Progress In Electromagnetics Research, *PIER 24, 97–118, 1999*

MULTI-DIMENSIONAL GENERALIZATION IN SPACE AND TIME DOMAINS FOR MIDDLETON'S STUDY IN STOCHASTIC EVALUATION OF CORRELATIVE MANY EM NOISE PROCESSES

M. Ohta

Honorary Professor
Hiroshima University
1106, 1-7-10 Matoba-Cho, Minami-Ku
Hiroshima City, 732-0824, Japan

Y. Mitani

Faculty of Engineering
Fukuyama University
Sanzo, 1 Gakuen-Cho
Fukuyama City, 729-0292, Japan

H. Ogawa

Hiroshima National College of Maritime Technology
4272-1 Higashino-Cho, Toyota-Gun
Hiroshima Prefecture, 725-0200, Japan

1. Introduction
2. A Correlative Series Model of Random Walks in Multi-Dimensional Signal Space for EM Interference
3. Derivation of Hankel Transform Type Joint Characteristic Function in a Multi-Dimensional Signal Space
4. Explicit Expressions of Joint PDF for Correlative Series of EM Fluctuation Wave
5. Experimental Consideration
5.1 Acoustic Type Simulation Experiment
5.1 Actual Application to EM Environment

6. Conclusion
References

1. INTRODUCTION

It is well-known that there are various kinds of random fluctuations of non-Gaussian type, in the field of man-made and natural EM interferences, due to diversified space and/or time causes of the fluctuation. Accordingly, it is essential to find some unified statistical analysis method for general EM environment which can evaluate and predict EM interference caused by these non-Gaussian type noises. Nowadays, many studies on such a man-made and natural noise can be found from various viewpoints (e.g., see [1, 2]). In particular, D. Middleton reported systematically series of valuable fundamental researches (e.g., [3]) which should still now be referred to. However, in his theory, the basic assumptions postulated mutually independent type Poisson distributions of source location in space and emissions in time. Furthermore, since a narrow band receiver with a mathematically defined envelope of sinusoidal type has been employed, D. Middleton's stochastic envelope model is established only in a two-dimensional space.

However, he intuitively indicated, "No restriction on the specific character of the statistics of the source parameters are as yet introduced." It seems natural that he got some restricted result, because he started his research based on the physical structure or the physical specialty (e.g., Poisson distribution, the statistical independency and the envelope definition of sinusoidal type), only from a forward way of view point in one way communication from source side to detector side.

As we reported previously [4], the essential problem was focused on how to analyze systematically an arbitrary EM fluctuation noise with a finite arbitrary frequency bandwidth through the receiver with mean square operation of analog memory type, especially not from the viewpoint of 'forward direction' (i.e., from source to detector) but from the viewpoint of 'backward direction' (i.e., from detector to source). This is because of the fact that every type EM interferences are in reality observed at the final stage of detection. So, the corresponding mathematical framework of unified analysis had to be first established, by the detector characteristic without losing the generality of stochastic EM input, based on the above forward way of viewpoint. It is noteworthy that the functional elements to dominantly generalize the above mathematical framework of pdf form were a non-linear part and a frequency bandwidth of detector input. More concretely, in our previous paper, some model of statistical interference fluctuating arbitrarily in a time domain was theoretically constructed (in close connection with D. Middleton's great research) by considering a random walk problem in an N-dimensional signal space related to Shannon's sampling theorem in a time domain. Specifically, a Hankel transform type characteristic function of arbitrary order was newly introduced, because it is suitable for consideration on the probability problem of N-dimensional random walks in the analysis of stochastic EM environment. Finally, the unified expression of probability distribution was explicitly derived for the EM interference fluctuation of a power scaled variable (or an effective value) measured by a receiver with mean square operation, and its validity is partly verified by showing the agreement with D. Middleton's canonical formula of lower order Hankel transform type as a special case. However, as a whole study style, the generality of analysis in the previous paper was introduced as how to meet the arbitrariness of input EM interference fluctuation in a time domain.

In this paper, for the purpose of finding a more unified research method on the stochastic evaluation of EM environment, the arbitrariness of correlative random fluctuations at many observation points in a space domain is taken into consideration in addition to the arbitrariness of random fluctuation in a time domain as reported in the previous paper. More concretely, a multivariate Hankel transform type joint characteristic function of arbitrary order is first introduced, because it is suitable for consideration in the stochastic analysis of correlative EM fluctuation waves observed at many measuring points, once after newly establishing the joint probability model for many series of correlative multi-dimensional random walks in a space domain. Hereupon, each time constant of mean squaring type detectors and each arbitrary frequency bandwidth of EM input waves are reflected in the dimension number of corresponding signal space. Furthermore, the space location and time emission of many EM interference sources are reflected in the form of stochastic property only in a time domain as the probability distribution parameters.

Finally, the validity and effectiveness of the proposed theory are experimentally confirmed through first an acoustic type simulation experiment (taken as the same wave motion type environment as EM interference) and then an actual application to an EM environment

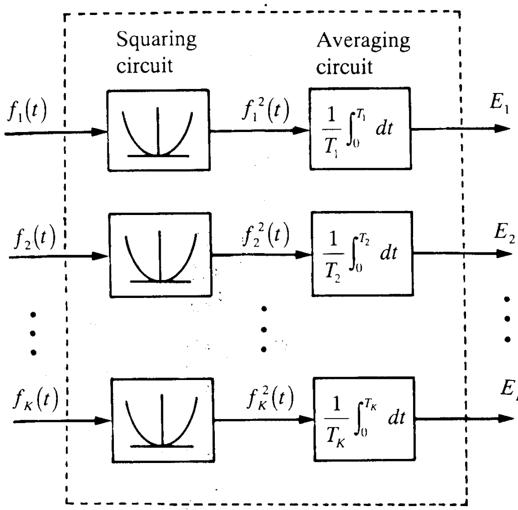


Figure 1. A functional sketch of K kinds of EM interference receivers.

leaked from a VDT of a personal computer and a television in the actual working situation.

2. A CORRELATIVE SERIES MODEL OF RANDOM WALKS IN MULTI-DIMENSIONAL SIGNAL SPACE FOR EM INTERFERENCE

Let us first consider a more generalized case with correlative many EM noises or interferences environment than that in the previous study. That is, we consider a general case when K kinds of EM noise waves $f_h(t)$ $(h = 1, 2, \cdots, K)$ with each arbitrary frequency bandwidth W_h are observed on a power scale, after passing through K kinds of square law detectors with each averaging time T_h (see Fig. 1).

Here, the above incident EM noise wave $f_h(t)$ can be expressed especially based on the time sampling, following to the Shannon's sampling theorem [5] in an information theory as follows:

$$f_h(t) = \sum_{i=0}^{2T_hW_h} f_i\left(\frac{i}{2W_h}\right)\frac{\sin(2\pi W_h t - i\pi)}{2\pi W_h t - i\pi} \quad (h = 1, 2, \cdots, K). \quad (1)$$

Accordingly, the observed power fluctuation after passing through each

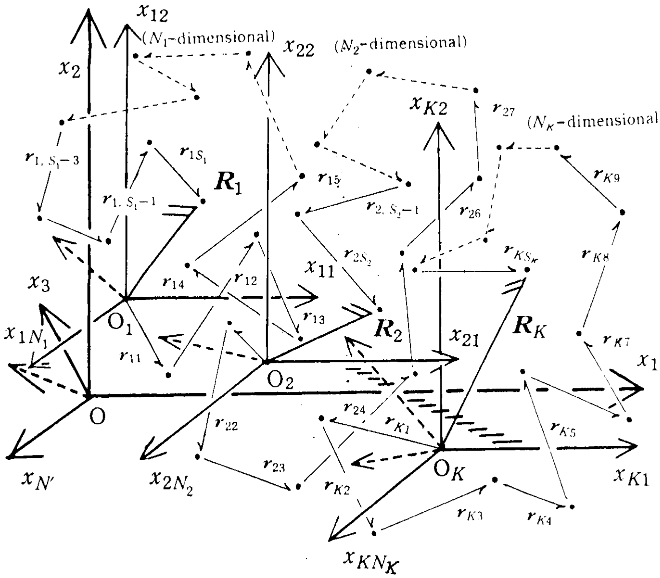


Figure 2. K series of random walks in N-dimensional signal space ($\max[N_1, N_2, \cdots, N_K] \leq N \leq N_1 + N_2 + \cdots + N_K$).

square law detector can be explicitly expressed as follows:

$$\frac{1}{T_h}\int_0^{T_h} f_h^2(t)\,dt\Big(=E_h\Big) = \sum_{i=0}^{2T_hW_h}\left(\frac{1}{\sqrt{2T_hW_h}}f\left(\frac{i}{2W_h}\right)\right)^2 = \sum_{i=0}^{2T_hW_h} X_{hi}^2 \tag{2}$$

with $X_{hi} = \frac{1}{\sqrt{2T_hW_h}}f\left(\frac{i}{2W_h}\right)$.

In the problem of multi-dimensional random walks, as shown in Fig. 2, from the fundamental relationship $\mathtt{r}_{hj} = \sum_{i=1}^{N_h} \mathtt{e}_i x_{hji}$ (where $\mathtt{e}_i$ is a unit vector of x_i axis in the N_h-dimensional vector space), a composition of S_h random vectors:

$$\mathtt{R}_h = \sum_{j=1}^{S_h} \mathtt{r}_{hj} \tag{3}$$

can be reduced to a spherical sum in an N-dimensional vector space with $N_1 = N_2 = \cdots = N_K (\equiv N)$ as:

$$R_h^2 = \sum_{i=1}^{N} x_{hi}^2 \quad (\text{with } x_{hi} = \sum_{j=1}^{S_h} x_{hji}). \tag{4}$$

Therefore, in a special case when all of T_h and W_h have equal values of T and W, (2) corresponds to (4) with $N = 2TW + 1$ (for the large number of T and/or W, $N \cong 2TW$). In (1), since the randomness of $f_h(t)$ is reflected in each $f(i/2W_h)$ of time sampling, our present problem can be also recognized as a problem to derive the probability distribution of the composition R_h ($\equiv |\mathtt{R}_h|$, or $E_h \equiv R_h^2$: non-negative random variable) by use of the statistical property of each x_{hi} (or $f(i/2W_h)$), in the Shannon's multi-dimensional signal space.

3. DERIVATION OF HANKEL TRANSFORM TYPE JOINT CHARACTERISTIC FUNCTION IN A MULTI-DIMENSIONAL SIGNAL SPACE

The present problem is: what is the postern for the probability calculation in an N_h-dimensional vector space?

[I] First, with no use of detailed information on the internal mechanism of (2) and/or (4) in each N_h-dimensional signal space, let us introduce a joint characteristic function $F(\lambda_1, \lambda_2, \cdots, \lambda_K)$ in the form of Hankel transform of arbitrary order applicable to respective probability problem of K correlative physical quantities R_h's $(h = 1, 2, \cdots, K)$ fluctuating only in a non-negative region, and derive the joint pdf $P(R_1, R_2, \cdots R_k)$.

In general, the joint K variates cumulative distribution function (abbr., cdf):

$$\begin{aligned} &Q(R_1, R_2, \cdots, R_K) \\ &\quad = \int_0^{R_1}\int_0^{R_2}\cdots\int_0^{R_K} P(R_1, R_2, \cdots, R_K)\ dR_1 dR_2 \cdots dR_K \end{aligned} \tag{5}$$

can be expressed by

$$\begin{aligned} &Q(R_{10}, R_{20}, \cdots, R_{K0}) \\ &\quad = \int_0^{\infty}\int_0^{\infty}\cdots\int_0^{\infty} P(R_1, R_2, \cdots, R_K) \prod_{h=1}^{K} D_h(R_h)\ dR_h, \end{aligned} \tag{6}$$

$$D_h(R_h) = \begin{cases} 1 & (R_h < R_{h0}), \\ 0 & (R_h > R_{h0}), \end{cases} \tag{7}$$

By using, instead of $D_h(R_h)$, the discontinuous integral due to Weber-Schafheitlin [6]:

$$R_{h0}^{m_h} \int_0^\infty J_{m_h}(R_{h0}\lambda_h) \frac{J_{m_h-1}(R_h\lambda_h)}{R_{h0}^{m_h-1}}\ d\lambda_h = \begin{cases} 1 & (R_h < R_{h0}), \\ 0 & (R_h > R_{h0}), \end{cases} \tag{8}$$

we can directly obtain:

$$\begin{aligned} Q(R_1, R_2, \cdots, R_K) &= \left(\frac{1}{2}\right)^{\sum^K m_h - K} \prod_{h=1}^K \frac{R_h^{m_h}}{\Gamma(m_h)} \\ &\times \int_0^\infty \int_0^\infty \cdots \int_0^\infty \left\{ \prod_{h=1}^K \lambda_h^{m_h-1} J_{m_h}(\lambda_h R_h) \right\} \\ &\qquad F(\lambda_1, \lambda_2, \cdots, \lambda_K)\ d\lambda_1 d\lambda_2 \cdots d\lambda_K, \end{aligned} \tag{9}$$

$$\begin{aligned} P(R_1, R_2, \cdots, R_K) &= \left(\frac{1}{2}\right)^{\sum^K m_h - K} \prod_{h=1}^K \frac{R_h^{m_h}}{\Gamma(m_h)} \\ &\times \int_0^\infty \int_0^\infty \cdots \int_0^\infty \left\{ \prod_{h=1}^K \lambda_h^{m_h} J_{m_h-1}(\lambda_h R_h) \right\} \\ &\qquad F(\lambda_1, \lambda_2, \cdots, \lambda_K)\ d\lambda_1 d\lambda_2 \cdots d\lambda_K, \end{aligned} \tag{10}$$

with

$$F(\lambda_1, \lambda_2, \cdots, \lambda_K) = \left\langle \prod_{h=1}^K 2^{m_h-1} \Gamma(m_h) \frac{J_{m_h-1}(\lambda_h R_h)}{(\lambda_h R_h)^{m_h-1}} \right\rangle \tag{11}$$

and $m_h \geq 1/2 (h = 1, 2, \cdots, K)$ or more explicitly:

$$\begin{aligned} Q(R_1, R_2, \cdots, R_K) = \left(\prod_{h=1}^K R_h^{m_h} \right) \int_0^\infty \int_0^\infty \cdots \int_0^\infty \left\{ \prod_{h=1}^K J_{m_h}(\lambda_h R_h) \right\} \\ \left\langle \prod_{h=1}^K R_h^{1-m_h} J_{m_h-1}(\lambda_h R_h) \right\rangle\ d\lambda_1 d\lambda_2 \cdots d\lambda_K, \end{aligned} \tag{12}$$

$$P(R_1, R_2, \cdots, R_K) = \left(\prod_{h=1}^{K} R_h^{m_h}\right) \int_0^\infty \int_0^\infty \cdots \int_0^\infty \left\{ \prod_{h=1}^{K} \lambda_h J_{m_h-1}(\lambda_h R_h) \right\} \left\langle \prod_{h=1}^{K} R_h^{1-m_h} J_{m_h-1}(\lambda_h R_h) \right\rangle d\lambda_1 d\lambda_2 \cdots d\lambda_K. \tag{13}$$

However, $F(\lambda_1, \lambda_2, \cdots, \lambda_K) = \prod_{h=1}^{K} F(\lambda_h)$ when $P(R_1, R_2, \cdots, R_K) = \prod_{h=1}^{K} P(R_h)$, i.e., $R_1, R_2, \cdots, R_K$ are statistically independent of each other.

[II] On the contrary, especially by use of detailed information on the internal mechanism of (2) and/or (4) in each N_h-dimensional signal space, let us derive a joint characteristic function $F(\lambda_1, \lambda_2, \cdots, \lambda_K)$ in the form of Hankel transform applicable to respective probability problems of K correlative physical quantities R_h 's $(h = 1, 2, \cdots, K)$, and express the joint pdf $P(R_1, R_2, \cdots, R_K)$.

Through the same complicated calculation process based on the transformation to N_h-dimensional polar coordinates as in the previous paper, as the result, we can derive the following expression:

$$F(\lambda_1, \lambda_2, \cdots, \lambda_K) = \frac{1}{\prod_{h=1}^{K} S_{(N_h)}} \int_{S_{(N_1)}} \cdots \int_{S_{(N_2)}} \cdots \int_{S_{(N_K)}} \int \prod_{h=1}^{K} dS_{(N_h)} \times \left[F\Big(\mu_{11,}\mu_{12}, \cdots, \mu_1 N_1; \cdots; \mu_{K_1}, \mu_{K_2}, \cdots, \mu_{K N_K}\Big)\right]\Bigg|_{\left(\mu_{h_1},\mu_{h_2},\cdots,\mu_{h_{N_h}}\right) \to \left(\lambda_h, \varphi_{h_1}, \cdots, \varphi_{h_{N_h-1}}\right)(\forall h)}, \tag{14}$$

$$F\Big(\mu_{11,}\mu_{12}, \cdots, \mu_{1N_1}; \cdots; \mu_{K_1}, \mu_{K_2}, \cdots, \mu_{K N_K}\Big) = = \left\langle \exp\left(i \left[\sum_{h=1}^{K} \sum_{j=1}^{N_h} \mu_{hj} x_{hj} \right] \right) \right\rangle, \tag{15}$$

where

$$S_{(N_h)} = \frac{(\sqrt{\pi})^{N_h} N_h}{\Gamma\left(\frac{N_h}{2}+1\right)} \quad \text{and} \quad dS_{(N_h)} = \prod_{j=1}^{N_h-1} (\sin\varphi_{hj})^{N_h-1-j} \; d\varphi_{hi}$$
$$(h = 1, 2, \cdots, K) \tag{16}$$

mean respectively a surface area and a surface element of an N_h-dimensional unit hypersphere, and $\left(\mu_{h_1}, \mu_{h_2}, \cdots, \mu_{h_{N_h}}\right) \to \left(\lambda_h, \varphi_{h_1}, \cdots, \varphi_{hN_h-1}\right)$ denotes the transformation to N_h-dimensional polar coordinates. After a further troublesome calculation, we can find that (14) agrees with $F(\lambda_1, \lambda_2, \cdots, \lambda_K)$ in (11), by use of an inversion formula of Hankel transformation, and also $P(R_1, R_2, \cdots, R_K)$ in (10) can be derived from (14) by application of the transformation to N_h-dimensional polar coordinates, $\left(x_{h1}, x_{h2}, \cdots, x_{hN_h}\right) \to \left(R_h, \theta_{h1}, \theta_{h2}, \cdots, \theta_{hN_h-1}\right)$ with $m_h = N_h/2$.

In a special case with $K = 1$, we easily have:

$$P(R) = \frac{R^m}{2^{m-1}\,\Gamma(m)} \int_0^\infty F(\lambda)\lambda^m J_{m-1}(\lambda R)\, d\lambda, \tag{17}$$

$$F(\lambda) = \left\langle 2^{m-1}\,\Gamma(m)\frac{J_{m-1}(\lambda R)}{(\lambda R)^{m-1}} \right\rangle \tag{18}$$
$$= 1 + \sum_{n=1}^{\infty} \frac{(-1)^n\,\Gamma(m)\Omega_n}{2^{2n} n!\,\Gamma(m+n)}\,\lambda^{2n} \quad \left(\Omega_n = \langle R^{2n}\rangle\right),$$

$$F(\lambda) = \frac{1}{S_{(N)}} \int\int_{S_{(N)}} \cdots \tag{19}$$
$$\cdots \int \left[F(\mu_1, \mu_2, \cdots, \mu_N)\right]\Big|_{(\mu_1,\mu_2,\cdots,\mu_N)\to(\lambda,\phi_1,\cdots,\phi_{N-1})} ds_{(N)}$$

with

$$F(\mu_1, \mu_2, \cdots, \mu_N) = \left\langle \exp\left(i \sum_{K=1}^{N} x_K \mu_K\right) \right\rangle . \tag{20}$$

Here, the above expressions agree completely with the result reported in the previous study for a simplified special case with $h = 1$, $N_1 = N$ and $m_1 = N/2$.

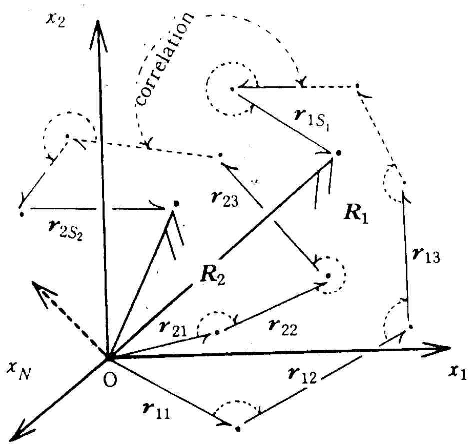


Figure 3. Two correlative series of random walks in the same N-dimensional signal space.

4. EXPLICIT EXPRESSIONS OF JOINT PDF FOR CORRELATIVE SERIES OF EM FLUCTUATION WAVE

Let use consider a special case with very large value of S_h in two correlative series of such random walks: $\mathbf{R}_h = \sum_{j=1}^{S_h} \mathbf{r}_{hj}$ $(h = 1, 2)$ in the same N-dimensional signal space as shown in Fig. 3. As a result, under the particular conditions: $\Omega'_{h0} = \frac{1}{m}\Omega_{h0} = \frac{2}{N}\sum_{j=1}^{S_h}\left\langle \mathbf{r}_{hj}^2 \right\rangle$ $(h = 1, 2)$ and $N_1 = N_2 = N = 2m$, we can asymptotically derive an explicit expression of joint pdf as follows:

$$P(R_1, R_2) = \frac{4(R_1 R_2)^{N/2}}{\Gamma\left(\frac{N}{2}\right)\Omega'_{10}\Omega'_{20}(1-\rho_E)} \left\{\frac{1}{\Omega'_{10}\Omega'_{20}\rho_E}\right\}^{(N/2)-1} e^{-\frac{1}{1-\rho_E}\left\{\frac{R_1^2}{\Omega'_{10}}+\frac{R_2^2}{\Omega'_{20}}\right\}} I_{(N/2)-1}\left(\frac{2\sqrt{\rho_E}\, R_1 R_2}{(1-\rho_E)\sqrt{\Omega'_{10}\Omega'_{20}}}\right) \tag{21-A}$$

$$= P(R_1)P(R_2)\left\{1 + \sum_{n=1}^{\infty} \rho_E n B\left(\frac{N}{2}, n\right) L_n^{(N/2)-1}\left(\frac{R_1^2}{\Omega'_{10}}\right) L_n^{(N/2)-1}\left(\frac{R_2^2}{\Omega'_{20}}\right)\right\} \quad (R_1, R_2 > 0),$$

$$= 0 \quad (R_1, R_2 \leq 0) \tag{21-B}$$

with a Beta function: $B(m,n) = \Gamma(m)\Gamma(n)/\Gamma(m+n)$ and

$$F(\lambda_1,\lambda_2) = \frac{\Gamma\left(\frac{N}{2}\right) 2^{N-2}(\lambda_1\lambda_2)^{1-N/2}}{\left(\sqrt{\rho_E \Omega'_{10}\Omega'_{20}}\right)^{(N/2)-1}} \; e^{-\frac{1}{4}\left[\Omega'_{10}\lambda_1^2 + \Omega'_{20}\lambda_2^2\right]}$$
$$I_{(N/2)-1}\left(\frac{1}{2}\sqrt{\rho_E \Omega'_{10}\Omega'_{20}}\;\lambda_1\lambda_2\right), \tag{22}$$

after employing the same troublesome calculation process based on the well-known saddle point method as in the previous paper. Here, $P(\bullet)$'s express respectively:

$$P(R_h) = \frac{2\left(\frac{N}{2}\right)^{N/2}}{\Gamma\left(\frac{N}{2}\right)\Omega_{h0}^{N/2}} \; R_h^{N-1} e^{-(NR_h^2/2\Omega_{h0})} \quad (h = 1, 2), \tag{23}$$

$$P(E_h) = \frac{1}{\Gamma\left(\frac{N}{2}\right)\Omega_{h0}^{N/2}} \; E_h^{N/2-1} e^{-(NE_h/2\Omega_{h0})} \quad (E_h = R_h^2) \tag{24}$$

and ρ_E denotes the correlation coefficient between R_1^2 and R_2^2. In the above derivation, we have used the following integrals [6]:

$$\int_0^\infty \lambda_1 e^{-(\Omega/4)\lambda_1^2} I_{m-1}\left(\frac{C}{2}\lambda_1\lambda_2\right) J_{m-1}(\lambda R_1)\; d\lambda_1$$
$$= \frac{2}{\Omega}\exp\left\{\frac{1}{\Omega}\left[\left(\frac{C}{2}\lambda_2\right)^2 - R_1^2\right]\right\} \; J_{m-1}\left(\frac{C}{\Omega}\lambda_2 R_1\right) \tag{25}$$

and

$$\int_0^\infty x e^{-Ax^2} J_\nu(Bx) J_\nu(Rx)\; dx$$
$$= \frac{1}{2A}\exp\left\{-\frac{1}{4A}(B^2 + R^2)\right\} \; I_\nu\left(\frac{1}{2A}BR\right). \tag{26}$$

Thus, we can directly obtain the joint pdf $P(E_1, E_1)$ of two series of detector output fluctuations on a power scale, as follows:

$$P(E_1,E_2) = \frac{1}{\Gamma(m) S_1 S_2 (1-\rho_E)} \left(\sqrt{\frac{E_1 E_2}{S_1 S_2 \rho_E}}\right)^{m-1}$$

$$\exp\left\{-\frac{1}{1-\rho_E}\left(\frac{E_1}{S_1}+\frac{E_2}{S_2}\right)\right\} I_{m-1}\left(\frac{2}{1-\rho_E}\sqrt{\frac{\rho_E E_1 E_2}{S_1 S_2}}\right) \tag{27-A}$$

$$= P(E_1)P(E_2)\left\{1+\sum_{n=1}^{\infty} n\rho_E^n B(m,n) L_n^{(m-1)}\left(\frac{E_1}{S_1}\right) L_n^{(m-1)}\left(\frac{E_2}{S_2}\right)\right\} \tag{27-B}$$

with

$$P(E_h) = \frac{1}{\Gamma(m)S_h}\, E_h^{m-1} e^{-E_h/S_h} \quad (h=1,2), \tag{27-C}$$

$$\begin{aligned} &E_h = R_h^2, \quad \sigma_{E_h}^2 = \left\langle (E_h - \langle E_h\rangle)^2 \right\rangle, \quad S_h = m/\Omega_{h0} = \Omega_{h0}/\sigma_{E_h}^2, \\ &m = N/2(=TW), \quad \Omega_{h0} = \langle E_h \rangle \qquad (h=1,2). \end{aligned} \tag{28}$$

Thus, we can easily have:

$$\begin{aligned} P(E_1|E_2) = &\frac{1}{S_1(1-\rho_E)}\left(\sqrt{\frac{S_2 E_1}{\rho_E S_1 E_2}}\right)^{m-1} \\ &\exp\left\{-\frac{1}{1-\rho_E}\left(\frac{E_1}{S_1}+\rho_E\frac{E_2}{E_1}\right)\right\} I_{m-1}\left(\frac{2}{1-\rho_E}\sqrt{\frac{\rho_E E_1 E_2}{S_1 S_2}}\right). \end{aligned} \tag{29}$$

In the parameters of this conditional pdf expression, the equivalent bandwidth W of the input noise and the time constant T of receiver (or detector) with mean squaring operation are reflected in m with the spatial correlation ρ_E between two observation points. As a result, regardless of the engineering implications of each parameter, its representation form coincides with the Bessel distribution already found in the other areas of interest.

With a view to study hierarchically the mutual spatial correlation effect in the form of linear, second, third, ..., orders instead of a total viewpoint, the statistical Laguerre series expansion expressions of the Bessel distribution can be found:

$$\begin{aligned} P(E_1|E_2) = &\frac{E_1^{m-1}}{\Gamma(m)S_1^m} e^{-E_1/m} \\ &\left\{1+\sum_{n=1}^{\infty} n\rho_E^n B(m,n) L_n^{(m-1)}(E_2/S_2) L_n^{(m-1)}(E_1/S_1)\right\}. \end{aligned} \tag{30}$$

This expression can also be written in the following form:

$$P(E_1|E_2) = \frac{E_1^{m-1}}{\Gamma(m)S_1^m} e^{-E_1/m} \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} A_{n_1 n_2} L_{n_1}^{(m-1)}\left(\frac{E_1}{S_1}\right) L_{n_2}^{(m-1)}\left(\frac{E_2}{S_2}\right) \tag{31}$$

with

$$A_{n_1 n_2} = \left\langle \prod_{i=1}^{2} \frac{n_i!\,\Gamma(m)}{\Gamma(m+n_i)} L_{n_i}^{(m-1)}\left(\frac{E_i}{S_i}\right)\right\rangle$$

$$= \prod_{i=1}^{2} \frac{n_i!\,\Gamma(m)}{\Gamma(m+n_i)} \int_0^{\infty}\int_0^{\infty} L_{n_1}^{(m-1)}\left(\frac{E_1}{S_1}\right) L_{n_2}^{(m-1)}\left(\frac{E_2}{S_2}\right) P(E_1, E_2)\; dE_1 dE_2 \tag{32-A}$$

$$= n\rho_{E}^{n} B(m,n)\ , \tag{32-B}$$

where some complicated calculation on a modified Bessel function of the 1st order has been employed by using (27-A) as $P(E_1, E_2)$ in (32-A).

Another conditional pdf expression identical to the Bessel distribution in (30) but with a different representation is the following well-known noncentral χ^2 distribution:

$$P(E_1|E_2) = e^{-\varepsilon}\sum_{n=0}^{\infty} \frac{\varepsilon^n}{n!} \frac{E_1^{n+m-1} e^{-E_1/S_1(1-\rho_E)}}{\Gamma(n+m)\left\{S_1(1-\rho_E)\right\}^{n+m}}, \tag{33}$$

where $\varepsilon\,(\equiv \rho_E E_2/S_2(1-\rho_E))$ is the noncentrality parameter. Hence, the convergence of this expansion is extremely good and only an employment of the first few expansion terms are sufficient in practice.

These three types of probability distribution expressions are mathematically equivalent although their representation styles are different. Hence, depending on the research object or engineering requirement, an appropriate one can be chosen among these three type expressions on $P(E_1|E_2)$ – a closed explicit representation, an orthogonal representation without hierarchical redundancy and a non-orthogonal but quickly converging expansion representation. As a specialized limiting case with $\rho_E \to 0$, the above three expressions, (29), (30), and (33), on pdf tend to:

$$P(E_1|E_2) = P(E_1) = \frac{E_1^{m-1}}{\Gamma(m) S_1^m} e^{-E_1/S_1} \ . \tag{34}$$

This implies that if the fluctuations at two observation points are mutually uncorrelated, this ensures independence property of fluctuations, and the corresponding distribution representation is a well-known Gamma pdf at only one observation point. As another specialized limiting case with $\rho_E \to 1$, (two observation points approach), (29), (30), and (33) become

$$P(E_1|E_2) = \delta(E_1 - E_2) \tag{35}$$

and express a delta function type pdf which has a value only when the intensity is equal to the conditioned level.

5. EXPERIMENTAL CONSIDERATION

5.1 Acoustic Type Simulation Experiment

In this paper, owing to the page limitation, we have placed emphasis on a new methodological trial of systematic EM evaluation mainly from the theoretical research viewpoint. Accordingly, in the corresponding experimental consideration, we have made only an abstract of principle statement for partly confirming first the validity and then the actual effectiveness of the proposed theory.

First, for the purpose of partly confirming the validity of (Bessel or related types) three conditional pdfs in (29), (30), and (33) by using a simulation technique, the proposed method is applied to the actual acoustic wave experiment with joint space-time non-Poisson type arbitrary interference process in the same field of wave motion type environment as EM interference, since the usual computer simulation is nothing but an artificial confirmation of consistency of the theory. The white noise of 1/3 octave band at the center frequency 200 Hz has been excited in a reverberation room having many image sources (simulating a joint space-time non-Poisson type arbitrary location of many image sources) as shown in Fig. 4. So, the noise data after a

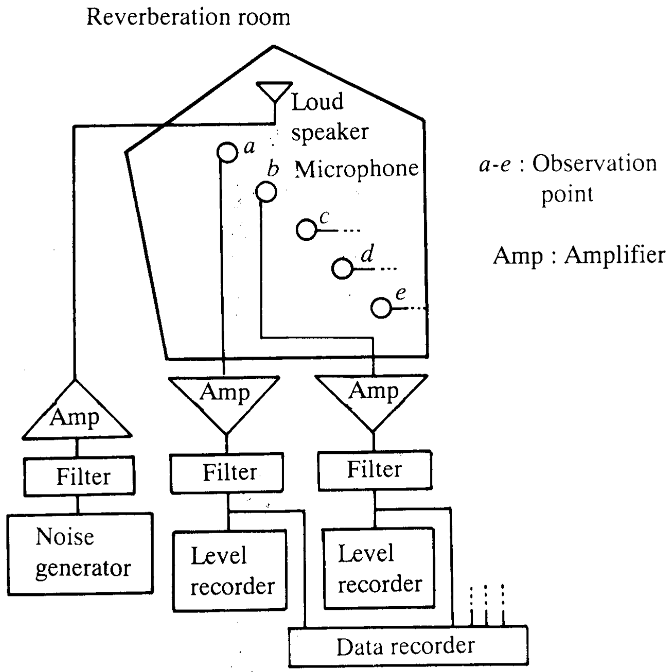
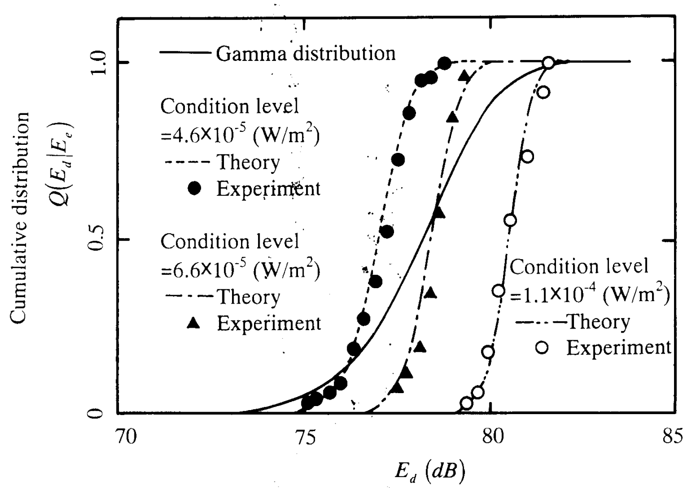


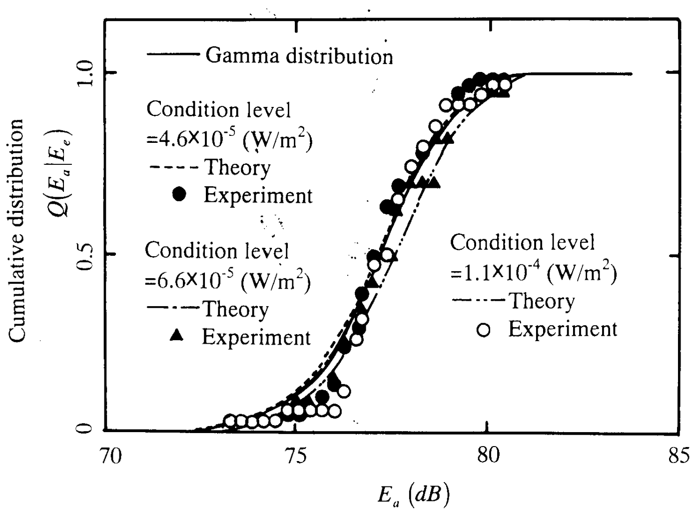
Figure 4. A block diagram of the experimental arrangement in an acoustic reverberation room.

large number of sound reflections with many image sources have been measured by a sound level meter with an averaging time $T = 0.125$ sec of standard type (i.e., Fast). This is quite similar to the measuring situation of EM environment under consideration (notice $T = 6$ minutes or 1 sec in EM interference detector measurement). Then, as a trial, the nominal bandwidth of 1/3 octave band has been employed as an equivalent bandwidth W. The microphones have been placed at a height of 1.2 m (according to Japanese Industrial Standard) from the floor with a separation of 0.2 m each at five locations. The noise level was sampled at every 0.2 sec in consideration of time constant of 0.125 sec of the sound level meter.

The conditioned level has been varied and the experimental results are shown in Fig. 5. Here, the measured E data at five measuring points, a, b, c, d and e are defined respectively as $E_a, E_b, E_c, E_d,$ and E_e. Although a number of conditioned levels are arbitrarily employed, the theoretically estimated cdf $Q(E_1|E_2)$ explains the measured re-



(a)



(b)

Figure 5. A comparison between the experimentally sampled points and the theoretically predicted curves for the conditional cdf of sound intensity fluctuation. Distance between observation: points: (a) 0.2 m and (b) 0.8 m.

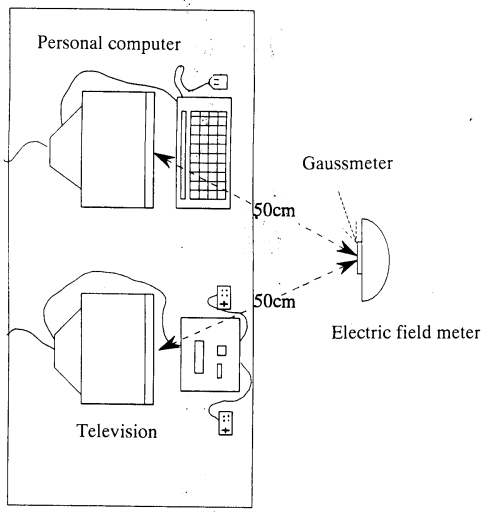


Figure 6. A schematic drawing of the EM experiment.

sults well. In this case, the correlation is strong as the distance between two microphones is very small. Hence, the pdf approaches the delta distribution which has a sharp rise cdf as shown in Fig. 5(a).

Next, to study the effect of the distance between observation points, the experiment for the distances, 0.4, 0.6 and 0.8 m between microphones is employed. As the distance increases, the theoretical probability distribution exhibits a smaller correlation. Hence, the mutual correlation of the fluctuations between two points decreases. In line with the theoretical representation in (34), the results in Fig. 5(b) approach that of the one (i.e., Gamma pdf) measured with only one microphone regardless of the conditioned level. At any rate, it must be noticed that regardless of the distance between microphones, the theoretical curves always explain the measured data.

5.2 Actual Application to EM Environment

Next, for the purpose of partly confirming the actual effectiveness of three conditional pdfs in (29), (30) and (33), the proposed method is applied to an evaluation problem of a probability distribution of electric and magnetic fields leaked from a VDT and a television under the actual situation of playing television games in the room. Figure 6 shows a schematic drawing of this EM experiment carried out in our laboratory. The rms values of electric and magnetic field fluctuation radiated have been measured coincidentally by use of a Holaday's electric field survey meter and a Narda's gaussmeter. The slowly fluctuating 720 data of nonstationary type for each stochastic variable have been sampled with a sampling interval of 5 sec. The measured data up to 500 has been first used for finding the conditional pdf $P(E_1|E_2)$ after regarding respectively the magnetic field on a power scale as E_1 and the electric field on a power scale as E_2. Then, the remaining nonstationary new data of E_2 have been used for predicting the response frequency distribution of $P(E_1)$ by theoretically finding many sample data of E_1 after substituting these E_2 data into the above-learned conditional expression $\langle E_1|E_2\rangle$, as follows:

$$\begin{aligned}\langle E_1|E_2\rangle &\left(= \int_0^\infty E_1 P(E_1|E_2)\, dE_1\right)\\ &= \sum_{m=0}^{\infty}\sum_{n=0}^{1} C_n A_{mn} \sqrt{\frac{m!\Gamma(m_2)}{\Gamma(m_2+m)}}\; L_m^{(m_2-1)}\left(\frac{E_2}{S_2}\right)\end{aligned} \tag{36}$$

with

$$A_{mn} = \left\langle \sqrt{\frac{\Gamma(m_2)m!}{\Gamma(m_2+m)}}\; L_m^{(m_2-1)}\left(\frac{E_2}{S_2}\right) \sqrt{\frac{\Gamma(m_1)n!}{\Gamma(m_1+n)}} L_n^{(m_1-1)}\left(\frac{E_1}{S_1}\right)\right\rangle . \tag{37}$$

In (36), each coefficient C_n (i.e., C_0 and C_1) is in advance given by the orthogonal expansion of arbitrary functions of E_1 based on the associated Laguerre polynomial $L_n^{(m_1-1)}\left(\frac{E_1}{S_1}\right)$, as follows:

$$C_0 = m_1 S_1, \quad C_1 = -\sqrt{m_1 S_1} \tag{38}$$

with $m_i = \mu_i^2/\sigma_{E_i}^2$ and $S_i = \sigma_E^2/\mu_{E_i}$ $(i = 1, 2)$. Here, μ_{E_i} and $\sigma_{E_i}^2$ denote respectively the mean value and the variance with respect to each stochastic variable $E_i (i = 1, 2)$.

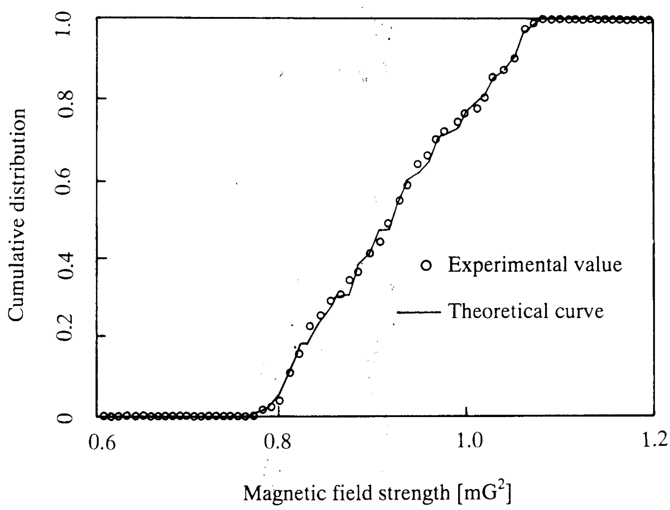


Figure 7. A comparison between the experimentally sampled points and the theoretically predicted curve for the cdf of magnetic field on a power scale.

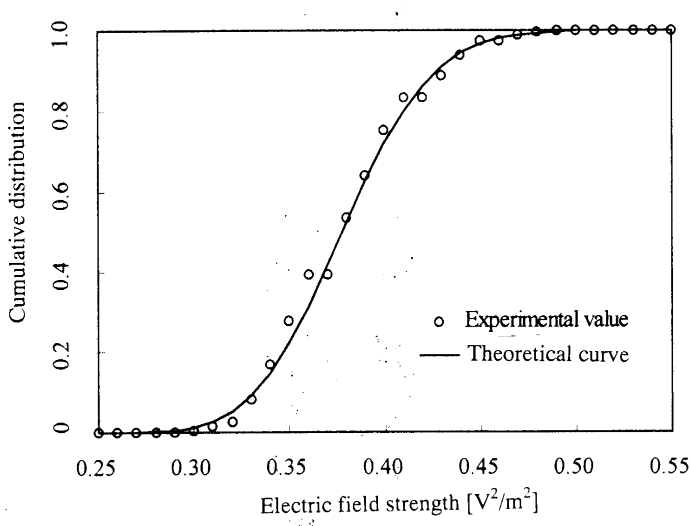


Figure 8. A comparison between the experimentally sampled points and the theoretically predicted curve for the cdf of electric field on a power scale.

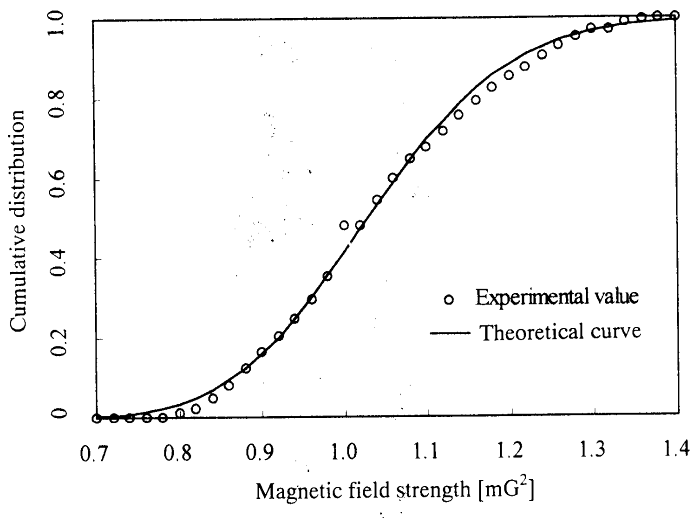


Figure 9. A comparison between the experimentally sampled points and the theoretically predicted curve for the cdf of magnetic field on a power scale.

Figure 7 shows a comparison between the theoretically predicted curve (i.e., the second approximation of the expansion series) by use of the proposed method and the experimentally sampled points for the cdf of magnetic field strength on a power scale. It is obvious that the theoretical curve is in good agreement with the experimental values.

On the other hand, as mentioned in the theoretical consideration, the theoretical result for one dimensional expression agrees completely with the result reported in the previous paper. In this case, the estimated results (i.e., the first term of the expansion series) for the cdfs of electric and magnetic field strengths on a power scale are respectively shown in Figs. 8 and 9. From these figures, it is obvious that the theoretical curves are also in good agreement with the experimental values.

6. CONCLUSION

For the purpose of finding a more unified research method on the stochastic evaluation of EM environment, in this paper, the arbitrariness of correlative random fluctuations at many observation points in a space domain has been taken into consideration in addition to the arbitrariness of random fluctuation in a time domain as reported in the previous paper. First, a multivariate Hankel transform type joint characteristic function of arbitrary order has been first introduced, because it is suitable for consideration in the stochastic analysis of correlative EM fluctuation waves observed at many measuring points, once after newly establishing the join probability model for many series of correlative multi-dimensional random walks. Hereupon, each time constant of mean squaring type detectors and each arbitrary frequency bandwidth of EM input waves have been reflected in the dimension number of corresponding signal space related to Shannon's information theory. Furthermore, the space location and time emission of many EM interference sources have been reflected in the form of stochastic property as the probability distribution parameters.

Finally, the validity and effectiveness of the proposed theory have been experimentally confirmed through first an acoustic type (stimulation) experiment (which is the same wave motion type environment as EM interference) and then an actual application to an EM environment leaked from a VDT of a personal computer and a television in the actual working situation.

This research is obviously at an early stage of development. There still remain many future problems to be solved, such as:

1) applying this unified fundamental theory to many other actual EM environments,
2) finding some simplified expression through the approximation of this theory,
3) finding an appropriate way to determine the optimal order of the proposed series expansion.

ACKNOWLEDGMENT

The authors would like to express their cordial thanks to Prof. K. Hatakeyama, Prof. A. Ikuta, Miss. Y. Sato, Mr. Y. Fujii, Mr. K. Morita and Mr. Y. Kashima for their helpful discussions.

REFERENCES

1. White, D. R. J., *EMC Handbook,* Don White Consultants, 1971.
2. Skomal, E. N., *Man-made Radio Noise,* Van Nostrand Reinhold, 1978.

3. Middleton, D., "Statistical-physical models of electromagnetic interference," *IEEE Trans. Electromagnetic Compatibility,* Vol. EMC-19, 106–127, 1977.
4. Ohta, M., Y. Mitani, and N. Nakasako, "A fundamental study on the statistical evaluation of receivers response for an electromagnetic wave environment in multi-dimensional signal space - theory and basic experiment," *Journal of Electromagnetic Waves and Applications,* Vol. 12, 677–699, 1998.
5. Goldman, S., *Information Theory,* Prentice-Hall, 1953.
6. Yoshida, K., T. Amemiya, K. Ito, S. Matsushita, and S. Furuya, *Handbook of Applied Mathematics,* Maruzen, Tokyo, 1967 (in Japanese).