = -\int_{S-S_n} \vec{J}_S \cdot \vec{E}_n^m ds \tag{16}$$

The right hand of eq. (6) will be taken as residual, and is expressed as $< \vec{E}_{null}, \vec{J}_S >$. This expression is also used in [9] and [11], where it has been proved that the residual can be neglected, and eq. (6) becomes

$$\int_{S_n} \vec{J}_S \cdot \vec{E}_n^m ds + \int_{S_n} \vec{J}_n^m \cdot \vec{E}^i ds = 0 \tag{17}$$

Suppose S_n is divided into $2M + 1$ patches, and approximate the integration in eq. (8) by a simple summation, we have

$$\sum_{j=-M}^{M} \vec{J}_s(\vec{r}_{i+j}) \cdot \vec{E}_n^m(\vec{r}_{i+j}) \Delta S_{i+j} + \sum_{j=-M}^{M} \vec{E}^i(\vec{r}_{i+j}) \cdot \vec{J}_n^m(\vec{r}_{i+j}) \Delta S_{i+j} = 0 \tag{18}$$

where $\vec{r}_i$ is the center of the ith patch. Rewrite the above equation into a more general form, we obtain

$$\sum_{j=-M}^{M} \vec{A}_j(\vec{r}_i) \cdot \vec{J}_S(\vec{r}_{i+j}) + \sum_{j=-M}^{M} \vec{B}_j(\vec{r}_i) \cdot \vec{E}^i(\vec{r}_{i+j}) = 0 \tag{19}$$

which is the MEI equation with coefficients to be determined. Since the current density and electrical field on the object surface are vectors, the MEI coefficients are represented in matrices $\vec{A}_j$ and $\vec{B}_j$. After finding the MEI coefficients, the surface current is directly obtained from eq. (19), as soon as the incidence field is given.

Eq. (19) indicates that the current on the conducting surface satisfies a differential equation and the incidence field is its exciting source. Rewrite eq. (19) as

$$\overline{\overline{A}} \bullet \overline{J_s} = \overline{\overline{B}} \bullet \overline{E^i} \tag{20}$$

and we have known the matrix of the MoM method can be depicted as

$$\overline{\overline{Z}} \bullet \overline{J_s} = \overline{E^i} \tag{21}$$

It is clear that the full matrix $\overline{\overline{Z}}$ in the MoM method is substituted by two sparse matrix $\overline{\overline{A}}$ and $\overline{\overline{B}}$ in the MEI method.

4. THE SOURCE-CONDITIONS-MEASURING STRATEGY FOR WIRE STRUCTURES

For the structure depicted in figure 3, let's consider the equation

$$-j\omega\mu_0\varepsilon_0 E_z^i = \left(\frac{d^2}{dz^2} + k^2\right) A_z \tag{22}$$

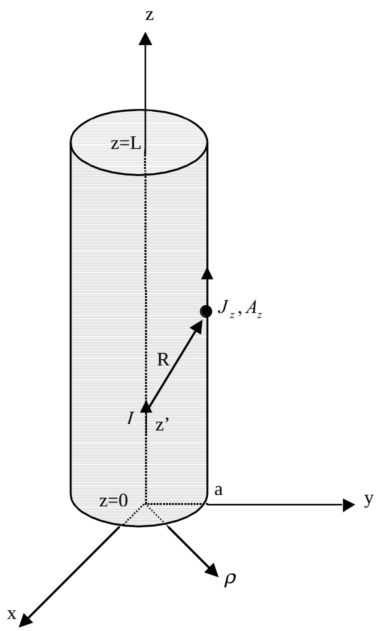


Figure 3. Wire structure in the cylindrical coordinate system.

then eq. (19) can be rewritten as

$$M_1 J_z(z) + M_2 A_z(z) = 0 \tag{23}$$

where A_z is the vector potential function in z-direction, and

$$M_1 = \sum_{i=-M}^{i=M} a_i \Delta_i \tag{24}$$

$$M_2 = \sum_{i=-N}^{i=N} b_i \Delta_i \tag{25}$$

$$\Delta_i f(z) = f(z + i\Delta z) \tag{26}$$

To obtain eq. (23), we have approximated eq. (22) by a difference formulation. For the real solutions, eq. (10) becomes

$$\begin{cases} M_1 J_z + M_2 A_z = 0 \\ J_z(z = 0, L) = 0 \\ A_z(z) = c\cos(kz) + d\sin(kz) - \dfrac{i\mu_0}{\eta_0} \displaystyle\int_0^z E_z^{inc}(z') \sin(k(z - z'))dz' \end{cases} \tag{27}$$

where E_z^{inc} is the real incidence field on the wire surface, c and d are unknowns. To obtain the third equation in (27), the surface boundary condition and the source condition have been included [14]. When a series of surface currents $\{J_{zm}\}$ are selected, the MEI operator $M = \overline{M_1 M_2}$ can be determined by

$$\begin{cases} M_1 J_{zm} + M_2 A_{zm} = 0 \\ J_{zm}(z = 0, L) = 0, m = 1, 2, 3, \ldots \\ A_{zm}(z) = \mu_0 \displaystyle\int_0^L G(z, z') I_m(z') dz' \end{cases} \tag{28}$$

where $G(z, z') = \frac{e^{-jkR}}{4\pi R}$ is the Green's function in free space. The select of the measuring surface currents $\{J_{zm}\}$ is arbitrary, except that they must satisfy the ends-condition as depicted in the second equation of (28). From the Pocklinton's integral equation [14], we have that

$$\int_0^L I_m(z) \left(k^2 + \frac{d^2}{dz^2} \right) G(z, z') dz' = -j\omega\varepsilon_0 E_{zm}^i \tag{29}$$

Hence we can see that when we directly select a series of surface currents to measure the MEI operator, we have in fact selected a corresponding series of incidence fields to induce those surface currents. Hence the method depicted in (28) is a source-conditions-measuring strategy (SCMS).

5. THE BOUNDARY-CONDITIONS-MEASURING STRATEGY FOR WIRE STRUCTURES

The second term in eq. (19) is dependent on the incidence field, we can simply represent it by a source term $V(\vec{r}_i)$,

$$\sum_{j=-M}^{M} \alpha_j(z_i) I(z_{i+j}) = V(z_i) \tag{30}$$

And according to eq. (10), the problem becomes

$$\begin{cases} M\left(I(z)\right) = V(z) \\ I = 0|_{z=0,L} \end{cases} \tag{31}$$

where the source term V is taken as known. The strategy to find the operator M can be depicted as

$$\begin{cases} M(I_m(z)) = V(z) \\ I_m + \begin{bmatrix} \Omega_{1m}|_{z=0} \\ \Omega_{2m}|_{z=L} \end{bmatrix} \dfrac{\partial}{\partial z} I_m = 0 \end{cases} \tag{32}$$

where $\{\Omega_{1m}\}$ and $\{\Omega_{2m}\}$ are related to the impedance loaded at the two ends of the wire, and $\Omega_1 = \Omega_2 = 0$ represents the real ends condition. From eq. (32), we can see that when we adjust the loads at the two ends, the exciting source V keeps invariant, i.e., the same incidence field E_z^{inc} in eq. (28) illuminates on the wire. Figure 4 schemes out the measurement with loads loaded at the two ends of the wire as adjusters.

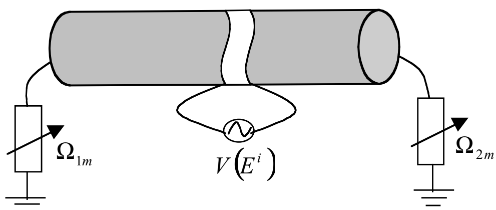


Figure 4. The sketch of the measuring scheme in the BCMS.

6. PRACTICE OF MEASURING THE MEI OPERATOR

The practice of the source-conditions-measuring strategy is the same as the original MEI method and the On-surface MEI method. When we apply eq. (28) to find the MEI coefficients, only measuring functions $\{A_{zm}\}$ need to be integrated from the selected metrons, and the measuring functions $\{\vec{J}_{zm}\}$ are directly equal to the metrons. The metrons should satisfy the zero-current conditions at the two ends of the wire, hence the sinusoidal functions are selected as metrons. Since the current varies rapidly in the feed-region [14], we have applied high order sinusoidal functions to characterize this behavior in the set of metrons. One technique for choosing the metrons is that we need not cover the whole region of m from the lowest to the highest order, which means we can continuously sample metrons in a low order region, and sample few metrons at intervals in a higher order region. This tech-

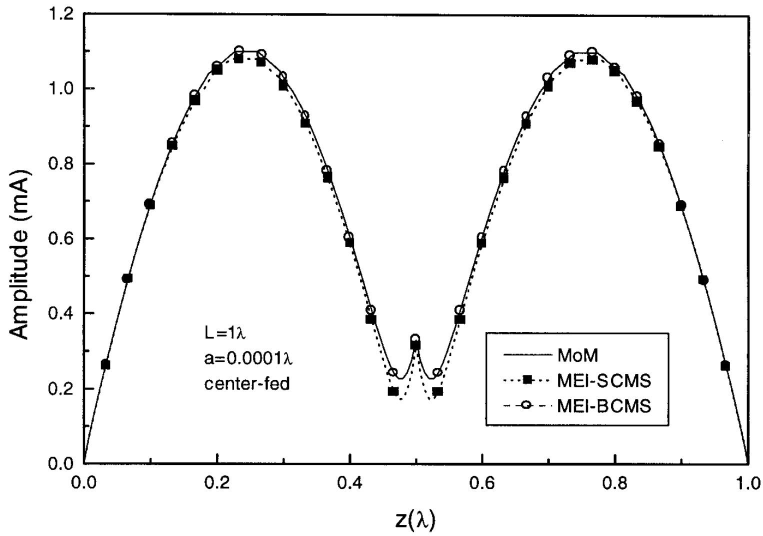
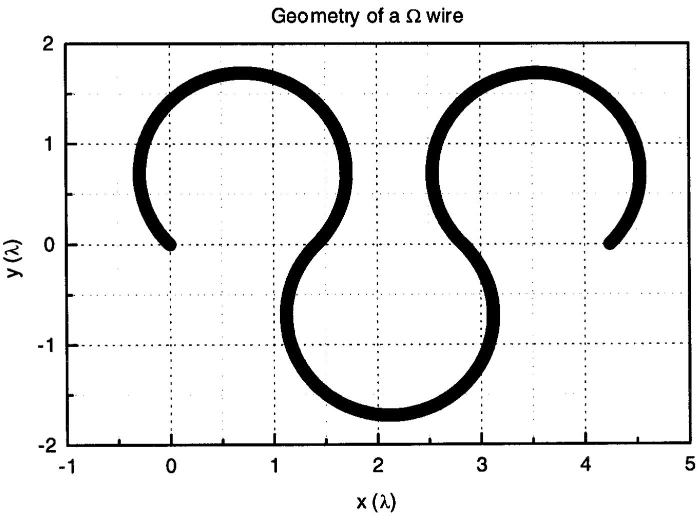


Figure 5. Current distribution on an linear wire antenna with $L = \lambda, a = 0.0001\lambda$.

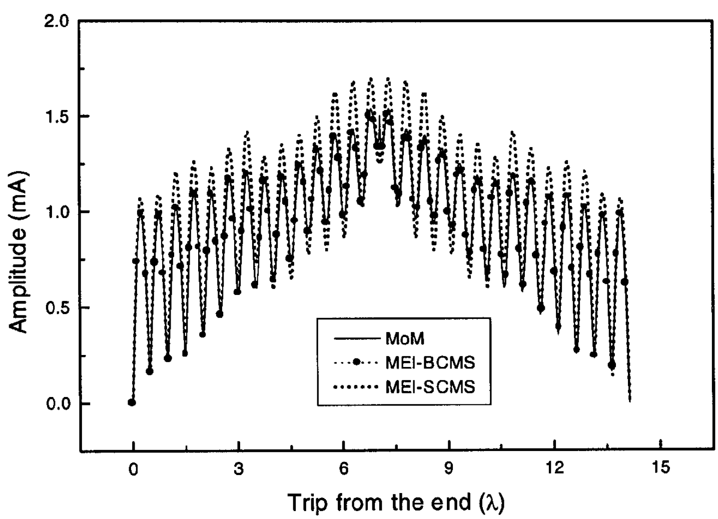
nique makes the set of metrons satisfy above requirements with the least numbers.

For the practice of the boundary-conditions-measuring strategy in the present problem we studied here, the source condition V as shown in eq. (31) can not be given explicitly, hence it is also determined by the MEI method. A series of $\{\Omega_{1m}\}$ paired with $\{\Omega_{2m}\}$ (measuring loads) is selected, and the corresponding currents $\{I_m\}$ can be given by the MoM method. When the series of $\{I_m\}$ is substituted into the first equation in (27), the MEI operator can be determined by the least square method, the source term, V, is also given at the same time. The real solution of $\{I\}$ is obtained by solving a highly sparse matrix system based on eq. (30), and the real current I distributed on the wire surface is then determined. It is found that only a small sequential change between the pairs of the measuring loads, Ω_{1m} and Ω_{2m}, can bring stable results. Hence when the MoM is utilized to give the set of measuring functions $\{I_m\}$, the perturbation technique for matrix equations [15], is developed to speed up this solution.

Above discussion is directly extended to the case of a general curved wire, as long as the generalized Green's function is used [14].

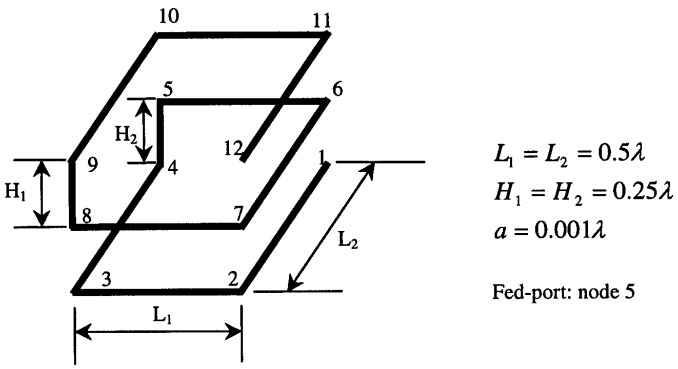


(a) Geometry of a Ω wire with $a = 0.001\lambda$.

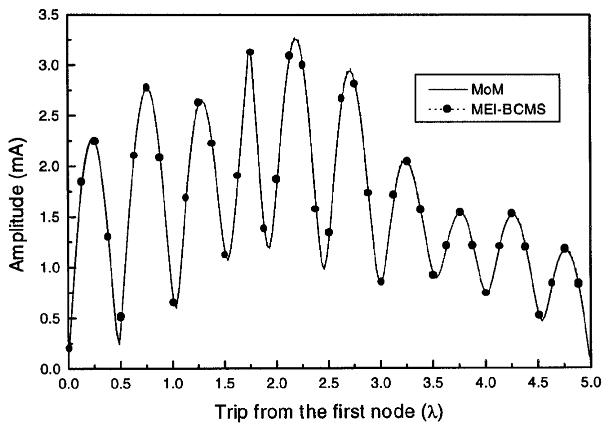


(b) Current distribution on the center-fed Ω wire antenna.

Figure 6. Ω wire antenna and its current distribution.



(a) Geometry of a helix wire.



(b) Current distribution.

Figure 7. Helix wire and its current distribution.

7. NUMERICAL RESULTS

Figure 5 gives the current distribution on a wire antenna with $a = 0.0001\lambda$, $L = 1\lambda$, driven by a perfect delta-gap voltage source at the center of the wire. For the SCMS, $N = 2$ is used, which means 10 MEI coefficients need to be determined in eq. (26). For the BCMS, $N = 1$ is used, which means 3 MEI coefficients are used in eq. (31). Compared with the MoM results, excellent agreement is obtained for the BCMS. Since the current in the feed-region varies rapidly [14], the selected metrons has to characterize this property, hence the accuracy of the SCMS is less than that of the BCMS.

Figure 6 (a) gives a Ω wire antenna with a fed port at the center of the wire and $a = 0.001\lambda$. Figure 6 (b) gives the current distribution computed by the MoM, SCMS and BCMS respectively. The agreement between the BCMS and the MoM is great, but the computing error of the SCMS becomes distinct.

The MEI operator can be measured out by using two test branches at the ends of the wire. Figure 7 (a) depicts a helix constructed by linear branches with $a = 0.001\lambda$, the current on the wire is given in figure 7 (b). To realize the adjustable boundary conditions at the two ends of the helix, two short test branches have been attached at the nodes *1* and *12*. The test branches have the length of 0.05λ, and are loaded with different impedance. Based on transmission line theory, the effect of the test branches can be equivalent to lumped loads, Ω_{1m} and Ω_{2m}, at the two ends.

8. DISCUSSION

The essence of the MEI method is discussed in this paper. Based on this discussion, it is pointed out that either the source conditions (SCMS) or the boundary conditions (BCMS) can be used as adjusted parameters to measure the MEI operator. The key of the strategies is the correct division of the source conditions and boundary conditions. Both kinds of the strategies are studied for wire antennas. In the BCMS, impedance loaded at the two ends of the wire is utilized as adjuster for measuring the MEI operator.

This paper indicates that the current on a conducting surface satisfy a differential equation with the incidence field as exciting source. The MEI method can be used to approximate this differential equation. In the antenna problems, the SCMS has to characterize the rapid variation of the current in the fed-region. In the BCMS presented in this paper, the MoM has been utilized to obtain measuring functions. A more applicable BCMS which is independent on the MoM method, is been developing and will be submitted in our another paper.

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