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FAR-FIELD DECOUPLED BASIS FOR THE METHOD OF MOMENTS-2D CASE

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1. INTRODUCTION

Computational techniques for solving electromagnetic scattering and radiation problems involving large complex bodies have been intensely studied by many researchers. We will focus on the different techniques for solving integral equations using the *method of moments* (MoM) [1]. Integral equation solvers result in dense matrices that would require $O\left(N^3\right)$ operations and $O\left(N^2\right)$ memory storage to obtain a direct solution where N is the number of unknowns; so the computational labor may be excessive and even prohibitive when dealing with large scatters. Most efforts have attempted to reduce this computational cost by different methods. An iterative approach like the *conjugate gradient* (CG) requires $O\left(N^2\right)$ arithmetic operations (corresponding to a matrix-vector multiply) for each step. In [2–5], the *fast multipole method* (FMM) has been proposed as a feasible alternative to

reduce the computational complexity of the matrix-vector multiply to $O\left(N^{3/2}\right)$, calculating the electromagnetic interactions in three different stages: aggregation, translation and disaggregation. Two level schemes can also be performed, to reduced the operation count down to $O\left(N^{4/3}\right)$. An extension of this technique is the *multilevel FMM* [6, 7], which is indeed a nested FMM, reaching an accurate solution in $O\left(N \log N\right)$ each step at the finest level. It is also noteworthy the *ray propagation fast multipole algorithm* (RPFMA) [8], that speeds up the FMM through radiation beam ideas to window the translation stage and scales up the computational complexity as $O\left(N^{4/3}\right)$. An analogous windowing procedure has been performed in [9], where the same computational cost is achieved evaluating asymptotically in the high-frequency limit the plane wave translation operator of the FMM. A two-level approach, based on an integral representation of the Green's functions and termed the *fast steepest descent path algorithm* (FASDPA), was proposed in [10]. The FASDPA performs the translation through basis of plane waves and a spatial distribution of the subscatters into different groups with $O\left(N^{4/3}\right)$ counts.

The methods based on specific basis transformations are also noteworthy. Canning [11, 12] has developed the *impedance matrix location method* (IML), which generates a sparse MoM matrix by constructing a set of basis that provide highly directional beams. Otherwise, in [13], the *singular value decomposition* (SVD) is applied to diagonalize the MoM matrix changing the standard basis. Another approach to get a sparse matrix consists of using wavelet transforms [14–16]. The degree of sparsity reached is closely related to the reduction of the crunching, resulting in an $O\left(N \log N\right)$ matrix-vector multiply. Recently, the *generalized sparse matrix reduction* (GSMR) technique that generates a sparse matrix choosing a set of current distributions over the entire scatter surface, was presented in [17].

Regarding the present work, a new set of basis functions characterized by its decoupled far-field radiated power is built up. We hope the discussion here may offer some insights into the physical interpretation of these set of decoupled basis. These basis are achieved by decomposing the power radiated by standard basis (e.g., pulse functions) via matrix algebra. These new basis are decoupled, so the summation of the power radiated by each basis is just the power due to the scattered field. The reduction of the number of basis (up to approximately 2 basis per wavelength instead of the typical 10 basis per wavelength

used in the conventional MoM formulation) together with the highly sparse system matrix, suppose to decrease, considerably, both the computational cost and the memory requirements without losing accuracy to characterize the scattered field. This paper is restricted to two-dimensional problems (2D); although, the proposed formulation can be straightforward extended to three-dimensional cases.

The present paper is organized as follows: In *Section 2* the radiated power coupling between the different basis is presented in a compact formulation that will be described in detail in *Appendix A* for pulse basis functions. The formulation of the proposed method, based on the *singular value decomposition* (SVD), is given in *Section 3*. Some examples and computations are presented in *Section 4* in order to illustrate the capabilities of the proposed technique. To conclude, *Section 5* summarizes the paper and shows some conclusions achieved.

2. RADIATED POWER COUPLING BETWEEN BASIS FUNCTIONS

The *electric field integral equation* (EFIE) is commonly used to solve electromagnetic scattering problems. Imposing the boundary condition on the surface (S) of a 2D perfectly electric conducting (PEC) scatter, its formulation is given by

$$-\hat{n} \times \frac{\omega\mu}{4} \int_S \mathbf{J}\left(\boldsymbol{\rho}'\right) H_0^{(2)}\left(k\left|\boldsymbol{\rho}-\boldsymbol{\rho}'\right|\right) dl' = -\hat{n} \times \mathbf{E}^i\left(\boldsymbol{\rho}\right) \tag{1}$$

where $\mathbf{E}^i\left(\boldsymbol{\rho}\right)$ is the incident field at a point $\boldsymbol{\rho} \in S$, $\mathbf{J}\left(\boldsymbol{\rho}'\right)$ is the induced current radiating in free space, $H_0^{(2)}$ is the second kind Hankel function with order zero and $\hat{n}$ is an outward unit normal vector to the surface. The total electric field, $\mathbf{E}^t\left(\boldsymbol{\rho}\right)$ can be expressed as the summation of the incident field, $\mathbf{E}^i\left(\boldsymbol{\rho}\right)$, and the scattered field, $\mathbf{E}^s\left(\boldsymbol{\rho}\right)$, which can be expressed as a function of the induced currents as,

$$\mathbf{E}^s\left(\boldsymbol{\rho}\right) = -\frac{\omega\mu}{4} \int_S \mathbf{J}\left(\boldsymbol{\rho}'\right) H_0^{(2)}\left(k\left|\boldsymbol{\rho}-\boldsymbol{\rho}'\right|\right) dl' \tag{2}$$

Discretizing (1), and using pulse basis functions for expanding $\mathbf{J}$ together with a point matching procedure, yields to the MoM matrix formulation,

$$Zx = e \tag{3}$$

where Z is an $N \times N$ impedance matrix, x is an $N \times 1$ vector representing the unknown current distribution, and e is an $N \times 1$ vector describing the incident field.

The power associated to the scattered field can be computed integrating the scattered power density over a close surface around the scatter. The scattered field power can be decomposed as a function of the scattered field due to each basis function. For a $\hat{z}$-polarized wave (TM polarization) this decomposition is given by

$$\begin{aligned} P_{rad} &= \frac{1}{2\eta_0} \int_0^{2\pi} \left(\sum_i E_i^z(\rho,\phi) \right)^* \cdot \left(\sum_j E_j^z(\rho,\phi) \right) \rho d\phi \\ &= \frac{1}{2\eta_0} \int_0^{2\pi} \left(\sum_i x_i f_i(\rho,\phi) \right)^* \left(\sum_j x_j f_j(\rho,\phi) \right) \rho d\phi \\ &= \frac{1}{2\eta_0} \int_0^{2\pi} \left(x^H \cdot f^H(\rho,\phi) \cdot f(\rho,\phi) \cdot x \right) \rho d\phi \\ &= x^H \cdot \left\{ \frac{1}{2\eta_0} \int_0^{2\pi} f^H(\rho,\phi) \cdot f(\rho,\phi) \rho d\phi \right\} \cdot x \\ &= x^H \cdot A \cdot x \end{aligned} \tag{4}$$

where η_0 is the intrinsic impedance, $E_i^z(\rho,\phi)$ represents the z-component of the electric field due to the i-basis, x_i is the coefficient of the i-current constituting the x vector, $f_i(\rho,\phi)$ defines the electric field radiated by the i-basis (constituting the f vector) and A is an $N \times N$ square matrix describing the coupling between primary patterns of the basis functions. The quadratic form (4) permits to manipulate easily the relationship between basis and power through matrix algebra decompositions.

An analogous formulation can be derived for the TE case. A more detailed description, restricted to pulse basis functions, is performed in *Appendix A* for both polarizations (TM and TE).

3. DECOUPLING BASIS FUNCTIONS

The proposed method is based on the use of a set of decoupled basis functions accomplished by the SVD of the radiated power coupling matrix, A, which is clearly related to the real part of the impedance matrix. The characteristics of the resultant matrices $(U,\ \Sigma,\ V)$ make

this particular decomposition method specially suitable to obtain decoupled basis and to simplify the calculation of the MoM solution. The SVD of the coupling matrix is given by

$$A = U\Sigma V^H \tag{5}$$

where $U = (u_1, ..., u_N)$ and $V = (v_1, ..., v_N)$ are matrices with orthonormal columns and $\Sigma = \mathrm{diag}(\sigma_1, ..., \sigma_N)$ is a diagonal matrix whose elements are ordered decreasingly.

The above mentioned matrix, A, is hermitian and definite positive ($x^H Ax > 0$) with real and positive diagonal terms. Taking into account these properties and the uniqueness of the SVD, it can be proved that U equals to V. Defining

$$L = V^H\,, \tag{6}$$

the power coupling matrix can be written as

$$A = L^H \Sigma L\,. \tag{7}$$

Substituting (7) into (4), the following expression is achieved

$$P_{rad} = x^H Ax = x^H L^H \Sigma L x = w^H \Sigma w \tag{8}$$

A set of basis functions (w) are considered to be decoupled if their associated radiated power can be expressed as a diagonal matrix. (The elements outside the diagonal would represent the power interactions between nonself-terms, so a diagonal matrix means decoupling between different basis). Note that the power coupling matrix associated with this equivalent basis is just the diagonal matrix Σ. So it can be concluded that the new set of decoupled basis is given by,

$$w = Lx \tag{9}$$

Considering the orthonormal property of matrix L, the standard MoM basis, x, could be achieved as a function of the new basis, w, as

$$L^{-1} = L^H \Rightarrow x = L^H w = Vw \tag{10}$$

substituting (10) into (3), and following a Galerkin procedure, an equivalent matrix representation is obtained,

$$V^H Z V w = V^H e \tag{11}$$

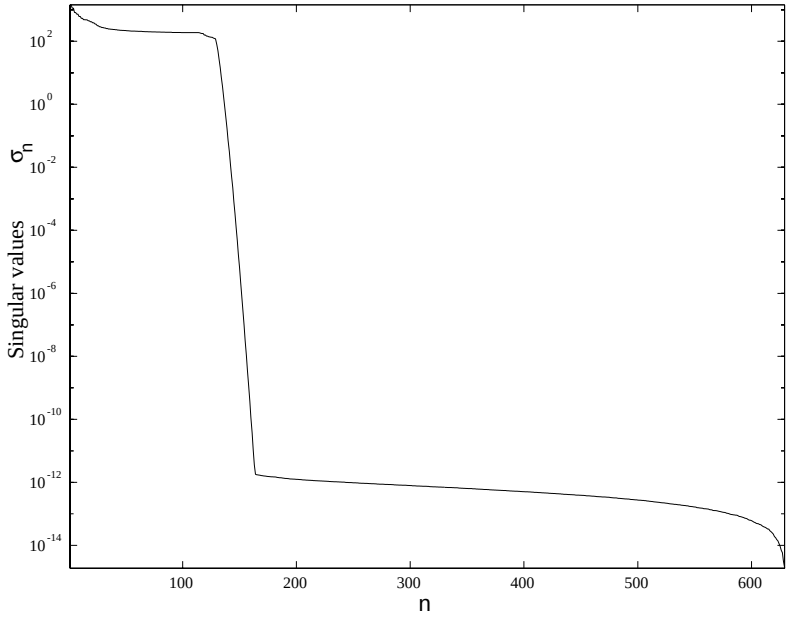

Figure 1. Singular values of the power coupling matrix, A.

It is possible to generate the decoupled-domain matrix elements by direct integration in the standard moment method fashion avoiding matrix-matrix multiplication. If the sparsity structure of the decoupled-matrix may be predicted in advance, it may be predicted in advance, it may be possible to significantly reduce the fill time of the proposed method by coupiling only the significant matrix elements. Regarding the physical interpretation, the columns of the matrix V, v_i, represent the relation between the novel and the old basis, while the i-diagonal element of the matrix Σ is just the power radiated by the new i-basis. It is also noteworthy that these diagonal elements decrease as the index i increases (due to the special characteristics of the SVD); which means that the power radiation of the associated basis vanishes. Therefore, the last basis can be neglected and the solution of the MoM will not change appreciably. In practice, the distribution of singular values in the matrix Σ has an abrupt gap (see *Fig. 1*) which allows to identify the more influential decoupled basis over the scattered field. This leads to a significant reduction in the number of basis functions up to about 2 or 3 basis per wavelength without losing accuracy.

Therefore, the equation solved is a truncated version of (11),

$$V_M^H Z V_M w_M = V_M^H e \tag{12}$$

where the matrix V is substituted by a matrix formed by its own M first columns, V_M $(N \times M)$ and w_M is a vector $(M \times 1)$ containing the considered M first decoupled basis.

Equation (12) can be written in a compact form as

$$Z_{dec} w_M = e_{dec} \tag{13}$$

It can be notice that, using the above procedure, the new matrix Z_{dec} has a quasi-diagonal structure, that is, most elements are concentrated close to its diagonal. Such an structure permits the application of a thresholding procedure whereby the matrix is rendered sparse, in order to simplify the solution of the resulting linear system, practically without degradation in the accuracy.

The reduction in the computational cost is not obvious at this point. The SVD of the full coupling matrix A $(N \times N)$ requires $O\left(N^3\right)$ operations. This cost is comparable to solve the system in a straightforward manner, inverting the impedance matrix Z. In spite of the appropriate physical behavior of this new set of basis (they are decoupled), its computation is highly inefficient. However, for the sake of savings in terms of computational requirements some extra measures can be taken:

- The matrix A has a tendency to concentrate its main values close to its diagonal due to the augmented decoupling between the radiated fields generated by the basis as their relative distance increases. Therefore, the elements faraway from the diagonal are sufficiently reduced to be neglected. It can be experimentally proved that the coupling between basis faraway more than one or two wavelengths can be neglected, so only those interactions from the self-basis and a few neighboring basis must be computed (namely the coupling between their associated radiated fields). This leads to a highly sparse matrix A_s, whose SVD can be evaluated in $O\left(N^2\right)$ counts. Such an approximation does not influence the accuracy of the MoM, though the new basis are, now, slightly matched.

- The singular values are just the radiated power associated to the decoupled basis, therefore only the main singular values (first and higher diagonal terms) of the coupling matrix A will influence considerably the solution, enabling the performance of a partial decomposition (partial SVD, e.g., by means of Lanczos algorithm) instead of the complete SVD and relieving the computational burden.

4. NUMERICAL RESULTS

In this section, several results for both TE and TM polarizations are presented to validate the above described method.

The first example is a perfectly conducting circular cylinder of radius $r = 10\lambda$ illuminated by an incident plane wave. The choice of 10 unknowns per wavelength which is a typical parameter in conventional MoM formulations implies 629 basis to get accurate results for this particular geometry. The SVD of the power coupling matrix is performed in order to obtained the decoupled basis.

The closed form current solution [18] together with the current results achieved, due to a TM polarized incident plane wave, are shown in *Fig. 2*. As can be seen a good agreement is observed with only 134 basis vs. the original 629 basis what implies diminishing up to 2.13 the number of unknowns per wavelength. An analogous result is accomplished for TE polarization (see *Fig. 3*), achieving an exact current with only 2.27 basis per wavelength (143 basis). The normalized error has been defined as

$$Normalized\ error\ in\ current = \frac{\|x - x_M\|}{\|x\|}, \tag{14}$$

where x and x_M are column vectors containing, respectively, the current achieved by the closed form solution and the current due to M decoupled basis. The normalized error in the radiated fields is calculated analogously as,

$$Normalized\ error\ in\ radiation = \frac{\|\mathbf{E} - \mathbf{E}_M\|}{\|\mathbf{E}\|}, \tag{15}$$

being $\mathbf{E}$ the electric field generated analytically and $\mathbf{E}_M$ the electric field due to M decoupled basis. Both errors are depicted in *Fig. 4* for both polarizations (TE and TM), observing a similar behavior. The

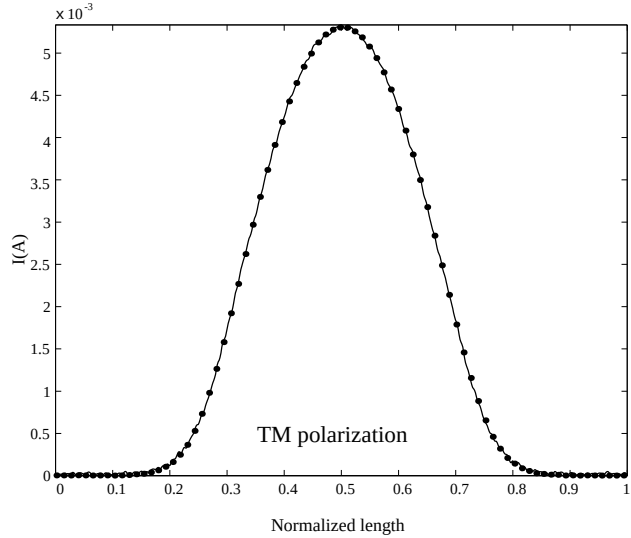

Figure 2. Current over a perfectly conducting cylinder of radius 10λ illuminated by a TM polarized plane wave. Decoupled basis solution in solid line, conventional MoM solution in dots.

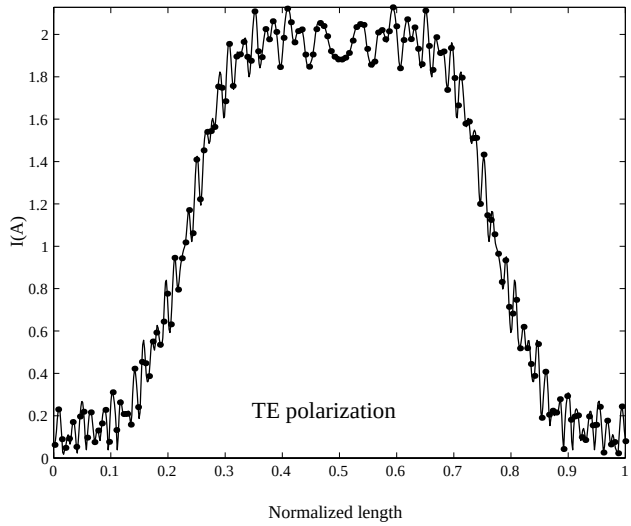

Figure 3. Current over a perfectly conducting cylinder of radius 10λ illuminated by a TE polarized plane wave. Decoupled basis solution in solid line, conventional MoM solution in dots.

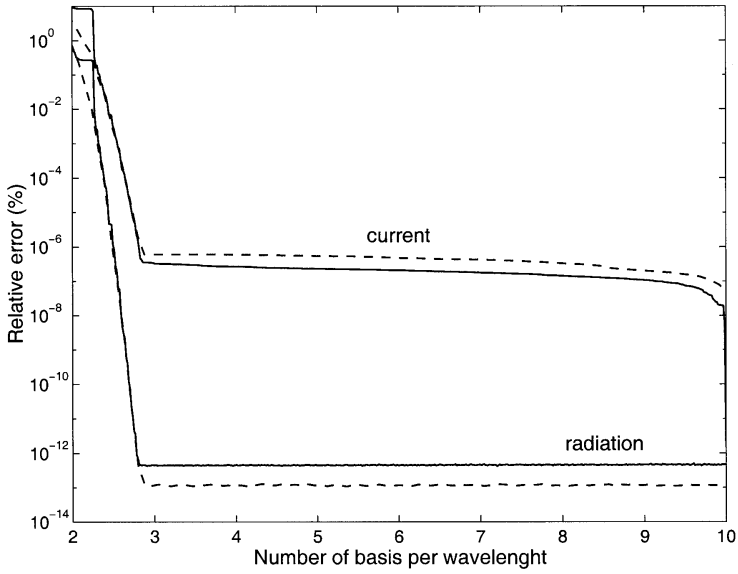

Figure 4. Relative error in current and radiated field vs. the number of basis per wavelength. Solid line: TM polarization. Dashed line: TE polarization.

normalized error in current is always greater than the error in the radiation, however both errors are negligible when the number of basis per wavelength is greater than 2.2. These results can be explained by means of the power associated to the decoupled basis, which is plotted in *Figs. 5–6* (TM and TE polarization, respectively). The power radiated by the last decoupled basis vanishes, concluding that only the basis corresponding to the main singular values influence the result.

In order to prove the ability of the proposed method to deal with more complex geometries, a second example is presented. It consists on a perfectly conducting circular sector ($r = 30\lambda,\ 120^\circ$) in front of a current line. According to the above mentioned MoM formulation, a conventional solution of this electromagnetic problem would involve 629 basis. The special characteristics of the radiated power coupling matrix allows us to apply the decomposition process to an approximate banded matrix (truncated version of the power coupling matrix whose elements are those closest to the diagonal, being the distance limit 1.4λ) without losing accuracy in the solution. *Figs. 7–10* show

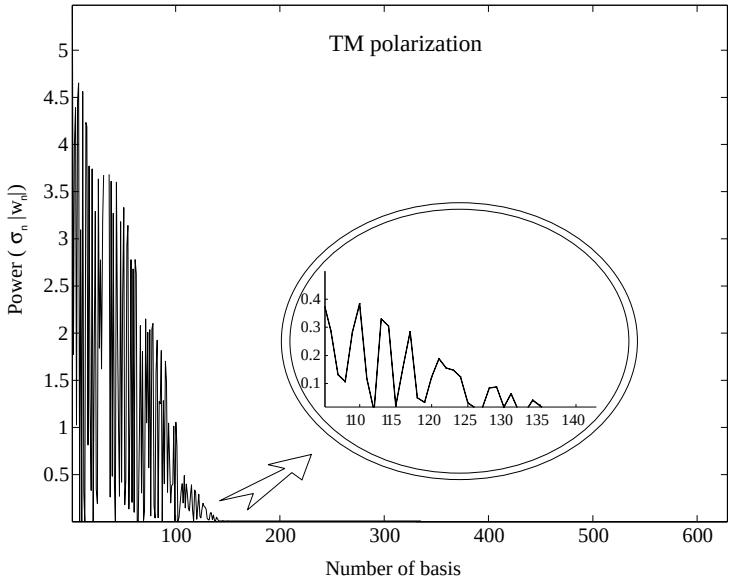

Figure 5. Power associated to the decoupled basis. TM polarization.

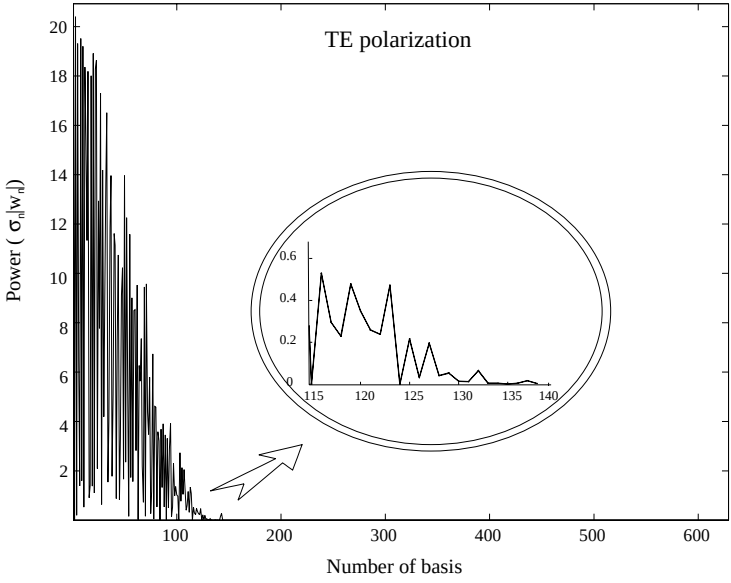

Figure 6. Power associated to the decoupled basis. TE polarization.

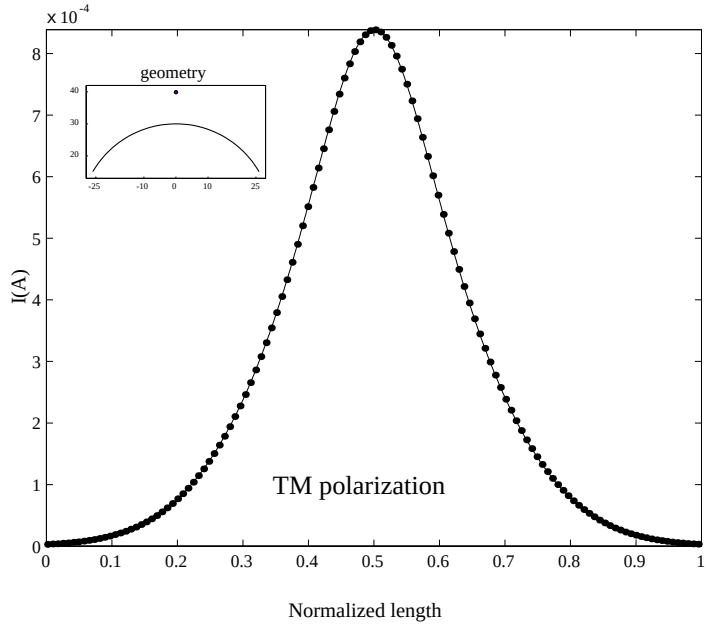

Figure 7. Current over a perfectly conducting geometry illuminated by a TM polarized current line. Power coupling matrix approximated by a banded matrix. Decoupled basis solution in solid line, conventional MoM solution in dots.

the current and normalized radiated field for TM and TE polarizations, compared with the reference MoM solution. A good agreement is achieved for TM polarization with only $n = 142$ basis in all cases, supposing a reduction in the number of basis per wavelength up to 2.25. Even better are the results for the TE polarized radiated field reaching a precise result for $n = 132$ basis (2.1 basis per wavelength). However, it must be pointed out that the error in current stays around 1% up to nearly 9 basis per wavelength due to the oscillating behavior of the current noticed on the borders.

The percentage of elements below a given threshold is depicted in *Fig. 11* together with the normalized error achieved in the currents, when this threshold is applied to render some elements of new MoM matrix to zero. Analyzing both graphics we can establish a suitable threshold and the resulting MoM matrix will be rendered sparse, enabling the numerical solution. The same behavior is observed for TE polarization as can be seen in *Fig. 12* providing a significant reduction

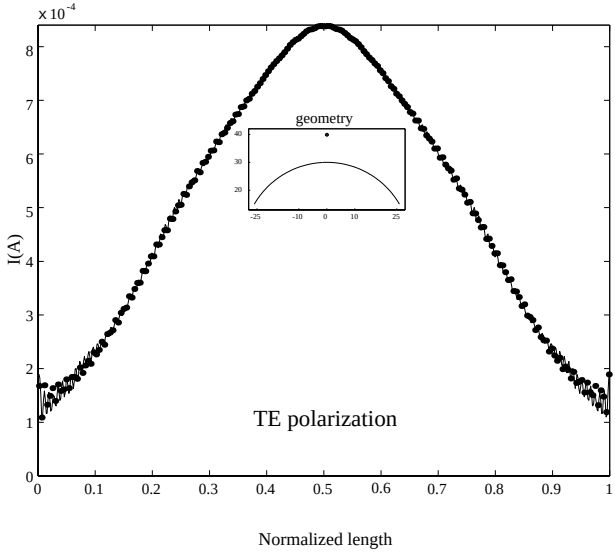

Figure 8. Normalized radiated field due to a perfectly conducting geometry illuminated by a TM polarized current line. Power coupling matrix approximated by a banded matrix. Decoupled basis solution in solid line, conventional MoM solution in dots.

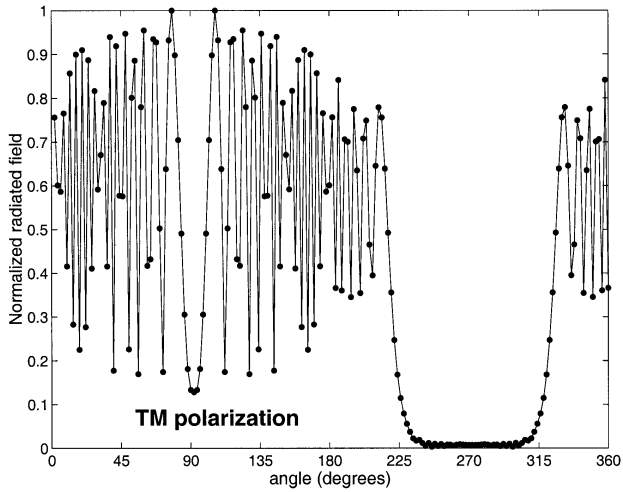

Figure 9. Current over a perfectly conducting geometry illuminated by a TE polarized current line. Power coupling matrix approximated by a banded matrix. Decoupled basis solution in solid line, conventional MoM solution in dots.

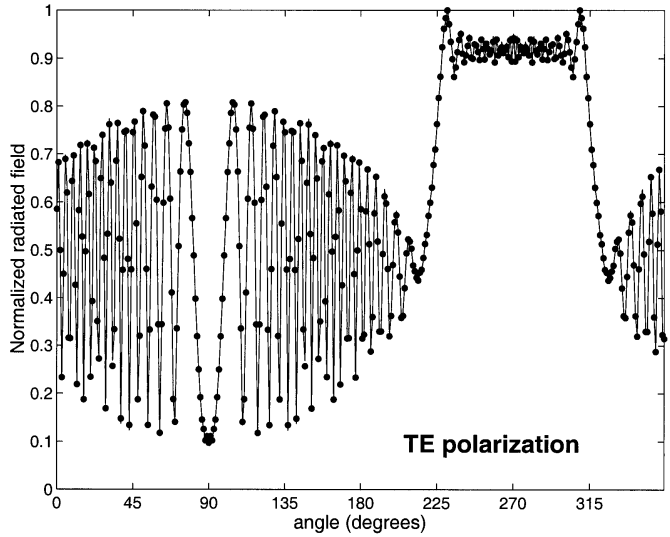

Figure 10. Normalized radiated field due to a perfectly conducting geometry illuminated by a TE polarized current line. Power coupling matrix approximated by a banded matrix. Decoupled basis solution in solid line, conventional MoM solution in dots.

in the computational cost and storage requirements.

Some numerical results have been shown to illustrate the capabilities of the proposed method. For the examples considered, the number of basis required to get accurate results has been reduced up to 2–3 basis per wavelength regarding the characteristic 10 basis per wavelength of the conventional MoM formulation. This reduction has been shown to be valid even when the starting power coupling matrix is an approximate banded matrix.

It is worth noting that the proposed method keeps on working when dealing with nonsmooth scatters. Though the method does not consider some significant high frequency current components, it computes all those influencing radiation considerably.

5. CONCLUSIONS

In the present paper, a new set of basis has been introduced as a feasible alternative to diminish the unknowns required to solve electromagnetic scattering problems when using the method of moments. By applying

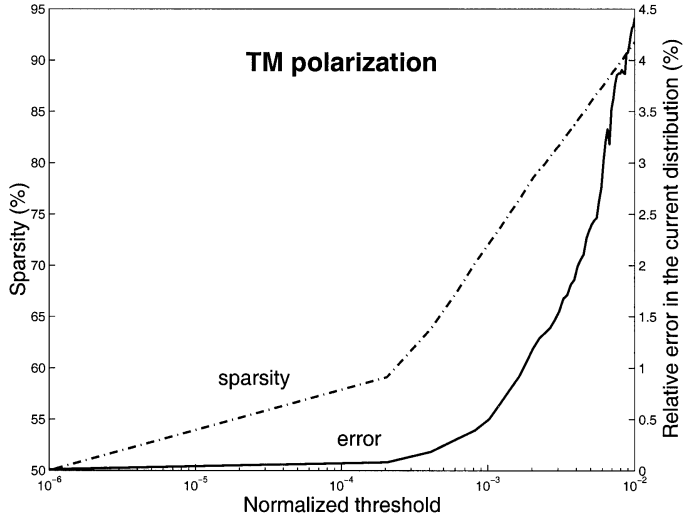

Figure 11. Degree of sparsity vs. a given threshold together with the error in current distribution when these thresholds are considered to render the impedance matrix sparse. TM polarization.

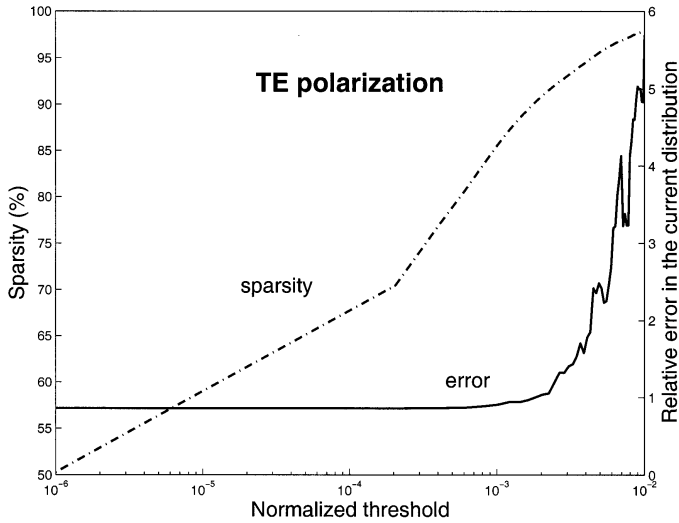

Figure 12. Degree of sparsity vs. a given threshold together with the error in current distribution when these thresholds are considered to render the impedance matrix sparse. TE polarization.

the SVD to a matrix A, defined as the radiated power coupling matrix, a basis transformation is performed, and the standard MoM basis are replaced by a set of decoupled basis. Furthermore, the SVD facilitates a systematic reduction of the impedance matrix to a submatrix of smaller rank. However, given the excessive computational cost of the SVD method, $O\left(N^3\right)$, it can not be handled yet for large problems. Therefore, some extra measures must be taken for the sake of saving computer requirements.

These measures take advantage of the special characteristics of the coupling matrix A, yielding a sparse representation of the impedance MoM matrix Z without considerable loss of accuracy. The matrix A tends to concentrate its main values close to its diagonal, which allows to neglect the elements faraway from the diagonal, leading to a highly sparse matrix A_s whose SVD can be evaluated in $O\left(N^2\right)$ counts, without significant loss of precision. A feasible alternative could be to apply the Lanczos algorithm [19] to the coupling matrix A. This algorithm is an iterative procedure, which achieves three matrices (U', Σ', V') with analogous properties to those given by the SVD (U, Σ, V) but where Σ' is a tridiagonal matrix. It involves $(2k+8)\,i$ flops in a single step, being k the average number of nonzeros per row and i the current iteration. The number of iterations, n, required to get accurate results uses to be many fewer than N, so the computational cost associated to a dense matrix is $O\left(Nn^2\right)$ and $O\left(n^2\right)$ when a sparse matrix (such as the banded power coupling matrix, A_s) is being considered. Therefore, when applying the Lanczos procedure, a reduction in CPU time is achieved at the expense of allowing some coupling between the new basis, namely the basis do not interact only with themselves but with their closest neighbors (slightly coupled). Otherwise, the reduction of the number of unknowns would be similar to that obtained when using the complete SVD (up to 2–3 unknowns per wavelength).

Summing up, the proposed technique based on the SVD of the power coupling matrix A has demonstrated to provide appropriate solutions for the method of moments with only 2–3 basis per wavelength, relieving not only the computational burden but the storage requirements.

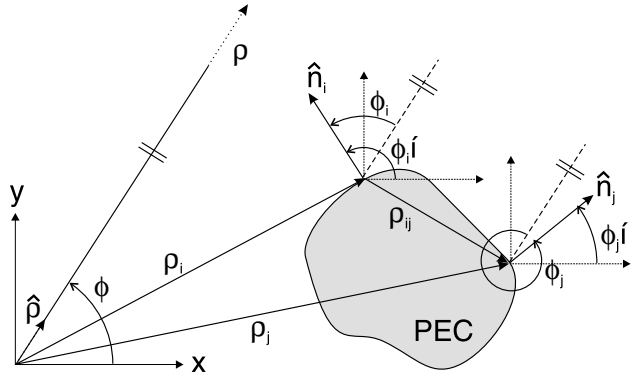

Figure 13. Coordinate system.

APPENDIX A. CLOSED FORM EXPRESSIONS FOR THE POWER COUPLING MATRIX

The coupling matrix, A, used in the body of the paper, considers the interaction between far-field patterns of the standard basis functions. Each element, $A_{i,j}$, represents the radiated power due to the interaction between the i and j basis (which is null if the basis are decoupling). In order to calculate this term the radiated power will be expressed as function of the associated basis coefficients.

Using the coordinate system depicted in *Fig. 13*, the following vector is defined

$$\boldsymbol{\rho}_i' = \boldsymbol{\rho} - \boldsymbol{\rho}_i \tag{16}$$

whose magnitude can be approximated in the far-field region as

$$\rho_i' \tilde{=} \rho - \boldsymbol{\rho}_i \hat{\rho} \quad for\ phase\ terms \tag{17}$$

$$\rho_i' \tilde{=} \rho \quad for\ amplitude\ terms \tag{18}$$

TM and TE polarizations will be treated separately using pulse basis functions.

A. TM Case

The $\hat{z}$ TM-polarized electric field radiated over a remote point by the i-pulse function (with associated current $I_i = \Delta C_i J_i$; being ΔC_i the width of the basis and J_i is the current density) can be expressed

as [1]

$$E_i^z(\boldsymbol{\rho}) = \frac{-k\eta_0}{4} I_i H_0^{(2)}\left(k\rho_i'\right) \tag{19}$$

Substituting the asymptotic expansion of the Hankel function, $H_0^{(2)}$:

$$H_0^{(2)}(k\rho_i') \cong \sqrt{\frac{2}{\pi k\rho}} e^{-jk\rho_i'} e^{j\pi/4} \tag{20}$$

and (17), in (19), becomes

$$E_i^z(\boldsymbol{\rho}) \simeq \frac{-k\eta_0}{4} I_i \sqrt{\frac{2}{\pi k\rho}} e^{-jk(\rho - \boldsymbol{\rho}_i\hat{\rho})} e^{j\pi/4} \tag{21}$$

where

$$\hat{\rho} = \hat{x}\cos\phi + \hat{y}\sin\phi \tag{22}$$

The power generated by the impressed currents can be computed as the coherent summation of the electric field radiated by all the basis functions:

$$\begin{aligned} P_{rad} &= \frac{1}{2\eta_0}\int_0^{2\pi} \left[E^z(\rho,\phi)\right]^* \cdot \left[E^z(\rho,\phi)\right] \rho d\phi \\ &= \frac{1}{2\eta_0}\int_0^{2\pi} \left[\sum_i E_i^z(\rho,\phi)\right]^* \cdot \left[\sum_j E_z^z(\rho,\phi)\right] \rho d\phi \end{aligned} \tag{23}$$

Substituting eq. (21) in eq. (23) and interchanging the integration and the summation then

$$P_{rad} = \frac{k\eta_0}{16\pi} \sum_i \sum_j \Delta C_i \Delta C_j J_i^* J_j \int_0^{2\pi} e^{jk(\boldsymbol{\rho}_j - \boldsymbol{\rho}_i)\hat{\rho}} d\phi \tag{24}$$

which can be written as a quadratic form as:

$$P_{rad} = J^H A J \tag{25}$$

where J is a column vector containing the current density of each pulse function, and A is the *power coupling matrix* whose elements are given by

$$A_{i,j} = \frac{k\eta_0}{16\pi} \Delta C_i \Delta C_j \int_0^{2\pi} e^{jk(\boldsymbol{\rho}_j - \boldsymbol{\rho}_i)\hat{\rho}} d\phi \tag{26}$$

which can be solved applying the integral representations of the Bessel functions [20]:

$$A_{i,j} = \frac{k\eta_0}{8}\Delta C_i \Delta C_j J_0\left(k\rho_{ij}\right) \tag{27}$$

B. TE Case

In this case, an analogous procedure is followed using the magnetic field instead of the electric field. The $\hat{z}$ polarized magnetic field in the far-field region can be expressed as [1]

$$H_i^z\left(\boldsymbol{\rho}\right) = \frac{jk\Delta C_i}{4} J_i \cos(\phi_i) H_1^{(2)}\left(k\rho_i'\right) \tag{28}$$

where ϕ_i is the angle between $\hat{\rho}$ and $\hat{n}_i$, the normal vector in the i-pulse position. Using the Hankel function, $H_1^{(2)}$ for large arguments,

$$H_1^{(2)}(k\rho_i') \tilde{=} \sqrt{\frac{2}{\pi k\rho}} e^{-jk\rho_i'} e^{j3\pi/4} \tag{29}$$

and eq. (17), eq. (28) becomes

$$H_i^z\left(\boldsymbol{\rho}\right) = \frac{jk\Delta C_i}{4} J_i \cos(\phi_i) \sqrt{\frac{2}{\pi k\rho}} e^{-jk(\rho - \boldsymbol{\rho}_i\hat{\rho})} e^{j3\pi/4} \tag{30}$$

The outgoing power is calculated as

$$P_{rad} = \frac{\eta_0}{2}\int_0^{2\pi}\left[\sum_i H_i^z\left(\rho,\phi\right)\right]^* \cdot \left[\sum_j H_j^z\left(\rho,\phi\right)\right]\rho d\phi \tag{31}$$

Substituting (30) in (31) and interchanging the integration and the summation then

$$P_{rad} = \sum_i \sum_j \frac{\eta_0 k \Delta C_i \Delta C_j}{16\pi} J_i^* J_j \int_0^{2\pi} \cos(\phi_i)\cos(\phi_j) e^{jk(\boldsymbol{\rho}_j - \boldsymbol{\rho}_i)\hat{\rho}} d\phi \tag{32}$$

The elements of the power coupling matrix, A, will be achieved in a straightforward manner through the quadratic form (25),

$$A_{i,j} = \frac{\eta_0 k^2 \Delta C_i \Delta C_j}{32} \int_0^{2\pi} \cos(\phi_i)\cos(\phi_j) \frac{2}{\pi k}\frac{1}{\rho} e^{jk(\boldsymbol{\rho}_j - \boldsymbol{\rho}_i)\hat{\rho}} \rho d\phi$$

and applying the integral representations of the Bessel functions [20], obtaining:

$$A_{i,j} = \frac{\eta_0 k}{16} \Delta C_i \Delta C_j \left(\cos(\phi_j^{'} - \phi_i^{'}) J_0(k\rho_{ij}) - \cos\left(2\phi_{ij} - (\phi_i^{'} + \phi_j^{'})\right) J_2\left(k\rho_{ij}\right) \right) \tag{33}$$

where $\phi_i^{'}$ and $\phi_j^{'}$ are the angles between $\hat{n}_i$ and $\hat{n}_j$ and the $\hat{x}$ axis, and ϕ_{ij} is the angle between $\boldsymbol{\rho}_{ij}$ vector and the $\hat{x}$ axis.

It is worth pointing out that the matrix terms obtained are real numbers for both cases and that the matrix is symmetric due to the properties of the power coupling between basis.

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