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THE INFLUENCE OF SPATIAL DISTRIBUTIONS OF INHOMOGENEITIES ON EFFECTIVE DIELECTRIC PROPERTIES OF COMPOSITE MATERIALS (EFFECTIVE FIELD APPROACH)

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1. INTRODUCTION

This work is a continuation of the previous work [1] of the authors devoted to the application of the effective field method to the problem of electromagnetic wave propagation through dielectric composites consisting of a homogeneous background medium (matrix) and a set of isolated inclusions. In this paper the attention is focused on studying the influence of spatial distributions of inclusions on the mean wave (electric) field in composites and therefore on the effective electrodynamic properties of such materials. As in [1] we consider the propagation of monochromatic electromagnetic waves through a matrix composite with a random set of spherical homogeneous inclusions.

The effective field method allows us to take into account spatial distributions of inclusions via a special two point correlation function of random field of inhomogeneities (see [1]). Such a function is present in the final equation for the mean wave field as well as in the dispersion equations that describe various types of waves that can propagate through inhomogeneous media. This function may be built for random sets of inhomogeneities as well as for regular lattices of inclusions. In the last case the corresponding correlation function is obtained by averaging the original regular lattice over all possible spatial translations of the latter. The resulting mean wave field may be interpreted as averaging of the detailed field over such translations.

The class of random distributions of inhomogeneities considered in the work is described by a generalized Boolean statistical model of inclusions. The simplest type of such distributions (one scale models) was studied in [2, 3] and used [1, 4] in application to the problem of electromagnetic wave propagation through inhomogeneous media. The one scale Boolean models are characterized by two independent parameters: the size of inclusions and the intensity of the Poisson process of generation of the inclusions centers. A generalization of the one scale Boolean statistical model was proposed in [5] and was called the two scale Boolean model. Such a model is a superposition of two one scale models with different values of the parameters. It is worth to note that there is an additional order in spatial distributions of inclusions corresponding to the two scale models. Thus one may expect some distinctions in behavior of the composites with the one scale and two scale Boolean distributions of inclusions in dynamics.

Another class of composite materials considered in the work is the composites with a regular lattice of identical spherical homogeneous inclusions in the matrix. In recent years, such composites are in focus of interests of many authors because of an important area of their engineering applications. The existence of so called “photonic bandgaps” in the composites with regular microstructure allows to create new types of wave filters on the basis of such artificial materials (see, e.g., [6, 7] where some other areas of applications of these materials are mentioned). In distinction to the case of random fields of inhomogeneities, one can obtain in principle an exact solution of the plane wave propagation problem for such a medium. But computational difficulties are essential even for the problem of construction of dispersion curves for such composites (see [8, 9]). The effective field method allows us to build an approximate solution of the considered problem in a wide region of the frequencies of the exiting field and parameters of the composites. The difficulties of the numerical realization of the methods are much less than by using known numerical methods of the exact solution of the problem. But this advantage is connected with some simplifying hypotheses concerning the wave field structure in the composite that are accepted in the effective field method. It is difficult to estimate *adhoc* the precision and area of application of these hypotheses. The application of the effective field method to the analysis of wave propagation through composites with periodic sets of isolated inclusions has two important aims. On the one hand one may examine the ability of the method to describe qualitatively right the main physical effects of wave propagation through such composites. On the other hand a possibility appear here to estimate the precision of the method by the comparison of its predictions with exact or numerical solutions when the latter are available.

The plan of the article is the following. In Section II the review of the results of application of the effective field method to the problem of electromagnetic wave propagation through the medium with isolated inclusions is presented. In section III we propose an approximate solution of the one particle problem that is used farther for the analysis of the mean wave field in the composites. In this approximation, the wave field inside an arbitrary inclusion is assumed to be a plane wave with the wave vector that coincides with the wave vector of the mean wave field in the composite. The unknown amplitude of this field is found on the basis of a variational formulation of the diffraction problem for an isolated inclusion (the plane wave approximation).

Section IV is devoted to the construction of the two point special correlation functions for regular structures and for two scale Boolean models of random sets of inhomogeneities.

In Section V we study the application of the effective field method to the calculation of static dielectric properties of composites with regular and random microstructures.

In Section VI the wave propagation through the medium with regular cubic lattice of spherical inclusions is considered. In this case the solution of the corresponding dispersion equation has an infinite set of different branches. It is shown that physically correct results in the framework of the effective field method (with the existence of non attenuating waves and narrow bands of attenuation in the vicinity of Bragg's frequencies) may be obtained if the plane wave approximation for the solution of the one particle problem is used.

Section VII is devoted to the analysis of the one and two scale Boolean models. We show that for these models one can point out a main ("acoustic") branch of the solution of the dispersion equation and a finite number of additional branches. The main branch corresponds to the wave with a minimal attenuation factor at least in the long wave region. The wave numbers and attenuation factors of the waves corresponding to the additional branches are very sensitive to the details of the behavior of correlation functions. In particular the attenuation of these waves is much less for two scale Boolean models than for the one scale ones. We consider also some family of model two points correlation functions with an increasing correlation radius. The analysis of the solutions of the dispersion equation for such functions allows us to predict what happens in the system if the order in the positions of inclusions increases. On the one hand there appear new branches of the solution of the dispersion equation; on the other hand the attenuation of the corresponding waves decreases as the order in the system increases.

The discussion of the obtained results and some details of the application of the effective field method to the composites with the considered microstructures are presented in the Conclusion.

2. THE EFFECTIVE FIELD METHOD IN THE PROBLEM OF ELECTROMAGNETIC WAVE PROPAGATION THROUGH A MEDIUM WITH ISOLATED INCLUSIONS

Let a plane electromagnetic wave of frequency ω propagate through a homogeneous medium with a random array of isolated inclusions that occupy region V with the characteristic function $V(x)$ ($V(x) = 1,\ x \in V;\ V(x) = 0,\ x \notin V$), $x(x_1, x_2, x_3)$ is a point of the 3D-space. The medium and the inclusions are dielectrics with tensors of dielectric properties $\boldsymbol{\varepsilon}_0$ and $\boldsymbol{\varepsilon}$, respectively. The amplitude $\mathbf{E}(x)$ of the electric field $\mathbf{E}(x,t) = \mathbf{E}(x)e^{i\omega t}$ in the inhomogeneous medium satisfies the following integral equation [1, 10]

$$\mathbf{E}(x) - \int \mathbf{G}(x - x') \cdot \boldsymbol{\varepsilon}_1 \cdot \mathbf{E}(x')V(x')dx' = \mathbf{E}_0(x), \quad \boldsymbol{\varepsilon}_1 = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_0, \quad (1)$$

where the integration is spread over all 3D-space, $\mathbf{E}_0(x)$ is an exciting field that would have existed in the medium with given sources of the field and $\boldsymbol{\varepsilon}_1 = 0$. A dot is a scalar product of vectors and tensors. For monochromatic plane waves of a unit amplitude the field $\mathbf{E}_0(x)$ takes the form

$$\mathbf{E}_0(x) = \mathbf{e}_0 e^{-i\mathbf{k}_0 \cdot x}, \quad \mathbf{k}_0 = k_0\mathbf{m}, \quad |\mathbf{m}| = 1, \quad |\mathbf{e}_0| = 1, \quad (2)$$

where $\mathbf{k}_0$ is the wave vector and $\mathbf{e}_0$ is the polarization vector of the electric field $\mathbf{E}_0(x)$ propagating through the homogeneous background medium.

For an isotropic medium ($\boldsymbol{\varepsilon}_0 = \varepsilon_0\mathbf{1}$ and ε_0 is a scalar) the tensor $\mathbf{G}(x)$ in Eq. (1) takes the form

$$\mathbf{G}(x) = k_0^2 g(x)\mathbf{1} + \nabla \otimes \nabla g(x), \quad g(x) = \frac{e^{-ik_0|x|}}{4\pi\varepsilon_0|x|}, \quad k_0^2 = \omega^2\varepsilon_0 \quad (3)$$

and vectors $\mathbf{e}_0$ and $\mathbf{k}_0$ in Eq. (2) are orthogonal. $\mathbf{G}(x)$ is a generalized function which regularization is considered in [10, 11]. The Fourier transform $\tilde{\mathbf{G}}(\mathbf{k})$ of this function has the form

$$\tilde{\mathbf{G}}(\mathbf{k}) = \frac{k_0^2}{\varepsilon_0(k^2 - k_0^2)}\left[\mathbf{1} - \frac{\mathbf{k} \otimes \mathbf{k}}{k_0^2}\right], \quad (4)$$

where $\mathbf{k}$ is the vector parameter of the Fourier transform, $k = |\mathbf{k}|$.

In order to calculate effective dielectric properties of composite materials one has to estimate the mean wave field in the composite $<\mathbf{E}(x)>$. This mean is the solution of Eq. (1) averaged over ensemble realizations of a random set of inclusions. For a general random set of inclusions this problem cannot be solved exactly and only some approximations are available. Here we use the effective field method in order to build such an approximate solution.

The main hypotheses of the effective field method concern the local external field (effective field) $\mathbf{E}_*(x)$ that acts on every inclusion in the composite material. In the version of the method developed in [1], this field is assumed to be in essence a plane wave with unknown wave vector $\mathbf{k}_*$ and amplitude $\mathbf{e}_*$. The mean wave field $< \mathbf{E}(x) >$ is expressed via the effective field $\mathbf{E}_*(x)$ and therefore the latter is the main unknown of the problem.

The hypotheses of the method formulated in [1] allow us to construct a closed integral equation for the effective field and for the mean electric field in an inhomogeneous medium. These equations turn to be convolution equations. As a result the Fourier transforms of the effective field $\mathbf{E}_*(\mathbf{k})$ and the mean electric field $< \mathbf{E}(\mathbf{k}) >$ are connected by the relation

$$\mathbf{E}_*(\mathbf{k}) = \boldsymbol{\Pi}(\mathbf{k}) \cdot < \mathbf{E}(\mathbf{k}) >$$

and the field $< \mathbf{E}(\mathbf{k}) >$ satisfies the following algebraic equation

$$< \mathbf{E}(\mathbf{k}) > = \mathbf{E}_0(\mathbf{k}) + p\tilde{\mathbf{G}}(\mathbf{k}) \cdot \boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\Lambda}_0(\mathbf{k}_*) \cdot \boldsymbol{\Pi}(\mathbf{k}) \cdot < \mathbf{E}(\mathbf{k}) > . \tag{5}$$

Here p is the volume concentration of inclusions, $\mathbf{E}_0(\mathbf{k})$ is the Fourier transform of the exciting field $\mathbf{E}_0(x)$, $\tilde{\mathbf{G}}(\mathbf{k})$ has the form (4).

The tensor $\boldsymbol{\Lambda}_0(\mathbf{k}_*)$ is defined from the solution of the one particle problem (diffraction of the plane effective field $\mathbf{E}_*(x) = \mathbf{e}_* e^{-i\mathbf{k}_* \cdot x}$ on an isolated inclusion that occupies the volume v with the center at point $x = 0$ in a homogeneous matrix). The solution of the latter problem gives us the expression of the electric field $\mathbf{E}(x)$ inside the inclusion in the form

$$\mathbf{E}(x) = \boldsymbol{\lambda}(x, \mathbf{k}_*) \cdot \mathbf{E}_*(x), \quad x \subset v,$$

where $\boldsymbol{\lambda}(x, \mathbf{k}_*)$ is a second rank tensor function. For spherical inclusions this function has the form of the series which terms are expressed via spherical vector harmonics that present in the Mie solution of the diffraction problem for a sphere (see [1, 12]). The tensor $\boldsymbol{\Lambda}_0(\mathbf{k}_*)$ is connected with the function $\boldsymbol{\lambda}(x, \mathbf{k}_*)$ by the relation

$$\boldsymbol{\Lambda}_0(\mathbf{k}_*) = \frac{1}{v_0} \int_v \boldsymbol{\lambda}(x, \mathbf{k}_*) dx, \tag{6}$$

where v_0 is the volume of the inclusion.

The function $\boldsymbol{\Pi}(\mathbf{k})$ in Eq. (5) has the form

$$\boldsymbol{\Pi}(\mathbf{k}) = [\mathbf{1} + p\mathbf{G}_\Phi(\mathbf{k}) \cdot \boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\Lambda}_0(\mathbf{k}_*)]^{-1}, \quad \mathbf{G}_\Phi(\mathbf{k}) = \int \mathbf{G}(x)\Phi(x)e^{i\mathbf{k}\cdot x}dx. \tag{7}$$

Here $\mathbf{G}(x)$ is the Green function (3) and $\Phi(x)$ is a pair correlation function of the random set of inhomogeneities. This function is the following conditional mean

$$\Phi(x' - x) = 1 - \frac{1}{p} < V(x'; x)|x >, \tag{8}$$

where $V(x'; x)$ is the characteristic function (with argument x') of the region V_x defined by the relation

$$V_x = \bigcup_{i \neq k} v_i, \quad \text{when } x \in v_k. \tag{9}$$

and $v_k(x)$ is the characteristic function of the region v_k occupied by the inclusion with number k, $< V(x'; x)|x >$ is the averaging over the ensemble realizations of the random function $V(x'; x)$ by the condition $x \subset V$, $(V = \cup_k v_k)$. It is assumed that the random set of inclusions is homogeneous in space and therefore the mean $< V(x'; x)|x >= 1 - p\Phi(x' - x)$ depends only on the difference $x' - x$.

The function $\Phi(x)$ has an evident property that is the consequence of its definition (8), (9)

$$\Phi(0) = 1. \tag{10}$$

If the correlation in the inclusions positions disappears when $| x |\to \infty$ one has $\Phi(x) \to 0$ in this limit.

In the framework of the considered version of the effective field method, the information about a spatial distribution of inclusions is present in Eq. (5) via the correlation function $\Phi(x)$ or to be exact via the integral $\mathbf{G}_\Phi(\mathbf{k})$ in Eq. (7).

After multiplying Eq. (5) by the tensor

$$\mathbf{L}(\mathbf{k}) = \tilde{\mathbf{G}}(\mathbf{k})^{-1} = \varepsilon_0 \left[\frac{k - k_0^2}{k_0^2} (\mathbf{1} - \frac{\mathbf{k} \otimes \mathbf{k}}{k^2}) - \frac{\mathbf{k} \otimes \mathbf{k}}{k^2} \right], \; k = |\mathbf{k}|, \; \mathbf{k} = k\mathbf{m}$$

and taking into account the fact that $\mathbf{L}(\mathbf{k}) \cdot \mathbf{E}_0(\mathbf{k}) = 0$ (here $\mathbf{E}_0(\mathbf{k})$ has the form (2)) we get the equation for the mean wave field in the form ($\overline{\boldsymbol{\varepsilon}}_{\mathbf{1}} = \frac{\mathbf{1}}{\boldsymbol{\varepsilon}_{\mathbf{0}}}\boldsymbol{\varepsilon}_{\mathbf{1}}$)

$$\left[\frac{(k^2-k_0^2)}{k_0^2}(\mathbf{1}-\mathbf{m}\otimes\mathbf{m})-\mathbf{m}\otimes\mathbf{m}-p\overline{\boldsymbol{\varepsilon}}_1\cdot\boldsymbol{\Lambda}_0(\mathbf{k}_*)\cdot\boldsymbol{\Pi}(\mathbf{k})\right] \cdot <\mathbf{E}(\mathbf{k})>= 0. \quad (11)$$

The dispersion equation for the wave vectors of the mean wave field $\mathbf{k}_*$ is the consequence of this equation and takes the form ($\mathbf{k} = \mathbf{k}_* = k_*\mathbf{m}$)

$$\det\left[\frac{(k_*^2-k_0^2)}{k_0^2}(\mathbf{1}-\mathbf{m}\otimes\mathbf{m})-\mathbf{m}\otimes\mathbf{m}-p\overline{\boldsymbol{\varepsilon}}_1\cdot\boldsymbol{\Lambda}_0(\mathbf{k}_*)\cdot\boldsymbol{\Pi}(\mathbf{k}_*)\right] = 0. \quad (12)$$

where det is the determinant of a two rank tensor.

The mean wave field from a point source (the mean Green function $<\mathbf{G}(x)>$) may be also constructed from Eq. (5). If one accepts that $\mathbf{E}_0(x) = \mathbf{G}(x)$, where $\mathbf{G}(x)$ has the form (3) the mean wave field coincides with the mean Green function and for the Fourier transform of this function $<\tilde{\mathbf{G}}(\mathbf{k})>$ we obtain the following equation

$$<\tilde{\mathbf{G}}(\mathbf{k})>= \left[\mathbf{L}(\mathbf{k})-p\overline{\boldsymbol{\varepsilon}}_1\cdot\boldsymbol{\Lambda}_0(\mathbf{k}_*)\cdot\boldsymbol{\Pi}(\mathbf{k})\right]^{-1}. \quad (13)$$

In order to apply these results to the construction of the mean wave fields in specific composite materials it is necessary to build the correlation function $\Phi(x)$ for the given microstructure and to calculate the integral $\mathbf{G}_\Phi$ in Eq. (7). After that we can go to the analysis of the solutions of the dispersion equation (12).

3. AN APPROXIMATE SOLUTION OF THE ONE PARTICLE PROBLEM

The one particle problem in the framework of the effective field method is the solution of the integral equation similar to Eq. (1)

$$\mathbf{E}(x) - \int \mathbf{G}(x-x')\cdot\boldsymbol{\varepsilon}_1\cdot\mathbf{E}(x')v(x')dx' = \mathbf{E}_*(x), \quad (14)$$

where $v(x)$ is a characteristic function of the spherical region with the center at $x = 0$ and radius a, $\mathbf{E}_*(x)$ is the effective field that acts on every inclusion in the composite. This field is a plane wave with unknown amplitude $\mathbf{e}_*$ and wave vector $\mathbf{k}_*$ ($\mathbf{k}_* \neq \mathbf{k}_0$)

$$\mathbf{E}_*(x) = \mathbf{e}_* e^{-i\mathbf{k}_*\cdot x}, \qquad \mathbf{k}_* = k_*\mathbf{m}.$$

For an isotropic spherical inclusion the tensor $\boldsymbol{\Lambda}_0(\mathbf{k}_*)$ has the following structure

$$\boldsymbol{\Lambda}_0(\mathbf{k}_*) = \Lambda_t(k_*)(\mathbf{1} - \mathbf{m}\otimes\mathbf{m}) + \Lambda_l(k_*)(\mathbf{m}\otimes\mathbf{m}), \tag{15}$$

where $\Lambda_t(k_*), \Lambda_l(k_*)$ are the transversal and longitudinal parts of $\boldsymbol{\Lambda}_0$.

Using the technique of the solution of Mie's problem it is possible to obtain the exact expression for the coefficient $\Lambda_t(k_*)$ in the form [1]

$$\Lambda_t = \frac{3}{2}\sum_{n=1}^{\infty}(2n+1)$$
$$\cdot\left\{C_n g_{0n}(k_*,k) + D_n[(n+1)\frac{j_n(k_*)j_n(k)}{k_*k} - g_{1n}(k_*,k)]\right\} + \frac{k_0^2-k_*^2}{k^2-k_*^2},$$

$$g_{0n}(k_*,k) = \frac{1}{k_*^2-k^2}[k_*j_{n+1}(k_*)j_n(k) - kj_{n+1}(k)j_n(k_*)],$$
$$g_{1n}(k_*,k) = \frac{1}{k_*^2-k^2}[k_*j_n(k_*)j_{n+1}(k) - kj_n(k)j_{n+1}(k_*)].$$

$$C_n = \frac{k(k_0^2-k^2)}{k_*(k_*^2-k^2)}\frac{[k_0\xi_n'(k_0)\psi_n(k_*) - k_*\psi_n'(k_*)\xi_n(k_0)]}{[k_0\xi_n'(k_0)\psi_n(k) - k\psi_n'(k)\xi_n(k_0)]}, \tag{16}$$

$$D_n = \frac{k(k_0^2-k^2)}{k_*(k_*^2-k^2)}\frac{[k_*\xi_n'(k_0)\psi_n(k_*) - k_0\psi_n'(k_*)\xi_n(k_0)]}{[k\xi_n'(k_0)\psi_n(k) - k_0\psi_n'(k)\xi_n(k_0)]},$$

$$\psi_n(k) = kj_n(k), \quad \xi_n(k) = kh^{(2)}(k), \quad f'(k) = df/dk.$$

Here $j_n(k)$ and $h_n^{(2)}(k)$ are the spherical Bessel and Hankel functions of order n, k is the wave number of the material of the inclusions. (The radius of inclusions is assumed to be equal to one $(a=1)$ and here $k,\ k_0,\ k_*$ are non dimensional wave numbers $ka,\ k_0a,\ k_*a$). It is known that this series converge very slowly in the regions of middle and short waves where their lengths is comparable with the sizes of inclusions [12].

Note that if $\mathbf{E}_*(x) = \mathbf{E}_0(x)$, $(k_* = k_0,\ \mathbf{e}_* = \mathbf{e}_0)$ Eq. (14) coincides with the classical Mie problem. For this case the function Q

$$Q = -\frac{4}{3}k_0\overline{\varepsilon}_1\Lambda_t \tag{17}$$

is proportional to the forward amplitude $\mathbf{F}(\mathbf{m})$ of the field scattered on the inclusion and the imaginary part of this function coincides with the total normalized extinction scattering cross-section of the considered inclusion (see [1, 12, 13])

$$Q = -\frac{4}{k_0}\mathbf{e}_0\cdot\mathbf{F}(\mathbf{m}).$$

Here $\mathbf{e}_0$ is the amplitude of the exciting field scattered on the inclusion (see Eq. 2).

Let us consider approximate solutions of the one particle problem. It is possible to demonstrate [13] that the wave field inside inclusion (the solution of the integral equation (14)) is a stationary point of the following functional

$$JQ(\mathbf{E}) = \frac{vk_0}{\pi\varepsilon_0}\left[(\mathbf{E},\boldsymbol{\varepsilon}_1\cdot\overline{\mathbf{E}}) - (\mathbf{G}\boldsymbol{\varepsilon}_1\mathbf{E},\boldsymbol{\varepsilon}_1\cdot\overline{\mathbf{E}}) - (\mathbf{E}_*,\boldsymbol{\varepsilon}_1\cdot\overline{\mathbf{E}}) - (\overline{\mathbf{E}}_*,\boldsymbol{\varepsilon}_1\cdot\mathbf{E})\right], \tag{18}$$

where

$$(f,\phi) = \frac{1}{v}\int_v f(x)\cdot\phi(x)dx,$$

$$(\mathbf{G}\boldsymbol{\varepsilon}_1\mathbf{E})(x) = \int_v \mathbf{G}(x-x')\cdot\boldsymbol{\varepsilon}_1\cdot\mathbf{E}(x')dx'$$

and the line over functions in Eq. (18) is the complex conjugation. Note that if $\mathbf{E}_*(x) = \mathbf{E}_0(x)$ the value of the functional JQ on the exact solution of the Eq. (14) coincides with the function Q in Eq. (17)

$$JQ(\mathbf{E}) = Q. \tag{19}$$

Thus for the exact solution of the diffraction problem the functional JQ is proportional to the forward amplitude of the scattering field and the imaginary part of JQ is the total normalized extinction cross-section of the inclusion.

Let us consider the diffraction of a monochromatic plane effective wave $\mathbf{E}_*(x) = \mathbf{e}_* e^{-i\mathbf{k}_*\cdot x}$ on an isolated spherical inclusion with the center at point $x = 0$ in an infinite homogeneous medium. In order to build an approximate solution of the Eq. (14) we assume that the electric field $\mathbf{E}(x)$ inside the inclusion is a plane wave with the wave vector of the effective field $\mathbf{k}_*$

$$\mathbf{E}(x) = \mathbf{E}e^{-i\mathbf{k}_*\cdot x}, \qquad \mathbf{k}_* = k_*\mathbf{m}, \tag{20}$$

and with an unknown vector amplitude $\mathbf{E}$.

After substituting this approximation for $\mathbf{E}(x)$ into the functional JQ and using the Ritz scheme we get the following equation for the constant vector $\mathbf{E}$

$$\mathbf{E} - \mathbf{I}_G \cdot \boldsymbol{\varepsilon}_1 \cdot \mathbf{E} = \mathbf{e}_*, \tag{21}$$

$$\mathbf{I}_G = \mathbf{I}_G(k_0, \mathbf{k}_*) = \int \mathbf{G}(x) e^{i\mathbf{k}_* \cdot x} f(x) dx,$$
$$f(x) = \frac{1}{v} \int v(x + x') v(x') dx',$$

where $v(x)$ is the characteristic function of the region occupied by the inclusion with the center at point $x = 0$, v is the volume of the inclusion.

The same equation for $\mathbf{E}$ may be obtained in the framework of Galerkin's scheme if we substitute Eq. (20) into Eq. (14), multiply its both parts on $e^{i\mathbf{k}_* \cdot x}$ and then average the result over the volume of the inclusion.

For a spherical isotropic inclusion with a the unit radius $a = 1$ and an isotropic background medium we have ($| x |= r$)

$$f(x) = f(r) = 1 - \frac{4}{3} r + \frac{1}{16} r^3, \; r < 2; \qquad f(r) = 0, \; r \geq 2, \tag{22}$$

$$\mathbf{I}_G(k_0, \mathbf{k}_*) = \frac{1}{\varepsilon_0} \left\{ [k_0^2 q(k_0, k_*) + K_1(k_0, k_*)] \mathbf{1} + K_2(k_0, k_*) \mathbf{m} \otimes \mathbf{m} \right\},$$
$$q(k_0, k_*) = \int_0^\infty e^{-ik_0 r} f(r) j_0(k_* r) r dr,$$
$$K_1(k_0, k_*) = \int_0^\infty e^{-ik_0 r} \left\{ f'(r) j_0(k_* r) + \left[r f''(r) - f'(r) \right] \frac{j_1(k_* r)}{k_* r} \right\} dr,$$
$$K_2(k_0, k_*) = -\int_0^\infty e^{-ik_0 r} \{ [r f''(r) - f(r)] j_2(k_* r) + 2 r f'(r) k j_1(k_* r)$$
$$+ r f(r) k_*^2 j_0(k_* r) \} dr.$$

Here $j_i(k)$ $(i = 0, 1, 2)$ are spherical Bessel functions, $f' = \frac{d}{dr} f$.

Note that q, K_1 and K_2 are some integrals that may be calculated in explicit analytical forms and are some combinations of polynomials, exponential functions and exponential integrals (see [1]).

Thus the electric field inside the inclusion takes the form

$$\mathbf{E}(x) = \boldsymbol{\Lambda}_0 \cdot \mathbf{E}_*(x), \qquad \boldsymbol{\Lambda}_0 = \left[\mathbf{1} - \mathbf{I}_G(k_0, \mathbf{k}_*) \cdot \boldsymbol{\varepsilon}_1\right]^{-1}. \tag{23}$$

Note that a similar approximation was used in [14] for the solution of the elastic wave propagation problem through polycrystalline materials.

In order to understand the quality of this approximation let us consider an isolated spherical inclusion of a unit radius in a infinite dielectric medium. For this problem the plane wave approximation is the assumption that the wave field inside the inclusion takes the form

$$\mathbf{E}(x) = \boldsymbol{\Lambda}_0 \cdot \mathbf{e}e^{-i\mathbf{k}_0 \cdot x}, \qquad \boldsymbol{\Lambda}_0 = \left[\mathbf{1} - \mathbf{I}_G(k_0, \mathbf{k}_0) \cdot \boldsymbol{\varepsilon}_1\right]^{-1}.$$

The results of the calculations of the real and imaginary parts of the function $Q(k_0)$ or functional $JQ(k_0)$ for the inclusion with the dielectric permittivity $\varepsilon = 5$ ($\varepsilon_0 = 1$) are presented in Fig. 1. Here the solid lines are the exact dependences of $Re[JQ(k_0)]$ and $Im[JQ(k_0)]$ (the results of Mie's theory), the lines with dots were obtained by substituting the plane wave approximation (23) into functional JQ in Eq. (18) ($\mathbf{E}_*(x) = \mathbf{E}_0(x)$). Note that $Im[JQ(k_0)]$ coincides with the total scattering cross-section of the considered inclusion. As it can be seen the plane wave approximation describes only a general trend of the function $JQ(k_0)$ but it cannot describe small scale oscillations of this function in the middle and short wave regions.

4. INTEGRAL $\mathbf{G}_\Phi$ FOR RANDOM SETS AND REGULAR LATTICES OF INCLUSIONS

Let us consider the construction of the function $\Phi(x)$ for one scale and two scale Boolean random fields of inclusions. The Boolean model V is obtained by implantation of random grains V'_k on the points x_k of a Poisson point process [2]:

$$V = \cup_k V'_{k,x_k}$$

In the present case, a one scale model of spheres is obtained by selecting for $V'_k = v_k$ the sphere of radius $a = 1$ with the center at point x_k. Since overlaps are allowed in this model, the resulting structure is not strictly speaking made of isolated spheres. However, for medium

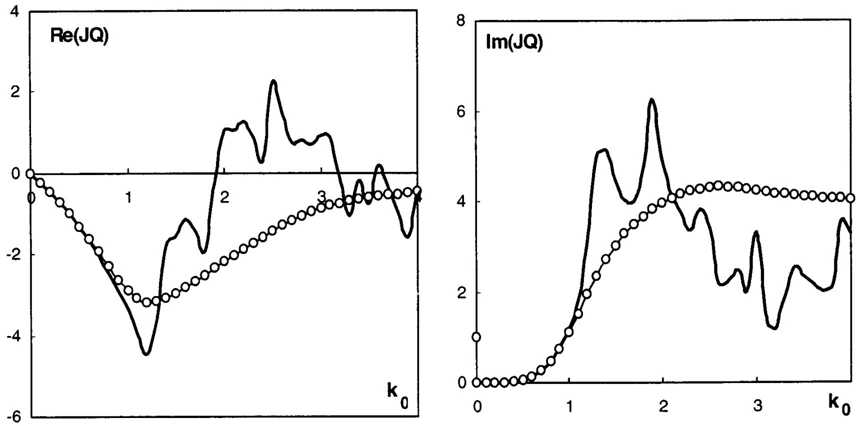

Figure 1. The dependences of the real and imaginary parts of the functional JQ on k_0. Solid lines correspond to the exact solution of the diffraction problem (Mie's solution) for an inclusion of a unit radius ($\varepsilon_0 = 1$, $\varepsilon = 5$), the lines with dots correspond to the plane wave approximation.

volume fractions, the volume fraction of overlaps can be neglected. This is illustrated in Fig. 2 where one can see a simulation of a section of a Boolean model of spheres with $p = 0.3$.

For the one scale model, the function $\Phi(\mathbf{r})$ has the form [1–4]:

$$\Phi(\mathbf{r}) = \Phi(r) = \frac{1}{p^2}\left\{pf(r) + (1-p)^2\left[1-(1-p)^{-f(r)}\right]\right\}, \quad |\mathbf{r}| = r \tag{24}$$

We consider now a random set V made of the intersection of two independent random sets V_1 and V_2, with volume fractions p_1 and p_2 and covariances $C_1(r)$ and $C_2(r)$:

$$V = V_1 \cap V_2$$

$$C_j(\mathbf{r}) = P\{x \in V_j, x+\mathbf{r} \in V_j\}, \quad j = 1, 2;$$

where $P\{x \in V_j, x+\mathbf{r} \in V_j\}$ is the probability for the points x and $x+\mathbf{r}$ to be inside the random set V_j.

We have for this model

$$p = p_1 p_2,$$
$$C(\mathbf{r}) = P\{x \in V, x+\mathbf{r} \in V\} = C_1(r)C_2(r).$$

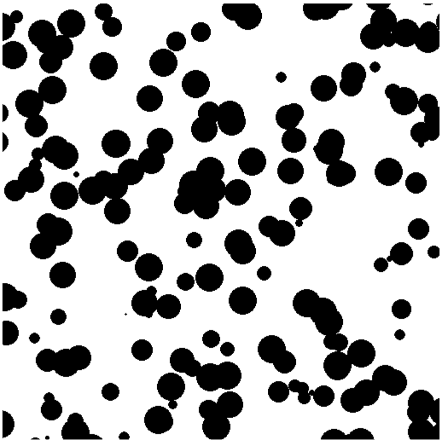

Figure 2. Simulation of a section of a Boolean model of spheres with $p = 0.3$.

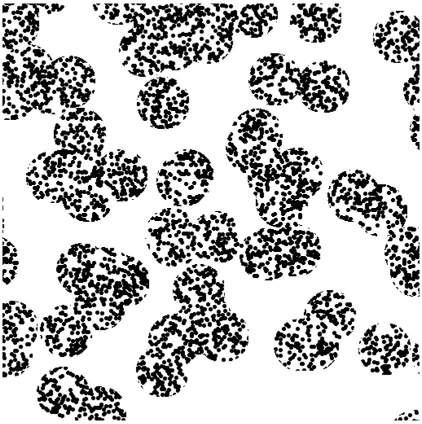

Figure 3. Simulation of a two scale model obtained in 2D by the intersection of two Boolean models of discs ($p_1 = p_2 = 0.5$; $p = 0.25$).

An interesting example is obtained by the intersection of two Boolean models with spherical primary grains (with radii $R_2 \gg R_1 = 1$). This situation is illustrated in Fig. 3 in the case of discs in two dimensions. This model presents two separate scale:

- on a microscopic scale (corresponding to V_1), we have a standard Boolean model of spheres (with few overlaps at a low volume fraction)
- on a larger scale (corresponding to V_2), we observe fluctuations of volume fraction, with particles inside V_2 and matrix inside $V_2^c = V \cap V_2$. This larger scale appears in $C(r)$, and consequently in $\Phi(r)$.

For two scale Boolean models we get [5]

$$\Phi(r) = 1 - \frac{1}{p^2}\left[C(r) - pf(r)f(r/R_2)\right], \quad p = p_1 p_2, \tag{25}$$

$$C(r) = \left[2p_1 - 1 + (1-p_1)^{(2-f(r))}\right]\left[2p_2 - 1 + (1-p_2)^{(2-f(r/R_2))}\right]$$

The functions $\Phi(x)$ for the two models are spherically symmetrical. The integral $\mathbf{G}_\Phi$ (7) for such functions after applying the Gauss theorem takes the form

$$\mathbf{G}_\Phi(\mathbf{k}) = \frac{1}{\varepsilon_0}\left[G_t(k_0,k)(\mathbf{1} - \mathbf{m}\otimes\mathbf{m}) + G_l(k_0,k)(\mathbf{m}\otimes\mathbf{m})\right], \quad \mathbf{k} = k\mathbf{m}, \tag{26}$$

$$\begin{aligned}
G_t(k_0,k) &= k_0^2 q_t(k_0,k) + J_t(k_0,k),\\
q_t(k_0,k) &= \int_0^\infty e^{-ik_0 r}\Phi(r) j_0(kr) r\,dr,\\
J_t(k_0,k) &= \int_0^\infty e^{-ik_0 r}\left\{\Phi'(r) j_0(kr) + [r\Phi''(r) - \Phi'(r)]\frac{j_1(kr)}{kr}\right\}dr,\\
G_l(k_0,k) &= -\int_0^\infty e^{-ik_0 r}\{[r\Phi''(r) - \Phi(r)] j_2(kr) + 2r\Phi'(r) k j_1(kr)\\
&\qquad + r\Phi(r)k^2 j_0(kr)\}dr.
\end{aligned}$$

For numerical calculations of the integrals G_t and G_l in Eq. (26) we have to use some approximations of the functions $\Phi(r)$. Here we use piecewise exponential approximations in the form

$$\Phi(r) = (1+\rho_0)e^{-\rho_1 r}\cos(b_1 r) - \rho_0, \quad r \le r_0, \tag{27}$$

$$\Phi(r) = \left[(1+\rho_0)e^{-\rho_1 r_0}\cos(b_1 r_0) - \rho_0\right] e^{-\rho_2(r-r_0)}\cos[b_2(r-r_0)],\ r > r_0.$$

This function is continuous together with the first derivative if

$$\rho_2 = \frac{(1+\rho_0)(\rho_1\cos(b_1 r_0) + b_1\sin(b_1 r_0))\exp(-\rho_1 r_0)}{(1+\rho_0)\exp(-\rho_1 r_0)\cos(b_1 r_0) - \rho_0}.$$

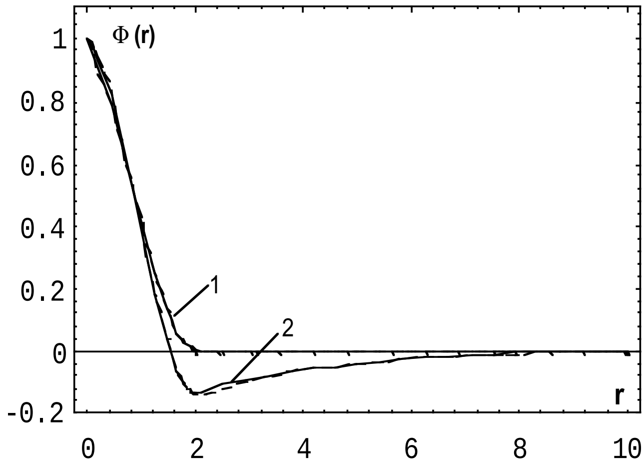

Figure 4. The function $\Phi(r)$ for Boolean random sets of inclusions; line 1 corresponds to the one scale model for $p = 0.2$, line 2 corresponds to the two scale model with the parameters $p_1 = 0.25,\ R_1 = 1,\ p_2 = 0.8,\ R_2 = 5$. The dashed lines are approximations (27) of these functions. For the one scale model ($\rho_0 = 0,\ \rho_1 = 0.3717,\ b_1 = 0.9264,\ \rho_2 = 3.233,\ b_2 = 2.2025\ , r_0 = 1.3577$), for the two scale model ($\rho_0 = -0.3,\ \rho_{01} = 0.2136,\ b_1 = 1.4382,\ \rho_2 = 0.4379,\ b_2 = 0.1331,\ r_0 = 2.1491$) .

The other parameters ($\rho_{0,}\ \rho_1,\ b_1,\ b_2$ and r_0) are chosen from minimizing the square mean error in comparison with the exact functions (24), (25).

The integrals (26) for approximations (27) of Boolean correlation functions may by calculated in an explicit analytical forms and are some finite combinations of power functions and exponential integrals [1].

The comparison of the correlation function $\Phi(x)$ for the one scale (line 1) and two scale (line 2) Boolean models is presented in Fig. 4 for the same total volume concentration of inclusions $p = 0.2$. For the two scale model (line 2) we chose $p_1 = 0.25,\ R_1 = 1,\ p_2 = 0.8,\ R_2 = 5$. The dashed lines in Fig. 4 are the approximations (27) of this functions.

Let us go to regular lattices of identical inclusions in a homogeneous matrix. In this case $V(x)$ is a characteristic function of the region occupied by a set of identical spherical inclusions of a unit radius which centers compose an infinite regular lattice in 3D space. If $\mathbf{q}$ is the vector of this lattice the function $V(x)$ takes the form

$$V(x) = \sum_{\mathbf{q}} v(x+\mathbf{q}), \quad \mathbf{q} = j\mathbf{a}_1 + s\mathbf{a}_2 + t\mathbf{a}_3, \quad j,s,t = 0, \pm 1, \pm 2, \ldots, \tag{28}$$

where $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$ are vectors of an elementary cell of the lattice, $v(x)$ is a characteristic function of the region occupied by a sphere of a unit radius with the center at point $x = 0$.

If $\mathbf{r}$ is a stochastic vector with a homogeneous distribution in space the realizations of the random function $V(x+\mathbf{r})$ are various translations of the original regular lattice. The second moment of $V(x+\mathbf{r})$ ($< V(\mathbf{r})V(x+\mathbf{r}) >$) is a periodic function. After averaging over the vector $\mathbf{r}$ we get

$$< V(\mathbf{r})V(x+\mathbf{r}) >= v_0 \sum_{\mathbf{q}} f(x+\mathbf{q}), \quad f(x) = \frac{1}{v_0}\int v(x+x')v(x')dx', \tag{29}$$

where v_0 is the volume of a unit sphere, the function $f(x)$ has the explicit form (22).

Using the definition (8) of the function $\Phi(x)$ we obtain for regular structures the following result

$$\Phi(x) = 1 - \frac{1}{p}\sum_{\mathbf{q}}{}' f(x+\mathbf{q}). \tag{30}$$

Here the prime over the summation sign denotes omitting the term with $\mathbf{q} = \mathbf{0}$.

The integral $\mathbf{G}_\Phi(\mathbf{k})$ in Eq. (7) for this function $\Phi(x)$ takes the form

$$\mathbf{G}_\Phi(\mathbf{k}) = \int \mathbf{G}(x)\Phi(x)\exp(i\mathbf{k}\cdot x)dx = \mathbf{G}^0_\Phi(\mathbf{k}) + \mathbf{G}^1_\Phi(\mathbf{k}), \tag{31}$$

$$\mathbf{G}^0_\Phi(\mathbf{k}) = \frac{1}{p}\int \mathbf{G}(x)f(x)\exp(i\mathbf{k}\cdot x)dx,$$

$$\mathbf{G}^1_\Phi(\mathbf{k}) = -\frac{1}{v_0}\sum_{\boldsymbol{\mu}}{}' \tilde{f}(\mu)\tilde{\mathbf{G}}(\mathbf{k}-\boldsymbol{\mu}).$$

Here $\tilde{f}(k)$ and $\tilde{\mathbf{G}}(\mathbf{k})$ are Fourier transforms of the functions $f(x)$ and $\mathbf{G}(x)$ defined in Eqs. (22), (3), $\boldsymbol{\mu}$ is the vector of the inverse lattice in respect to the original one,

$$f^*(\mu) = 12\pi\frac{j_1^2(\mu)}{\mu^2}, \quad j_1(\mu) = \frac{\sin(\mu)}{\mu^2} - \frac{\cos(\mu)}{\mu}, \quad \mu = \mid \boldsymbol{\mu} \mid . \qquad (32)$$

The prime over the summation sign in Eq. (31) for $\mathbf{G}_\Phi^1(\mathbf{k})$ denotes omitting the term with $\boldsymbol{\mu} = \mathbf{0}$. In order to obtain this expression for $\mathbf{G}_\Phi^1(\mathbf{k})$ we have to go to Fourier transforms of the integrand functions.

5. EFFECTIVE STATIC DIELECTRIC PROPERTIES OF COMPOSITES

Let us consider the propagation of the electromagnetic waves in a homogeneous dielectric medium with the tensor of dielectric properties $\boldsymbol{\varepsilon}_*$. The Fourier transform of the electric field $\mathbf{E}(\mathbf{k})$ in such a medium satisfies the following equation [1, 10]

$$\left[k^2(\mathbf{1} - \mathbf{m} \otimes \mathbf{m}) - \omega^2 \boldsymbol{\varepsilon}_*\right] \cdot \mathbf{E}(\mathbf{k}) = 0. \qquad (33)$$

Eq. (11) for the mean electric field in the composite medium may be rewritten in the form

$$\left[k^2(\mathbf{1} - \mathbf{m} \otimes \mathbf{m}) - k_0^2\mathbf{1} - p\, k_0^2 \overline{\boldsymbol{\varepsilon}}_1 \cdot \boldsymbol{\Lambda}_0(\mathbf{k}_*) \cdot \boldsymbol{\Pi}(\mathbf{k})\right] \cdot < \mathbf{E}(\mathbf{k}) >= 0,$$
$$k_0^2 = \omega^2 \varepsilon_0.$$

Comparing this two equations one can note that the last one describes propagation of waves in the medium with the effective tensor of dielectric properties $\boldsymbol{\varepsilon}_*$ that has the following form

$$\boldsymbol{\varepsilon}_*(\mathbf{k}) = \boldsymbol{\varepsilon}_0 + p\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\Lambda}_0(\mathbf{k}_*) \cdot \boldsymbol{\Pi}(\mathbf{k}). \qquad (34)$$

Let us consider the static limit of this tensor when $\omega, \mathbf{k} \to 0$. (The limit $\mathbf{k} \to 0$ means that the exciting field is a constant.) In this limit we have for spherical homogeneous inclusions

$$\boldsymbol{\varepsilon}_*^s = \boldsymbol{\varepsilon}_0 + p\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\Lambda}_0^s \cdot \boldsymbol{\Pi}^s. \qquad (35)$$

$$\boldsymbol{\Lambda}_0^s = \boldsymbol{\Lambda}_0(0) = \frac{3\varepsilon_0}{3\varepsilon_0 + \varepsilon_1}\mathbf{1}, \quad \boldsymbol{\Pi}^s = \boldsymbol{\Pi}(0) = \left[\mathbf{1} + p\boldsymbol{\varepsilon}_1 \cdot \mathbf{G}_\Phi^s \cdot \boldsymbol{\Lambda}_0^s\right]^{-1},$$

$$\mathbf{G}_\Phi^s = \int \mathbf{G}^s(x)\Phi(x)dx, \quad \mathbf{G}^s(x) = \bigtriangledown \otimes \bigtriangledown \left(\frac{1}{4\pi\varepsilon_0|x|}\right).$$

For isotropic and homogeneous random set of inclusions the function $\Phi(x)$ depends only on $\mid x \mid = r$ and the integral $\mathbf{G}^s_\Phi$ takes the form

$$\mathbf{G}^s_\Phi = \frac{1}{\varepsilon_0} J^s \mathbf{1}, \tag{36}$$

$$J^s = \int_0^\infty \{\Phi'(r) + \frac{1}{3}[r\Phi''(r) - \Phi'(r)]\} dr = -\frac{1}{3}\Phi(0) = -\frac{1}{3}.$$

Here the property (10) of $\Phi(x)$ is used.

Thus the value of this integral does not depend on detailed behavior of the function $\Phi(r)$ and the effective dielectric permittivity of the composite takes the form of the Maxwell Garnet formula

$$\varepsilon^s_* = \varepsilon_0 \left[1 + \frac{3p\varepsilon_1}{3\varepsilon_0 + (1-p)\varepsilon_1} \right]. \tag{37}$$

A generalization of this result may be obtained for the case when Φ is not a spherically symmetric one but is anisotropic (for instance it might present an a ellipsoidal symmetry but the procedure of construction of such a distribution of inclusions is unknown). Some anisotropic distributions of spherical inclusions can be obtained as follows: replace the standard Poisson point process for the implantation of spheres by a non isotropic point process; this can be built in two steps: first consider Poisson lines with the intensity $\theta(\omega)$ (equal to the average number of lines per unit area orthogonal to the plane with a normal of orientation ω); secondly, we generate on every line $D(\omega)$ a Poisson point process with the intensity $\lambda(\omega)$. The correlation function Φ of this model can be calculated as a function of $\theta(\omega)$ and $\lambda(\omega)$, which control the anisotropy of the medium. In this case the tensor $\boldsymbol{\varepsilon}^s_*$ is also non isotropic and this anisotropy is caused by the spatial distribution of inclusions.

Let us assume that there is a linear transformation A of x-space ($y = Ax$) that converts the function Φ in a spherically symmetric one: $\Phi(A^{-1}y) = \Phi(\mid y \mid)$. For this case the integral $\mathbf{G}^s_\Phi$ also does not depend on detailed behavior of Φ and takes the form [15]

$$\mathbf{G}^s_\Phi = g_1 \mathbf{e}_1 \otimes \mathbf{e}_1 + g_2 \mathbf{e}_2 \otimes \mathbf{e}_2 + g_3 \mathbf{e}_3 \otimes \mathbf{e}_3, \tag{38}$$

$$g_i = \frac{a_1 a_2 a_3}{\varepsilon_0} \int_0^\infty \frac{d\sigma}{(a_i^2 + \sigma) H(\sigma)}, \quad H(\sigma) = \sqrt{(a_1^2 + \sigma)(a_2^2 + \sigma)(a_3^2 + \sigma)}.$$

Here $\mathbf{e}_i$ and a_i $(i = 1, 2, 3)$ are the main directions and the corresponding main semiaxes of the ellipsoid ($x \cdot Ax = 1$). The tensor $\boldsymbol{\varepsilon}_*$ in this case has an orthorhombic symmetry

$$\boldsymbol{\varepsilon}_*^s = \varepsilon_*^{(1)}\mathbf{e}_{1\otimes}\mathbf{e}_1 + \varepsilon_*^{(2)}\mathbf{e}_{2\otimes}\mathbf{e}_2 + \varepsilon_*^{(3)}\mathbf{e}_{3\otimes}\mathbf{e}_3, \tag{39}$$

$$\varepsilon_*^{(i)} = \varepsilon_0 + p\varepsilon_1 \frac{\Lambda_0}{1 + p\varepsilon_1 g_i \Lambda_0}, \quad i = 1, 2, 3, \quad \Lambda_0 = \frac{3\varepsilon_0}{3\varepsilon_0 + \varepsilon_1}.$$

For regular lattices of inclusions the integral $\mathbf{G}_\Phi$ has the form of Eq. (31) and for the tensor $\boldsymbol{\varepsilon}_*^s$ we get the following expression

$$\boldsymbol{\varepsilon}_*^s = \boldsymbol{\varepsilon}_0 + p\boldsymbol{\varepsilon} z_1 \cdot [1 + p\boldsymbol{\varepsilon}_1 \cdot \boldsymbol{\Gamma}]^{-1}, \tag{40}$$

$$\boldsymbol{\Gamma} = -\frac{1}{v_0} {\sum_{\boldsymbol{\mu}}}' \tilde{f}(\mu)\tilde{\mathbf{G}}^s(\boldsymbol{\mu}), \quad \tilde{\mathbf{G}}^s(\boldsymbol{\mu}) = -\frac{1}{\varepsilon_0}\frac{\boldsymbol{\mu}\otimes\boldsymbol{\mu}}{\mu^2}.$$

Here the tensor $\boldsymbol{\Gamma}$ has the symmetry of the regular lattice. For instance in the case of an orthorhombic lattice this tensor takes the form

$$\boldsymbol{\Gamma} = \alpha_1\mathbf{e}_1\otimes\mathbf{e}_1 + \alpha_2\mathbf{e}_2\otimes\mathbf{e}_2 + \alpha_3\mathbf{e}_3\otimes\mathbf{e}_3,$$

$$\alpha_j = \frac{3\lambda_1\lambda_2\lambda_3}{(2\pi)^2\varepsilon_0} {\sum_{i_1,i_2,i_3}}' \frac{j_1^2\left(\sqrt{(\lambda_1 i_1)^2 + (\lambda_2 i_2)^2 + (\lambda_3 i_3)^2}\right)}{\left[(\lambda_1 i_1)^2 + (\lambda_2 i_2)^2 + (\lambda_3 i_3)^2\right]^2}(\lambda_j i_j)^2,$$
$$\lambda_j = \frac{2\pi}{L_j}.$$

Here $L_j\mathbf{e}_j (j = 1, 2, 3)$ are the vectors of an elementary cell of the lattice ($|\ \mathbf{e}_j\ | = 1$), $i_j = 0, \pm 1, \pm 2, ...,$ a prime over the sum means omitting the term with $i_1 = i_2 = i_3 = 0$.

For a cubic lattice $(L_1 = L_2 = L_3 = L_0)$ this tensor is isotropic and takes the form

$$\boldsymbol{\Gamma} = \frac{1}{\varepsilon_0}\alpha_0(\lambda_0)\mathbf{1}, \tag{41}$$

$$\alpha_0(\lambda_0) = \frac{3\lambda_0}{(2\pi)^2} {\sum_{i,j,m}}' \frac{j_1^2(\lambda_0\sqrt{i^2 + j^2 + m^2})}{i^2 + j^2 + m^2},$$

$$\lambda_0 = \frac{2\pi}{L_0}, \qquad i, j, m = 0, \pm 1, \pm 2, \ldots,$$

where L_0 is the distance between the center of inclusions. For inclusions of a unit radius the parameter λ_0 is connected to the volume concentration of inclusions p by the relations

$$\lambda_0 = \left(6\pi^2 p\right)^{\frac{1}{3}}.$$

The graph of the function $\alpha(p)$ is presented in Fig. 5. The corresponding dependence of the effective dielectric permittivity of the composite with a cubic lattice of inclusion is presented in Fig. 6 ($\varepsilon = 8, \varepsilon_0 = 1$) by solid line. Note that the dependence $\varepsilon_*^s(p)$ for the cubic lattice almost coincides with the same dependence for an isotropic distributions inclusions in space given by Eq. (34) (the point line in Fig. 6). The deviation from the exact values of the effective dielectric permittivity is observed only for rather high values of the volume concentration of inclusions ($p > 0.4$). The exact values of $\varepsilon_*^s(p)$ for a cubic lattice of inclusions were built in [16] and the dashed line in Fig.6 corresponds to this exact solution.

As it was obtained in [1] the long wave limit of the attenuation factor γ of the mean wave field in the composites has the form

$$\gamma = \frac{k_0^4(\varepsilon_*^s - \varepsilon_0)^2}{9p\varepsilon_0\sqrt{\varepsilon_*^s\varepsilon_0}} \left[1 - n_0 \int \Phi(x)dx\right]. \tag{42}$$

This expression describes the attenuation connected with the Rayleigh wave scattering on inclusions. Here n_0 is the numerical concentration of inclusions, integration is spread over all 3-D space. In the case of a spherically symmetric correlation function ($\Phi(x) = \Phi(r)$) the integral takes the form

$$\int \Phi(x)dx = 4\pi \int_0^\infty \Phi(r)r^2 dr.$$

For periodic structures the function $\Phi(x)$ has the form (30). Note that for the function $f(x)$ in Eq. (22) we get

$$\int f(x + \mathbf{q})dx = \frac{4\pi}{3}.$$

For spheres of a unit radius $p = \frac{4\pi}{3}n_0$ and the volume of an elementary cell of the lattice is equal to n_0^{-1}. Thus the integrals over all the elementary cells disappear except the cell with $\mathbf{q} = \mathbf{0}$ because the

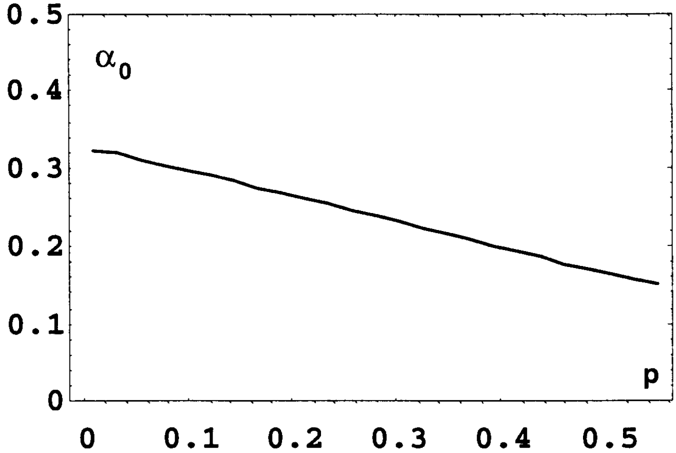

Figure 5. The dependence of the coefficient α_0 in Eq. (41) for a cubic lattice of inclusions on their volume concentration p.

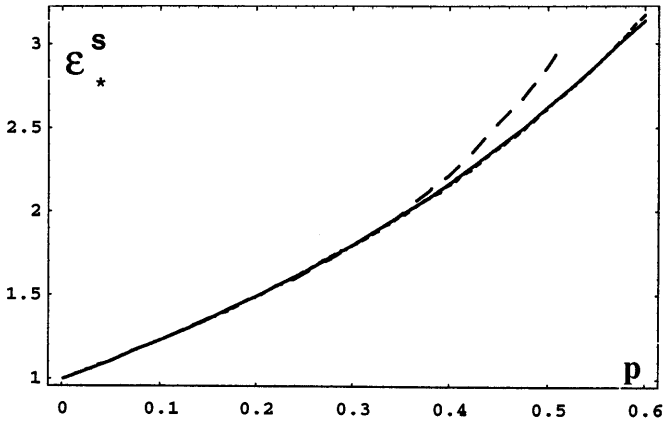

Figure 6. The dependence of the effective static dielectric permittivity ε_*^s of the composite with a cubic lattice of spherical inclusions on their volume concentration p $(\varepsilon = 10, \varepsilon_0 = 1)$. Point line is the same dependence for isotropic distribution of inclusions (Eq. (37)); dashed line is the exact values of the effective permittivity obtained in [16].

corresponding term is not present in the sum (30). The integral over this cell is equal to n_0^{-1} and we have finally the result

$$\int \Phi(x)dx = n_0^{-1}.$$

Thus for periodic structures the long wave limit (42) of the attenuation factor is equal to zero and γ is of order at least higher than k_0^4. Therefore, there is no Rayleigh scattering of waves in this case. This fact is well-known for periodic structures.

On the other hand in the short wave limit the effective field method gives the following asymptotic value of the effective wave number k_* of the composite [1]

$$k_* = k_0 - i\gamma,$$

where k_0 is the wave number of the matrix and the attenuation factor γ is the solution of the following equation

$$\gamma(1 - pI_\Phi(\gamma)) = \frac{3}{4}p, \quad I_\Phi(\gamma) = \int e^{\gamma|x|}\Phi(x)dx.$$

For the function $\Phi(x)$ in the form (30) and $\gamma > 0$ the integral $I_\Phi(\gamma)$ diverges and we get $\gamma \to 0$ in the short wave limit. Thus the effective field methods gives us physically correct results for the short wave limit also: the velocities of very short waves coincide with their velocities in the matrix and they propagate through the medium without attenuation.

6. THE SOLUTION OF THE DISPERSION EQUATIONS

Let us consider wave propagation through the inhomogeneous materials we have discussed above and start with a cubic lattice of spherical inclusions of a unit radius. For this structure the tensor $\mathbf{G}_\Phi(\mathbf{k})$ takes the form (31)

$$\mathbf{G}_\Phi(\mathbf{k}) = \mathbf{G}_\Phi^0(\mathbf{k}) + \mathbf{G}_\Phi^1(\mathbf{k}), \tag{43}$$

$$\mathbf{G}_\Phi^0(\mathbf{k}) = \frac{1}{p}\left[G_t^0(k_0,k)(\mathbf{1} - \mathbf{m}\otimes\mathbf{m}) + G_l^0(k_0,k)\mathbf{m}\otimes\mathbf{m}\right], \quad \mathbf{k} = k\mathbf{m},$$

$$G_t^0(k_0,k) = \frac{1}{\varepsilon_0}\left[k_0^2 q_t(k_0,k) + J_t(k_0,k)\right],$$

$$G_l^0(k_0,k) = G_t^0(k_0,k) + \frac{1}{\varepsilon_0}G_l(k_0,k),$$

where integrals $q_t(k_0,k), J_t(k_0,k), G_l(k_0,k)$ are defined in Eq. (26). The tensor $\mathbf{G}^1_\Phi(\mathbf{k})$ in the basis of the unit vectors $\mathbf{e}_i (i = 1,2,3)$ directed along the sides of a cubic elementary cell takes the form

$$\mathbf{G}^1_\Phi(\mathbf{k}) = (G_\Phi)_{jl}\mathbf{e}_j \otimes \mathbf{e}_l,$$

$$(G_\Phi)_{jj} = -\frac{9k_0^2}{\varepsilon_0}\sum_{r,s,t}{}' \frac{j_1^2(\mu(r,s,t))}{\mu^2(r,s,t)\left[|\mathbf{k}-\boldsymbol{\mu}(r,s,t)|^2 - k_0^2\right]} \cdot\left[1 - \frac{(k_j - \mu_j(r,s,t))^2}{k_0^2}\right],$$

$$(j = l = 1,2,3; \qquad r,s,t = 0, \pm 1, \pm 2, \pm 3, \dots)$$

$$(G_\Phi)_{jl} = \frac{9}{\varepsilon_0}\sum_{r,s,t}{}' \frac{j_1^2(\mu(r,s,t))}{\mu^2(r,s,t)\left[|\mathbf{k}-\boldsymbol{\mu}(r,s,t)|^2 - k_0^2\right]} \cdot (k_j - \mu_j(r,s,t))(k_l - \mu_l(r,s,t)),$$

$$j,l = 1,2,3; \quad j \neq l.$$

Here k_j and $\mu_j(r,s,t)$ are the components of the vectors $\mathbf{k}$ and $\boldsymbol{\mu}(r,s,t) = \lambda_0(r\mathbf{e}_1 + s\mathbf{e}_2 + t\mathbf{e}_3)$ in the chosen basis, $\mu(r,s,t) = \lambda_0\sqrt{r^2+s^2+t^2}$, the prime over the sum sign means omitting the term with $r = s = t = 0$.

If the vector $\mathbf{k}$ is directed along a side of the elementary cell (i.e., $\mathbf{k} = k\mathbf{e}_1$) the non diagonal terms of the matrix $(G_\Phi)_{jl}$ disappear and the diagonal ones take the forms

$$(G_\Phi)_{11} = G_l(k_0,k) = -\frac{9}{\varepsilon_0}\sum_{r,s,t}{}' \frac{j_1^2(\mu(r,s,t))\left[k_0^2 - k^2 + 2k\lambda_0 r - \lambda_0^2 r^2\right]}{\mu^2(r,s,t)\left[k^2 + \mu^2(r,s,t) - k_0^2 - 2k\lambda_0 r\right]}, \tag{45}$$

$$(G_\Phi)_{22} = (G_\Phi)_{33} = G_t(k_0,k) = -\frac{9}{2\varepsilon_0}\sum_{r,s,t}{}' \frac{j_1^2(\mu(r,s,t))\left[2k_0^2 - \mu^2(r,s,t) + \lambda_0^2 r^2\right]}{\mu^2(r,s,t)\left[k^2 + \mu^2(r,s,t) - k_0^2 - 2k\lambda_0 r\right]},$$

The functions $\boldsymbol{\Pi}(\mathbf{k})$ in Eq. (7) for this case takes the form

$$\boldsymbol{\Pi}(\mathbf{k}) = \Pi_t(k)(\mathbf{1} - \mathbf{e}_1 \otimes \mathbf{e}_1) + \Pi_l(k)\mathbf{e}_1 \otimes \mathbf{e}_1, \tag{46}$$

$$\Pi_t(k) = \left[1 + p\overline{\varepsilon}_1 \Lambda_t(k_0,k_*)G_t(k_0,k)\right]^{-1},$$

$$\Pi_l(k) = \left[1 + p\overline{\varepsilon}_1 \Lambda_l(k_0,k_*)G_l(k_0,k)\right]^{-1}.$$

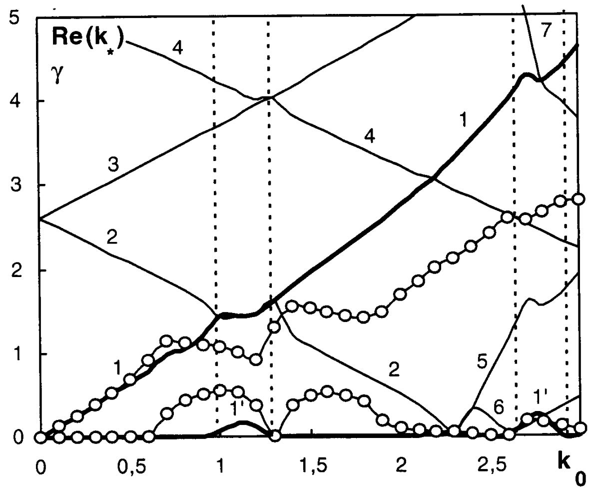

Figure 7. The dispersion curves (dependences $\mathrm{Re}[k_*(k_0)]$) and attenuation factors ($\gamma = \mathrm{Im}[k_*(k_0)]$) of the mean wave field in the composite with a regular cubic lattice of spherical inclusions of a unit radius ($\varepsilon_0 = 1;\ \varepsilon = 5, p = 0.3$).

The dispersion equation (12) is divided in two equations for the transversal part of the mean wave field ($\mathbf{k} = \mathbf{k}_* = k_*\mathbf{e}_1$)

$$k_*^2 - k_0^2 - pk_0^2\overline{\varepsilon}_1[\Lambda_t(k_0,k_*)^{-1} + p\overline{\varepsilon}_1 G_t^0(k_0,k_*) + p\overline{\varepsilon}_1 G_t^1(k_0,k_*)]^{-1} = 0 \quad (47)$$

and for the longitudinal part

$$1 + p\overline{\varepsilon}_1[\Lambda_l(k_0,k_*)^{-1} + p\overline{\varepsilon}_1 G_l^0(k_0,k_*) + p\overline{\varepsilon}_1 G_l^1(k_0,k_*)]^{-1} = 0. \quad (48)$$

Here $\Lambda_t(k_0,k_*)$ and $\Lambda_l(k_0,k_*)$ are transversal and longitudinal parts of the tensor $\boldsymbol{\Lambda}_0(k_*)$ in Eq. (16).

Let us consider the transversal part of the mean wave field that propagates along the vector $\mathbf{e}_1$. We emphasize that this field is the detail field averaged over all the translations of the regular lattice. For the coefficient $\Lambda_t(k_0,k_*)$ one can use the exact solution (16) or the approximate solution (23). If the approximate solution is used (the wave field inside every inclusion is a plane wave with the wave vector of the effective field) the dispersion equation is simplified dramatically and take the forms

$$k_*^2 = k_0^2 + pk_0^2\overline{\varepsilon}_1\Gamma_t(k_0,k_*), \quad \Gamma_t(k_0,k_*) = [1 + p\overline{\varepsilon}_1 G_t^1(k_0,k_*)]^{-1}, \quad (49)$$

$$1 + p\bar{\varepsilon}_1\Gamma_l(k_{0,}k_*) = 0, \quad \Gamma_l(k_{0,}k_*) = [1 + p\bar{\varepsilon}_1 G_l^1(k_0, k_*)]^{-1}. \tag{50}$$

Here we take into account that for the plane wave approximation

$$\boldsymbol{\Lambda}_0^{-1}(\mathbf{k}_*) = \mathbf{1} - \mathbf{I_G}(\mathbf{k_0}, \mathbf{k}_*) \cdot \boldsymbol{\varepsilon}_\mathbf{1} = \mathbf{1} - \varepsilon_1 p \mathbf{G}_\Phi^\mathbf{0}(\mathbf{k}_*). \tag{51}$$

The numerical analysis of the equation (49) shows the existence of an infinite set of different branches of its solution. The branches (1–7) that are found inside the area ($0 \leq k_0 \leq 3,\ 0 \leq \mathrm{Re}(k_*) \leq 5$) are presented in Fig. 7 ($\varepsilon = 5,\ \varepsilon_0 = 1,\ p = 0.3$).

Let us consider the main ("acoustic") branch (1) of this equation. The wave numbers k_* that correspond to this branch are real if the wave number k_0 (non dimensional frequency) is out of the narrow strips between vertical dashed lines in Fig. 7. The corresponding waves propagate through the medium without attenuation. Inside these strips (the bands of attenuations) the roots of the dispersion equation (k_*) that correspond to the branches 1 and 2 are complex numbers. (Note that inside the band of attenuation the branch 1 and 2 are coincide and they are deviated only outside this band.) For the branch 1 (and 2) the corresponding attenuation factor is presented in the Fig. 7 by the curve $1'$. Other branches corresponds to some different types of waves that can propagate through this inhomogeneous medium. Note that the bands of attenuation are located in the vicinities of the Bragg's frequencies (wave numbers) that are defined by the equation

$$k_0 = \frac{1}{2}\lambda_0 j, \qquad j = 1, 2, 3, \ldots \tag{52}$$

and $\lambda_0 = 2.609$ for $p = 0.3$.

In order to better understand the input of different branches into the mean wave field let us consider the mean Green function defined in Eq. (13) or the mean wave field from a concentrated source of wave in the considered medium. The general expression for the Fourier transform of the Green function (13) takes the form

$$< \tilde{\mathbf{G}}(\mathbf{k}) >= \tilde{g}_t(k)(\mathbf{1} - \mathbf{e}_1 \otimes \mathbf{e}_1) - \tilde{g}_l(k)\mathbf{e}_1 \otimes \mathbf{e}_1, \tag{53}$$

$$\tilde{g}_t(k) = \frac{k_0^2}{\varepsilon_0 \left[k^2 - k_0^2 - p\bar{\varepsilon}_1 k_0^2 \Gamma_t(k, k_0)\right]}, \quad \tilde{g}_l(k) = \frac{1}{\varepsilon_0 \left[1 + p\bar{\varepsilon}_1 \Gamma_l(k, k_0)\right]},$$

where $\tilde{g}_t(k)$, $\tilde{g}_l(k)$ are transversal and longitudinal parts of the Fourier transform of the mean Green function. After application of the inverse Fourier transform operator and integration over the unit sphere we get for $< \mathbf{G}(x) >$

$$< \mathbf{G}(x) >= \mathbf{G}_t(x) + \mathbf{G}_l(x), \tag{54}$$

$$\begin{aligned}\mathbf{G}_t(x) =& \frac{1}{4\pi^2 ri}\int_{-\infty}^{\infty} g_t(k)e^{ikr}kdk\mathbf{1}\\ &+ \boldsymbol{\nabla}\otimes\boldsymbol{\nabla}\left[\frac{1}{4\pi^2 ri}\int_{-\infty}^{\infty} g_t(k)e^{ikr}\frac{dk}{k}\right],\\ \mathbf{G}_l(x) =& \boldsymbol{\nabla}\otimes\boldsymbol{\nabla}\left[\frac{1}{4\pi^2 ri}\int_{-\infty}^{\infty} g_l(k)e^{ikr}\frac{dk}{k}\right].\end{aligned}$$

For the calculation of these integrals the residual theory may be applied.

Let k_0 be small. In this case the poles of the function $g_t(k)$ are located at point $k = k_* = k_0\sqrt{\varepsilon_*^s}$ and close to the points $k = k_s^{\pm} = \lambda_0 s \pm k_0$, $s = 1, 2, 3, \ldots$, some other poles of the function $g_t(k)$ and poles of $g_l(k)$ are located close to the points

$$\begin{aligned}&k = k_s^l = \lambda_0(s - i\sqrt{j^2 + m^2}), \qquad s, j = 1, 2, 3, \ldots;\\ &m = 0, 1, 2, 3, \ldots, \qquad i = \sqrt{-1}.\end{aligned}$$

As a result the expression for $< \mathbf{G}(x) >$ takes the following form

$$\begin{aligned}< \mathbf{G}(x) >=& \frac{k_*^2}{4\pi r\varepsilon_*^s}e^{-ik_* r} + \nabla\otimes\nabla\left(\frac{1}{4\pi r\varepsilon_*^s}e^{-ik_* r}\right)\\ &+ \frac{k_0^2}{4\pi r\varepsilon_0}\sum_{s=1}^{\infty}\left(R_s^+ e^{-ik_s^+ r} + R_s^- e^{-ik_s^- r}\right)\\ &+ \frac{k_0^2}{4\pi\varepsilon_0}\nabla\otimes\nabla\left(\frac{1}{r}\sum_{s=1}^{\infty}\left(R_s^+ e^{-ik_s^+ r} + R_s^- e^{-ik_s^- r}\right)\right)\\ &+ \ldots\end{aligned} \tag{55}$$

Here $R_s^{\pm}$ are residuals of the functions $g_t(k)$ at points $k_s^{\pm}$. For large s the numbers $R_s^{\pm}$ may be estimated as

$$| R_s^{\pm} | \simeq \frac{9}{2}\frac{p^2\varepsilon_1^2 k_0^2}{\varepsilon_0^2(\lambda_0 s)^5} \mid \cos(\lambda_0 s) \mid .$$

Thus the picture of the mean wave field that propagates from a point source in the medium with a cubic lattice of inclusions has the following structure. The first two terms in Eq. (55) describe the propagation of waves in the homogeneous medium with the effective static properties ε_*^s of the composite (compare with the Eq. (3) for $\mathbf{G}(x)$). This is the main wave that corresponds to the branch 1 in Fig. 7. The other waves that are generated in the medium have wave numbers $k_s^{\pm}$, their amplitudes are proportional to $R_s^{\pm}$ and much less than the amplitude of the main branch 1. For large s the amplitudes of these waves rapidly turn to zero. The terms that are not written in Eq. (55) attenuate exponentially with attenuation factors of order 1 and more. Thus the corresponding waves almost disappear beyond the length L_0 (the distance between inclusions).

Note that this picture of the mean wave field in the composite with a cubic lattice of inclusions was obtained by using the plane wave approximation for the solution of the one particle problem. If the exact solution (16) of this problem is used the results will change. In this case strictly speaking there are no non attenuating waves. The attenuation exists for all the frequencies but in the vicinities of Bragg's frequencies the attenuation factors are in two-there orders higher than in the region out of the attenuation bands (The lines with dots in Fig. 7 correspond to the branch 1 if the solution (16) of the one particle problem is used). It is possible to say that the plane wave approximation is compatible with the effective field method in the case of regular structures. Only in the framework of such an approximation one can get physically correct results: the existence of the non attenuating waves and bands of attenuation in the vicinities of Bragg's frequencies.

Let us study the solutions of the dispersion equation for the Boolean random sets of inclusions. The dispersion equation for transversal waves has the form

$$k_*^2 = k_0^2 + pk_0^2\bar{\varepsilon}_1[\Lambda_t(k_0, k_*)^{-1} + p\bar{\varepsilon}_1 G_t(k_0, k_*)]^{-1}, \qquad (56)$$

where $G_t(k_0, k_*)$ is defined in Eq. (26), $\Lambda_t(k_0, k_*)$ has the form in Eq. (16) if the exact solution of the one particle problem is used and

$$\Lambda_t(k_0, k_*) = \left[1 - \bar{\varepsilon}_1[k_0^2 q(k_0, k_*) + K_1(k_0, k_*)]\right]^{-1}$$

if the plane wave approximation (23) is used. Here $q(k_0, k_*)$ and $K_1(k_0, k_*)$ are defined in Eq. (22).

For the one scale model ($\varepsilon = 5,\ \varepsilon_0 = 1,\ p = 0.2$) strictly speaking one can find three different branches of the solutions of the dispersion equation (see lines with dots in Fig.8). But in the long wave region only the main branch (1) is essential. The branches 2 and 3 have very high attenuation factors ($\gamma \approx 3$) and the corresponding waves disappear on the length of the diameter of inclusions. The attenuation of the wave that corresponds to the branch 3 decreases in the short wave region where this wave should be taken into account.

Let us consider the two scale Boolean model of the random field of inclusions with the parameters

$$p_1 = 0.25, \quad p_2 = 0.8, \quad R_1 = a = 1, \quad R_2 = 5, \quad p = p_1 p_2 = 0.2.$$

The solution of the corresponding dispersion equation has also three branches (see solid lines in Fig. 8). The main branch 1 is similar to the similar branch in the case of the one scale model. But the attenuations of the additional waves that correspond to the branches 2 and 3 are much less then for the one scale model (see the graphs in the right hand side of Fig. 8). It is possible to explain this fact as a result of additional order in the positions of inclusions in the two scale model in comparison with the one scale one.

Let us consider a model correlation function $\Phi(r)$ in the form

$$\Phi(r) = e^{-\rho r} \cos(0.5r). \tag{57}$$

Parameter ρ^{-1} here may be interpreted as a correlation length in the random field of inhomogeneities.. This length decreases as ρ increases. (The construction of random sets corresponding to this correlation function is not known.)

The results of the solution of the dispersion equation for $\rho = 0.1$; 0.3; 0.7 ($\varepsilon = 5, \varepsilon_0 = 1, p = 0.2$) are presented in Fig. 9–11. In all these cases one finds three branches of the solutions of the dispersion equation. The main branch 1 in the long wave region does not depend essentially on the parameter ρ. But the value of the attenuation factor of this branch in the short wave region as well as attenuation factors and positions of the branches 2 and 3 in all considered region of frequencies strongly depend on the value of ρ. The attenuation of the additional waves increases as parameter ρ increases, which corresponds to less ordered distributions of inclusions.

All the calculations in this work were done with the help of the “Mathematica” package [17].

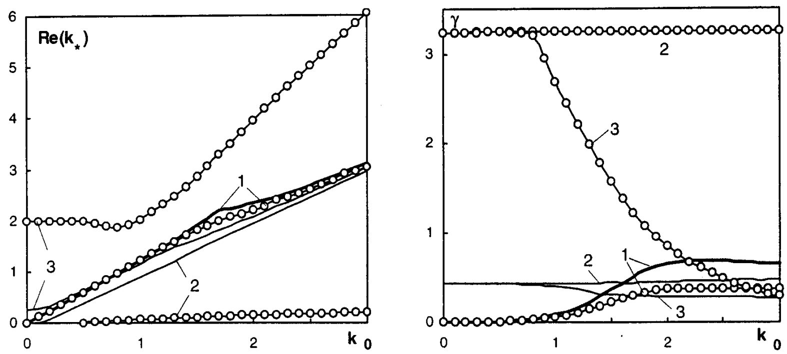

Figure 8. The dispersion curves and attenuation factors for the one scale (lines with dots) and two scale (solid lines) Boolean random sets of inclusions ($\varepsilon_0 = 1;\ \varepsilon = 5,\ p = 0.2$).

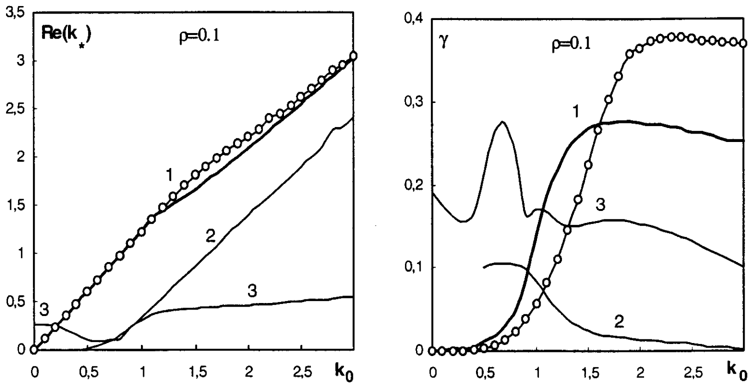

Figure 9. The disperse curves and attenuation factors for the mean wave field in the composite with the model correlation function (57) and $\rho = 0.1$ ($\varepsilon_0 = 1;\ \varepsilon = 5,\ p = 0.2$). Lines with dots are one scale Boolean model for $p = 0.2$.

7. CONCLUSION

The version of the effective field method developed in [1] and in this work allows us to describe the influence of the peculiarities of spatial distributions of inclusions in matrix composites on the effective dynamic properties of the latter. For composites with regular lattices of inclusions the method gives the right symmetry (anisotropy) of the

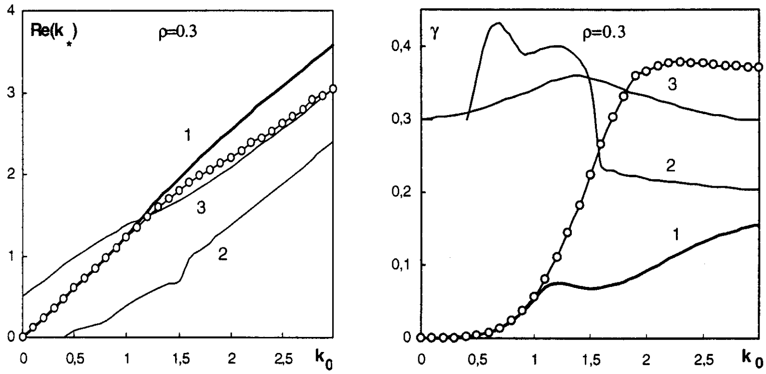

Figure 10. The same graphs as in Fig. 9 for $\rho = 0.3$.

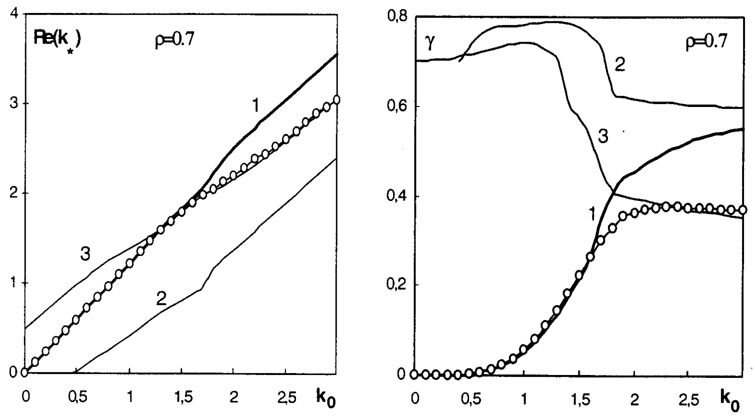

Figure 11. The same graphs as in Fig. 9, 10 for $\rho = 0.7$.

effective properties in statics as well as in dynamics. The method predicts the existence of the bands of attenuations for regular composites in the vicinities of Bragg's frequencies.

For composites with homogeneous and isotropic distributions of inhomogeneities the method predicts the existence of a main (acoustic) branch and two additional branches of the solution of the dispersion equation for transversal electromagnetic waves. The attenuations of waves that correspond to the additional branches are higher than the attenuation of the waves corresponding to the main branch in the long wave region. But the value of the attenuation factors of the additional branches decreases as the correlation radius of the random field of inhomogeneities increases. This results allow us to expect that if the order in the position of inclusions in space increases, there appear new branches of the solution of the dispersion equation or new types of waves that may propagate through the system. For homogeneous and isotropic random fields of inclusions all the waves attenuate but the corresponding attenuation factors decrease as the order in the system increases. In the limit when one goes to a regular lattice of inclusions the attenuation takes place only inside narrow bands of frequencies. In the short wave limit, the method gives the absence of attenuation of waves in regular composites. This result seems to be physically correct for the following reasons. In this limit the wave field may be considered as a set of independent direct beams. Because of the existence of direct lines in the matrix that do not intersect inclusions that compose a regular lattice, these beams may propagate through the medium without attenuation and their velocity coincides with the velocity of waves in the matrix.

Note that for regular structures the method gives physically correct results only if the plane wave approximation of the solution of the one particle problem is used.

The important question is the estimation of the precision of the method. In statics the results of the method are close to the known exact solutions of the problem and to experimental data if the volume concentration of inclusions p does not exceed the value 0.4. In dynamics the area of application of the method has not been investigated properly yet. The comparison with available experimental data that are usually obtained for composites with small volume concentrations of inclusions shows a good agreement with the predictions of the method [1]. But the lack of such data in a wide region of properties of inclusions, their volume concentrations and frequencies of the exciting fields does not allow to estimate precisely the limits of the application of the method.

The main source of the possible errors of the method is the assumption that the field that acts on every inclusion in composite is a plane wave that is the same for all inclusions. For periodic composites this field is definitely not a plane wave and the method cannot describe the fine structure of the dispersion curves in the region of middle and short wave lengths. In particular the structure of the band-gaps in this region of frequencies may hardly be investigated by the method. But in the region of rather long waves that correspond to the first Bragg frequencies one may expect good agreement with the exact solutions of the problem.

The comparison with the exact solutions for the periodic composite may help us to estimate the area of application of the method. But the method is more useful in application to random composites when exact solutions are not available. The comparison with experimental data in statics shows that the method needs corrections for composites with very contrast components if the volume concentration of inclusions exceeds 0.4. The version of the method developed in this work is a simple one. The method may be modified in order to take into account more precisely the interaction between inclusions. In statics of elastic composites such a more complex version of the method was developed in ([15], Chapt. 7). This modification may widen the limits of application of the method. But the price to pay for such widening is due to technical difficulties of the solution of the one particle problem.

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