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DIFFRACTION OF ELECTROMAGNETIC WAVES BY AN OPEN ENDED PARALLEL PLATE WAVEGUIDE CAVITY WITH IMPEDANCE WALLS

B. A. Çetiner

Yıldız Technical University
Faculty of Electrical and Electronics Engineering
80750, Beşiktaş, İstanbul, Turkey

A. Büyükaksoy

Gebze Institute of Technology
Department of Mathematics
P.O. Box 141, Gebze, 41400
Kocaeli, Turkey

F. Güneş

Yıldız Technical University
Faculty of Electrical and Electronics Engineering
80750, Beşiktaş, İstanbul, Turkey

1. INTRODUCTION

The analysis of the electromagnetic wave scattering by open ended parallel plate waveguide cavity configuration is an important topic in diffraction theory from both theoretical and engineering points of view. Such an analysis is useful for dealing with radar cross sections (RCS) analysis and EM penetration problems. This problem also serves as a simple model of duct structures such as jet engine intakes. Up to now, in the open literature many cavity scattering problems have been analysed by using a variety of analytical and numerical techniques. The most used two typical methods are the waveguide modal approach [1, 2] and the high frequency ray techniques [3, 4]. However, the solutions obtained by these methods remain valid depending on the length of the cavity size. There are only few investigations of the diffraction by open-ended cavities valid for arbitrary cavity apertures. The most extensive treatment is that of Kobayashi [5] who used rigorous function-theoretic method. In the work of Kobayashi, the walls of the cavity are assumed to be perfectly conducting. The main objective of this work is to extend the analysis carried out by Wiener-Hopf technique to the more general case where the walls forming the waveguide cavity are characterised by impedance boundary conditions. From mathematical point of view the extension of the analysis related to the perfectly conducting case to the impedance case is not straightforward and merits a detailed investigation. Furthermore, this analysis will enable us to reveal influence of the impedance to the RCS of the cavity geometry

The geometry is uniform in the z-direction and the cavity is assumed to be illuminated by an electrical line source located in free space parallel to the z axis passing through the point, $P(x_0, y_0, 0)$ (See Fig. 1). Hence, this problem is a two dimensional one that consists of finding am explicit expression of scattered field. Firstly, by using the "image bisection principle" [6] the excitation of the original problem may be regarded as a superposition of four cylindric incident waves (See Figs. 2a, 2b) or their equivalences shown in Figs. 2c, 2d. Thus, the original problem can be converted into two simpler ones (See Figs. 2c, 2d). Once the solutions of the configurations depicted in Fig. 2c and Fig. 2d are obtained we simply add them together to get the desired solution for the original problem (See Fig. 1). Since this approach separates the symmetrical and asymmetrical components of the total field at the very beginning, the formulation of the Wiener-Hopf equation is comparatively easier.

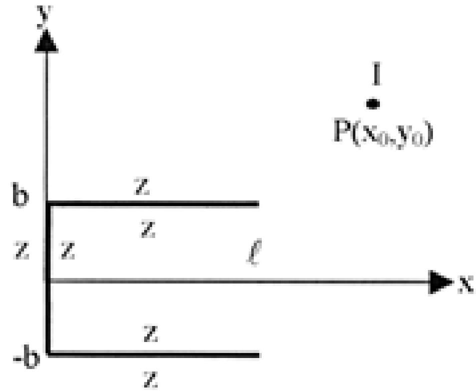


Figure 1. Geometry of the diffraction problem.

The present method is based on the Fourier integral and series representations of the scattered field in appropriate regions. The total field inside the waveguide region is expanded into a series of normal modes and Fourier transform technique is used elsewhere. Then, applying the boundary conditions the problem is formulated in terms of Modified Wiener-Hopf equation for each excitation. Application of the decomposition and factorisation procedures to these W-H equations and taking into account the well known Liouville theorem together with the edge condition leads to a Fredholm integral equation of the second kind that can be solved approximately by the method of successive approximations. The solution contains a set of infinite number of constants satisfying an infinite system of linear algebraic equations.

An e^{-iwt} time dependence for the EM fields will be assumed and suppressed in the analysis that follow.

2. ANALYSIS

The geometry of the diffraction problem considered in this work is given in Fig. 1 and is defined as $S = \{(x, y, z) : x \in (0, l),\ y \in (-b, b)\,,\ z \in (-\infty, \infty)\}$. The geometry is illuminated by an electrical line source I as shown in Fig. 1 and the problem at hand is to find an explicit expression of the scattered field both inside and outside of the waveguide region. The walls of the cavity are assumed to be characterised by a constant surface impedance $Z = \eta Z_0$ with $Z_0 = \sqrt{\mu_0/\varepsilon_0}$

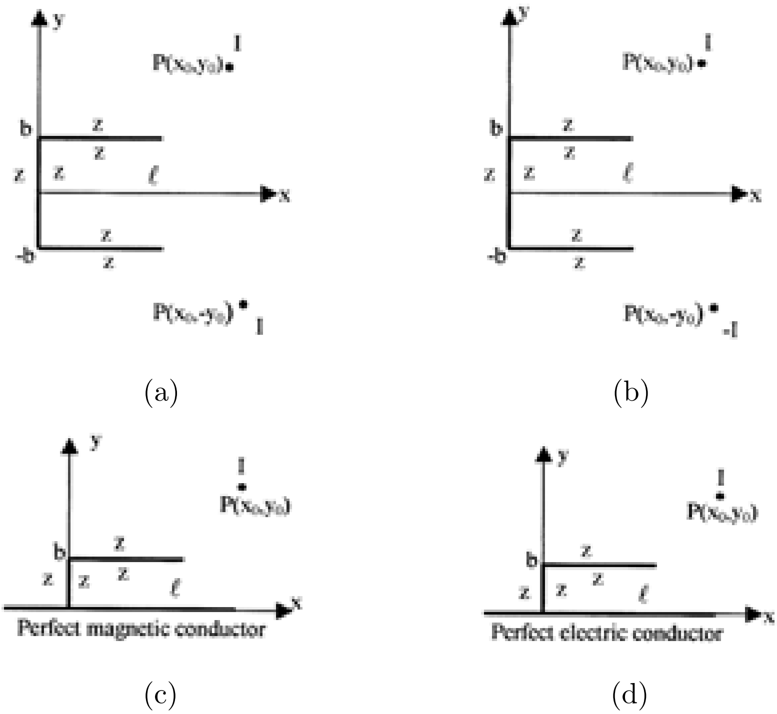


Figure 2. (a) Symmetrical excitation (b) asymmetrical excitation (c) equivalent to (a) (d) equivalent to (c).

being the characteristic impedance of the free space. For the sake of analytical convenience we will assume that the surface impedance is purely reactive $\eta = i\xi$; $\xi \in \Re$ where ξ denotes the relative surface reactance.

Instead of attacking the original diffraction problem shown in Fig. 1, it is preferable to use the image bisection principle and to separate this geometry into two simpler ones depicted in Fig. 2c and Fig. 2d The solution of the original geometry is then the superposition of the solutions related to the geometries shown in Fig. 2c and Fig. 2d. In what follows the even and odd excitations will be treated separately.

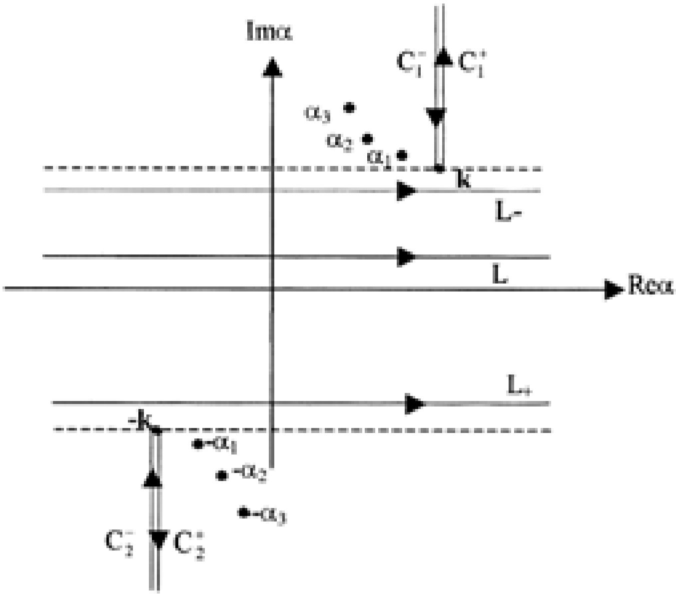


Figure 3. The complex α-plane.

2.1 Formulation for the Even Excitation Case

We first consider the configuration shown in Fig. 2c which is equivalent to the even excitation case. For analysis purposes, we will consider the following regions B_n separately.

$$\begin{aligned} B_0 &= \{(x,y) : y > y_0,\ x \in (-\infty,\infty)\} \\ B_1 &= \{(x,y) : y \in (b,y_0),\ x \in (-\infty,\infty)\} \\ B_2 &= \{(x,y) : y \in (0,b),\ x \in (-\infty,0)\} \\ B_3 &= \{(x,y) : y \in (0,b),\ x \in (0,l)\} \\ B_4 &= \{(x,y) : y \in (0,b),\ x \in (l,\infty)\} \end{aligned} \tag{1}$$

The expression of the scattered field inside the region B_n will be denoted by $u_n^e(x,y)$. These functions satisfy the Helmholtz equation.

$$\Delta u_n + k^2 u_n = 0, \quad (x,y) \in B_n, \quad n = 1,2,3,4 \tag{2}$$

Here k denotes the wave number of the free space which is temporarily assumed to have a small imaginary part. The solution related to the lossless medium is obtained by letting $\mathrm{Im}(k) \to 0$ at the end of the analysis.

The appropriate expression of the function $u_0^e(x,y)$ that satisfy both the Eqn. 2 and the following radiation condition,

$$\sqrt{r}(\partial u/\partial r - iku) \to 0, \qquad r \to \infty \tag{3}$$

can be written as follows:

$$u_0(x,y) = \frac{1}{2\pi}\int_L A^e(\alpha)e^{iK(\alpha)y}e^{-i\alpha x}d\alpha. \tag{4}$$

Here $A^e(\alpha)$ is a yet unknown spectral coefficient to be determined through the boundary conditions while $K(\alpha)$ is given by

$$K(\alpha) = \sqrt{k^2-\alpha^2} \tag{5}$$

The square root function $K(\alpha)$ is defined in the complex α-plane cut as shown in Fig. 3 such that $K(0)=k$. By considering the following asymptotic behaviour of $u_0^e(x,y)$,

$$u_0^e(x,y) = O\left(e^{ik|x|}/\sqrt{x}\right) \tag{6}$$

for y=constant and $|x|\to\infty$, we see that $A^e(\alpha)e^{iK(\alpha)y}$ appearing in (4) is regular in the strip $\mathrm{Im}(-k) < \mathrm{Im}(\alpha) < \mathrm{Im}(k)$.

For the function $u_1^e(x,y)$, which is defined in the region $y \in (b, y_0)$ and $x \in (-\infty,\infty)$ we can

$$u_1^e(x,y) = \frac{1}{2\pi}\int_L \left\{B^e(\alpha)e^{iK(\alpha)(y-b)} + C(\alpha)e^{-iK(\alpha)(y-b)}\right\}e^{-i\alpha x}d\alpha \tag{7}$$

The Fourier transform of $u_1^e(x,y)$ which will be denoted by $\hat{u}_1^e(\alpha,y)$, reads

$$\begin{aligned}\hat{u}_1^e(\alpha,y) &= B^e(\alpha)e^{iK(\alpha)(y-b)} + C(\alpha)e^{-iK(\alpha)(y-b)}\\ &= F_-^e(\alpha,y) + F_+^e(\alpha,y)e^{i\alpha l} + F_1^e(\alpha,y)\end{aligned} \tag{8a}$$

where

$$\begin{aligned}F_-^e(\alpha,y) &= \int_{-\infty}^{0} u_1^e(x,y)e^{i\alpha x}dx,\\ F_1^e(\alpha,y) &= \int_0^l u_1^e(x,y)e^{i\alpha x}dx,\\ F_+^e(\alpha,y)e^{i\alpha l} &= \int_l^\infty u_1^e(x,y)e^{i\alpha x}dx\end{aligned} \tag{8b}$$

In (8a) and (8b) $F_-^e(\alpha,y)$ and $F_+^e(\alpha,y)$ are regular in the half-planes $\mathrm{Im}(\alpha)<\mathrm{Im}(k)$ and $\mathrm{Im}(\alpha)>\mathrm{Im}(-k)$ respectively, whereas $F_1^e(\alpha,y)$ is an entire function. We shall henceforth use the subscripts $(-)$ and $(+)$ to denote the functions which are regular in the lower and upper halves of the complex α-plane.

Let us consider the function $u_2^e(x,y)$ defined in the region $y\in(0,b)$ and $x\in(-\infty,0)$. By defining its analytical continuation into the region, $y\in(0,b)$ and $x>0$, as $u_2^e(x,y)\equiv 0$ ant taking the Fourier transform of (4) we get

$$\left[\frac{d^2}{dy^2}+K^2(\alpha)\right]G_-^e(\alpha,y)=-\left[\frac{\partial u_2^e(0,y)}{\partial x}-i\alpha u_2^e(0,y)\right] \tag{9a}$$

with

$$G_-^e(\alpha,y)=\int_{-\infty}^{0}u_2^e(x,y)e^{i\alpha x}dx \tag{9b}$$

Inside the waveguide region, $y\in(0,b)$ and $x\in(0,l)$, we can express $u_3^e(x,y)$ in terms of normal modes.

$$u_3^e(x,y)=\sum_{n=1}^{\infty}D_n^e\left[e^{i\beta_n^e x}-\frac{\left(1+\beta_n^e\dfrac{\eta}{k}\right)}{\left(1-\beta_n^e\dfrac{\eta}{k}\right)}e^{-i\beta_n^e x}\right]\cos\tau_n^e y \tag{10a}$$

with

$$\tau_n^e=\sqrt{k^2-(\beta_n^e)^2},\quad \cot g(\tau_n^e b)=-\frac{\eta}{ik}\tau_n^e \tag{10b}$$

D_n^e appearing in (10a) is yet unknown.

The expression of $u_4^e(x,y)$ can be obtained by a procedure similar to the one outlined above for $u_2^e(x,y)$. Indeed, considering $u_4^e(x,y)\equiv 0$ for the region $y\in(0,b)$ and $x<l$ we obtain

$$\left[\frac{d^2}{dy^2}+K^2(\alpha)\right]H_+^e(\alpha,y)=\left[\frac{\partial u_4^e(l,y)}{\partial x}-i\alpha u_4^e(l,y)\right] \tag{11a}$$

where $H_+^e(\alpha,y)$ stands for

$$H_+^e(\alpha,y)=\int_{l}^{\infty}u_4^e(x,y)e^{i\alpha(x-l)}dx \tag{11b}$$

In order to determine the unknown spectral coefficients appearing in the expression of the scattered field given above, one must take into account the following boundary and continuity relations as well as the definition of the source.

$$u_0^e(x,y_0)-u_1^e(x,y_0)=0, \qquad x\in(-\infty,\infty) \tag{12a}$$

$$\frac{\partial}{\partial y}\left[u_0^e(x,y_0)-u_1^e(x,y_0)\right]=ikIZ_0\delta(x-x_0), \quad x\in(-\infty,\infty) \tag{12b}$$

$$\left(1+\frac{\eta}{ik}\frac{\partial}{\partial y}\right)u_1^e(x,b)=0, \qquad x\in(0,l) \tag{12c}$$

$$u_1^e(x,b)-u_2^e(x,b)=0, \qquad x\in(-\infty,0) \tag{12d}$$

$$\frac{\partial}{\partial y}\left[u_1^e(x,b)-u_2^e(x,b)\right]=0, \qquad x\in(-\infty,0) \tag{12e}$$

$$u_1^e(x,b)-u_4^e(x,b)=0, \qquad x\in(l,\infty) \tag{12f}$$

$$\frac{\partial}{\partial y}\left[u_1^e(x,b)-u_4^e(x,b)\right]=0, \qquad x\in(l,\infty) \tag{12g}$$

$$\frac{\partial u_2^e(x,0)}{\partial y}=0, \qquad x\in(-\infty,0) \tag{12h}$$

$$\frac{\partial u_3^e(x,0)}{\partial y}=0, \qquad x\in(0,l) \tag{12i}$$

$$\frac{\partial u_4^e(x,0)}{\partial y}=0, \qquad x\in(l,\infty) \tag{12j}$$

$$\left(1-\frac{\eta}{ik}\frac{\partial}{\partial y}\right)u_3^e(x,b)=0, \qquad x\in(0,l) \tag{12k}$$

$$\left(1-\frac{\eta}{ik}\frac{\partial}{\partial x}\right)u_2^e(0,y)=0, \qquad y\in(0,b) \tag{12l}$$

$$\left(1+\frac{\eta}{ik}\frac{\partial}{\partial x}\right)u_3^e(0,y)=0, \qquad y\in(0,b) \tag{12m}$$

$$u_3^e(l,y)-u_4^e(l,y)=0, \qquad y\in(0,b) \tag{12n}$$

$$\frac{\partial}{\partial x}\left[u_3^e(l,y)-u_4^e(l,y)\right]=0, \qquad y\in(0,b) \tag{12o}$$

By considering (4) and (7), $C(\alpha)$ can immediately be solved from (12a) and (12b) to give

$$C(\alpha)=\frac{kIZ_0e^{i\alpha x_0}}{2K(\alpha)}e^{iK(\alpha)(y_0-b)} \tag{13}$$

The Fourier transform of (12d) is

$$F_1^e(\alpha,b)+\frac{\eta}{ik}\dot{F}_1^e(\alpha,b)=0 \tag{14}$$

where the dots means $\partial/\partial y$. If we consider (7), (8a), and (8b) together with (14), we get the following expression.

$$F^e_-(\alpha,b)+\frac{\eta}{ik}\dot{F}^e_-(\alpha,b)+e^{i\alpha l}\left[F^e_+(\alpha,b)+\frac{\eta}{ik}\dot{F}^e_+(\alpha,b)\right]$$
$$=\left[1+\frac{\eta}{k}K(\alpha)\right]B^e(\alpha)+\left[1-\frac{\eta}{k}K(\alpha)\right]C(\alpha)\quad(15)$$

Now, the unknown coefficient $B^e(\alpha)$ can be solved from (15).

Substituting the boundary conditions given by (12h) and (12l) into (9a) and using the following definition

$$-\frac{\partial u^e_2(0,y)}{\partial x}=f^e(y)\quad(16)$$

we get

$$G^e_-(\alpha,y)=P^e(\alpha)\cos Ky+\left[\left(1-\frac{\eta\alpha}{k}\right)/K\right]\int_0^y f^e(t)\sin[K(y-t)]dt\quad(17)$$

In a similar way, substituting (12j) into (11a) and using the following definitions

$$\frac{\partial u^e_4(l,y)}{\partial x}=r^e(y),\qquad iu_4(l,y)=s^e(y)\quad(18)$$

one obtains

$$H^e_+(\alpha,y)=Q^e(\alpha)\cos Ky+\frac{1}{K}\int_0^y[r^e(t)-\alpha s^e(t)]\sin[K(y-t)]dt\quad(19)$$

The integration constants, $P^e(\alpha)$, $Q^e(\alpha)$, appearing in (17) and (19) can uniquely be obtained via the boundary conditions (12d), (12e), (12f), and (12g). Taking Fourier transform of these boundary conditions we arrive at

$$\begin{aligned}F^e_-(\alpha,b)&=G^e_-(\alpha,b), & \dot{F}^e_-(\alpha,b)&=\dot{G}^e_-(\alpha,b),\\ F^e_+(\alpha,b)&=G^e_+(\alpha,b), & \dot{F}^e_+(\alpha,b)&=\dot{G}^e_+(\alpha,b)\end{aligned}\quad(20)$$

Now, by introducing the functions $T^{(e)}_\pm(\alpha)$ via

$$F^e_-(\alpha,b)+\frac{\eta}{ik}\dot{F}^e_-(\alpha,b)=T^e_-(\alpha),\quad F^e_+(\alpha,b)+\frac{\eta}{ik}\dot{F}^e_+(\alpha,b)=T^e_+(\alpha)\quad(21)$$

and substituting them into (17) and (19) one obtains

$$T_-^e(\alpha) = P^e(\alpha)X^e(\alpha) + \frac{1}{K}\left(1 - \frac{\eta\alpha}{k}\right)\int_0^b f^e(t)\chi^e(\alpha, t)dt \quad (22a)$$

$$T_+^e(\alpha) = Q^e(\alpha)X^e(\alpha) + \frac{1}{K}\int_0^b \left[r^e(t) - \alpha s^e(t)\right]\chi^e(\alpha, t)dt \quad (22b)$$

with

$$\begin{aligned} X^e(\alpha) &= \cos Kb - \frac{\eta}{ik}K\sin Kb, \\ \chi^e(\alpha, t) &= \sin K(b - t) + \frac{\eta}{ik}K\cos K(b - t) \end{aligned} \quad (23)$$

After solving $P^e(\alpha)$ and $Q^e(\alpha)$ from (22a) and (22b) and then substituting them into (17) and (19) we get, respectively

$$\begin{aligned} G_-^e(\alpha, y) &= \frac{\cos Ky}{X^e(\alpha)}\left[T_-^e(\alpha) - \frac{1}{K}\left(1 - \frac{\eta\alpha}{k}\right)\int_0^b f^e(t)\chi^e(\alpha, t)dt\right] \\ &\quad + \frac{1}{K}\left(1 - \frac{\eta\alpha}{k}\right)\int_0^y f^e(t)\sin K(y - t)dt \end{aligned} \quad (24a)$$

$$\begin{aligned} H_+^e(\alpha, y) &= \frac{\cos Ky}{X^e(\alpha)}\left[T_+^e(\alpha) - \frac{1}{K}\int_0^b \left[r^e(t) - \alpha s^e(t)\right]\chi^e(\alpha, t)dt\right] \\ &\quad + \frac{1}{K}\int_0^y \left[r^e(t) - \alpha s^e(t)\right]\sin K(y - t)dt \end{aligned} \quad (24b)$$

Note that the left hand side of (24a) and (24b) are regular in the lower $\mathrm{Im}(\alpha) < \mathrm{Im}(k)$ and upper $\mathrm{Im}(\alpha) > \mathrm{Im}(-k)$ halves of the complex α-plane, respectively. Therefore their right hand sides must also be regular in the respective regions of regularity. This requirement is satisfied if the following relations hold.

$$\begin{aligned} &T_-^e(-\alpha_m^e) \\ &= \frac{\sin K_m^e b}{2K_m^e}\left[1 - \left(\frac{\eta K_m^e}{k}\right)^2\right]\left(b + \frac{\eta}{ik}\sin^2 K_m^e b\right)\left(1 + \alpha_m^e\frac{\eta}{k}\right)f_m^e \end{aligned} \quad (25a)$$

$$\begin{aligned} &T_+^e(\alpha_m^e) \\ &= \frac{\sin K_m^e b}{2K_m^e}\left[1 - \left(\frac{\eta K_m^e}{k}\right)^2\right]\left(b + \frac{\eta}{ik}\sin^2 K_m^e b\right)\left(r_m^e - \alpha_m^e s_m^e\right) \end{aligned} \quad (25b)$$

Here α_m^e are the zeros of $X^e(\alpha)$ given by (23) and $K_m^e = K(\pm\alpha_m^e)$

$$\begin{bmatrix} f_m^e \\ r_m^e \\ s_m^e \end{bmatrix} = \frac{2}{\left(b + \dfrac{\xi}{k}\sin^2 K_m b\right)} \int_0^b \begin{bmatrix} f^e(t) \\ r^e(t) \\ s^e(t) \end{bmatrix} \cos K_m^e t dt \tag{26}$$

From (26) one can express $f^e(t)$, $r^e(t)$ and $s^e(t)$ as follows

$$\begin{bmatrix} f^e(t) \\ r^e(t) \\ s^e(t) \end{bmatrix} = \sum_{m=1}^{\infty} \begin{bmatrix} f_m^e \\ r_m^e \\ s_m^e \end{bmatrix} \cos K_m^e t \tag{27}$$

Consequently, by taking into account (8a), (15), (20), (21), (24a), (24b), (27) and then carrying out some algebraic manipulations, we arrive at the following modified Wiener-Hopf equation of the third kind.

$$\begin{aligned} &\left[T_-^e(\alpha) + e^{i\alpha l} T_+^e(\alpha)\right] \frac{\tau(\alpha)}{\Delta^e(\alpha)} \\ &\quad = 2ik\tau(\alpha)C(\alpha) + \dot{F}_1^e(\alpha, b) \\ &\qquad + \sum_{m=1}^{\infty} \left[\left(1 - \alpha\frac{\eta}{k}\right) f_m^e + e^{i\alpha l}\left(r_m^e - \alpha s_m^e\right)\right] \frac{K_m^e \sin K_m^e b}{\alpha^2 - (\alpha_m^e)^2} \end{aligned} \tag{28}$$

with

$$\Delta^e(\alpha) = -\frac{ie^{iKb}}{k}\left[\cos Kb - \frac{\eta}{ik}K(\alpha)\sin Kb\right], \ \tau(\alpha) = \frac{K(\alpha)}{k + \eta K(\alpha)} \tag{29}$$

This Wiener-Hopf equation permits us to solve the functions $T_\pm^e(\alpha)$ in terms of the unknown Fourier coefficients f_m^e, s_m^e, and r_m^e.

2.2 Approximate Solution to the MWHE

Let us consider the MWHE obtained in (28). The first step in solving (28) is to perform the factorization of the functions $\tau(\alpha)$ and $\Delta^e(\alpha)$ in the following form

$$\tau(\alpha) = \tau_-(\alpha)\tau_+(\alpha), \qquad \Delta^e(\alpha) = \Delta_-^e(\alpha)\Delta_+^e(\alpha) \tag{30}$$

Where, $\tau_+(\alpha)$, $\Delta_+^e(\alpha)$ and $\tau_-(\alpha)$, $\Delta_-^e(\alpha)$ are regular and free of zeros in the upper and lower half α-planes defined by $\mathrm{Im}(\alpha) > \mathrm{Im}(-k)$ and $\mathrm{Im}(\alpha) < \mathrm{Im}(k)$, respectively. The explicit expression of $\Delta_\pm^e(\alpha)$ are given as follows [7]:

$$\Delta_+^e(\alpha) = \sqrt{\frac{-i}{k}}\left(\cos kb - \frac{\eta}{i}\sin kb\right)^{1/2} \exp\left\{\frac{Kb}{\pi}\ln\left(\frac{\alpha+iK}{k}\right) + \frac{i\alpha b}{\pi}\left(1 - C + \ln\frac{2\pi}{kb} + \frac{i\pi}{2}\right)\right\} \times \prod_{n=1}^{\infty}\left(1+\frac{\alpha}{\alpha_n^e}\right)e^{\frac{i\alpha b}{n\pi}} \tag{31a}$$

$$\Delta_-^e(\alpha) = \Delta_+^e(-\alpha) \tag{31b}$$

As to the split functions $\tau_\pm(\alpha)$, they can be expressed explicitly in terms of the Maliuzhinetz functions [8].

$$\tau_+(k\cos\phi) = \frac{4}{\sqrt{\eta}}\sin\frac{\phi}{2}\left\{\frac{M_\pi(3\pi/2-\phi-\theta)M_\pi(\pi/2-\phi+\theta)}{M_\pi^2(\pi/2)}\right\}^2 \times\left\{\left[1+\sqrt{2}\cos\left(\frac{\pi/2-\phi+\theta}{2}\right)\right] \cdot\left[1+\sqrt{2}\cos\left(\frac{3\pi/2-\phi-\theta}{2}\right)\right]\right\}^{-1} \tag{32a}$$

$$\tau_-(k\cos\phi) = \tau_+(-k\cos\phi) \tag{32b}$$

with

$$\sin\theta = \frac{1}{\eta}, \quad M_\pi(z) = \exp\left\{-\frac{1}{8\pi}\int_0^z \frac{\pi\sin u - 2\sqrt{2}\sin(u/2) + 2u}{\cos u}du\right\} \tag{32c}$$

Now, multiplying both sides of (28) by $\Delta_+^e(\alpha)/\tau_+(\alpha)$ and applying the well known decomposition procedure we get

$$T_-^e(\alpha)\frac{\tau_-(\alpha)}{\Delta_-^e(\alpha)} - \frac{1}{2\pi i}\int_{L_-} T_+^e(\beta)\frac{\tau_-(\beta)}{\Delta_-^e(\beta)}\frac{e^{i\beta l}}{(\beta-\alpha)}d\beta - I_-^e(\alpha) - \Omega_m^{-e}(\alpha) - \omega_m^{-e}(\alpha) = -\frac{1}{2\pi i}\int_{L_+} T_+^e(\beta)\frac{\tau_-(\beta)}{\Delta_-^e(\beta)}\frac{e^{i\beta l}}{(\beta-\alpha)}d\beta + I_+^e(\alpha) + \Omega_m^{+e}(\alpha) + \omega_m^{+e}(\alpha) + \dot{F}_1^e(\alpha,b)\frac{\Delta_+^e(\alpha)}{\tau_+(\alpha)} \tag{33}$$

with

$$I^e_{\pm}(\alpha) = \pm\,\frac{1}{2\pi i}\int_{L_\pm}\frac{2ik\tau_-(\beta)\Delta^e_+(\beta)C(\beta)}{\beta-\alpha}d\beta,$$

$$\Omega^{-e}_m(\alpha) = -\sum_{m=1}^{\infty}\frac{\Delta^e_+(\alpha^e_m)}{\tau_+(\alpha^e_m)}\left(1-\alpha^e_m\frac{\eta}{k}\right)f^e_m\frac{K^e_m\sin K^e_m b}{2\alpha^e_m(\alpha^e_m-\alpha)} \tag{34a}$$

$$\Omega^{+e}_m(\alpha) = \sum_{m=1}^{\infty}\frac{\Delta^e_+(\alpha^e_m)}{\tau_+(\alpha^e_m)}\left(1-\alpha^e_m\frac{\eta}{k}\right)f^e_m\frac{K^e_m\sin K^e_m b}{2\alpha^e_m(\alpha^e_m-\alpha)} + \frac{\Delta^e_+(\alpha)}{\tau_+(\alpha)}\left(1-\alpha\frac{\eta}{k}\right)f^e_m\frac{K^e_m\sin K^e_m b}{\alpha^2-(\alpha^e_m)^2} \tag{34b}$$

$$\omega^{-e}_m(\alpha) = -\sum_{m=1}^{\infty}\frac{\Delta^e_+(\alpha^e_m)}{\tau_+(\alpha^e_m)}\left(r^e_m-\alpha^e_m s^e_m\right)e^{i\alpha^e_m l}\frac{K^e_m\sin K^e_m b}{2\alpha^e_m(\alpha^e_m-\alpha)} \tag{34c}$$

$$\omega^{+e}_m(\alpha) = \sum_{m=1}^{\infty}\frac{\Delta^e_+(\alpha^e_m)}{\tau_+(\alpha^e_m)}\left(r^e_m-\alpha^e_m s^e_m\right)e^{i\alpha^e_m l}\frac{K^e_m\sin K^e_m b}{2\alpha^e_m(\alpha^e_m-\alpha)} + \frac{\Delta^e_+(\alpha)}{\tau_+(\alpha)}\left(r^e_m-\alpha^e_m s^e_m\right)e^{i\alpha l}\frac{K^e_m\sin K^e_m b}{\alpha^2-(\alpha^e_m)^2} \tag{34d}$$

The functions on the right hand sides of (33) are regular in the upper half-plane, whereas those appearing in the left hand side are regular in the lower half plane. Therefore, by analytic continuation principle, (33) defines an entire function. By using the following edge conditions,

$$\frac{\partial u^e_1(x,b)}{\partial y} = O\left(x^{-1/3}\right), \qquad x\to 0; \qquad u^e_1(x,b) = O\left(x^{2/3}\right), \qquad x\to 0 \tag{35a}$$

$$\frac{\partial u^e_1(x,b)}{\partial y} = O\left[(x-l)^{-1/2}\right], \qquad x\to l; \qquad u^e_1(x,b) = O\left[(x-l)^{1/2}\right], \qquad x\to l \tag{35b}$$

and the well known Tauberian theorems, it can be shown that, $T^e_{\pm}(\alpha)$ have their following asymptotic behaviour in the respective region of regularity.

$$T^e_+(\alpha) = O\left(\alpha^{-3/2}\right), \qquad T^e_-(\alpha) = O\left(\alpha^{-5/3}\right) \tag{36}$$

Furthermore, we can easily show that we have

$$\frac{\tau_\pm(\alpha)}{\Delta^e_\pm(\alpha)} = O(1) \tag{37}$$

By considering (36), (37) and applying the Liouville theorem, one can show that the above mentioned entire function is identically zero. Then we may write

$$\begin{aligned} T^e_-(\alpha)\frac{\tau_-(\alpha)}{\Delta^e_-(\alpha)} &= \frac{1}{2\pi i}\int_{L_-} T^e_+(\beta)\frac{\tau_-(\beta)}{\Delta^e_-(\beta)}\frac{e^{i\beta l}}{(\beta-\alpha)}d\beta \\ &\quad + I^e_-(\alpha) + \Omega^{-e}_m(\alpha) + \omega^{-e}_m(\alpha) \end{aligned} \tag{38}$$

Now, multiplying both sides of (28) with $\frac{\Delta^e_-(\alpha)}{\tau_-(\alpha)}e^{i\alpha l}$ and performing the similar procedure to those described above, we obtain

$$\begin{aligned} T^e_+(\alpha)\frac{\tau_+(\alpha)}{\Delta^e_+(\alpha)} &= -\frac{1}{2\pi i}\int_{L_+} T^e_-(\beta)\frac{\tau_+(\beta)}{\Delta^e_+(-\beta)}\frac{e^{-i\beta l}}{(\beta-\alpha)}d\beta \\ &\quad + \widetilde{I}^e_+(\alpha) + \Theta^{+e}_m(\alpha) + \widetilde{\Theta}^{+e}_m(\alpha) \end{aligned} \tag{39}$$

with

$$\begin{aligned} \widetilde{I}^e_+(\alpha) &= \frac{k}{\pi}\int_{L_+}\frac{\tau_+(\beta)\Delta^e_-(\beta)C(\beta)e^{-i\beta l}}{\beta-\alpha}d\beta; \\ \widetilde{\Theta}^{+e}_m(\alpha) &= -\sum_{m=1}^{\infty}\frac{\Delta^e_+(\alpha^e_m)}{\tau_+(\alpha^e_m)}(r^e_m+\alpha^e_m s^e_m)\frac{K^e_m\sin K^e_m b}{2\alpha^e_m(\alpha+\alpha^e_m)} \end{aligned} \tag{40a}$$

$$\Theta^{+e}_m(\alpha) = -\sum_{m=1}^{\infty}\frac{\Delta^e_+(\alpha^e_m)}{\tau_+(\alpha^e_m)}\left(1+\alpha^e_m\frac{\eta}{k}\right)f^e_m e^{i\alpha^e_m l}\frac{K^e_m\sin K^e_m b}{2\alpha^e_m(\alpha+\alpha^e_m)} \tag{40b}$$

$T^e_\pm(\alpha)$ will be determined through the equations (38) and (39) which constitute a pair of coupled integral equations. These integral equations can be solved by iterations. When kl is large, the free terms lying in the right hand sides of (38) and (39) gives the first order solutions. Second order solutions can then be obtained by replacing the unknown functions appearing in the integrands by their first order approximations. Thus, one can write

$$T^e_\pm(\alpha) \cong T^{e1}_\pm(\alpha) + T^{e2}_\pm(\alpha) \tag{41}$$

where superscripts 1 and 2 indicates the first and second order approximations, respectively. In (41) $T_{\pm}^{e1}(\alpha)$ are given as follows

$$T_{+}^{e1}(\alpha) = \frac{\Delta_{+}^{e}(\alpha)}{\tau_{+}(\alpha)} \left[\widetilde{I}_{+}^{e}(\alpha) + \sum_{m=1}^{\infty} \frac{\theta_m^{+e}}{\alpha_m^e + \alpha} \right],$$
$$T_{-}^{e1}(\alpha) = \frac{\Delta_{-}^{e}(\alpha)}{\tau_{-}(\alpha)} \left[I_{-}^{e}(\alpha) + \sum_{m=1}^{\infty} \frac{\psi_m^{-e}}{\alpha_m^e - \alpha} \right] \tag{42}$$

with

$$\psi_m^{-e} = -\frac{\Delta_{+}^{e}(\alpha_m^e)}{\tau_{+}(\alpha_m^e)} \Big[(r_m^e - \alpha_m^e s_m^e) e^{i\alpha_m^e l} + \left(1 - \alpha_m^e \frac{\eta}{k}\right) f_m^e \Big] \frac{K_m^e \sin K_m^e b}{2\alpha_m^e} \tag{43a}$$

$$\theta_m^{+e} = -\frac{\Delta_{+}^{e}(\alpha_m^e)}{\tau_{+}(\alpha_m^e)} \Big[(r_m^e + \alpha_m^e s_m^e) + \left(1 + \alpha_m^e \frac{\eta}{k}\right) f_m^e e^{i\alpha_m^e l} \Big] \frac{K_m^e \sin K_m^e b}{2\alpha_m^e} \tag{43b}$$

The explicit expressions of the functions $I_{-}^{e}(\alpha)$ and $\widetilde{I}_{+}^{e}(\alpha)$ appearing in (42) can be obtained asymptotically by substituting,

$$x_0 - l = \rho_0 \cos\phi_0, \qquad y_0 - b = \rho_o \sin\phi_0;$$
$$x_0 = \rho_0' \cos\phi_0', \qquad y_0 - b = \rho_0' \sin\phi_0'; \qquad \beta = -k\cos t \tag{44}$$

and using saddle point technique. The result is:

$$I_{-}^{e}(\alpha) \approx \frac{Ik^2 Z_0}{\sqrt{2\pi}} e^{-i\pi/4} \frac{\tau_{-}(k\cos\phi_0')\Delta_{+}^{e}(k\cos\phi_0')}{\alpha - k\cos\phi_0'} \frac{e^{ik\rho_0'}}{\sqrt{k\rho_0'}} + i\frac{k^2 I Z_0}{K(\alpha)} \tau_{-}(\alpha)\Delta_{+}^{e}(\alpha) e^{i\alpha x_0} e^{iK(y_0 - b)} H\left[\mathrm{Im}(\alpha - k\cos\phi_0')\right] \tag{45a}$$

$$\widetilde{I}_{-}^{e}(\alpha) \approx \frac{-Ik^2 Z_0}{\sqrt{2\pi}} e^{-i\pi/4} \frac{\tau_{+}(k\cos\phi_0)\Delta_{+}^{e}(k\cos\phi_0)}{\alpha - k\cos\phi_0} \frac{e^{ik\rho_0}}{\sqrt{k\rho_0}} + i\frac{k^2 I Z_0}{K(\alpha)} \tau_{+}(\alpha)\Delta_{-}^{e}(\alpha) e^{i\alpha(x_0 - l)} e^{iK(y_0 - b)} H\left[\mathrm{Im}(k\cos\phi_0 - \alpha)\right] \tag{45b}$$

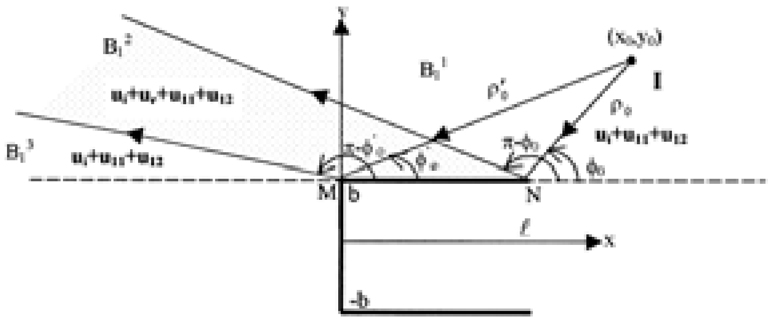


Figure 5. Field terms observed in the region for $y > b$.

Here, H denotes the unit step function and the meanings of ρ_0, ρ_0', ϕ_0, and ϕ_0' are indicated in Fig. 5.

The second order solutions are given as follows:

$$T_+^{e2}(\alpha) = \frac{\Delta_+^e(\alpha)}{\tau_+(\alpha)} J^e(\alpha), \qquad T_-^{e2}(\alpha) = \frac{\Delta_+^e(\alpha)}{\tau_-(\alpha)} \widetilde{J}^e(\alpha) \tag{46}$$

where $J^e(\alpha)$ and $\widetilde{J}^e(\alpha)$ stand for,

$$\begin{aligned} J^e(\alpha) &= -\frac{1}{2\pi i} \int_{L_+} T_-^{e1}(\beta) \frac{\tau_+(\beta)}{\Delta_+^e(\beta)} \frac{e^{-i\beta l}}{(\beta-\alpha)} d\beta, \\ \widetilde{J}^e(\alpha) &= -\frac{1}{2\pi i} \int_{L_-} T_+^{e1}(\beta) \frac{\tau_-(\beta)}{\Delta_-^e(\beta)} \frac{e^{i\beta l}}{(\beta-\alpha)} d\beta \end{aligned} \tag{47}$$

According to Jordan's lemma the integration line L_+ and L_- in (47) can be deformed on to the brunch cuts $C_2^+ + C_2^-$ lying in the lower half plane and $C_1^+ + C_1^-$, lying in the upper half plane, respectively.

The resulting brunch cut integrals can be evaluated asymptotically. Omitting the details, we write

$$J^e(\alpha) = J_1^e(\alpha) + J_2^e(\alpha), \qquad \widetilde{J}^e(\alpha) = \widetilde{J}_1^e(\alpha) + \widetilde{J}_2^e(\alpha) \tag{48}$$

where, $J_{1,2}^e(\alpha)$ and $\widetilde{J}_{1,2}^e(\alpha)$ are given as follows;

$$J_1^e(\alpha) = \sqrt{\frac{2}{\pi}} \frac{k^2}{\eta^2} \frac{e^{-i\pi/4}}{(1+T)} \left[\frac{\Delta_+^e(k)}{\tau_+(k)}\right]^2 \frac{e^{ikl}}{\sqrt{kl}} \left\{ \widetilde{\widetilde{I}}{}_-^e(-k) \left\{ b_1[1 - F[kl(1+\cos\phi_0')]] + b_2[1 - F[kl(1+\alpha/k)]] + b_3[1 - F[kl(1-T)]] \right\} + \frac{1}{(\alpha + kT)} \sum_{m=1}^{\infty} \frac{\psi_m^{-e}}{(\alpha_m^e + k)} \left\{ F[kl(1-T)] - F[kl(1+\alpha/k)] \right\} \right\} \quad (49)$$

with

$$b_1 = \frac{1}{k(\cos\phi_0' + T)(k\cos\phi_0' - \alpha)}, \quad b_2 = \frac{1}{(\alpha + kT)(\alpha - k\cos\phi_0')}, \quad b_3 = \frac{1}{k(\alpha + kT)(\cos\phi_0' + T)} \quad (50a)$$

$$T = \left(1 - 1/\eta^2\right)^{1/2}, \qquad \widetilde{\widetilde{I}}{}_-^e(\alpha) = I_-^e(\alpha)(\alpha - k\cos\phi_0') \quad (50b)$$

$$J_2^e(\alpha) = \sum_{n=1}^{\infty} \frac{(K_n^e)^2 \sin K_n^e b \left[\Delta_+^e(\alpha_n^e)/\tau_+(\alpha_n^e)\right]^2}{\alpha_n^e b \left[X^o(\alpha_n^e) + \frac{\eta}{ikb}\sin K_n^e b\right](\alpha_n^e + \alpha)} \cdot \left\{ \left(\sum_{m=1}^{\infty} \frac{\psi_m^{-e}}{(\alpha_m^e + \alpha_n^e)} + I_-^e(-\alpha_n^e) \right) \right\} e^{i\alpha_n^e l} + \frac{k^2 e^{kb/\xi} \left[\Delta_+^e(kT)/\tau_+(kT)\right]^2}{\xi^3 T(kT + \alpha) X^e(kT)} \cdot \left\{ \left(I_-^e(-kT) + \sum_{m=1}^{\infty} \frac{\psi_m^{-e}}{(\alpha_m^e + kT)} \right) e^{ikTl} \right\} \quad (51)$$

and

$$\widetilde{J}_1^e(\alpha) = \sqrt{\frac{2}{\pi}} \frac{k^2}{\eta^2} \frac{e^{-i\pi/4}}{(1+T)} \left[\frac{\Delta_+^e(k)}{\tau_+(k)}\right]^2 \frac{e^{ikl}}{\sqrt{kl}} \cdot \left\{ \widetilde{\widetilde{I}}{}_+^e(k) \{ a_1[1 - F[kl(1 - \cos\phi_0)]] + a_2[1 - F[kl(1 - \alpha/k)]] \right.$$

$$+ a_3[1 - F[kl(1-T)]]\} + \frac{1}{(\alpha - kT)} \sum_{m=1}^{\infty} \frac{\theta_m^{+e}}{(\alpha_m^e + k)}$$
$$\cdot \{F[kl(1-T)] - F[kl(1-\alpha/k)]\}\Big\} \tag{52}$$

$$\widetilde{J}_2^e(\alpha) = \sum_{n=1}^{\infty} \frac{(K_n^e)^2 \sin K_n^e b \left[\Delta_+^e(\alpha_n^e)/\tau_+(\alpha_n^e)\right]^2}{\alpha_n^e b \left[X^o(\alpha_n^e) + \frac{\eta}{ikb} \sin K_n^e b\right] (\alpha_n^e - \alpha)}$$
$$\cdot \left\{ \left(\sum_{m=1}^{\infty} \frac{\theta_m^{+e}}{(\alpha_m^e + \alpha_n^e)} + \widetilde{I}_+^e(\alpha_n^e) \right) \right\} e^{i\alpha_n^e l}$$
$$+ \frac{k^2 e^{kb/\xi} \left[\Delta_+^e(kT)/\tau_+(kT)\right]^2}{\xi^3 T(kT - \alpha) X^e(kT)}$$
$$\cdot \left\{ \left(\widetilde{I}_+^e(kT) + \sum_{m=1}^{\infty} \frac{\theta_m^{+e}}{(\alpha_m^e + kT)} \right) e^{ikTl} \right\} \tag{53}$$

with

$$a_1 = \frac{1}{k(\cos\phi_0 - T)(k\cos\phi_0 - \alpha)},$$
$$a_2 = \frac{1}{(\alpha - kT)(\alpha - k\cos\phi_0)}, \tag{54a}$$
$$a_3 = \frac{1}{k(\alpha - kT)(\cos\phi_0 - T)}$$

$$\widetilde{\widetilde{I}}_+^e(\alpha) = \widetilde{I}_+^e(\alpha)(\alpha - k\cos\phi_0) \tag{54b}$$

In the above expression $F(z)$ is the modified Fresnel integral defined by [9].

$$F(z) = -2i\sqrt{z}e^{-iz} \int_{\sqrt{z}}^{\infty} e^{it^2} dt \tag{55}$$

For $T_{\pm}^{e2}(\alpha)$, following expression can be written

$$T_+^{e2}(\alpha) = \frac{\Delta_+^e(\alpha)}{\tau_+(\alpha)} \left[J_{11}^e(\alpha) + J_{12}^e(\alpha) + J_{21}^e(\alpha) + J_{22}^e(\alpha)\right] \tag{56a}$$

$$T_-^{e2}(\alpha) = \frac{\Delta_-^e(\alpha)}{\tau_-(\alpha)} \left[\widetilde{J}_{11}^e(\alpha) + \widetilde{J}_{12}^e(\alpha) + \widetilde{J}_{21}^e(\alpha) + \widetilde{J}_{22}^e(\alpha)\right] \tag{56b}$$

By considering (41), (42) and (56a), (56b) we may write

$$T_{+}^{e}(\alpha) \cong \frac{\Delta_{+}^{e}(\alpha)}{\tau_{+}(\alpha)} \left[\sum_{m=1}^{\infty} \frac{\theta_{m}^{+e}}{\alpha_{m}^{e} + \alpha} + \widetilde{I}_{+}^{e}(\alpha) \right. \\ \left. + J_{11}^{e}(\alpha) + J_{12}^{e}(\alpha) + J_{21}^{e}(\alpha) + J_{22}^{e}(\alpha) \right] \quad (57a)$$

$$T_{-}^{e}(\alpha) \cong \frac{\Delta_{-}^{e}(\alpha)}{\tau_{-}(\alpha)} \left[\sum_{m=1}^{\infty} \frac{\psi_{m}^{-e}}{\alpha_{m}^{e} - \alpha} + I_{-}^{e}(\alpha) \right. \\ \left. + \widetilde{J}_{11}^{e}(\alpha) + \widetilde{J}_{12}^{e}(\alpha) + \widetilde{J}_{21}^{e}(\alpha) + \widetilde{J}_{22}^{e}(\alpha) \right] \quad (57b)$$

2.3 Determination of the Fourier Coefficients

At this stage our aim is to solve the unknown Fourier coefficients, f_m^e, s_m^e, r_m^e appearing in the expressions of $T_{\pm}^{e}(\alpha)$ and D_n^e appearing in $u_3^e(x,y)$. If one apply the boundary conditions (12n), (12o) to (30a) that $u_3^e(x,y)$ must satisfies and then use the definitions given by (18) with (27) the relationships between unknowns s_m^e, r_m^e, and D_n^e is obtained as follows:

$$\sum_{n=1}^{\infty} D_n^e \left[e^{i\beta_n^e l} - \frac{\left(1 + \beta_n^e \frac{\eta}{k}\right)}{\left(1 - \beta_n^e \frac{\eta}{k}\right)} e^{-i\beta_n^e l} \right] \cos \tau_n^e y = -2\pi i \sum_{m=1}^{\infty} s_m^e \cos K_m^e y \quad (58a)$$

$$\sum_{n=1}^{\infty} \beta_n^e D_n^e \left[e^{i\beta_n^e l} + \frac{\left(1 + \beta_n^e \frac{\eta}{k}\right)}{\left(1 - \beta_n^e \frac{\eta}{k}\right)} e^{-i\beta_n^e l} \right] \cos \tau_n^e y = -2\pi \sum_{m=1}^{\infty} r_m^e \cos K_m^e y \quad (58b)$$

Now by taking into account (25a), (25b), (57a), (57b) together with (58a) and (58b) the problem of finding the unknown coefficients f_m^e, s_m^e, r_m^e and D_n^e can be reduced into a system of linear algebraic equations with infinite unknowns. By considering the convergence of the solution, a truncation number, N, can be found depending on the sizes of the diffraction geometry. Finally, these unknowns can be determined with high accuracy by using this truncated system.

2.4 Odd Excitation

The solution for odd excitation is similar to that of the even excitation case. Indeed, by considering the configuration shown in Fig. 2d which is equivalent to the odd excitation case and assuming a representation similar to even case with the superscript "e" being replaced by "o", it can be seen that all the boundary and continuity relations in (12a)–(12o) remain valid for the odd excitation case as well, except (12h), (12i), and (12j) which are to be changed as

$$u_2^o(x,o) = 0, \qquad x \in (-\infty, 0) \tag{59a}$$

$$u_3^o(x,o) = 0, \qquad x \in (0, l) \tag{59b}$$

$$u_4^o(x,o) = 0, \qquad x \in (l, \infty) \tag{59c}$$

Taking into account these three different boundary conditions and performing similar procedure in case of even excitation, one obtains the following modified Wiener-Hopf equation related to the odd excitation case,

$$\begin{aligned}\Big[T_-^o(\alpha) + e^{i\alpha l} T_+^o(\alpha)\Big] \frac{\tau(\alpha)}{\Delta^o(\alpha)} \\ = 2ik\tau(\alpha)C(\alpha) + \dot{F}_1^o(\alpha, b) + \sum_{m=1}^{\infty} \left[\left(1 - \alpha\frac{\eta}{k}\right) f_m^o \right. \\ \left. + e^{i\alpha l}(r_m^o - \alpha s_m^o)\right] \frac{K_m^o \cos K_m^o b}{\alpha^2 - (\alpha_m^o)^2}\end{aligned} \tag{60}$$

with

$$\Delta^o(\alpha) = -\frac{e^{iKb}}{k} X^o(\alpha); \quad X^o(\alpha) = \sin Kb + \frac{\eta}{ik} K \cos Kb \tag{61}$$

and $\pm\alpha_m^o$ are the roots of $X^o(\alpha) = 0$ .For the odd excitation case the relations given by (25a) and (25b) must be changed as follows

$$\begin{aligned}T_-^o(-\alpha_m^o) \\ = \frac{\cos K_m^o b}{2K_m^o} \left[\left(\frac{\eta K_m^o}{k}\right)^2 - 1\right] \left(b + \frac{\eta}{ik}\cos^2 K_m^o b\right) \left(1 + \alpha_m^o \frac{\eta}{k}\right) f_m^o\end{aligned} \tag{62a}$$

$$\begin{aligned}T_+^o(\alpha_m^o) \\ = \frac{\cos K_m^o b}{2K_m^o} \left[\left(\frac{\eta K_m^o}{k}\right)^2 - 1\right] \left(b + \frac{\eta}{ik}\cos^2 K_m^o b\right) (r_m^o + \alpha_m^o s_m^o)\end{aligned} \tag{62b}$$

where,

$$K_m^o = K(\pm\alpha_m^o) \tag{63a}$$

and

$$\begin{bmatrix} f_m^o \\ r_m^o \\ s_m^o \end{bmatrix} = \frac{2}{\left(b + \dfrac{\xi}{k}\cos^2 K_m^o b\right)} \int_0^b \begin{bmatrix} f^o(t) \\ r^o(t) \\ s^o(t) \end{bmatrix} \sin K_m^o t dt \tag{63b}$$

The modal representation of $u_3^o(x,y)$ inside the waveguide region is now given by

$$u_3^o(x,y) = \sum_{n=1}^{\infty} D_n^o \left[e^{i\beta_n^o x} - \frac{\left(1 + \beta_n^o \dfrac{\eta}{k}\right)}{\left(1 - \beta_n^o \dfrac{\eta}{k}\right)} e^{-i\beta_n^o x} \right] \sin \tau_n^o y; \tag{64a}$$

with

$$\tau_n^o = \sqrt{k^2 - (\beta_n^o)^2}; \qquad tg\tau_n^o b = \frac{\eta}{ik}\tau_n^o \tag{64b}$$

The pair of coupled integral equations to be used to determine $T_{\mp}^o(\alpha)$ by iteration are easily obtained as,

$$T_-^o(\alpha)\frac{\tau_-(\alpha)}{\Delta_-^o(\alpha)} = \frac{1}{2\pi i}\int_{L_-} T_+^o(\beta)\frac{\tau_-(\beta)}{\Delta_-^o(\beta)}\frac{e^{i\beta l}}{(\beta-\alpha)}d\beta + I_-^o(\alpha) + \Omega_m^{-o}(\alpha) + \omega_m^{-o}(\alpha) \tag{65a}$$

$$T_+^o(\alpha)\frac{\tau_+(\alpha)}{\Delta_+^o(\alpha)} = -\frac{1}{2\pi i}\int_{L_+} T_-^o(\beta)\frac{\tau_+(\beta)}{\Delta_+^o(\beta)}\frac{e^{-i\beta l}}{(\beta-\alpha)}d\beta + \widetilde{I}_+^o(\alpha) + \Theta_m^{+o}(\alpha) + \widetilde{\Theta}_m^{+o}(\alpha) \tag{65b}$$

with

$$\Delta_+^o(\alpha) = \frac{i}{\sqrt{k}}\sqrt{\alpha+k}\left(\frac{\sin kb}{k} + \frac{\eta}{ik}\cos kb\right)^{1/2} e^{\left[\frac{Kb}{\pi}\ln\left(\frac{\alpha+iK}{k}\right)\right]} \cdot e^{\left[\frac{i\alpha b}{\pi}\left(1-C+\ln\frac{2\pi}{kb}+\frac{i\pi}{2}\right)\right]} \times \prod_{n=1}^{\infty}\left(\frac{1+\alpha}{\alpha_n^o}\right) e^{\frac{i\alpha b}{n\pi}} \tag{66a}$$

$$\Delta_-^o(\alpha) = \Delta_+^o(-\alpha) \tag{66b}$$

$$I_-^o(\alpha) = \frac{Ik^2 Z_0}{\sqrt{2\pi}} e^{-i\pi/4} \frac{\tau_-(k\cos\phi_0')\Delta_+^o(k\cos\phi_0')}{\alpha - k\cos\phi_0'} \frac{e^{ik\rho_0'}}{\sqrt{k\rho_0'}} + i\frac{k^2 I Z_0}{K(\alpha)}\tau_-(\alpha)\Delta_+^o(\alpha) e^{i\alpha x_0} e^{iK(y_0-b)}$$

$$\cdot H\left[\mathrm{Im}(\alpha - k\cos\phi_0')\right] \tag{66c}$$

$$\widetilde{I}_+^o(\alpha) = -\frac{Ik^2Z_0}{\sqrt{2\pi}}e^{-i\pi/4}\frac{\tau_+(k\cos\phi_0)\Delta_-^o(k\cos\phi_0)}{\alpha - k\cos\phi_0}\frac{e^{ik\rho_0}}{\sqrt{k\rho_0}} + i\frac{k^2IZ_0}{K(\alpha)}\tau_+(\alpha)\Delta_-^o(\alpha)e^{i\alpha(x_0-l)}e^{iK(y_0-b)} \cdot H\left[\mathrm{Im}(k\cos\phi_0 - \alpha)\right] \tag{66d}$$

$$\Omega_m^{-o}(\alpha) = -\sum_{m=1}^{\infty}\frac{\Delta_+^o(\alpha_m^o)}{\tau_+(\alpha_m^o)}\left(1 - \alpha_m^o\frac{\eta}{k}\right)f_m^o\frac{K_m^o\cos K_m^o b}{2\alpha_m^o(\alpha_m^o - \alpha)} \tag{66e}$$

$$\omega_m^{-o}(\alpha) = -\sum_{m=1}^{\infty}\frac{\Delta_+^o(\alpha_m^o)}{\tau_+(\alpha_m^o)}\left(r_m^o - \alpha_m^o s_m^o\right)e^{i\alpha_m^o l}\frac{K_m^o\cos K_m^o b}{2\alpha_m^o(\alpha_m^o - \alpha)} \tag{66f}$$

$$\Theta_m^{+o}(\alpha) = -\sum_{m=1}^{\infty}\frac{\Delta_+^o(\alpha_m^o)}{\tau_+(\alpha_m^o)}\left(1 + \alpha_m^o\frac{\eta}{k}\right)f_m^o e^{i\alpha_m^o l}\frac{K_m^o\cos K_m^o b}{2\alpha_m^o(\alpha + \alpha_m^o)} \tag{66g}$$

$$\widetilde{\Theta}_m^{+o}(\alpha) = -\sum_{m=1}^{\infty}\frac{\Delta_+^o(\alpha_m^o)}{\tau_+(\alpha_m^o)}\left(r_m^o + \alpha_m^o s_m^o\right)\frac{K_m^o\cos K_m^o b}{2\alpha_m^o(\alpha + \alpha_m^o)} \tag{66h}$$

The first order solutions of $T_\pm^o(\alpha)$, namely $T_\pm^{o1}(\alpha)$ are

$$\begin{aligned} T_+^{o1}(\alpha) &= \frac{\Delta_+^o(\alpha)}{\tau_+(\alpha)}\left[\widetilde{I}_+^o(\alpha) + \sum_{m=1}^{\infty}\frac{\theta_m^{+o}}{\alpha_m^o + \alpha}\right], \\ T_-^{o1}(\alpha) &= \frac{\Delta_-^o(\alpha)}{\tau_-(\alpha)}\left[I_-^o(\alpha) + \sum_{m=1}^{\infty}\frac{\psi_m^{-o}}{\alpha_m^o - \alpha}\right] \end{aligned} \tag{67}$$

with

$$\psi_m^{-o} = -\frac{\Delta_+^o(\alpha_m^o)}{\tau_+(\alpha_m^o)}\left[(r_m^o - \alpha_m^o s_m^o)e^{i\alpha_m^o l} + \left(1 - \alpha_m^o\frac{\eta}{k}\right)f_m^o\right]\frac{K_m^o\cos K_m^o b}{2\alpha_m^o} \tag{68a}$$

$$\theta_m^{+o} = -\frac{\Delta_+^o(\alpha_m^o)}{\tau_+(\alpha_m^o)}\left[(r_m^o + \alpha_m^o s_m^o) + \left(1 + \alpha_m^o\frac{\eta}{k}\right)f_m^o e^{i\alpha_m^o l}\right]\frac{K_m^o\cos K_m^o b}{2\alpha_m^o} \tag{68b}$$

Finally, let us write the second order solution, $T_\pm^{o2}(\alpha)$;

$$T_+^{o2}(\alpha) = \frac{\Delta_+^o(\alpha)}{\tau_+(\alpha)}J^o(\alpha), \quad T_-^{o2}(\alpha) = \frac{\Delta_-^o(\alpha)}{\tau_-(\alpha)}\widetilde{J}^o(\alpha) \tag{69}$$

Here $J^o(\alpha)$ and $\widetilde{J}^o(\alpha)$ can be written as a sum of the two terms,

$$J^o(\alpha) = J_1^o(\alpha) + J_2^o(\alpha), \quad \widetilde{J}^o(\alpha) = \widetilde{J}_1^o(\alpha) + \widetilde{J}_2^o(\alpha) \tag{70}$$

where the terms $J_{1,2}^o(\alpha)$ and $\widetilde{J}_{1,2}^o(\alpha)$ are given by

$$\begin{aligned} J_1^o(\alpha) = &\sqrt{\frac{2}{\pi}}\frac{k^2}{\eta^2}\frac{e^{-i\pi/4}}{(1+T)}\left[\frac{\Delta_+^o(k)}{\tau_+(k)}\right]^2 \frac{e^{ikl}}{\sqrt{kl}}\Bigg\{\widetilde{\widetilde{I}}{}_-^o(-k) \\ &\cdot\{b_1[1-F[kl(1+\cos\phi_0')]]+b_2[1-F[kl(1+\alpha/k)]] \\ &+b_3[1-F[kl(1-T)]]\}+\frac{1}{(\alpha+kT)}\sum_{m=1}^{\infty}\frac{\psi_m^{-o}}{(\alpha_m^o+k)} \\ &\cdot\{F[kl(1-T)]-F[kl(1+\alpha/k)]\}\Bigg\} \end{aligned} \tag{71a}$$

$$\begin{aligned} J_2^o(\alpha) = &\sum_{n=1}^{\infty}\frac{(K_n^o)^2\cos K_n^o b\left[\Delta_+^o(\alpha_n^o)/\tau_+(\alpha_n^o)\right]^2}{\alpha_n^o b\left[X^e(\alpha_n^o)+\dfrac{\eta}{ikb}\cos K_n^o b\right](\alpha_n^o+\alpha)} \\ &\cdot\left\{\left(\sum_{m=1}^{\infty}\frac{\psi_m^{-o}}{(\alpha_m^o+\alpha_n^o)}+I_-^o(-\alpha_n^o)\right)\right\}e^{i\alpha_n^o l} \\ &+i\frac{k^2e^{kb/\xi}\left[\Delta_+^o(kT)/\tau_+(kT)\right]^2}{\xi^3T(kT+\alpha)X^o(kT)} \\ &\cdot\left\{\left(I_-^o(-kT)+\sum_{m=1}^{\infty}\frac{\psi_m^{-o}}{(\alpha_m^o+kT)}\right)e^{ikTl}\right\} \end{aligned} \tag{71b}$$

$$\begin{aligned} \widetilde{J}_1^o(\alpha) = &\sqrt{\frac{2}{\pi}}\frac{k^2}{\eta^2}\frac{e^{-i\pi/4}}{(1+T)}\left[\frac{\Delta_+^e(k)}{\tau_+(k)}\right]^2 \frac{e^{ikl}}{\sqrt{kl}}\Bigg\{\widetilde{\widetilde{I}}{}_+^e(k) \\ &\cdot\{a_1[1-F[kl(1-\cos\phi_0)]]+a_2[1-F[kl(1-\alpha/k)]] \\ &+a_3[1-F[kl(1-T)]]\}+\frac{1}{(\alpha-kT)}\sum_{m=1}^{\infty}\frac{\theta_m^{+e}}{(\alpha_m^e+k)} \\ &\cdot\{F[kl(1-T)]-F[kl(1-\alpha/k)]\}\Bigg\} \end{aligned} \tag{71c}$$

$$\begin{aligned} \widetilde{J}_2^o(\alpha) = &\sum_{n=1}^{\infty}\frac{(K_n^o)^2\cos K_n^o b\left[\Delta_+^o(\alpha_n^o)/\tau_+(\alpha_n^o)\right]^2}{\alpha_n^o b\left[X^e(\alpha_n^o)+\dfrac{\eta}{ikb}\cos K_n^o b\right](\alpha_n^o-\alpha)} \\ &\cdot\left\{\left(\sum_{m=1}^{\infty}\frac{\theta_m^{+o}}{(\alpha_m^o+\alpha_n^o)}+\widetilde{I}_+^o(\alpha_n^o)\right)\right\}e^{i\alpha_n^o l} \end{aligned}$$

$$+ i \frac{k^2 e^{kb/\xi} \left[\Delta_+^o(kT)/\tau_+(kT)\right]^2}{\xi^3 T(kT-\alpha) X^o(kT)}$$
$$\cdot \left\{ \left(I_+^o(-kT) + \sum_{m=1}^{\infty} \frac{\theta_m^{+o}}{(\alpha_m^o + kT)} \right) e^{ikTl} \right\} \tag{71d}$$

For $T_\pm^{e2}(\alpha)$, following expression can be written

$$T_+^{o2}(\alpha) = \frac{\Delta_+^o(\alpha)}{\tau_+(\alpha)} \left[J_{11}^o(\alpha) + J_{12}^o(\alpha) + J_{21}^o(\alpha) + J_{22}^o(\alpha)\right] \tag{72a}$$
$$T_-^{o2}(\alpha) = \frac{\Delta_-^o(\alpha)}{\tau_-(\alpha)} \left[\widetilde{J}_{11}^o(\alpha) + \widetilde{J}_{12}^o(\alpha) + \widetilde{J}_{21}^o(\alpha) + \widetilde{J}_{22}^o(\alpha)\right] \tag{72b}$$

In the above expression, $T_\pm^o(\alpha) \cong T_\pm^{o1}(\alpha) + T_\pm^{o2}(\alpha)$ and $u_3^o(x,y)$ contain unknown coefficients, f_m^o, s_m^o, r_m^o, and D_n^o, respectively. These unknowns can be determined as a similar way to that of the even case.

3. DIFFRACTED FIELDS AND NUMERICAL RESULTS

By considering (7) with (13), (15), and (21) the scattered field in the region for $y > b$, for even and odd excitations can be written as,

$$u_1^{e,o}(x,y) = u_{11}^{e,o}(x,y) + u_{12}^{e,o}(x,y) + u_{13}^{e,o}(x,y) + u_{14}^{e,o}(x,y) \tag{73}$$

with

$$u_{11}^{e,o}(x,y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} T_-^{e,o}(\alpha) \frac{e^{iK(\alpha)(y-b)}}{(1+\eta K(\alpha)/k)} e^{-i\alpha x} d\alpha \tag{74a}$$
$$u_{12}^{e,o}(x,y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} T_+^{e,o}(\alpha) \frac{e^{iK(\alpha)(y-b)}}{(1+\eta K(\alpha)/k)} e^{-i\alpha (x-l)} d\alpha \tag{74b}$$
$$u_{13}^{e,o}(x,y) = \frac{IkZ_0}{4\pi} \int_{-\infty}^{\infty} \frac{(1-\eta K(\alpha)/k)}{(1+\eta K(\alpha)/k)} \frac{e^{-2iK(\alpha)b}}{K(\alpha)}$$
$$\cdot e^{iK(\alpha)(y+y_0)} e^{-i\alpha(x-x_0)} d\alpha \tag{74c}$$
$$u_{14}^{e,o}(x,y) = \frac{IkZ_0}{4\pi} \int_{-\infty}^{\infty} \frac{e^{-iK(\alpha)(y-y_0)}}{K(\alpha)} e^{-i\alpha(x-x_0)} d\alpha \tag{74d}$$

According to the image bisection principle, for the region $y > b$, the field scattered from the original geometry, say $u(x,y)$, can be written in terms of even and odd solutions as,

$$u(x,y) = \frac{u^e(x,y) + u^o(x,y)}{2} \tag{75}$$

The asymptotic evaluation of the integrals in (74a)–(74d) can be accomplished through the saddle point technique. Firstly, let us consider $u_{13}^{e,o}(x,y)$ and $u_{14}^{e,o}(x,y)$. Their asymptotic evaluation gives,

$$\begin{aligned} u_{14}\left(\rho^{+},\phi^{+}\right) &= \left(u_{14}^{e}\left(\rho^{+},\phi^{+}\right)+u_{14}^{o}\left(\rho^{+},\phi^{+}\right)\right)/2 \\ &\approx \frac{IkZ_o}{2\sqrt{2\pi}}e^{-i\pi/4}\frac{e^{ik\rho^{+}}}{\sqrt{k\rho^{+}}} \\ &= u_i \end{aligned} \tag{76a}$$

$$\begin{aligned} u_{13}\left(\rho^{-},\phi^{-}\right) &= \left(u_{13}^{e}\left(\rho^{-},\phi^{-}\right)+u_{13}^{o}\left(\rho^{-},\phi^{-}\right)\right)/2 \\ &\approx -\frac{IkZ_0}{2\sqrt{2\pi}}e^{-i\pi/4}\frac{(1-\eta\sin\phi^{-})}{(1+\eta\sin\phi^{-})}e^{-ik\sin\phi^{-}}\frac{e^{ik\rho^{-}}}{\sqrt{k\rho^{-}}} \\ &= u_r \end{aligned} \tag{76b}$$

where (ρ^{+},ϕ^{+}) and (ρ^{-},ϕ^{-}) are defined by (See Fig. 4)

$$x-x_0=\rho^{+}\cos\phi^{+},\quad y-y_0=-\rho^{+}\sin\phi^{+},\quad \phi^{+}\in(0,\pi) \tag{77a}$$

$$x-x_0=\rho^{-}\cos\phi^{-},\quad y-y_0=\rho^{-}\sin\phi^{-},\quad \phi^{-}\in(0,\pi) \tag{77b}$$

It is easily seen that $u_{14}(\rho^{+},\phi^{+})$ denotes the dominant term in the asymptotic expansion of the incident field emanating by an electrical line source and $u_{13}(\rho^{+},\phi^{+})$ is the field reflected from the upper impedance wall of the waveguide cavity. The asymptotic evaluations of (74a) and (74b) can be made in a way similar to those of (74c) and (74d).

From (41) we have $T_{\pm}^{e,o}(\alpha)\cong T_{\pm}^{e1,o1}(\alpha)+T_{\pm}^{e2,o2}(\alpha)$, and consequently $u_{11}(\rho_1,\phi_1)$ and $u_{12}(\rho_2,\phi_2)$ can be written as follows,

$$\begin{aligned} u_{11}(\rho_1,\phi_1) &= u_{11}^{1}(\rho_1,\phi_1)+u_{11}^{2}(\rho_1,\phi_1), \\ u_{12}(\rho_2,\phi_12) &= u_{12}^{1}(\rho_2,\phi_2)+u_{12}^{2}(\rho_2,\phi_2) \end{aligned} \tag{78}$$

Due to the presence of unit step function in $I_{-}^{e,o}(\alpha)$ and $\widetilde{I}_{+}^{e,o}(\alpha)$ appearing in the expression of $u_{11}^{e1,o1}(x,y)$ and $u_{12}^{e1,o1}(x,y)$, respectively one must consider the cases $\phi_1<\pi-\phi_0'$, $\phi_1>\pi-\phi_0'$ for $u_{11}^{1}(\rho_1,\phi_1)$ and $\phi_2<\pi-\phi_0$, $\phi_2>\pi-\phi_0$ for $u_{12}^{1}(\rho_1,\phi_1)$ separately. The results for $u_{11}^{1}(\rho_1,\phi_1)$ are as follows;

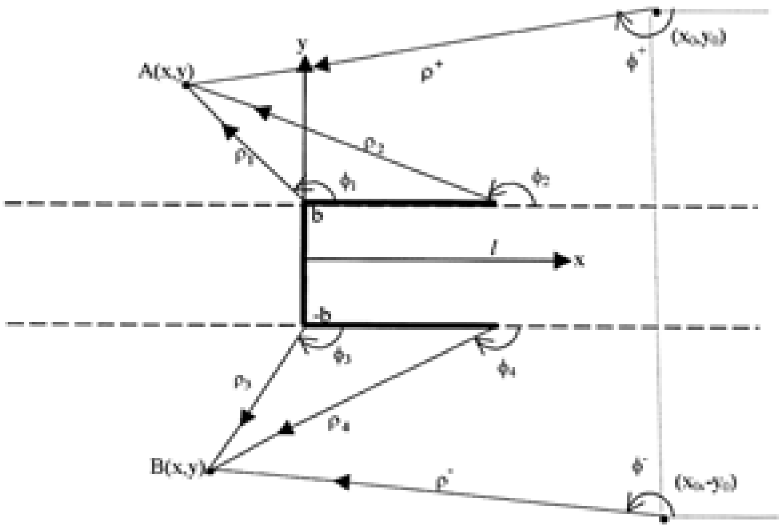


Figure 4. Various angles and distances.

The Case of $\phi_1 < \pi - \phi_0'$

In this case, since the argument of the unit step function appearing in $I_-^{e,o}(\alpha)$ is negative then the contribution of this term is zero. Taking into account this fact, the following result is obtained.

$$\begin{aligned} u_{11}^1(\rho_1, \phi_1) \\ &= \frac{u_{11}^{e1}(\rho_1, \phi_1) + u_{11}^{o1}(\rho_1, \phi_1)}{2} \\ &\approx u_i(M) D_{11}^{11}(\phi_1, \phi_0') \left[\sec \frac{1}{2}(\phi_1 + \phi_0') F\left(2k\rho_1 \cos^2 \frac{1}{2}(\phi_1 + \phi_0')\right) \right. \\ &\quad \left. + \sec \frac{1}{2}(\phi_1 - \phi_0') F\left(2k\rho_1 \cos^2 \frac{1}{2}(\phi_1 - \phi_0')\right) \right] \frac{e^{ik\rho_1}}{\sqrt{k\rho_1}} \\ &\quad + D_{11}^{12}(\phi_1) \frac{e^{ik\rho_1}}{\sqrt{k\rho_1}} \end{aligned} \tag{79}$$

where we have put

$$D_{11}^{11}(\phi_1,\phi_0') = \frac{\left(\begin{array}{l} ke^{i3\pi/4}\sin\phi_1\sin\phi_0' \\ \qquad \left[\Delta_+^e(k\cos\phi_1)\Delta_+^e(k\cos\phi_0')+\Delta_+^o(k\cos\phi_1)\Delta_+^o(k\cos\phi_0')\right]\end{array}\right)}{\sqrt{2\pi}(1+\eta\sin\phi_1)(1+\eta\sin\phi_0')\tau_+(k\cos\phi_1)\tau_+(k\cos\phi_0')} \tag{80a}$$

$$D_{11}^{12}(\phi_1) = \frac{ke^{-i\pi/4}\tau_-(k\cos\phi_1)}{2\sqrt{2\pi}}\left[\sum_{m=1}^{\infty}\frac{\Delta_+^e(k\cos\phi_1)}{(\alpha_m^e+k\cos\phi_1)}\psi_m^{-e} + \sum_{m=1}^{\infty}\frac{\Delta_+^o(k\cos\phi_1)}{(\alpha_m^o+k\cos\phi_1)}\psi_m^{-o}\right] \tag{80b}$$

and

$$u_i(M) = \frac{IkZ_0}{2\sqrt{2\pi}}e^{-i\pi/4}\frac{e^{ik\rho_0'}}{\sqrt{k\rho_0'}} \tag{80c}$$

is the incident field evaluated at the edge of $M(0,b)$ (See Fig. 5). (ρ_1,ϕ_1) are the cylindrical coordinates defined by (See Fig. 4)

$$x=\rho_1\cos\phi_1, \quad y-b=\rho_1\sin\phi_1, \quad \phi_1\in(0,\pi) \tag{81}$$

The Case of $\phi_1 > \pi-\phi_0'$

In this case, the argument of the unit step function is positive, and the contribution of this term gives the opposite of the reflected field. When this term is added to the terms obtained for the case $\phi_1<\pi-\phi_0'$, we get;

$$u_{11}^1 \approx \begin{cases} u_{11}^1, & \phi_1<\pi-\phi_0' \\ u_{11}^1-u_r, & \phi_1>\pi-\phi_0' \end{cases} \tag{82}$$

Similarly, the asymptotic solution of $u_{11}^2(\rho_1,\phi_1)$ can easily be obtained as,

$$u_{11}^2(\rho_1,\phi_1) \approx \frac{e^{-i\pi/4}}{2\sqrt{2\pi}}\frac{k\sin\phi_1}{(1+\eta\sin\phi_1)}\left[T_+^{e2}(-k\cos\phi_1) + T_+^{o2}(-k\cos\phi_1)\right]\frac{e^{ik\rho_1}}{\sqrt{k\rho_1}} \tag{83}$$

with $T_{\pm}^{e2,o2}(\alpha)$ being given by (56a), (56b), (72a), and (72b).

The asymptotic expressions of $u_{12}^1(\rho_2,\phi_2)$ and $u_{12}^2(\rho_2,\phi_2)$ can be obtained in a way similar to that described for $u_{11}^1(\rho_1,\phi_1)$ and $u_{11}^2(\rho_1,\phi_1)$, respectively. The result is,

$$u_{12}^1 \approx \begin{cases} u_{12}^1, & \phi_2 > \pi - \phi_0 \\ u_{12}^1 - u_r, & \phi_2 < \pi - \phi_0 \end{cases} \tag{84}$$

where,

$$\begin{aligned} u_{12}^1(\rho_2,\phi_2) & \\ \approx u_i(N) D_{12}^{11}(\phi_2,\phi_0) & \left[\sec\frac{1}{2}(\phi_2+\phi_0) F\left(2k\rho_2\cos^2\frac{1}{2}(\phi_2+\phi_0)\right)\right. \\ & \left. + \sec\frac{1}{2}(\phi_2-\phi_0) F\left(2k\rho_2\cos^2\frac{1}{2}(\phi_2-\phi_0)\right)\right]\frac{e^{ik\rho_2}}{\sqrt{k\rho_2}} \\ & + D_{12}^{12}(\phi_2)\frac{e^{ik\rho_2}}{\sqrt{k\rho_2}} \end{aligned} \tag{85a}$$

with

$$D_{12}^{11}(\phi_2,\phi_0) = \frac{\left(\begin{aligned} & ke^{i\pi/4}\sin\phi_2\sin\phi_0 \\ & \quad \left[\Delta_-^e(k\cos\phi_2)\Delta_-^e(k\cos\phi_0) + \Delta_-^o(k\cos\phi_2)\Delta_-^o(k\cos\phi_0)\right] \end{aligned}\right)}{\sqrt{2\pi}(1+\eta\sin\phi_2)(1+\eta\sin\phi_0)\tau_-(k\cos\phi_2)\tau_-(k\cos\phi_0)} \tag{85b}$$

$$\begin{aligned} D_{12}^{12}(\phi_2) &= \frac{ke^{-i\pi/4}\tau_+(k\cos\phi_2)}{2\sqrt{2\pi}}\left[\sum_{m=1}^{\infty}\frac{\Delta_-^e(k\cos\phi_2)}{(\alpha_m^e - k\cos\phi_2)}\theta_m^{+e}\right. \\ & \left. + \sum_{m=1}^{\infty}\frac{\Delta_-^o(k\cos\phi_2)}{(\alpha_m^o - k\cos\phi_2)}\theta_m^{+o}\right] \end{aligned} \tag{85c}$$

$$u_i(N) = \frac{IkZ_0}{2\sqrt{2\pi}} e^{-i\pi/4}\frac{e^{ik\rho_0}}{\sqrt{k\rho_0}} \tag{85d}$$

Here $u_i(N)$ is the incident field evaluated at the edge of $N(l,b)$. (ρ_2,ϕ_2) are defined by

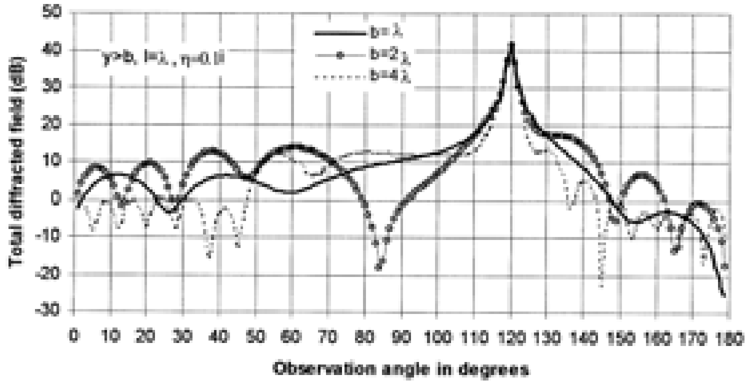
$$x - l = \rho_2\cos\phi_2, \quad y - b = \rho_2\sin\phi_2, \quad \phi_2 \in (0,\pi) \tag{86}$$

(See Fig. 4). Finally, the asymptotic expression of $u_{12}^2(\rho_2, \phi_2)$ is

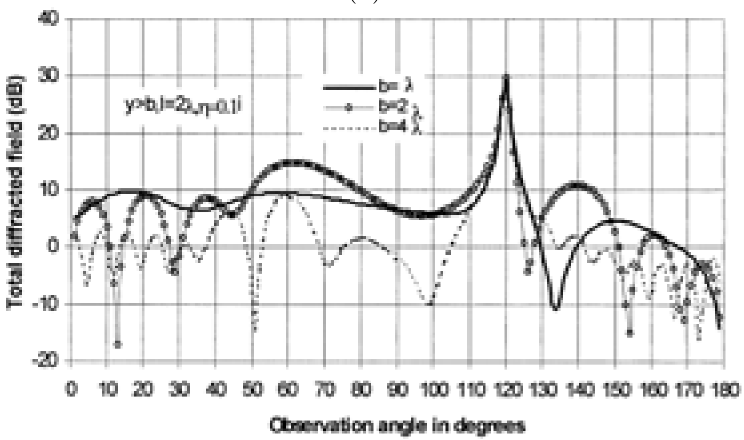
$$u_{12}^2(\rho_2, \phi_2) \approx \frac{e^{-i\pi/4}}{2\sqrt{2\pi}} \frac{k \sin\phi_2}{(1+\eta\sin\phi_2)} \left[T_+^{e2}(-k\cos\phi_2) + T_+^{o2}(-k\cos\phi_2)\right] \frac{e^{ik\rho_2}}{\sqrt{k\rho_2}} \tag{87}$$

According to the results obtained from the above analysis, the region $y > b$ can be divided into three subregions, B_1^1, B_1^2, and B_1^3, where different constituents of the total field are present (See Fig. 5). In the region B_1^1 and B_1^3, we observe the fields u_{11} and u_{12} which denote the diffracted fields emanating from the edge $N(l, b)$ and $M(0, b)$, respectively and u_i which denotes the field incident from the electrical line source. However, in the region B_1^2 in addition to the terms observed in B_1^1 and B_1^3, the field reflected from the upper impedance plane and denoted by u_r is also present as expected.

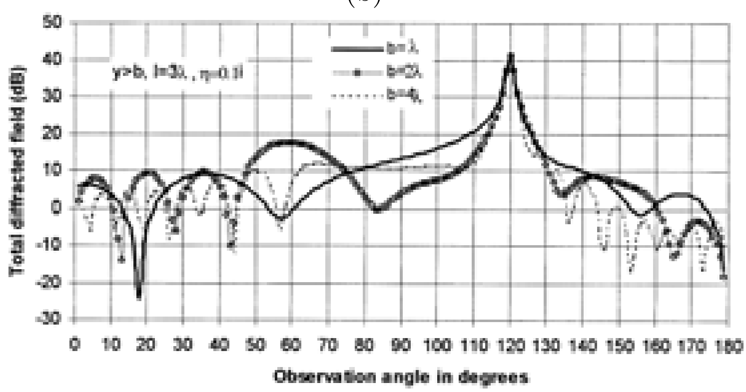
In order to illustrate the effects of the different cavity sizes and surface impedances on the scattering characteristics of the waveguide cavity representative numerical examples are presented. These figures shows the variation of the total diffracted field amplitude, $20\log_{10}|u_1\sqrt{k\rho}|$, with the observation angle. Figs. 6a–6c display the variation of $20\log_{10}|u_1\sqrt{k\rho}|$ with the observation angle for different values of the cavity apertures. In These figures the angle of incidence, ϕ_0, is fixed at $60°$, the surface impedance is chosen as $\xi = 0.1$. In Figs. 6a, 6b, and 6c the length of the cavity is set to $l = \lambda$, $l = 2\lambda$, and $l = 3\lambda$, respectively. Fig. 7 shows the diffracted field amplitude with the observation angle with different incidence angle, $\phi_0 = 120°$. The variation depicted in Fig. 8 is given for the different values of surface impedances for $\phi_0 = 60°$. It is seen that in all numerical examples, the total diffracted field has maximum values along the direction of the reflection boundary given by $\phi = 120°$ for $\phi_0 = 60°$ and $\phi = 60°$ for $\phi_0 = 120°$ as expected. The peaks along these reflection boundaries become sharper and the total diffracted field decreases with an increase of the cavity aperture, b, for a fixed l. On viewing the curves for $b = 4\lambda$ in Figs. 6a, 6b, 6c, and 7 it is found that there are no remarkable differences between the scattering characteristics within $0° < \theta < 50°$ and those within $130° < \theta < 180°$ and total diffracted field oscillates rapidly within these ranges. Whereas outside these ranges oscillation is slow and variation of the total diffracted field is smooth. Fig. 8 has also the same features with the figures



(a)



(b)



(c)

Figure 6. Total diffracted field versus the observation angle, ϕ, for $\phi_0 = 60°$, for different values of b.

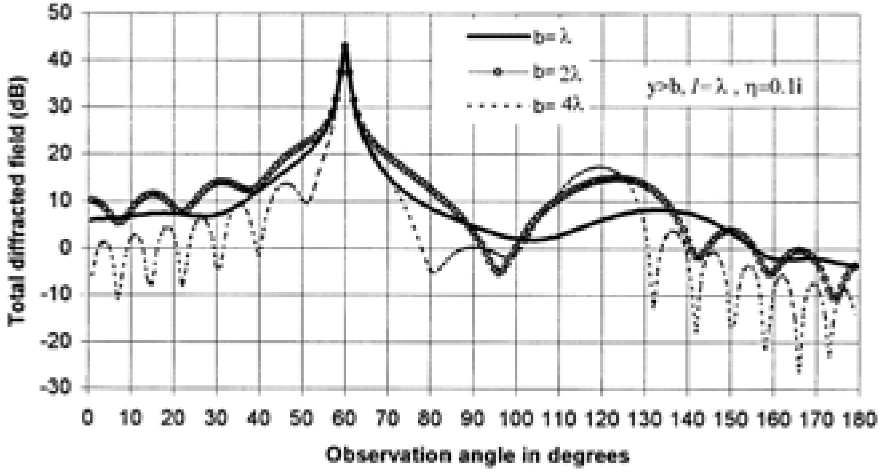


Figure 7. Total diffracted field versus the observation angle, ϕ, for $\phi_0 = 120°$, for different values of b.

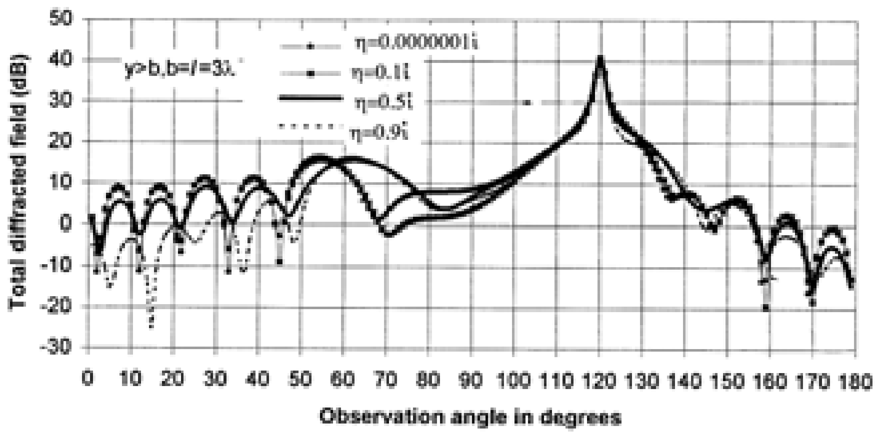


Figure 8. Total diffracted field versus the observation angle, ϕ, for $\phi_0 = 60°$, for different values of η.

considered above. In this figure, the value of $\eta = 0.000001i$ is chosen to represent perfectly conducting case, then it is possible to make a comparison between impedance case and perfectly conducting case. As it is seen the total diffracted field decreases with an increase of surface admittance, η.

4. CONCLUDING REMARKS

The diffraction of electromagnetic waves by an open ended parallel plate waveguide cavity with impedance walls has been investigated rigorously through the Wiener-Hopf technique. In order to get a scalar type Wiener-Hopf equation, we propose a hybrid method which consists of a combination of Fourier transform technique and modal expansion. Since this problem is important in several engineering applications, representative numerical examples have also been given. It has been shown that the total diffracted field can be reduced by coating the walls of the cavity with thin dielectric materials. Note that this problem can easily be extended to the case where the cavity is filled by a dielectric material and to the case where the inner and outer impedances of the cavity are different.

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