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ELECTROMAGNETIC WAVE SCATTERING AT DOUBLE FREQUENCY FROM ANTENNAS WITH NONLINEAR LOAD

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1. INTRODUCTION

The main cause of nonlinear effects in electromagnetic wave scattering by metallic constructions is the presence of different mechanical connections: welded junctions, adjoining contacts of different metals in places of antenna joints and mounting, etc. As far as is known, [1], the presence of such "internal" nonlinearities of the object leads to the spectral variation of the scattered signal that may be the reason of a high interference level at the radio reception and may have as well useful applications: recognition of objects at the presence of strong background reflections, detection of hidden defects in industrial products and so on [1, 2].

In most papers devoted to the problem of nonlinear radio wave scattering the authors investigate the processes using analytical methods based on the solution of equations for the equivalent chain (by Tevenin or Norton) or numerical ones [3–5]. In the present paper we consider stationary processes in the approximation of a weak nonlinearity that permits using only first harmonics in the field spectral representation as dependent on the type of nonlinearity. For example, for a nonlinear contact of two metals via an oxide film the highest intensity has the spectral component at the third harmonic and for the semiconductor nonlinear element at the second harmonic [6]. When studying processes of nonlinear radio wave scattering the most attention is paid to the nonlinear scatterer itself, to the processes in it and the influence of external factors on these processes [7]. The nonlinear scatterer may have other scatterers near-by including nonlinear ones and an account of their influence on it is of interest. The present paper deals with the electromagnetic wave scattering by the system of two nonlinear scatterers (dipoles or loops) placed in a free space. [1] As a nonlinear load we have used a semiconductor diode which current-voltage characteristic for a moderate values of voltage can be written in the form [8]:

$$I \simeq \frac{V + \beta V^2}{R_0}$$

where I is the diode current, V is its voltage, R_0 is the initial diode resistance (at $V = 0$), β is the nonlinearity coefficient. The electrical circuit of the diode connection in the scatterer has been given in detail in [9]. To investigate the signal scattering at the second harmonic we can restrict ourselves by the quadratic dependence of the current-voltage characteristic in the case of a diod load. Note, that in scattering of intensive waves a current-voltage characteristic of the diod may have another form [10].

This paper is organized in the following way. The second section investigates the electromagnetic wave scattering by thin metallic dipoles disposed in mutually orthogonal planes. Some polarization features of the field scattered at the double frequency which are absent in the field scattered at the basic signal frequency are discussed. The third section deals with electromagnetic wave back scattering by parallel oscillators containing nonlinear load (NL). The dependence of the double

[1] As a model of the simplest nonlinear scatterer it is often used dipoles or loops loaded on nonlinear elements.

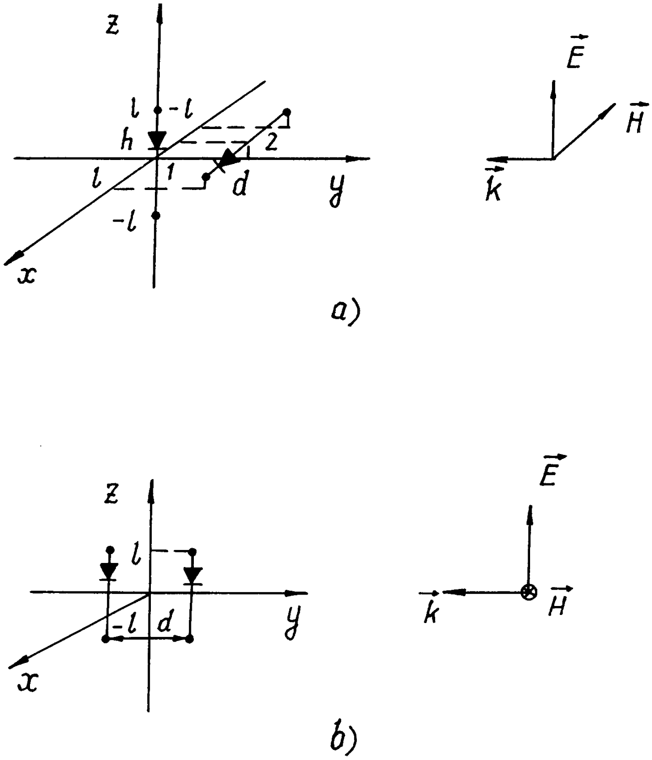

Figure 1. Geometry of the problem (see the text for the figure discussion in §2 and §3).

frequency scattered field on the distance between oscillators and NL location is investigated. The fourth section considers the electromagnetic wave back scattering by a system of two loops or dipoles in the case when NL is only in one of them. The main attention is paid here to the study of the back scattered field at the second harmonic as dependent on the distance between dipoles, NL location and dipole lengths. And finally the fifth section discusses some practical aspects of using the results obtained in the previous sections.

2. ELECTROMAGNETIC WAVE SCATTERING BY A SYSTEM OF ORTHOGONAL (CROSSED) DIPOLES CONTAINING NL

2.1.

In the beginning we start with the theoretical analysis of nonlinear electromagnetic wave scattering by a system of two half-wave dipoles with NL disposed in mutually orthogonal planes (see Fig. 1a).

Let a plane electromagnetic wave of a circular polarization be incident on a system of two crossed dipoles (it is evident this polarization of the incident wave gives the maximum amplitude of the scattered field at the second harmonic):

$$\vec{E} = E_0(\vec{x}^0 \cos\varphi_0 - i\vec{z}^0 \sin\varphi_0)e^{+i\omega t - iky} \tag{1}$$

The currents induced by the incident wave at frequency ω can be calculated by the method of induced electromotive force (E.M.F.) and written in the form:

$$\begin{aligned} I_\omega^{(1)} &= \frac{2E_0 \sin\varphi_0}{kS^\omega} \cos kz \\ I_\omega^{(2)} &= \frac{2iE_0 \cos\varphi_0 e^{ikd}}{kS^\omega} \cos kx \end{aligned} \tag{2}$$

where

$$\begin{aligned} S^\omega &= \frac{k}{4\pi\omega\varepsilon_0}\left\{D_i(2\pi) + iS_i(2\pi)\right\} + Z_\omega \\ D_i(x) &= \int_0^x \frac{1-\cos t}{t}dt, \qquad S_i(x) = \int_0^x \frac{\sin t}{t}dt \end{aligned} \tag{3}$$

Indices (1), (2) refer to the first and second dipoles, respectively, $Z_\omega = \frac{R_0}{1+i\omega cR_0}$ is the load impedance. The current at higher harmonics is excited due to the lumped E.M.F.

$$\mathcal{E}_{2\omega} = \beta Z_\omega^2 I_\omega^2(\xi)\,\delta(\xi - \xi_0) \tag{4}$$

caused by the presence of the local nonlinear load (more detail see in [10]). Here $I_\omega(\xi)$ is the current at fundamental frequency ω defined by formulas (2), ξ_0 is the coordinate of NL location. The current distribution at the double frequency depends essentially on NL location and is given as

$$I_{\omega}^{(1)}=\frac{\mathcal{E}_{2\omega}^{(1)}Z_{\omega}}{R_0}\begin{cases}\dfrac{\sin 2kz_0}{S^{2\omega}}\sin 2kz, & z_0\neq 0\\ \dfrac{i\pi\omega\varepsilon_0}{k\ln(2ka)}\sin 2k|z|, & z_0=0\end{cases}\tag{5}$$

$$I_{\omega}^{(2)}=\frac{\mathcal{E}_{2\omega}^{(2)}Z_{\omega}}{R_0}\begin{cases}\dfrac{\sin 2kx_0}{S^{2\omega}}\sin 2kx, & x_0\neq 0\\ \dfrac{i\pi\omega\varepsilon_0}{k\ln(2ka)}\sin 2k|x|, & x_0=0\end{cases}\tag{6}$$

$$S^{2\omega}=\frac{k}{4\pi\omega\varepsilon_0}\left\{D_i(4\pi)+iS_i(4\pi)\right\}+Z_{2\omega}$$

here $2a$ is the wire diameter, z_0, x_0 are the coordinates of NL at the first and second oscillators, respectively, $Z_{2\omega}=\frac{R_0}{1+i2\omega cR_0}$. The field scattered by two dipoles at the double frequency is the sum of radiation fields of currents $I_{2\omega}^{(1,2)}$: $\vec{E}_{2\omega}=\vec{E}_{2\omega}^{(1)}+\vec{E}_{2\omega}^{(2)}$ and in the wave zone it is described by the following expressions:

in the case $z_0\neq 0,\quad x_0\neq 0$

$$E_{z,2\omega}^{(1)}=\frac{iE_0^2\cos^2\varphi_0}{2\pi\omega\varepsilon_0 k}\frac{\beta Z_{2\omega}Z_{\omega}^2}{R_0S^{2\omega}(S^{\omega})^2}\cos^2 kz_0\sin 2kz_0\left[\frac{e^{-i2kr_{11}}}{r_{11}}-\frac{e^{-i2kr_{12}}}{r_{12}}\right]$$

$$E_{x,2\omega}^{(1)}=-\frac{iE_0^2\cos^2\varphi_0}{2\pi\omega\varepsilon_0 k}\frac{\beta Z_{2\omega}Z_{\omega}^2}{R_0S^{2\omega}(S^{\omega})^2}\cos^2 kz_0\sin 2kz_0$$
$$\times\frac{x}{x^2+y^2}\left[\frac{e^{-i2kr_{11}}}{r_{11}}(l+z)+\frac{e^{-i2kr_{12}}}{r_{12}}(l-z)\right]$$

$$E_{z,2\omega}^{(2)}=\frac{iE_0^2\sin^2\varphi_0}{2\pi\omega\varepsilon_0 k}\frac{\beta Z_{2\omega}Z_{\omega}^2}{R_0}\frac{e^{2ikd}}{(S^{\omega})^2S^{2\omega}}\cos^2 kx_0\sin 2kx_0$$
$$\times\frac{z-h}{(y-d)^2+(z-h)^2}\left[\frac{e^{-i2kr_{21}}}{r_{21}}(l+x)+\frac{e^{-i2kr_{22}}}{r_{22}}(l-x)\right]$$

$$E_{x,2\omega}^{(2)}=-\frac{iE_0^2\sin^2\varphi_0}{2\pi\omega\varepsilon_0 k}\frac{\beta Z_{2\omega}Z_{\omega}^2}{R_0}\frac{e^{2ikd}}{(S^{\omega})^2S^{2\omega}}\cos^2 kx_0\sin 2kx_0$$
$$\times\left[\frac{e^{-i2kr_{21}}}{r_{21}}-\frac{e^{-i2kr_{22}}}{r_{22}}\right]\tag{7}$$

in the case $z_0=0,\quad x_0=0$

$$E_{z,2\omega}^{(1)}=-\frac{Z_{2\omega}Z_{\omega}^2\beta E_0^2\sin^2\varphi_0}{2R_0\ln(2ka)k^2(S^{\omega})^2}\left[\frac{e^{-i2kr_{11}}}{r_{11}}+\frac{e^{-i2kr_{12}}}{r_{12}}+2\frac{e^{-i2kr_0}}{r_0}\right]$$

$$
\begin{aligned}
E^{(1)}_{x,2\omega} &= -\frac{Z_{2\omega} Z^2_\omega \beta E^2_0 \sin^2\varphi_0}{2R_0 \ln(2ka) k^2 (S^\omega)^2} \frac{x}{x^2+y^2} \\
&\quad \times \left[2z \frac{e^{-i2kr_0}}{r_0} + (l+z) \frac{e^{-i2kr_{11}}}{r_{11}} - (l-z) \frac{e^{-i2kr_{12}}}{r_{12}} \right] \\
E^{(2)}_{z,2\omega} &= \frac{Z_{2\omega} Z^2_\omega \beta E^2_0 e^{2ikd} \cos^2\varphi_0}{2R_0 \ln(2ka) k^2 (S^\omega)^2} \frac{z-h}{(z-h)^2+(y-d)^2} \\
&\quad \times \left[2x \frac{e^{-i2kr_{00}}}{r_{00}} + (l+x) \frac{e^{-i2kr_{21}}}{r_{21}} - (l-x) \frac{e^{-i2kr_{22}}}{r_{22}} \right] \\
E^{(2)}_{x,2\omega} &= \frac{Z_{2\omega} Z^2_\omega \beta E^2_0 e^{2ikd} \cos^2\varphi_0}{2R_0 \ln(2ka) k^2 (S^\omega)^2} \left[\frac{e^{-i2kr_{21}}}{r_{21}} + \frac{e^{-i2kr_{22}}}{r_{22}} + 2 \frac{e^{-i2kr_{00}}}{r_{00}} \right]
\end{aligned} \tag{8}
$$

where

$$
\begin{aligned}
r_{11} &= \sqrt{x^2+y^2+(z+l)^2}, \qquad r_{12} = \sqrt{x^2+y^2+(z-l)^2}, \\
r_{21} &= \sqrt{(y-d)^2+(z-h)^2+(x+l)^2}, \\
r_{22} &= \sqrt{(y-d)^2+(z-h)^2+(x-l)^2}, \\
r_0 &= \sqrt{x^2+y^2+z^2}, r_{00} = \sqrt{x^2+(y-d)^2+(z-h)^2}.
\end{aligned}
$$

Here d is the distance between dipoles, $2l$ is the length of the dipoles, h is the height of the dipole centers displacement relative to each other.

Formulas (7), (8) have been used to calculate the electric field component $E_{2\omega}$, in the plane (x, y) at an angle of $45°$ to axis x as dependent on distance d (φ_0 is also equal to $45°$). Figure 2a $(z_0 = x_0 = 0)$ and Figure 2b $(z_0/\lambda = x_0/\lambda = 0.125)$ show the dependencies of the back- scattered field at double frequency $|\vec{E}_{2\omega}|$ on the distance between dipoles d at the observational point $x/\lambda = 0.25, z/\lambda = 0.25, y/\lambda = 10, h = 0, a/\lambda = 1.5 \cdot 10^{-3}, 4l = \lambda$. These dependencies have a periodical character with period $T_d/\lambda = 0.125$. As expected, for a plane-polarized wave incident under angles $\varphi_0 = 0$ or $\varphi_0 = \pi/2$ the dipoles make a weak influence to each other at the double frequency and the dependence of the back-scattered field on distance d has a quasimonotone character.

2.2.

It should be noted that when an electromagnetic wave is incident on a system of two crossed dipoles containing NL we may have new

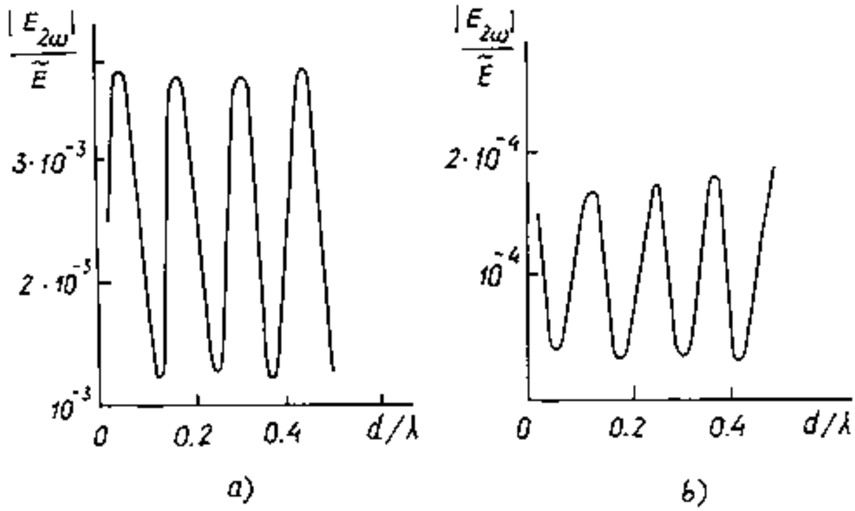

Figure 2. Distribution of the back scattered field $|E_{z,2\omega}(d)|^2$, over the distance between parallel dipoles. **a**) $z_0 = x_0 = 0$, **b**) $z_0/\lambda = x_0/\lambda = 0.125$.

polarization effects at higher harmonics which are absent at the fundamental frequency. Now we consider them more in detail. Let us take for simplicity the case when $h = z_0 = x_0 = 0$. Then from (8) at the observation point $x = z = 0, y \neq 0$ we have for a radial component $(\vec{r} = \vec{x}_0 \cos\varphi + \vec{z}_0 \sin\varphi)$ of the electromagnetic field at the double frequency the following expressions

$$\vec{E} = \vec{r}D\left\{e^{2ikd\cos\varphi\cos^2\varphi_0}\left[\frac{e^{-i2k\sqrt{(y-d)^2+l^2}}}{\sqrt{(y-d)^2+l^2}} + \frac{e^{-i2k(y-d)}}{y-d}\right] + \sin\varphi\sin^2\varphi_0\left[\frac{e^{-i2k\sqrt{y^2+l^2}}}{\sqrt{y^2+l^2}} + \frac{e^{-i2ky}}{y}\right]\right\} \tag{9}$$

where

$$D = -\frac{Z_{2\omega}Z_\omega^2\beta E_0^2}{R_0\ln(2ka)k^2(S^\omega)^2k^2}$$

For $y \gg \max(d,l)$ and $\varphi_0 = 45^\circ$ the expression (9) reduces to

$$|E_r^2| = \frac{|D|^2}{y^2}(1 - \sin 2\varphi\cos 4kd) \tag{10}$$

The change of polarity of one of dipoles leads to the change of sign in the round brackets of formula (10).

One has to remember that the polarization diagram is the dependence of received power on angle φ when the polarization plane of a linear receiving antenna is rotated. Let the initial position of the polarization plane of the incident wave be vertical, then as it follows from (10) the polarization of the received signal at frequency 2ω changes from vertical $(d = 0)$ to elliptic one $(d_1/\lambda_0 = 1/32)$, then circular $(d_2/\lambda_0 = 1/16)$, elliptic with another ellipse slope $(d_3/\lambda_0 = 3/32)$), horizontal (at $d_4/\lambda_0 = 1/8$) and so on. Hence, from (10) it follows that in a period of $T_d = \lambda_0/4$ the polarizations recurs completely. The change of the diode polarity, as it has been already mentioned, changes the polarization type to the opposite one (for example, the vertical to the horizontal). Therefore, the polarization diagram of the signal at the double frequency depends on the spatial configuration of dipoles (distance d) and in this way it may differ from the polarization of the incident field at the fundamental frequency. This has been also noted in [11].

3. ELECTROMAGNETIC WAVE SCATTERING BY PARALLEL DIPOLES WITH NL

In this section we consider back-scattering of a plane linearly-polarized electromagnetic wave by a system of two parallel dipoles with NL in the form of a semiconductor diode (see Figure 1b).

3.1.

So, let a wave of frequency ω $(\vec{E} = \vec{z}E_0 e^{+i\omega t - iky})$ be incident on a system of two parallel half-wave dipoles $(2l = \lambda/2)$. This wave induces in the dipoles both currents at frequencies ω and 2ω . The former are written in the following way:

$$
\begin{aligned}
I_\omega^{(1)} &= \frac{2E_0 e^{-ikd}}{kS^\omega}\left\{1 - \frac{ike^{i2kd}J(d)}{4\pi\omega\varepsilon_0 S^\omega}\right\}\cos kz \\
I_\omega^{(2)} &= \frac{2E_0 e^{-ikd}}{kS^\omega}\left\{e^{i2kd} - \frac{ikJ(d)}{4\pi\omega\varepsilon_0 S^\omega}\right\}\cos kz
\end{aligned}
\tag{11}
$$

where S^ω is determined as in §2 by formulas (3), function $J(d)$ is given by

$$J(d) = \int_{-l}^{l} \left[\frac{e^{-ik\sqrt{4d^2+(z+l)^2}}}{\sqrt{4d^2+(z+l)^2}} + \frac{e^{-ik\sqrt{4d^2+(z-l)^2}}}{\sqrt{4d^2+(z-l)^2}} \right] \cos(kz)dz \qquad (12)$$

The first terms in formula (11) describe as before the current induced by the incident field, the second ones do the current induced on the surface of the dipole by the other one. The currents at the second harmonic are induced due to the lumped EMF (see (4)) and are given in the following form:

$$\begin{aligned} I_{2\omega}^{(1,2)} &= \frac{\mathcal{E}_{1,2}}{1+i2\omega c R_0} \begin{cases} \dfrac{\sin 2kz_{1,2}}{S^{2\omega}} \sin 2kz, & z_{1,2} \neq 0, \\ \dfrac{i\pi\omega\varepsilon_0}{k\ln(2ka)} \sin 2k|z|, & z_{1,2} = 0, \end{cases} \\ S^{2\omega} &= \frac{k}{4\pi\omega\varepsilon_0} \left\{ D_i(4\pi) + iS_i(4\pi) \right\} + Z_{2\omega} \end{aligned} \qquad (13)$$

here $z_{1,2}$ are the coordinates of NL at the first and second dipoles, respectively, $S^{2\omega}$ is determined by (6) and $\mathcal{E}_{\infty,\in}$ is determined by formula (4). The field scattered by a system of two dipoles at the double frequency is the sum of radiation fields of currents $I_{2\omega}^{(1,2)}$. Below we give an expression for z-component of the scattered field ($E_{z,2\omega} = E_{z,2\omega}^{(1)} + E_{z,2\omega}^{(2)}$) which is valid in the wave zone
in the case $z_1 \neq 0, \quad z_2 \neq 0$

$$\begin{aligned} E_{z,2\omega}^{(1)} &= \frac{iE_0^2\beta Z_{2\omega} Z_\omega^2}{2\pi\omega\varepsilon_0 k R_0 (S^\omega)^2 S^{2\omega}} \cos^2 kz_1 \sin 2kz_1 e^{-2ikd} \\ &\times \left[1 - ik\frac{e^{2ikd}J(d)}{4\pi\omega\varepsilon_0 S^\omega} \right]^2 \left[\frac{e^{-i2kr_{11}}}{r_{11}} - \frac{e^{-i2kr_{12}}}{r_{12}} \right] \\ E_{z,2\omega}^{(2)} &= \frac{iE_0^2\beta Z_{2\omega} Z_\omega^2 e^{-2ikd}}{2\pi\omega\varepsilon_0 k R_0 (S^\omega)^2 S^{2\omega}} \cos^2 kz_2 \sin 2kz_2 \\ &\times \left[e^{2ikd} - \frac{ikJ(d)}{4\pi\omega\varepsilon_0 S^\omega} \right]^2 \left[\frac{e^{-i2kr_{11}^*}}{r_{11}^*} - \frac{e^{-i2kr_{12}^*}}{r_{12}^*} \right] \end{aligned} \qquad (14)$$

In the case $z_1 = z_2 = 0$

$$\begin{aligned} E^{(1)}_{z,2\omega} &= -\frac{Z_{2\omega} Z_\omega^2 \beta E_0^2 e^{-2ikd}}{2R_0 \ln(2ka) k^2 (S^\omega)^2} \left[1 - ik \frac{e^{2ikd} J(d)}{4\pi\omega\varepsilon_0 S^\omega}\right]^2 \\ &\quad \times \left[\frac{e^{-i2kr_{11}}}{r_{11}} + \frac{e^{-i2kr_{12}}}{r_{12}} + 2\frac{e^{-i2kr_0}}{r_0}\right] \\ E^{(2)}_{z,2\omega} &= -\frac{Z_{2\omega} Z_\omega^2 \beta E_0^2 e^{-2ikd}}{2R_0 \ln(2ka) k^2 (S^\omega)^2} \left[e^{2ikd} - \frac{ikJ(d)}{4\pi\omega\varepsilon_0 S^\omega}\right]^2 \\ &\quad \times \left[\frac{e^{-i2kr^*_{11}}}{r^*_{11}} + \frac{e^{-i2kr^*_{12}}}{r^*_{12}} + 2\frac{e^{-i2kr^*_0}}{r^*_0}\right] \end{aligned} \tag{15}$$

where

$$\begin{aligned} r_{11} &= \sqrt{x^2 + (y+d)^2 + (z+l)^2}, & r^*_{11} &= \sqrt{x^2 + (y-d)^2 + (z+l)^2} \\ r_{12} &= \sqrt{x^2 + (y+d)^2 + (z-l)^2}, & r^*_{12} &= \sqrt{x^2 + (y-d)^2 + (z-l)^2}, \\ r_0 &= \sqrt{x^2 + (y+d)^2 + z^2}, & r^*_0 &= \sqrt{x^2 + (y-d)^2 + z^2}. \end{aligned}$$

Figure 3 shows the results of numerical calculations by formulas (14), (15) of the dependence of the back-scattered field on the distance between dipoles d for different positions of NL $z_1 = z_2 = 0$ (Fig. 3a) and $z_1 = 0, z_2/\lambda = 0.125$ (Fig. 3b) for following values of parameters: $y/\lambda = 10, z = x = 0, l/\lambda = 0.5$. This dependence $|E_{z,2\omega}(d)|$ has a periodical character with a period of $T_d = 0.25\lambda$; evidently the oscillation amplitude is maximum at $z_1 = z_2 = 0$, for shifted positions of NL it decreases, the oscillation period remains the same. If NL direction of one of dipoles is changed to the opposite one (the polarity of the diode is changed) then the positions of maxima and minima will change their places. Solid lines in Figure 3a, b correspond to the same directions of NL of different dipoles, the dashed lines do to the opposite directions.

3.2.

Apart from the theoretical analysis of electromagnetic wave nonlinear scattering experimental investigations have also been carried out.

The nonlinear half-wave scatterers (dipoles) were placed on an open platform lifted above the ground at 2 m. The dipole was fixed at the same height and at a distance of 6.5 m from receiving and transmitting antennas. The horizontal polarization was used. Care was taken to decrease the influence of the ground on the measurement results. This

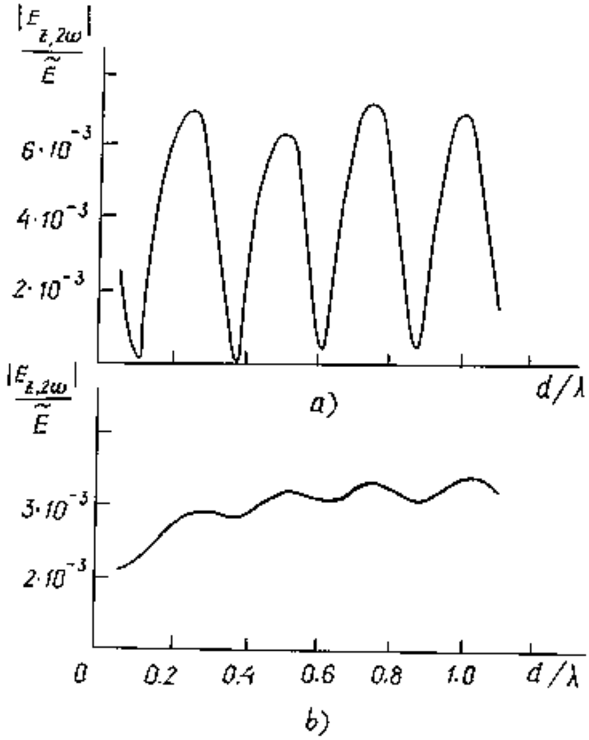

Figure 3. Distribution of the back scattered field $|E_{z,2\omega}(d)|$, over the distance between parallel dipoles. **a**) $z_1 = z_2 = 0$, **b**) $z_1 = 0$, $z_2/\lambda = 0.125$.

influence was controlled during the measurements in the following way: nonuniformity of the back-scattered field in the direction of the receiving antenna in the course of the nonlinear dipole displacement within the given range of d did not exceed 1 dB. The antenna gain was 10 dB, the signal frequency was 290 MHz, and the signal power was 7.5 W. Figure 4 shows the experimental curves of the receiving power scattered by parallel dipoles with nonlinear load at the second harmonic of the signal as dependent on the distance between dipoles the latter being of the same length and with a central position of the NL. Power values $P_s (P_s \sim A|E_z|^2)$ are given in dB relative to a receiver noises.

Comparing the experimental results with calculations it should be noted:

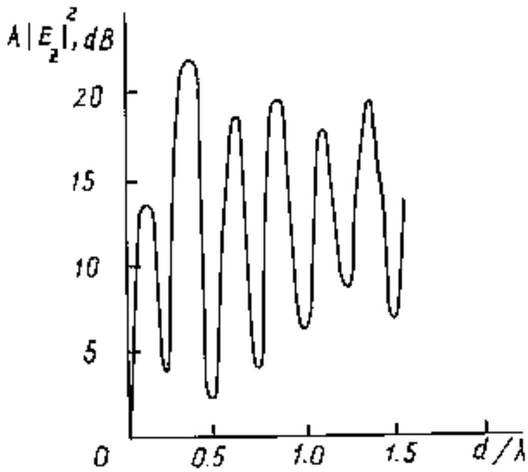

Figure 4. Distribution of the back scattered field $|E_{z,2\omega}(d)|^2$ over the distance d/λ between parallel dipoles (the results of the measurements) .

1. When both dipoles are in the immediate vicinity to each other $(d = 0)$ the scattered signals are mutually compensated by each of scatterers and therefore $P_s = 0$.
2. Dimensionless special period of P_s , is equal to 0.25 and it corresponds to the calculated one.
3. Disagreement in the amplitudes of P_s , oscillations with theoretical ones can be explained by some difference of the NL current-voltage characteristics taken for the calculations with the real one.

4. ELECTROMAGNETIC WAVE SCATTERING BY A SYSTEM OF TWO PARALLEL LOOPS (OR DIPOLES) IN THE CASE WHEN ONLY ONE OF THE SCATTERERS HAS NL

In this section we will present a more detail theoretical investigation of electromagnetic wave scattering by loops in the case when only one of them contains NL. As for the system of two parallel dipoles, we will give only a brief presentation.

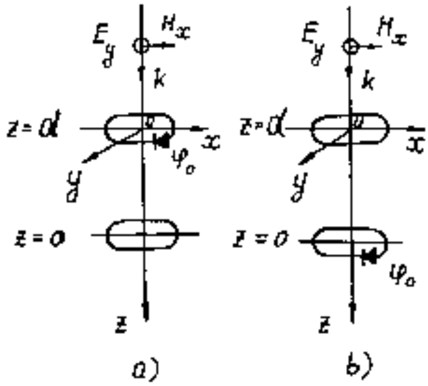

Figure 5. Space configurations of the half-wave loops.

4.1.

Let us first consider the configuration of scatterers shown in Figure 5a: half-wave loops placed in a free space parallel to each other are illuminated by a plane electromagnetic wave propagating along axis z. We call the reader's attention to the fact that the first loop contains NL. The field components of the incident plane linearly polarized electromagnetic wave are written in the form:

$$\vec{E} = \vec{y}_0 E_0 e^{-ikz+i\omega t}, \qquad \vec{H} = -\vec{x}_0 H_0 e^{-ikz+i\omega t}$$

The currents induced by the incident field are calculated by the method of induced E.M.F. as in §2 and at the fundamental frequency (only for the case of half-wave loops) they have the following form:

$$\begin{aligned} I_\omega^{(1)} &= -\frac{iE_0 b}{(\alpha_1 + 2Z_\omega/(\pi z_0))\, z_0}\left(1 - G_1(d)\frac{ie^{-ikd}}{\alpha_1}\right)\cos\varphi \\ I_\omega^{(2)} &= -\frac{ie^{ikd}E_0 b}{z_0\alpha_1}\left(1 - G_1(d)\frac{i}{\alpha_1 + 2Z_\omega/(\pi z_0)}\right)\cos\varphi \end{aligned} \tag{16}$$

where (r, φ, z) are the cylindrical coordinates, indices (1), (2) refer to the first and second loops respectively, b is the loop radius, Z_ω is impedance of the diod, $z_0 = \sqrt{\mu_0/\varepsilon_0}$, coefficients α are determined by the expressions

$$\alpha_N = -\mu_{1,N} + \frac{\varepsilon(Nk_0 b)^2}{2}(\mu_{2,N} + \mu_{0,N}) \tag{17}$$

Here coefficients $\mu_{m,N}$ are given in the following way:

under $m = 0$

$$\mu_{0,N} = \frac{1}{Nkb}\left\{\frac{1}{\pi}\ln\frac{8b}{a} - \frac{1}{2}\left(\int_0^{2Nkb}\Omega_0(x)dx + \ i\int_0^{2Nkb} J_0(x)dx\right)\right\}$$

under $m \neq 0$

$$\mu_{m,N} = \frac{1}{Nkb}\left\{\frac{1}{\pi}\left[K_0\left(\frac{ma}{b}\right)I_0\left(\frac{ma}{b}\right) + C_m\right] - \right.$$
$$\left.\frac{1}{2}\left(\int_0^{2Nkb}\Omega_{2m}(x)dx + i\int_0^{2Nkb} J_{2m}(x)dx\right)\right\}$$

$$C_m = \ln 4m + 0.577215 - 2\sum_{n=0}^{m-1}\frac{1}{2n+1}$$

where $J_m(\xi), \Omega_m(\xi)$ are Bessel and Lommel-Weber functions and $K_0(\xi), I_0(\xi)$ are modified Bessel functions, indices N equals to 1 or 2 dependence on the frequency. Function $G_N(d)$ is expressed by the formula

$$G_N(d) = \int_0^\infty\left\{\frac{\sqrt{\lambda^2 - N^2}}{N}J_1^2(k_0 b\lambda) - \frac{k_0^2 b^2\lambda^2 N}{\sqrt{\lambda^2 - N^2}}J_1'^2(k_0 b\lambda)\right\}$$
$$\times\exp\left\{-k_0 d\sqrt{\lambda^2 - N^2}\right\}\frac{d\lambda}{\lambda} \tag{18}$$

At the fundamental frequency $N = 1$. The first terms in formula (16) describe the current induced by the incident field, the second ones do the current induced on the surface of the loop by the other one. The current at the second harmonic 2ω in the first loop is induced due to the lumped E.M.F. (see (4)), the current at the double frequency in the second loop is induced by the field of the first loop at frequency 2ω. Thus, the following relations can be obtained for the currents at the double frequencies:

$$I_{2\omega}^{(1)} = \frac{\beta Z_\omega\left(I_\omega^{(1)}(\varphi = \varphi_0)\right)^2}{\pi\left(\alpha_2 + 2Z_{2\omega}/(\pi z_0)\right)}2\cos(\varphi - \varphi_0)$$
$$I_{2\omega}^{(2)} = -i\frac{I_{2\omega}^{(1)}G_2(d)}{\alpha_2}2\cos(\varphi - \varphi_0) \tag{19}$$

where φ_0 is the position of NL, α_2 and G_2 are determined by expressions (17), (18) respectively at $N = 2$,

It has been taken into account in (19) that the loop is the wave one $(2\pi b = \lambda)$ at the double frequency, so in the Fourier series for the current we have taken only the second term. We have to note that the current distribution at the double frequency for each loop depends on the NL position at the first loop. Below we give the expression for the azimuth component of the field $E_{\varphi,2\omega}$ at the double frequency produced by currents (19):

$$E_{\varphi,2\omega}(z) = \frac{4\beta R_0^2 \left(I_\omega^{(1)}(\varphi = \varphi_0)\right)^2}{b\pi\alpha_2} \times \left\{Q^{(1)}(z) - i\frac{G_2(d)}{\alpha_2}Q^{(2)}(z)\right\}\cos(\varphi - \varphi_0) \tag{20}$$

where

$$Q^{(1),(2)} = \pi\int_0^\infty \left\{\frac{\sqrt{\lambda^2-4}}{2}J_1(kb\lambda) - \frac{2k\lambda b}{\sqrt{\lambda^2-4}}J_1'(kb\lambda)\right\} \times e^{-k\sqrt{\lambda^2-4}|z|}\begin{cases}1 & \text{for} \quad Q^{(1)} \\ e^{-kd\sqrt{\lambda^2-4}} & \text{for} \quad Q^{(2)}\end{cases} d\lambda \tag{21}$$

Expressions (20) have been obtained under condition $Z_{N\omega}/z_0 \ll \alpha_N$

A similar problem on the back-scattered field at the double frequency can be solved also for the configuration shown in Figure 5b. In this case the field is expressed as

$$E_{\varphi,2\omega}(z) = \frac{4\beta}{b}\frac{R_0^2}{\pi}\frac{\left(I_\omega^{(2)}(\varphi = \varphi_0)\right)^2}{\pi\alpha_2} \times \left\{Q^{(2)}(z) - \frac{iG_2(d)}{\alpha_2}Q^{(1)}(z)\right\}\cos(\varphi - \varphi_0) \tag{22}$$

where $I_\omega^{(2)}(\varphi = \varphi_0)$ is obtained by the formula

$$I_\omega^{(2)}(\varphi_0) = -\frac{iE_0 b}{z_0}\frac{1}{\alpha_1 + 2Z_\omega/(\pi z_0)}\left\{1 - \frac{1}{\alpha_1}e^{ikd}G_1(d)\right\}\cos\varphi_0$$

Figures 6a, b show dependencies of the electric field azimuth component $E_{\varphi,2\omega}$ on distance d/λ between loops for the configurations presented in Figures 5a, b respectively. The curves are plotted for the following parametric values: $a/\lambda = 1.5\cdot 10^{-3}, z/\lambda = 10, \rho = 0, \varphi_0 = 0$. It

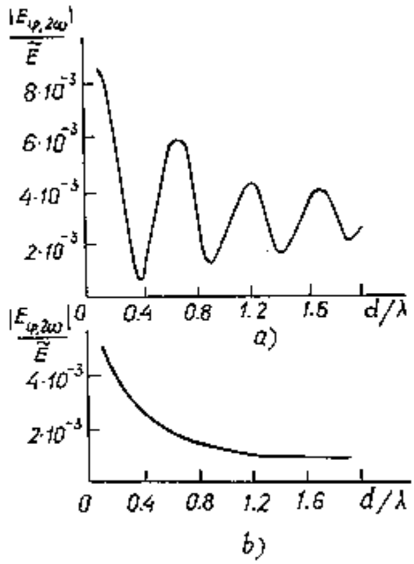

Figure 6. Dependencies of back scattered field $|E_{\varphi,2\omega}(d)|$, versus d for the configurations presented in Figure 5a, b.

is seen from Figures 6a,b that a behavior of the field pattern $|E_{\varphi,2\omega}(d)|$ depends on which loop is moved, i.e.:

a. the loop containing the load; in this case the field is oscillating with period $T_d = 0.5\lambda$ (see Figure 6a) or
b. the loop without NL; here oscillations are absent (see Figure 6b).

4.2.

Further to complete the consideration similar expressions for the backscattered field at the double frequency is given for a system of two parallel half-wave dipoles one of which has NL (see Figures 7a, b). For the configuration shown in Figure 7a we have ($z_1 = z_2 = 0$, the observation point lies on $x = 0, z = \lambda$)the expression for z-component of the scattering field in the form:

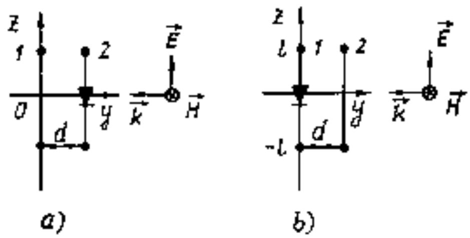

Figure 7. Space configurations of the half-wave dipoles.

$$E_{z,2\omega} = -\frac{\beta E_0^2}{2k^2 \ln(2ka)} \frac{R_0^2}{(S^\omega)^2} \left[1 - \frac{ie^{ikd} J(d)}{D_i(2\pi) + iS_i(2\pi)} \right]^2 \times \left[\frac{e^{-i2k\sqrt{y^2+4l^2}}}{\sqrt{y^2+4l^2}} + \frac{e^{-i2ky}}{y} + 2\frac{e^{-i2k\sqrt{y^2+l^2}}}{\sqrt{y^2+l^2}} \right]$$
$$E_{x,2\omega} = 0 \qquad (23)$$

For the configuration shown in Figure 7b (at the same observation point) the field components are defined by the following expressions

$$E_{z,2\omega} = -\frac{\beta E_0^2}{2k^2 \ln(2ka)} \frac{R_0^2}{(S^\omega)^2} \left[e^{ikd} - \frac{iJ(d)}{D_i(2\pi) + iS_i(2\pi)} \right]^2 \times \left[\frac{e^{-i2k\sqrt{(y-d)^2+4l^2}}}{\sqrt{(y-d)^2+4l^2}} + \frac{e^{i2k(y-d)}}{y-d} + 2\frac{e^{-i2k\sqrt{(y-d)^2+l^2}}}{\sqrt{(y-d)^2+l^2}} \right]$$
$$E_{x,2\omega} = 0 \qquad (24)$$

Formula (12) is used to find function $J(d)$ in (23), (24). Results of calculations by formulas (23), (24) are shown in Figures 8a, b respectively. It is seen from Figure 8a that field $E_{z,2\omega}$, grows steadily with d when the dipole without NL is moved in the direction of the antenna. When the dipole with NL is moved in the same direction the field $|E_{z,2\omega}(d)|$ at the observation point changes periodically with period $T_d/\lambda = 0.5$ (see Figure 8b) as in the case of loops (see section 4.1).

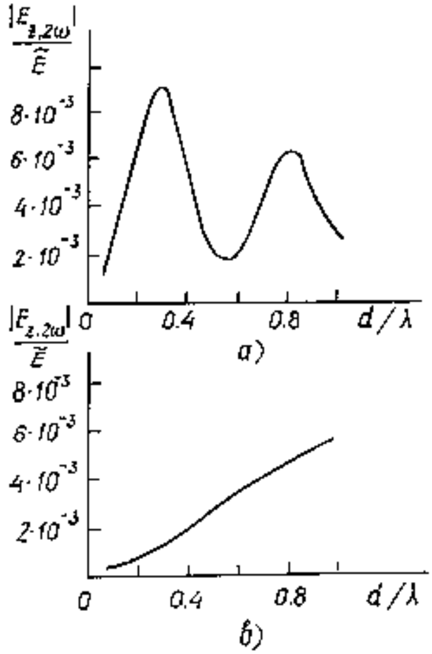

Figure 8. Dependencies of back scattered field $|E_{z,2\omega}(d)|$, versus d/λ for the configurations presented in Figure 7a, b.

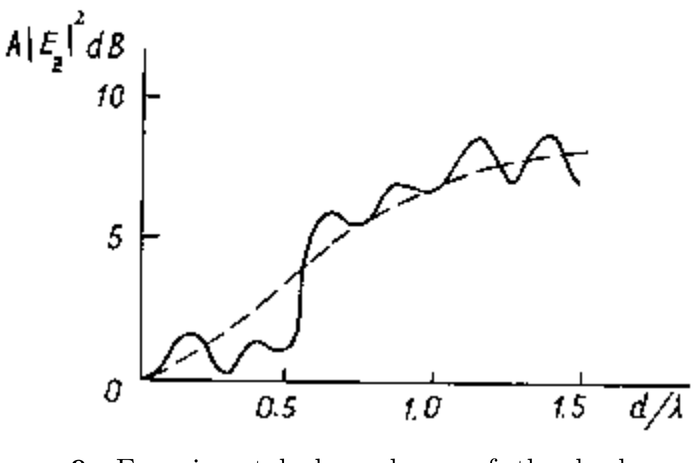

Figure 9. Experimental dependence of the back scattered field $|E_{z,2\omega}(d)|^2$ versus d for the configuration corresponding to Figure 7b.

Experimental investigations relevant to this section were also carried out at the set-up which description has been given above (see section 3.2). Figure 9b shows the dependence of the power of back-scattered signal P_s scattered by the system of two parallel dipoles in the direction of the receiving antenna on the displacement of the linear dipole (without NL) the nonlinear one being fixed at point $r = 6.5\,\mathrm{m}$. Figure 10 shows this dependence vice versa, i.e., the nonlinear dipole is moved whereas linear one is fixed at the same point. The data obtained can be explained in the following way:

1. When the linear dipole is between the nonlinear scatterer (NS) and antennas it plays the role of a screen which efficiency drops with the removal of the dipole from the nonlinear one.
2. When the linear dipole is behind NS it plays the role of a reflector strengthening the sounding signal at the position of NS and therefore strong oscillations are observed with the period corresponding to the calculated one.
3. Small oscillations of $P_s(d)$ in Figure 9 with periods $\lambda/4$ and $\lambda/2$ can be explained that the linear dipole also plays the role of the reflector with respect to nonlinear one but mainly at the second harmonic. These oscillations are absent in the calculated curve because for the sake of simplicity the corresponding terms in the series were not taken into account.

5. DISCUSSION

Now we dwell on a possible usage of the obtained results.

1. An essential new point is the fact that the polarization diagram of the signal at the double frequency depends on the spatial configuration of dipoles (distance d) and in this way it may differ from the polarization of the incident field at the fundamental frequency. In this case it is possible to produce the preassigned polarization if the signal at the double frequency using dipole space diversity and directivity of NL.
2. It has been shown in [12] that NS can be used for remote sensing of moving objects. Such a nonlinear scatterer being placed in the near zone of the linear scatterer (object) can retransmit information on the intensity distribution of the corresponding component of the object. This scatterer sensor can be made sufficiently small and it has no feeder so it does not distort the field sensed.

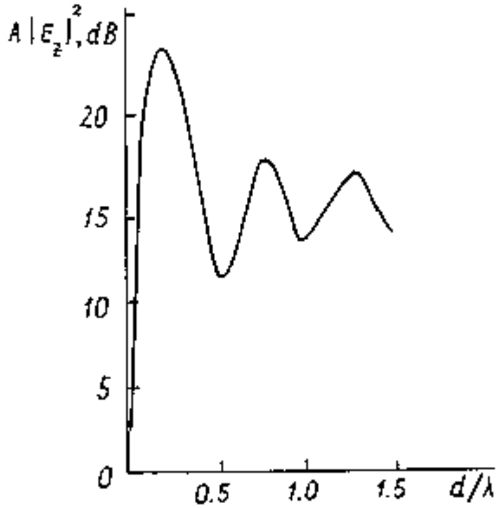

Figure 10. Experimental dependence of the back scattered field $|E_{z,2\omega}(d)|^2$ versus d for the configuration corresponding to Figure 7a.

3. It has been reported at the XXVth Assembly of URSI an interesting results on the influence of biological objects on nearby NS such as home electronic appliances. These results show that under definite conditions it is possible not only to fix the movement of objects but also to measure their physiological parameters in the presence of radio transparent obstacles. The application of arrays of simplest nonlinear antennas makes it possible to widen their functional abilities in remote sensing measurements.

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