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REFLECTION AND RETRODIRECTION EFFECTS IN THE PRESENCE OF CYLINDERS AND CORRUGATED SURFACES: THEORY

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1. INTRODUCTION AND PRELIMINARY CONSIDERATIONS

The interest in bi-anisotropic and bi-isotropic media [1–6], especially what is termed chiral (also known as "optically active") media, has been growing recently. In the simplest terms, isotropic chiral media are characterized by dyadic (or sometimes expressed in matric notation) constitutive parameters, manifesting rotation of the polarization vector along the trajectory of propagation. This can be visualized by a medium containing helix-like particles. Obviously right/left handed helices preserve their right/left handedness property even if flipped over, hence the phenomena depend on the chirality of the medium and the propagation vector only. Another class of bi-anisotropic media is afforded by gyrotropic media, e.g., a magnetized plasma. [1, 7, 8]. In this case, which is characterized by a Hermitian constitutive matrix, rotation of the polarization vector along the trajectory of propagation is manifested too, but its sense (clockwise or anti-clockwise) depends on the parallelness or anti-parallelness of the direction of propagation with respect to the preferred spatial direction (e.g., the direction of the external static magnetic field in magnetized plasmas, but this can also happen in certain acoustical systems, see for example [9, 10]).

Consequently, chiral and gyrotropic media manifest different phenomena on round trip propagation in the presence of (simple) reflection [1]: In the case of a chiral medium, the rotation of the polarization vector, acquired on the leg towards the mirror, is canceled (unraveled to zero) when the wave emerges after performing the return leg of the trip. In contradistinction, in the presence of gyrotropic media, the rotation is enhanced (doubled) after the round trip involving reflection [1]. On the other hand, if the mirror is a retrodirector, the opposite enhancement or cancellation phenomena happen.

It has been noticed [11] that on mirroring, in the vicinity of the Brewster angle, in the case of a chiral medium, the rotation of the polarization is enhanced when the wave emerges after performing the return leg of the trip. We have dubbed this mirroring mechanism as retrodirection [1], as opposed to (simple) reflection. This led to a quest for more general situations regarding reflection and retrodirection: It is then easy to show (merely by inspection of the graphical representations, [1]) that in the case of a gyrotropic medium and a retrodirector, the rotation is canceled. A consistent classification based on the number of mirroring events and the nature of the individual mirrors

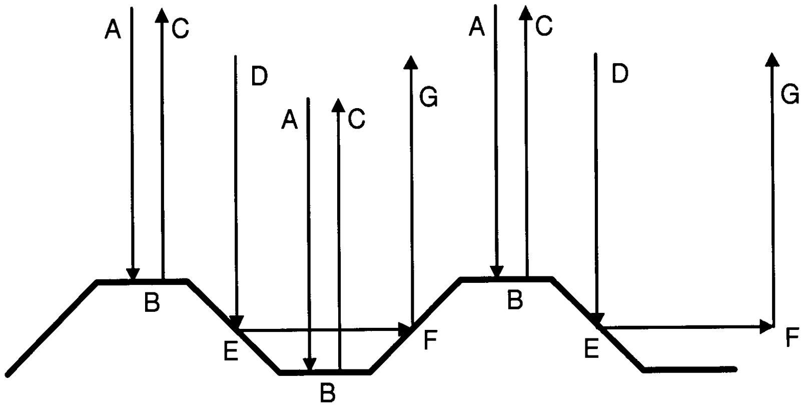

Figure 1. Specular mirroring mechanisms expected in the presence of corrugated and cylinder-lined surfaces. $A \to B \to C$ is a (simple reflection process, but $D \to E \to F \to G$ is a retrodirective process.

(depending on the number of reflectors and retrodirectors involved) determines the overall performance of the system, i.e., whether the rotation is enhanced or canceled.

We consider the effect of scattering by single cylinders as the relevant mirroring mechanism, whether they behave as reflectors and/or retrodirectors. Scattering by cylinders is a classical canonical boundary-value problem, hence analytic computation is feasible. The conclusions of the present study depend on signs of particular coefficients, and therefore the problem is outlined in some detail, to readily facilitate a re-examination of the results. The detailed calculations are deferred to Appendix A in order to leave the main body of the article more streamlined.

2. REFLECTION AND RETRODIRECTION FOR SPECULAR MIRRORS

To obviate repetition, direct references to [1] will be made, indicated by braces, e.g., thusly: Fig. $\{1\}$. The different cases of mirroring for fields in the plane of incidence and perpendicular to it are carefully classified [1], see Figs. $\{1-5\}$. Essentially, in simple reflection, either both the **E** and **H** fields retain their original direction, or both are spatially flipped over (reverse direction spatially). This is depicted in [1] in Figs. $\{6,7\}$ for the case of dielectric mirrors, and the ensuing effect on chiral and gyrotropic media, respectively, and in Figs. $\{8,9\}$ for magnetic mirrors. In Figs. $\{10,12,14,15\}$ retrodirection is displayed. Essentially, the two field components (say, of the **E** field), in the plane of incidence and perpendicular to it, undergo opposite processes: If one is flipped over, then the other retains its original direction. The difference between simple reflection and retrodirection can therefore be expressed in the following manner: In simple reflection the **E** or **H** fields of the incident and returned waves are either parallel or anti-parallel (flipped over). In retrodirection, the total **E** or **H** fields of the incident and returned waves are not parallel or anti-parallel, but symmetrical (or skew-symmetrical) with respect to the cylindrical axis of the system. This symmetry is demonstrated in [1] in Figs. $\{14,15\}$ for the specific cases discussed there.

Fig. 1 depicts the specular processes expected for cylinder-lined, or corrugated surfaces. The processes $A \to B \to C$ involve a single mirroring process, hence if the surface is a simple, dielectric or magnetic reflector, the result will be a simple reflection. On the other hand, the processes of type $D \to E \to F \to G$ involve two returns, which for individual simple reflections correspond to an overall retrodirective process, as in [1], Fig. $\{15\}$. Obviously the total output is in this case a mixture of the two modes. This raises the question of separation of those waves, as discussed below.

3. FILTERING OF REFLECTED AND RETRODIRECTED WAVES

In many of the processes presently described, the creation of simply reflected and retrodirected waves occurs simultaneously, and this raises the question of filtering and separating the two modes. Let us assume that the incident wave **f** is linearly polarized, such that either the electric **E**-field or the magnetic **H**-field is polarized in the y-z plane according to,

$$\mathbf{f} = (f_y\hat{\mathbf{y}} + f_z\hat{\mathbf{z}})e^{ikx-i\omega t}, \quad \mathbf{f} = \mathbf{E} \text{ or } \mathbf{H} \tag{1}$$

where $\hat{\mathbf{z}}$ is defined as the system's cylindrical axis. In (1) $k = \omega\sqrt{\mu\varepsilon}$ is the propagation vector magnitude, with the constitutive parameters μ, ε and (angular) frequency ω.

A simple reflection will identically affect the two field components, $f_y\hat{\mathbf{y}}$, $f_z\hat{\mathbf{z}}$ (1), depending on the nature of the reflector, and the field

($\mathbf{f} = \mathbf{E}$ or $\mathbf{H}$) under consideration, while a retrodirection inverts the sign of one of the components. Thus the mirror yields a reflected, retrodirected field,

$$\mathbf{f}_r = A(\pm f_y \hat{\mathbf{y}} \pm f_z \hat{\mathbf{z}}) e^{-ikx-i\omega t} \tag{2a}$$

$$\mathbf{f}_r = B(\pm f_y \hat{\mathbf{y}} \mp f_z \hat{\mathbf{z}}) e^{-ikx-i\omega t} \tag{2b}$$

given by (2a), (2b), respectively. In (2), A, B are complex coefficients whose magnitudes express the yield for the two processes, while their argument corresponds to the extra phase acquired in the scattering process. Obviously a spatial angular shift is introduced between the simply reflected and retrodirected components. To maximize the effect, one should strive to have a system in which $|f_{ry}| = |f_{rz}|$ which produces a $\pi/2$ spatial angle between the directions of polarizations. An output polarizer will therefore effectively separate the two wave modes.

4. SCATTERING BY A CYLINDER AT NORMAL INCIDENCE

Twersky [12] considered scattering of a plane harmonic wave incident normally with respect to the axis of a cylinder possessing an arbitrary finite cross section, and various representations of the scattered wave. The following discussion focuses on circular cylinders and normal incidence, which provide the simplest example.

We start with an incident plane harmonic wave, whose field $\mathbf{f}$ ($\mathbf{f} = \mathbf{E}$ or $\mathbf{H}$) is polarized along the cylindrical z-axis, propagating with an amplitude f_0 in the x-axis direction (cf. (1)),

$$\mathbf{f} = \hat{\mathbf{z}} f_0 e^{ikx-i\omega t} \tag{3}$$

The choice of the specific direction of propagation (3) does not impair the generality of the model. A cylinder with a finite cross section in the x-y plane is given. The incident wave (3) is now recast in terms of a Bessel-Fourier series in cylindrical (in the present case reducing to polar) coordinates, see for example Stratton [7],

$$\mathbf{f} = \hat{\mathbf{z}} f_0 \sum_{m=-\infty}^{\infty} i^m J_m(kr) e^{im\theta-i\omega t} \tag{4}$$

where $J_m(kr)$ denotes the non-singular Bessel functions of order m and argument kr. In (4), θ is the azimuthal angle. For simplicity, a circular cross-section and a simple (homogeneous) material are chosen to discuss the scattering problem. Accordingly we postulate the scattered field as a solution of the wave equation in cylindrical coordinates, represented in terms of a Hankel-Fourier series. In accordance with the assumption that the cylinder is made of an homogeneous medium, no depolarization effects occur, hence the scattered wave field is represented by

$$\mathbf{u} = \hat{\mathbf{z}} f_0 \sum_{m=-\infty}^{\infty} i^m a_m H_m(kr) e^{im\theta - i\omega t} \tag{5}$$

where $H_m^{(1)}$ denotes Hankel functions of the first kind, The choice of $H_m^{(1)}$ together with the time exponent in (5) guarantees outgoing scattered waves. Of course, this is a physical argument based on the notion of causality, and is not prescribed by the mathematics of the problem. The corresponding wave field in the internal domain is similarly given by

$$\mathbf{f}_i = \hat{\mathbf{z}} f_0 \sum_{m=-\infty}^{\infty} i^m b_m J_m(Kr) e^{im\theta - i\omega t} \tag{6}$$

where the nonsingular Bessel functions, (6), are appropriate inside the scatterer, subscript i indicates the internal domain, and $K = \omega\sqrt{\mu_i \varepsilon_i}$, involves the relevant constitutive parameters. Using the Sommerfeld integral representation for the cylindrical functions,

$$i^m H_m^{(1)}(\rho) = \frac{1}{\pi} \int_{-(\pi/2)-i\infty}^{+(\pi/2)+i\infty} e^{i\rho\cos(\tau) + im\tau} d\tau \tag{7}$$

(5) is recast as a sum of plane waves

$$\mathbf{u}(\mathbf{r}, t) = \hat{\mathbf{z}} \frac{f_0}{\pi} \int_{\theta-(\pi/2)+i\infty}^{\theta+(\pi/2)-i\infty} e^{ikr\cos(\theta-\tau) - i\omega t} g(\tau) d\tau \tag{8}$$

propagating in complex directions indicated by the complex angle τ, where the scattering amplitude, i.e., the weight function for each such plane wave, is given by the Fourier series,

$$g(\theta) = \sum_{m=-\infty}^{\infty} a_m e^{im\theta} \tag{9}$$

Exploiting the leading term of the asymptotic series for the Hankel functions [7], or the saddle point approximation for (8), in the far field the scattered field is represented by,

$$\mathbf{u} = \hat{\mathbf{z}} f_0 \sqrt{\frac{2}{i\pi kr}} e^{ikr - i\omega t} g(\theta) \tag{10}$$

The boundary conditions at the surface of the scatterer, namely the continuity of the tangential components of $\mathbf{F} = \mathbf{E}$ and $\mathbf{F} = \mathbf{H}$, where $\mathbf{F} = \mathbf{f} + \mathbf{u}$ denotes the total field in the external domain, can be expressed as

$$\hat{\mathbf{n}} \times \mathbf{F} = \hat{\mathbf{n}} \times \mathbf{f}_i \Big|_{\text{at the surface}} \tag{11}$$

$$\hat{\mathbf{n}} \times \nabla \times \mathbf{F} = \alpha \hat{\mathbf{n}} \times \nabla \times \mathbf{f}_i \Big|_{\text{at the surface}} \tag{12}$$

where $\hat{\mathbf{n}}$ is a unit normal vector, and (12) is obtained by substitution from Maxwell's equations. For $\mathbf{F} = \mathbf{E}, \mathbf{H}$ we have $\alpha = \mu/\mu_i,\ \varepsilon/\varepsilon_i$, respectively. For a circular cylinder of radius d in particular, (11), (12) become,

$$J_m(kd) + a_m H_m(kd) = b_m J_m(Kd) \tag{13}$$

$$J'_m(kd) + a_m H'_m(kd) = Z b_m J'_m(Kd) \tag{14}$$

respectively, where $Z = \frac{K\alpha}{k}$ and the prime denotes differentiation with respect to the argument. The solution of the simultaneous equations (13), (14) yields,

$$a_m = \frac{J_m(Kd) J'_m(kd) - Z J_m(kd) J'_m(Kd)}{Z H_m(kd) J'_m(Kd) - J_m(Kd) H'_m(kd)} \tag{15}$$

which must be carefully evaluated for thin cylinders under various conditions. See Appendix A.

Inasmuch as a circular cylinder is assumed, the scattered wave must be symmetrical with respect to the direction of incidence. In (9, 10) this means,

$$g(\theta) = g(-\theta) \tag{16a}$$

and therefore (9) can be rewritten as,

$$g(\theta) = \sum_{m=-\infty}^{\infty} a_m e^{im\theta} = a_0 + 2 \sum_{m=1}^{\infty} a_m \cos(m\theta) \tag{16b}$$

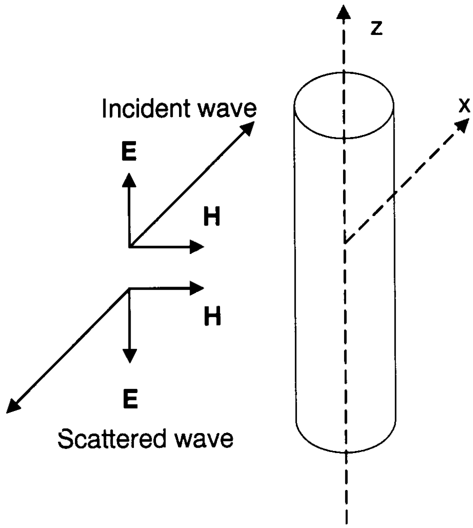

Figure 2. For thin highly conducting cylinders. Electric-type mirror. The **E**-field is flipped over, while the **H**-field retains its original direction.

The various leading terms for thin cylinders, both highly conducting and dielectric, are analyzed in Appendix A. The results will be used to examine backscattering processes in the presence of thin cylinders.

5. REFLECTION AND RETRODIRECTION BACKSCATTERING

For an incident wave (3) a scattered wave (5) is obtained, which in the far field is approximated by (10), and for circular cylinders and backscattering $\theta = \pi$, hence (10, 16) yield:

$$\mathbf{u} = \hat{\mathbf{z}} f_0 \sqrt{\frac{2}{i\pi kr}} e^{ikr - i\omega t} \left[a_0 + 2 \sum_{m=1}^{\infty} (-1)^m a_m \right] \tag{17}$$

For thin cylinders the predominant terms in brackets in (17) must be considered. The detailed analysis is given in Appendix A. It is noted that all coefficients a_m, $m = 0, 1, 2, \ldots$ in (17), possess a common factor $i\pi$, and of course all multipoles have the same phase determined by the square root and exponential factors in (17).

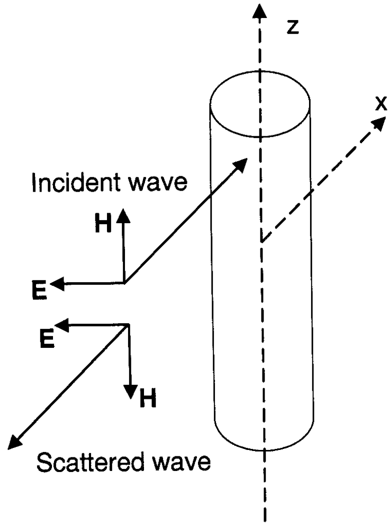

Figure 3. For thin highly conducting cylinders. Magnetic-type mirror. The **H**-field is flipped over, while the **E**-field retains its original direction.

Firstly, let us discuss perfectly conducting thin cylinders:

Consider an incident wave given by (1), and a scattered field as given in (17). The z-component in (1) is taken as an **E**-field, accordingly (17) applies to an **E**-field too. According to (A6), the monopole term is predominant, and suppressing phase factors common to all multipole terms, we refer to the monopole field in (17) as $-E_0^{(v)}$, and on account of the sign, use the phrase that the vertically polarized scattered **E**-field is flipped over compared to the incident vertical **E**-field. This means, of course, that the corresponding **H**-field preserves its direction. See Fig. 2.

Consider an incident wave given by (1), and a scattered field as given in (17), but now the y-component in (1) is taken as the **E**-field, and the z-component is therefore an **H**-field. Accordingly (17) applies to a z-component **H**-field too. According to (A8) and (A10), the monopole and dipole terms are now predominant. Furthermore, they are identical, except for sign. Let us consider first the monopole term $-H_0^{(v)}$. Similarly to the previous case of Fig. 2, this field is flipped over, because of the sign, and discarding phase factors common to all multipole terms, the results of Fig. 3 are obtained. This means that the

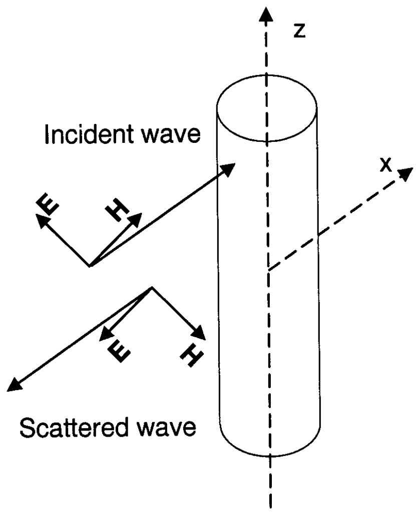

Figure 4. For thin highly conducting cylinders. Combines the effects of Figs. 2, 3, for an arbitrarily polarized incident wave, demonstrating the resulting retrodirective process.

E-field must retain its original direction, in order for the plane wave to maintain the proper relation of the fields and conform to backward direction of propagation.

By inspection of (A8), (A10), the monopole and dipole terms have opposite signs. However, for $m = 1$ in (17), we get another sign inversion, and the final effect is once again depicted by Fig. 3.

An arbitrarily polarized wave, Fig. 4, can be decomposed into **E**, **H** polarizations (referred to the z-axis), as depicted in Figs. 2, 3, respectively. By inspection of Figs. 2, 3, it is clear that Fig. 2 displays the electric-type mirror (like simple reflection from a highly conducting plane) property, while Fig. 3 corresponds to a simple reflection by a magnetic-type wall. It follows that the field which retains its original direction in one case, is flipped over in the other case. The total effect of Figs. 2, 3, is displayed in Fig. 4. We conclude therefore that thin highly conducting cylinders should act as retrodirectors.

We now turn our attention to thin dielectric cylinders. According to Appendix A, (A11), For $\mathbf{F} = \mathbf{E}$ polarization, we have a predominant monopole term, which appears with a positive sign, hence behaving as a magnetic wall mirror, see Fig. 5. For $\mathbf{F} = \mathbf{H}$ polarization (A15) predicts a leading dipole term having the same sign, which according to (17) and $m = 1$ acquires a sign change. Therefore the **H**-field is now flipped over, and the **E**-field retains its original polarization. This is once again behaving as a magnetic wall mirror, as depicted in Fig. 3. The combined effect is depicted in Fig. 6. Unlike the case of thin conducting cylinders, this phenomenon is a simple reflection from a magnetic wall.

The question of the conducting cylinder as the limit where $\varepsilon_i/\varepsilon \to \infty$, is discussed in Appendix A, and the apparent paradox is resolved. In essence, the present results for dielectric cylinders only hold for small arguments Kd for the nonsingular Bessel functions in the internal domain.

6. SUMMARY AND CONCLUDING REMARKS

The present discussion has been limited to two extreme cases: Large (with respect to wavelength) structures for which specular mirroring is expected, and single cylinders of circular cross section. In the latter case, analytical approximations for thin cylinders are available.

The corrugated, or thick cylinders lined surface, where specular mirroring is expected, involves both reflection and retrodirection, as discussed above, see Fig. 1. The analysis of scattering by thin cylinders propounds that a surface lined with non-interacting, thin, highly conducting cylinders, will act as a retrodirector. This is an interesting novel observation that might explain the behavior of existing structures, or suggest new methods for mirroring polarimetry, in systems where such a method is desired.

As the corrugation step length, or the cylinder radius, increases, the analytical results for the thin cylinder become invalid. In the case of a single cylinder, numerical simulations of (10), (15) might demonstrate the gradual transition to specular mirroring. It is also shown that moderately dielectric (as opposed to highly conducting) cylinders, behave as simple reflectors. Hence a surface lined with such cylinders will show predominantly simple reflection, until the regime where the size and number of cylinders will also involve double mirroring, as shown in Fig. 1. The separation of reflection and retrodirection contributions to the total field may be effected by appropriately oriented polarizers, as explained above.

Beyond the analysis presented here, there are many conjectures which require experimental verification: The feasibility of efficient separation of reflected and retrodirected products, for example. The

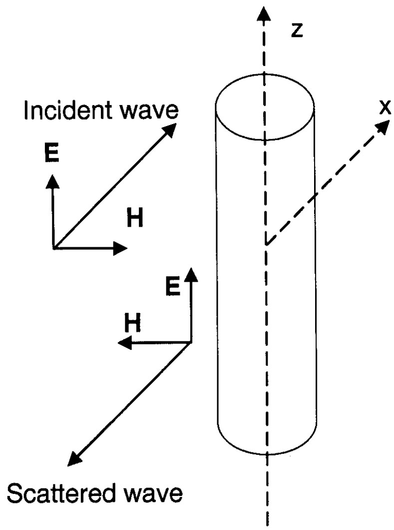

Figure 5. For thin dielectric cylinders. Magnetic-type mirror. The **H**-field is flipped over, while the **E**-field retains its original direction.

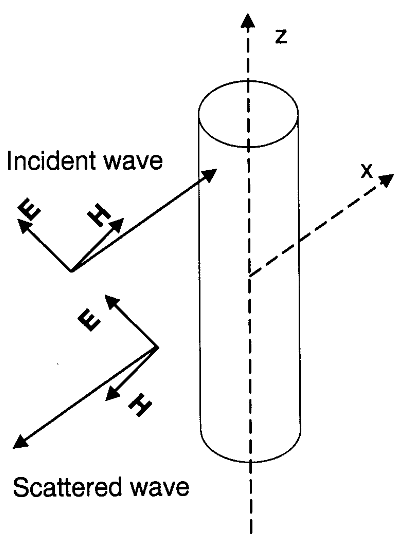

Figure 6. For thin dielectric cylinders. Magnetic-type mirror. Combines the effects of Figs. 5, 6, for an arbitrarily polarized incident wave, demonstrating the resulting retrodirective process.

behavior of surfaces lined with thin conducting cylinders requires experimentation: To what extent can the single cylinder properties found above be extrapolated to a surface lined by such cylinders? Additional analytic and numerical investigations are necessary for basic questions, for example, the effects produced by a plane wave obliquely incident on single thin cylinders of various materials, conducting or dielectric, or, the effect of non-circular cylinders.

APPENDIX A: SCATTERING BY THIN CIRCULAR CYLINDERS

Scattering by circular cylinders and various representations is given by Twersky [12]. For limiting cases involving thin cylinders see Twersky [12], but note that the present case is a special "single space" specialization of his general formulation. The results given here are sensitive to sign in predicting reflection or retrodirection.

We start with (13) specialized to perfectly conducting cylinders, which is tantamount to considering finite μ, μ_i and taking $\varepsilon_i \to \infty$, i.e., and therefore $Z \to \infty,\ 0$ for $\mathbf{f} = \mathbf{E}, \mathbf{H}$, respectively, and consequently (15) becomes,

$$a_m = \frac{-J_m(kd)}{H_m(kd)} \tag{A1}$$

$$a_m = \frac{-J'_m(kd)}{H'_m(kd)} \tag{A2}$$

for $\mathbf{f} = \mathbf{E}, \mathbf{H}$, respectively. If $\varepsilon_i \ll \varepsilon$, we assume the limiting case $\varepsilon_i \to 0$, and therefore $Z \to 0,\ \infty$ for $\mathbf{f} = \mathbf{E}, \mathbf{H}$, respectively, and (A2), (A1) to $\mathbf{f} = \mathbf{E}, \mathbf{H}$, respectively. Similarly for magnetic materials we consider first finite ε, ε_i, and take $\mu_i \gg \mu$, resulting in $Z \to 0,\ \infty$, and yielding (A2), (A1), for $\mathbf{f} = \mathbf{E}, \mathbf{H}$, respectively. Finally for $\mu_i \ll \mu$, we find $Z \to 0,\ \infty$, and (A1), (A2), for $\mathbf{f} = \mathbf{E}, \mathbf{H}$, respectively. To avoid the complications, let us consider only the first case above, i.e., perfectly conducting cylinders, and (A1), (A2), for $\mathbf{f} = \mathbf{E}, \mathbf{H}$, respectively.

For thin cylinders we exploit the leading terms of the series expansion for the nonsingular Bessel functions,

$$J_m(\rho) = \sum_{l=0}^{\infty} \frac{(-1)^l}{l!(m+l)!} \left(\frac{\rho}{2}\right)^{m+2l}, \quad m = 1, 2, \ldots \tag{A3}$$

and use the leading term $l = 0$, unless the expression vanishes, forcing us to use the next term $l = 1$. For the Hankel functions we use,

$$H_m(\rho) = J_m(\rho) + iN_m(\rho) \approx iN_m(\rho) \tag{A4}$$

noticing that near the origin $\rho \approx 0$ the Neumann function $N_m(\rho)$ is predominant, and given by [7],

$$N_0(\rho) \approx -\frac{2}{\pi} \ln\left(\frac{2}{\delta\rho}\right), \quad \delta = 1.781\ldots \tag{A5a}$$

$$N_m(\rho) \approx \frac{-(m-1)!}{\pi} \left(\frac{2}{\rho}\right), \quad m = 1, 2, \ldots \tag{A5b}$$

Note that for small radii, $\rho \approx 0$, both expressions (A5) retain their sign. Using (A3)–(A5a), we obtain for a_0 in (A1), for $\mathbf{F} = \mathbf{E}$,

$$a_0 \approx \frac{-i\pi}{2\ln(2/\delta kd)} \tag{A6}$$

To obtain higher terms in (A1) for $\mathbf{F} = \mathbf{E}$, use (A3), (A4), (A5b), This yields,

$$a_m \approx \frac{-i\pi m(kd/2)^{2m}}{(m!)^2}, \quad m = 1, 2, \ldots \tag{A7}$$

Evidently (e.g., use L'Hospital's rule), the monopole term a_0 is predominant with respect to higher multipole coefficients $m = 1, 2, , \ldots$ in (A7).

For $\mathbf{F} = \mathbf{H}$ we use (A2). For a_0 this prescribes the differentiation of (A3) for $m = 0$, The term $l = 0$ vanishes, therefore the term $l = 1$ must be retained. Alternatively, the relation $Z'_0 = -Z_1$, valid for any cylindrical function Z_m can be applied to (A3), yielding $a_0 = -J_1(kd)/H_1(kd)$, which upon expansion is approximated by,

$$a_0 \approx -i\pi(kd/2)^2 \tag{A8}$$

Higher multipole coefficients are obtained from (A2), using (A3), (A4), (A5b), and differentiating. with respect to the argument, yielding,

$$a_m \approx \frac{i\pi m(kd/2)^{2m}}{(m!)^2}, \quad m = 1, 2, \ldots \tag{A9}$$

It is seen that the dipole term,

$$a_1 \approx i\pi(kd/2)^2 \tag{A10}$$

is identical to the monopole term (A8), except for the sign, and (A8), (A10) are the predominant terms.

Next, consider dielectric cylinders with finite $\varepsilon/\varepsilon_i$. For $\mathbf{F} = \mathbf{E}$, we have $\alpha = \mu/\mu_i = 1$, hence the impedance is $Z = K/k$. Using in (15) $Z_0' = -Z_1$ for the relevant cylindrical functions, we get for the $l = 0$ approximation of (A3) $a_0 = 0$, so $l = 1$ must be retained. Using (A5), and noting (e.g., using L'Hospital's rule) that near the origin $N_1 \gg N_0$, i.e., N_1, is predominant, we finally get,

$$a_0 \approx i\pi \left(\frac{kd}{2}\right)^2 \left(\frac{\varepsilon_i}{\varepsilon} - 1\right) \tag{A11}$$

Note carefully that the transition from (A11) to (A6) by simply assuming that a conducting cylinder can be represented by $\varepsilon_i/\varepsilon \to \infty$ in (A11) is not justified. A more careful derivation, based on $Z_0' = -Z_1$, in which the actual limit $N_0(\rho)/N_1(\rho) \to \rho$ as $\rho \to 0$ is used, leads to (A11) divided by $1 - 2(Kd/2)^2$. Clearly, going to the limit $\varepsilon_i/\varepsilon \to \infty$ would violate the assumption that Kd is infinitesimal.

For a_1, the first order approximation of (A3) in the numerator of (13) vanishes, hence we keep the terms $l = 0, 1$ in the series (A3). With the appropriate approximations in the denominator we finally find the order of magnitude

$$a_1 \propto d^4 \tag{A12}$$

For higher terms we find,

$$a_m \propto d^{2m}, \quad m = 2, 3, \ldots \tag{A13}$$

showing that the monopole term in (A11) is dominant.

For $\mathbf{F} = \mathbf{H}$ polarization, we have $\alpha = \varepsilon/\varepsilon_i$, hence the impedance is $Z = K\alpha/k = k/K$. Consequently for a_0 the numerator in (15) vanishes for the $l = 0$ truncation of (A3), and upon retaining $l = 0, 1$ terms, and performing the appropriate approximations in the denominator, we find the order of magnitude,

$$a_0 \propto d^4 \tag{A14}$$

for the monopole term. For higher terms we get the order of magnitude (A13).

Specifically for a_1 we find in the present case,

$$a_1 \approx i\pi \left(\frac{kd}{2}\right)^2 \left(\frac{\varepsilon_i}{\varepsilon} - 1\right) / \left(\frac{\varepsilon_i}{\varepsilon} + 1\right) \tag{A15}$$

Obviously (A11) and (A15) are of same order of magnitude and sign.

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