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ELECTROMAGNETIC SCATTERING BY MULTILAYERED CHIRAL-MEDIA STRUCTURES: A SCATTERING-TO-RADIATION TRANSFORM

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1. INTRODUCTION

In electromagnetic wave theory, there exist two well-known distinct areas. One of them is the electromagnetic radiation due to an antenna located in an environment where scatterers of different geometries are present, and the other is the electromagnetic scattering of an electromagnetic plane wave by objects of different geometries. These two areas are almost parallelly developed. This paper aims at relating the scattering problem to the radiation problem by utilizing the volumetric distribution of an electric current source that is assumed to be located at infinity so as to generate the plane wave. Therefore, the scattering problem can be considered as a specific radiation problem where the radiated source is located at infinity, which forms the scattering-to-radiation transform.

During the past few decades, much attention has been focused on the interaction between chiral media and electromagnetic fields, as a result of numerous applications in the electromagnetic scattering, antenna radiation, and radio wave propagation. The historical background, and a general description, of the subject of electromagnetic chirality and its applications can be found, as examples, in literature [1–8]. Among the research work about the electromagnetic wave scattering by chiral objects, exact (or analytic) and asymptotic (or numerical) solutions to electromagnetic scattering by chiral sphere(s) [9–13], cylinder(s) [14–17], spherical shells [18] and a spheroid [19] are available in the literature. In the present paper, the method of vector wave eigenfunction expansion [10, 16, 19, 20] is utilized to calculate the electromagnetic scattering by a planarly stratified chiral medium structure, a multilayered chiral cylinder, and a multilayered chiral sphere. Electromagnetic fields in free space as well as in each layer of the multilayered chiral media of three geometries are expanded in terms of the corresponding vector eigenfunctions in their convenient coordinates systems, and the coefficients of the scattered electric field are derived by matching boundary conditions satisfied by the transmission coefficient matrices [21, 22] at all dielectric interfaces. By using the asymptotic forms for large-argument Hankel functions and spherical Hankel functions, the scattered electromagnetic fields in far-zone are obtained.

The dyadic Green's function (DGF) technique [23–26], is a very powerful and elegant method for solving boundary-value problems, and has been extensively developed and used to solve the fundamental electromagnetic chirality problems of various geometries [1, 2, 5, 27–29]. The developments and applications of dyadic Green's functions can be found from the existing published work involving different physical geometries, e.g., the unbounded chiral media [3, 30–35], the planarly stratified media [36–42], the spherically multilayered chiral media [36, 43–45], and the cylindrically multilayered media [46, 47].

Recently, Li et al. [48] presented an idea of making the plane wave in electromagnetic scattering equivalent to an electromagnetic field due to a point source located at infinity in a unbounded isotropic medium. Also, the method for deriving the volumetric current distribution is developed. Based on the dyadic Green's functions for chiral media of different geometries and the assumed volumetric current distribution, electromagnetic fields are formulated in terms of integrals consisting the derived volumetric current distribution located at infinity and the dyadic Green's functions. Both scattering theory and radiation theory are utilized in the paper for comparison. Good agreements between the results obtained from the two methods are observed. Hence, the scattering-to-radiation transform in three basic coordinates systems, i.e., rectangular, cylindrical and spherical coordinates systems, are established and verified.

2. THE CHIRAL-MEDIUM WAVE EQUATION

Consider an N-layered geometry of the chiral medium. The incident waves of both parallel and perpendicular polarizations are assumed to illuminate in the first layer towards the $(N-1)$-layered medium. In the following analysis the time dependence, $e^{-i\omega t}$, is assumed and is suppressed throughout the paper. A chiral medium of a layer is usually characterized by the following set of constitutive relations: [24, 35, 38, 47]

$$\boldsymbol{D}_f = \varepsilon_f \boldsymbol{E} + i\chi_f \boldsymbol{H}, \tag{1a}$$
$$\boldsymbol{B}_f = \mu_f \boldsymbol{H} - i\chi_f \boldsymbol{E}, \tag{1b}$$

where ε_f, μ_f, and χ_f are the medium's permittivity, permeability and chirality parameter, respectively, and $f = 1, 2, \cdots, N$ denoting the field region. If ε_f, μ_f or χ_f are complex, the medium is considered to be lossy. If χ_f =0, then (1a) and (1b) reduce to the constitutive relations for an achiral medium.

In a chiral medium without source distributions, the wave equations with constitutive relation are

$$\nabla^2 \begin{bmatrix} \boldsymbol{E_f} \\ \boldsymbol{H_f} \end{bmatrix} + [k]^2 \begin{bmatrix} \boldsymbol{E_f} \\ \boldsymbol{H_f} \end{bmatrix} = 0, \tag{2a}$$

where

$$[k] = \begin{bmatrix} -\omega\chi_f & -i\omega\mu_f \\ i\omega\epsilon_f & -\omega\chi_f \end{bmatrix}. \tag{2b}$$

By following Bohren or Lindell [9, 8], the coupling caused by $[k]$ in the wave equation can be removed by diagonalizing $[k]$ such that

$$[A]^{-1}[k][A] = \begin{bmatrix} -k_f^{(R)} & 0 \\ 0 & k_f^{(L)} \end{bmatrix}. \tag{3}$$

A simple form of $[A]$ is found to be

$$[A] = \begin{bmatrix} 1 & 1 \\ -\frac{i}{\eta_f} & \frac{i}{\eta_f} \end{bmatrix}, \tag{4}$$

with the achiral wave impedance given by :

$$\eta_f = \sqrt{\frac{\mu_f}{\epsilon_f}}. \tag{5}$$

The propagation constant k_f in each layer of the multilayered medium is designated generally as [47]:

$$k_f^2 = \omega^2(\mu_f \varepsilon_f - \chi_f^2), \tag{6}$$

and a symbol ξ_f is defined by

$$\xi_f = \omega \chi_f. \tag{7}$$

Hence, there are two circularly polarized modes present in the unbounded medium, i.e., the *r*ight- and *l*eft-handed *c*ircularly *p*olarized (RCP and LCP) waves. Their corresponding wave numbers in (3) are given by

$$k_f^{(R)} = \xi_f + \omega\sqrt{\mu_f \varepsilon_f}, \tag{8a}$$

$$k_f^{(L)} = -\xi_f + \omega\sqrt{\mu_f \varepsilon_f}. \tag{8b}$$

Define $(\boldsymbol{E}_f, \boldsymbol{H}_f)$ in terms of $(\boldsymbol{E}_f^R, \boldsymbol{E}_f^L)$, i.e.,

$$\begin{bmatrix} \boldsymbol{E_f} \\ \boldsymbol{H_f} \end{bmatrix} = [A] \begin{bmatrix} \boldsymbol{E_f^{(R)}} \\ \boldsymbol{E_f^{(L)}} \end{bmatrix} \tag{9}$$

where $\boldsymbol{E_f^{(R)}}$ and $\boldsymbol{E_f^{(L)}}$ are the electric fields of right and left circularly polarized waves with propagation constants $k_f^{(R)}$ and $k_f^{(L)}$. Thus the decoupled source-free wave equations in chiral media can be written as [16]:

$$\nabla^2 \begin{bmatrix} \boldsymbol{E_f^{(R)}} \\ \boldsymbol{E_f^{(L)}} \end{bmatrix} + \begin{bmatrix} {k_R}^2 \boldsymbol{E_f^{(R)}} \\ {k_L}^2 \boldsymbol{E_f^{(L)}} \end{bmatrix} = 0.$$

Subsequently, we will consider three general cases for the development and application of the scattering-to-radiation transform, namely spherical, cylindrical, and planar structures. Because the spherical coordinates system has the highest-degree symmetry, the plane wave scattering by a multilayered chiral sphere is considered firstly. The second case is the scattering by a multilayered chiral cylinder and the last one is the plane wave scattering by stratified planarly chiral media.

3. ELECTROMAGNETIC SCATTERING BY A MULTILAYERED CHIRAL SPHERE

In this section, two techniques are to be developed and compared. The first is the scattering theory and the other is the radiation theory.

3.1 Scattering Theory Using Eigenfunction Expansion Method

3.1.1 The Vector Wave Function Expansion

Consider two pairs of incident electromagnetic waves: parallel (I) and perpendicular (II) polarizations incident at an arbitrary angle on a sphere whose center o is at the origin of the Cartesian coordinates system. The incident wave fields are expressed by:

$$\boldsymbol{E}_I^i = E_I(\cos\alpha\widehat{\boldsymbol{x}} - \sin\alpha\widehat{\boldsymbol{z}})e^{ik_0(x\sin\alpha + z\cos\alpha)}, \tag{11a}$$

$$\boldsymbol{H}_I^i = \frac{k_0 E_I}{\omega\mu_0}\widehat{\boldsymbol{y}}e^{ik_0(x\sin\alpha + z\cos\alpha)}, \tag{11b}$$

and

$$\boldsymbol{E}_{II}^i = E_{II}\widehat{\boldsymbol{y}}e^{ik_0(x\sin\alpha + z\cos\alpha)}, \tag{12a}$$

$$\boldsymbol{H}_{II}^i = -\frac{k_0 E_{II}}{\omega\mu_0}(\cos\alpha\widehat{\boldsymbol{x}} - \sin\alpha\widehat{\boldsymbol{z}})e^{ik_0(x\sin\alpha + z\cos\alpha)}, \tag{12b}$$

where E_I and E_{II} are the amplitude of the incident electric field, α is the incident angle with respect to the $\widehat{\boldsymbol{z}}$-axis. It is assumed for convenience that the incident wave lies on the $\widehat{\boldsymbol{x}}\widehat{\boldsymbol{z}}$-plane, i.e., $\phi' = 0$. The incident electromagnetic fields can be expanded in terms of spherical vector wave eigenfunctions defined in the spherical coordinates system as follows [23]:

$$\begin{aligned}\boldsymbol{M}_{\substack{e\\o}mn}(k) &= \mp\frac{m z_n(kr)}{\sin\theta}P_n^m(\cos\theta)\frac{\sin}{\cos}m\phi\widehat{\boldsymbol{\theta}} \\ &\quad - z_n(kr)\frac{\partial P_n^m(\cos\theta)}{\partial\theta}\frac{\cos}{\sin}m\phi\widehat{\boldsymbol{\phi}},\end{aligned} \tag{13a}$$

$$\begin{aligned}\boldsymbol{N}_{\substack{e\\o}mn}(k) &= \frac{n(n+1)z_n(kr)}{kr}P_n^m(\cos\theta)\frac{\cos}{\sin}m\phi\widehat{\boldsymbol{r}} \\ &\quad + \frac{\partial[rz_n(kr)]}{kr\partial r}\frac{\partial P_n^m(\cos\theta)}{\partial\theta}\frac{\cos}{\sin}m\phi\widehat{\boldsymbol{\theta}} \\ &\quad \mp\frac{m}{\sin\theta}\frac{\partial[rz_n(kr)]}{kr\partial r}P_n^m(\cos\theta)\frac{\sin}{\cos}m\phi\widehat{\boldsymbol{\phi}},\end{aligned} \tag{13b}$$

where $z_n(kr)$ represents the spherical Bessel functions of order n, and $P_n^m(\cos\theta)$ identifies the associated Legendre function of the first kind with the order (n, m).

The incident waves under the two polarizations have, as introduced by Morrison and Cross [49], the following forms:

$$\boldsymbol{E}^i_{\substack{I\\II}} = \sum_{n=1}^{\infty}\sum_{m=0}^{n}\left[P^i_{\substack{o\\e}mn}\boldsymbol{M}^{(1)}_{\substack{o\\e}mn}(k_0) + Q^i_{\substack{e\\o}mn}\boldsymbol{N}^{(1)}_{\substack{e\\o}mn}(k_0)\right], \tag{14a}$$

$$\boldsymbol{H}^i_{\substack{I\\II}} = \frac{ik_0}{\omega\mu_0}\sum_{n=1}^{\infty}\sum_{m=0}^{n}\left[P^i_{\substack{o\\e}mn}\boldsymbol{N}^{(1)}_{\substack{o\\e}mn}(k_0) + Q^i_{\substack{e\\o}mn}\boldsymbol{M}^{(1)}_{\substack{e\\o}mn}(k_0)\right], \tag{14b}$$

where the spherical Bessel functions of the first kind, i.e., $z_n(k_0 r) = j_n(k_0 r)$, are used in the above vector wave functions, the orthogonal properties of $\boldsymbol{M}_{\substack{e\\o}mn}(k_0)$ and $\boldsymbol{N}_{\substack{e\\o}mn}(k_0)$ are considered, and the coefficients of the expanded incident electromagnetic wave, $P^i_{\substack{e\\o}mn}$ and $Q^i_{\substack{e\\o}mn}$, are given by [50, 51]:

$$P^i_{\substack{o\\e}mn} = i^n(2-\delta_{m0})\mathcal{N}_{mn}\left\{\begin{array}{c}\dfrac{mP_n^m(\cos\alpha)}{\sin\alpha}E_I\\ -\dfrac{\partial P_n^m(\cos\alpha)}{\partial\alpha}E_{II}\end{array}\right\}, \tag{15a}$$

$$Q^i_{\substack{e\\o}mn} = -i^{n+1}(2-\delta_{m0})\mathcal{N}_{mn}\left\{\begin{array}{c}\dfrac{\partial P_n^m(\cos\alpha)}{\partial\alpha}E_I\\ \dfrac{mP_n^m(\cos\alpha)}{\sin\alpha}E_{II}\end{array}\right\}, \tag{15b}$$

with δ_{mn} $(=1$ for $m=n$; and 0 for $m\neq n)$ denoting the Kronecker symbol and $\mathcal{N}_{mn}$ being the normalization coefficient given by

$$\mathcal{N}_{mn} = \frac{(2n+1)}{n(n+1)}\frac{(n-m)!}{(n+m)!}. \tag{16}$$

For simplicity, we make abbreviation as follows:

$$a = \frac{mP_n^m(\cos\alpha)}{\sin\alpha}, \tag{17a}$$

$$b = \frac{\partial P_n^m(\cos\alpha)}{\partial\alpha}. \tag{17b}$$

The electric field in a multilayered chiral sphere in the eigenfunction expansion form can be written as a superposition of right-handed and left-handed circularly polarized fields. And the right-handed and left-handed circularly polarized field modes can be expressed using vector wave functions as follows [16]:

$$\boldsymbol{E}_{Rmn}^{(p)} = \boldsymbol{M}_{\substack{e\\o}mn}^{(p)}(k^{(R)}) + \boldsymbol{N}_{\substack{e\\o}mn}^{(p)}(k^{(R)}), \tag{18a}$$

$$\boldsymbol{E}_{Lmn}^{(p)} = \boldsymbol{M}_{\substack{e\\o}mn}^{(p)}(k^{(L)}) - \boldsymbol{N}_{\substack{e\\o}mn}^{(p)}(k^{(L)}). \tag{18b}$$

where the superscript p equals 1 or 3 representing the first type of spherical Bessel function and the first kind of spherical Hankel function.

The electric field and magnetic field in f-th layer can be expressed in the following form:

$$\boldsymbol{E}_{mn,f}^{(p)} = \boldsymbol{E}_{Lmn,f}^{(p)} + \boldsymbol{E}_{Rmn,f}^{(p)}, \tag{19a}$$

$$\boldsymbol{H}_{mn,f}^{(p)} = -\frac{i}{\eta_f}\left(-\boldsymbol{E}_{Lmn,f}^{(p)} + \boldsymbol{E}_{Rmn,f}^{(p)}\right). \tag{19b}$$

Under the spherical coordinates, the electromagnetic fields usually consist of the radially outgoing- and incoming-propagation wave modes. Hence, the electric and magnetic fields in the layer f ranging from the second to the $(N-1)$-th region are expressed as:

$$\begin{aligned}\boldsymbol{E}_f = \sum_{m,n} \Big\{ & C_{\substack{e\\o}1f}\left[\boldsymbol{M}_{\substack{e\\o}mn}^{(3)}(k_f^{(R)}) + \boldsymbol{N}_{\substack{e\\o}mn}^{(3)}(k_f^{(R)})\right] \\ & + C_{\substack{e\\o}2f}\left[\boldsymbol{M}_{\substack{e\\o}mn}^{(3)}(k_f^{(L)}) - \boldsymbol{N}_{\substack{e\\o}mn}^{(3)}(k_f^{(L)})\right] \\ & + C_{\substack{e\\o}3f}\left[\boldsymbol{M}_{\substack{e\\o}mn}^{(1)}(k_f^{(R)}) + \boldsymbol{N}_{\substack{e\\o}mn}^{(1)}(k_f^{(R)})\right] \\ & + C_{\substack{e\\o}4f}\left[\boldsymbol{M}_{\substack{e\\o}mn}^{(1)}(k_f^{(L)}) - \boldsymbol{N}_{\substack{e\\o}mn}^{(1)}(k_f^{(L)})\right]\Big\}, \end{aligned} \tag{20a}$$

$$\begin{aligned}\boldsymbol{H}_f = -\frac{i}{\eta_f}\sum_{m,n} \Big\{ & C_{\substack{e\\o}1f}\left[\boldsymbol{M}_{\substack{e\\o}mn}^{(3)}(k_f^{(R)}) + \boldsymbol{N}_{\substack{e\\o}mn}^{(3)}(k_f^{(R)})\right] \\ & - C_{\substack{e\\o}2f}\left[\boldsymbol{M}_{\substack{e\\o}mn}^{(3)}(k_f^{(L)}) - \boldsymbol{N}_{\substack{e\\o}mn}^{(3)}(k_f^{(L)})\right] \\ & + C_{\substack{e\\o}3f}\left[\boldsymbol{M}_{\substack{e\\o}mn}^{(1)}(k_f^{(R)}) + \boldsymbol{N}_{\substack{e\\o}mn}^{(1)}(k_f^{(R)})\right] \\ & - C_{\substack{e\\o}4f}\left[\boldsymbol{M}_{\substack{e\\o}mn}^{(1)}(k_f^{(L)}) - \boldsymbol{N}_{\substack{e\\o}mn}^{(1)}(k_f^{(L)})\right]\Big\}. \end{aligned} \tag{20b}$$

While only the inward waves exist in the inner-most layer, the outward waves exist in the outer-most layer. Therefore, the scattering coefficients corresponding to the outgoing waves in the inner-most layer and to the incoming waves in the outer-most layer must vanish. The electric field in the out-most and inner-most layer are written, respectively, as follows:

$$\begin{aligned}\boldsymbol{E}_1 &= \boldsymbol{E}^i + \boldsymbol{E}^s \\ &= \boldsymbol{E}^i + \sum_{m,n}\Big\{C_{\substack{e\\o}11}\Big[\boldsymbol{M}^{(3)}_{\substack{e\\o}mn}(k_0) + \boldsymbol{N}^{(3)}_{\substack{e\\o}mn}(k_0)\Big] \\ &\quad + C_{\substack{e\\o}21}\Big[\boldsymbol{M}^{(3)}_{\substack{e\\o}mn}(k_0) - \boldsymbol{N}^{(3)}_{\substack{e\\o}mn}(k_0)\Big]\Big\}, \qquad (21a)\\ \boldsymbol{E}_N &= \sum_{m,n}\Big\{C_{\substack{e\\o}3N}\Big[\boldsymbol{M}^{(1)}_{\substack{e\\o}mn}(k_N^{(R)}) + \boldsymbol{N}^{(1)}_{\substack{e\\o}mn}(k_N^{(R)})\Big] \\ &\quad - C_{\substack{e\\o}4N}\Big[\boldsymbol{M}^{(1)}_{\substack{e\\o}mn}(k_N^{(L)}) - \boldsymbol{N}^{(1)}_{\substack{e\\o}mn}(k_N^{(L)})\Big]\Big\}, \qquad (21b)\end{aligned}$$

where k_0 is the wave number outside the multilayered sphere, given by

$$k_0 = \omega\sqrt{\mu_0\epsilon_0}. \qquad (22)$$

3.1.2 Determination of Scattering Coefficients by Boundary Conditions

The electric and magnetic fields satisfy the following boundary conditions at the spherical interfaces $r = a_j \quad (j = 1, 2, \cdots, N-1):$

$$\widehat{\boldsymbol{r}} \times \boldsymbol{E_f} = \widehat{\boldsymbol{r}} \times \boldsymbol{E}_{(f+1)}, \qquad (23a)$$

$$\widehat{\boldsymbol{r}} \times \boldsymbol{H_f} = \widehat{\boldsymbol{r}} \times \boldsymbol{H}_{(f+1)}. \qquad (23b)$$

Without any loss of generality of the problem, we extend (23a)–(23b) into a linear equation system. To simplify the complicated algebraic calculations, let us introduce the following operators

$$\Im_{im}^{(r,l)} = j_n(k_i^{(r,l)} a_m), \qquad (24a)$$

$$\hbar_{im}^{(r,l)} = h_n^{(1)}(k_i^{(r,l)} a_m), \qquad (24b)$$

$$\partial\Im_{im}^{(r,l)} = \left.\frac{1}{\rho}\frac{d[\rho j_n(\rho)]}{d\rho}\right|_{\rho=k_i^{(r,l)}a_m}, \qquad (24c)$$

$$\partial\hbar_{im}^{(r,l)} = \left.\frac{1}{\rho}\frac{d[\rho h_n^{(1)}(\rho)]}{d\rho}\right|_{\rho=k_i^{(r,l)}a_m}; \qquad (24d)$$

$$i = 1, 2, \cdots, N,$$

$$m = i - 1 \text{ and } i.$$

Writing the linear equation system of the coefficients in matrix form, we have the following equation

$$\mathbf{F}_f C_f = \mathbf{F}_{f+1} C_{f+1}, \tag{25}$$

where the parameter and coefficient matrices are defined as:

$$\mathbf{F}_f = \begin{bmatrix} \partial\hbar_{ff}^{(r)} & -\partial\hbar_{ff}^{(l)} & \partial\Im_{ff}^{(r)} & -\partial\Im_{ff}^{(l)} \\ \hbar_{ff}^{(r)} & \hbar_{ff}^{(l)} & \Im_{ff}^{(r)} & \Im_{ff}^{(l)} \\ \eta_f^{-1}\partial\hbar_{ff}^{(r)} & \eta_f^{-1}\partial\hbar_{ff}^{(l)} & \eta_f^{-1}\partial\Im_{ff}^{(r)} & \eta_f^{-1}\partial\Im_{ff}^{(l)} \\ \eta_f^{-1}\hbar_{ff}^{(r)} & -\eta_f^{-1}\hbar_{ff}^{(l)} & \eta_f^{-1}\Im_{ff}^{(r)} & -\eta_f^{-1}\Im_{ff}^{(l)} \end{bmatrix}, \tag{26a}$$

$$\mathbf{C}_f = \begin{bmatrix} C_{{}^e_o 1f} \\ C_{{}^e_o 2f} \\ C_{{}^e_o 3f} \\ C_{{}^e_o 4f} \end{bmatrix}, \tag{26b}$$

$$\mathbf{F}_{f+1} = \begin{bmatrix} \partial\hbar_{(f+1)f}^{(r)} & -\partial\hbar_{(f+1)f}^{(l)} & \partial\Im_{(f+1)f}^{(r)} & -\partial\Im_{(f+1)f}^{(l)} \\ \hbar_{(f+1)f}^{(r)} & \hbar_{(f+1)f}^{(l)} & \Im_{(f+1)f}^{(r)} & \Im_{(f+1)f}^{(l)} \\ \eta_{(f+1)}^{-1}\partial\hbar_{(f+1)f}^{(r)} & \eta_{(f+1)}^{-1}\partial\hbar_{(f+1)f}^{(l)} & \eta_{(f+1)}^{-1}\partial\Im_{(f+1)f}^{(r)} & \eta_{(f+1)}^{-1}\partial\Im_{(f+1)f}^{(l)} \\ \eta_{(f+1)}^{-1}\hbar_{(f+1)f}^{(r)} & -\eta_{(f+1)}^{-1}\hbar_{(f+1)f}^{(l)} & \eta_{(f+1)}^{-1}\Im_{(f+1)f}^{(r)} & -\eta_{(f+1)c}^{-1}\Im_{(f+1)f}^{(l)} \end{bmatrix}, \tag{26c}$$

$$\mathbf{C}_{{}^e_o f+1} = \begin{bmatrix} C_{{}^e_o 1(f+1)} \\ C_{{}^e_o 2(f+1)} \\ C_{{}^e_o 3(f+1)} \\ C_{{}^e_o 4(f+1)} \end{bmatrix}, \tag{26d}$$

with

$$\eta_f^{-1} = \sqrt{\frac{\varepsilon_f}{\mu_f}}, \tag{27a}$$

$$\eta_{(f+1)}^{-1} = \sqrt{\frac{\varepsilon_{(f+1)}}{\mu_{(f+1)}}}. \tag{27b}$$

To simplify the derivation of the coefficients, we found the inverse of $\mathbf{F}_{f+1}$ by using commercially available softwares with symbolic calculations such as Mathematica.

We can rewrite the linear equation (25) into the following form

$$C_{f+1} = \mathbf{T}_f C_f \tag{28}$$

where the transmission matrix in the eigen-expansion domain is given by:

$$\mathbf{T}_f = \mathbf{F}_{f+1}^{-1}\mathbf{F}_f = \left[T_{j\ell}^f\right]_{4\times 4}. \tag{29}$$

For convenience and simplicity, let us assume that

$$\mathbf{T}^{(k)} = \left[\mathcal{T}_{j\ell}^{(k)}\right]_{4\times 4} = [\mathbf{T}_{N-1}][\mathbf{T}_{N-2}]\cdots[\mathbf{T}_{k+1}][\mathbf{T}_k]. \tag{30}$$

According to equation (28), we have the relation

$$\mathbf{C}_N = [\mathbf{T}_{N-1}\mathbf{T}_{N-2}\cdots\mathbf{T}_2\mathbf{T}_1]\,\mathbf{C}_1 \tag{31}$$

which can be re-written as:

$$\mathbf{C}_N = \mathbf{T}^{(1)}\mathbf{C}_1 \tag{32}$$

where

$$\mathbf{C}_N = \begin{bmatrix} 0 \\ 0 \\ C_{\substack{e\\o}3N} \\ C_{\substack{e\\o}4N} \end{bmatrix}, \tag{33a}$$

$$\mathbf{T}^{(1)} = \begin{bmatrix} \mathcal{T}_{11}^{(1)} & \mathcal{T}_{12}^{(1)} & \mathcal{T}_{13}^{(1)} & \mathcal{T}_{14}^{(1)} \\ \mathcal{T}_{21}^{(1)} & \mathcal{T}_{22}^{(1)} & \mathcal{T}_{23}^{(1)} & \mathcal{T}_{24}^{(1)} \\ \mathcal{T}_{31}^{(1)} & \mathcal{T}_{32}^{(1)} & \mathcal{T}_{33}^{(1)} & \mathcal{T}_{34}^{(1)} \\ \mathcal{T}_{41}^{(1)} & \mathcal{T}_{42}^{(1)} & \mathcal{T}_{43}^{(1)} & \mathcal{T}_{44}^{(1)} \end{bmatrix}, \tag{33b}$$

$$\mathbf{C}_1 = \begin{bmatrix} C_{\substack{e\\o}11} \\ C_{\substack{e\\o}21} \\ C_{\substack{e\\o}31} \\ C_{\substack{e\\o}41} \end{bmatrix}. \tag{33c}$$

Based on the equations (14) and (20a), we obtain :

$$C_{o31}^{I} = \frac{P_{omn}^{i}}{2}, \qquad C_{e31}^{I} = \frac{Q_{emn}^{i}}{2}, \tag{34a}$$

$$C_{o41}^{I} = \frac{P_{omn}^{i}}{2}, \qquad C_{e41}^{I} = -\frac{Q_{emn}^{i}}{2}, \tag{34b}$$

$$C_{o31}^{II} = \frac{Q_{omn}^{i}}{2}, \qquad C_{e31}^{II} = \frac{P_{emn}^{i}}{2}, \tag{34c}$$

$$C_{o41}^{II} = -\frac{Q_{omn}^{i}}{2}, \qquad C_{e41}^{II} = -\frac{P_{emn}^{i}}{2}, \tag{34d}$$

From Eqs. (29) to (33), we can derive the scattering coefficients, $C_{\substack{e\\o}11}$ and $C_{\substack{e\\o}12}$, as follows:

$$C_{\substack{e\\o}11}^{I} = \begin{bmatrix} Q_{emn}^{i} \\ P_{omn}^{i} \end{bmatrix} \frac{\mathcal{T}_{13}^{(1)}\mathcal{T}_{22}^{(1)} - \mathcal{T}_{23}^{(1)}\mathcal{T}_{12}^{(1)}}{2\left[\mathcal{T}_{12}^{(1)}\mathcal{T}_{21}^{(1)} - \mathcal{T}_{11}^{(1)}\mathcal{T}_{22}^{(1)}\right]} + \begin{bmatrix} -Q_{emn}^{i} \\ P_{omn}^{i} \end{bmatrix} \frac{\mathcal{T}_{14}^{(1)}\mathcal{T}_{22}^{(1)} - \mathcal{T}_{24}^{(1)}\mathcal{T}_{12}^{(1)}}{2\left[\mathcal{T}_{12}^{(1)}\mathcal{T}_{21}^{(1)} - \mathcal{T}_{11}^{(1)}\mathcal{T}_{22}^{(1)}\right]}, \tag{35a}$$

$$C_{\substack{e\\o}21}^{I} = \begin{bmatrix} Q_{emn}^{i} \\ P_{omn}^{i} \end{bmatrix} \frac{\mathcal{T}_{11}^{(1)}\mathcal{T}_{23}^{(1)} - \mathcal{T}_{13}^{(1)}\mathcal{T}_{21}^{(1)}}{2\left[\mathcal{T}_{12}^{(1)}\mathcal{T}_{21}^{(1)} - \mathcal{T}_{11}^{(1)}\mathcal{T}_{22}^{(1)}\right]} + \begin{bmatrix} -Q_{emn}^{i} \\ P_{omn}^{i} \end{bmatrix} \frac{\mathcal{T}_{11}^{(1)}\mathcal{T}_{24}^{(1)} - \mathcal{T}_{14}^{(1)}\mathcal{T}_{21}^{(1)}}{2\left[\mathcal{T}_{12}^{(1)}\mathcal{T}_{21}^{(1)} - \mathcal{T}_{11}^{(1)}\mathcal{T}_{22}^{(1)}\right]}, \tag{35b}$$

$$C_{\substack{e\\o}11}^{II} = \begin{bmatrix} P_{emn}^{i} \\ Q_{omn}^{i} \end{bmatrix} \frac{\mathcal{T}_{13}^{(1)}\mathcal{T}_{22}^{(1)} - \mathcal{T}_{23}^{(1)}\mathcal{T}_{12}^{(1)}}{2\left[\mathcal{T}_{12}^{(1)}\mathcal{T}_{21}^{(1)} - \mathcal{T}_{11}^{(1)}\mathcal{T}_{22}^{(1)}\right]} + \begin{bmatrix} P_{emn}^{i} \\ -Q_{omn}^{i} \end{bmatrix} \frac{\mathcal{T}_{14}^{(1)}\mathcal{T}_{22}^{(1)} - \mathcal{T}_{24}^{(1)}\mathcal{T}_{12}^{(1)}}{2\left[\mathcal{T}_{12}^{(1)}\mathcal{T}_{21}^{(1)} - \mathcal{T}_{11}^{(1)}\mathcal{T}_{22}^{(1)}\right]}, \tag{35c}$$

$$C_{\substack{e\\o}21}^{II} = \begin{bmatrix} P_{emn}^{i} \\ Q_{omn}^{i} \end{bmatrix} \frac{\mathcal{T}_{11}^{(1)}\mathcal{T}_{23}^{(1)} - \mathcal{T}_{13}^{(1)}\mathcal{T}_{21}^{(1)}}{2\left[\mathcal{T}_{12}^{(1)}\mathcal{T}_{21}^{(1)} - \mathcal{T}_{11}^{(1)}\mathcal{T}_{22}^{(1)}\right]} + \begin{bmatrix} P_{emn}^{i} \\ -Q_{omn}^{i} \end{bmatrix} \frac{\mathcal{T}_{11}^{(1)}\mathcal{T}_{24}^{(1)} - \mathcal{T}_{14}^{(1)}\mathcal{T}_{21}^{(1)}}{2\left[\mathcal{T}_{12}^{(1)}\mathcal{T}_{21}^{(1)} - \mathcal{T}_{11}^{(1)}\mathcal{T}_{22}^{(1)}\right]}. \tag{35d}$$

Hence, the scattered electric field can be written as follows:

$$\boldsymbol{E}_I^s = \sum_{n=1}^{\infty}\sum_{m=0}^{n} C^I_{\substack{e\\o}M}\boldsymbol{M}^{(3)}_{\substack{e\\o}mn}(k_0) + C^I_{\substack{e\\o}N}\boldsymbol{N}^{(3)}_{\substack{e\\o}mn}(k_0), \tag{36a}$$

$$\boldsymbol{E}_{II}^s = \sum_{n=1}^{\infty}\sum_{m=0}^{n} C^{II}_{\substack{e\\o}M}\boldsymbol{M}^{(3)}_{\substack{e\\o}mn}(k_0) + C^{II}_{\substack{e\\o}N}\boldsymbol{N}^{(3)}_{\substack{e\\o}mn}(k_0). \tag{36b}$$

where

$$C^{I,II}_{\substack{e\\o}M} = C^{I,II}_{\substack{e\\o}11} + C^{I,II}_{\substack{e\\o}21}, \tag{37a}$$

$$C^{I,II}_{\substack{e\\o}N} = C^{I,II}_{\substack{e\\o}11} - C^{I,II}_{\substack{e\\o}21}. \tag{37b}$$

So far, we have derived the scattered electric field using the conventional scattering theory where the vector wave function expansion technique and boundary condition matching are applied. In the next section, we shall use the radiation theory to reconsider this same problem and compare the results by the different techniques.

3.2 The Method by Dyadic Green's Function

The total electric field $\boldsymbol{E}(\boldsymbol{r})$ of an electromagnetic wave everywhere is governed by the Maxwell's equations. The vector wave equation can be expressed as follows if suppressing the time factor $e^{-i\omega t}$:

$$\begin{aligned}\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r}) - 2\xi_f\boldsymbol{\nabla}\times\boldsymbol{E}(\boldsymbol{r}) - k^2\boldsymbol{E}(\boldsymbol{r})&\\ = i\omega\mu\boldsymbol{J}(\boldsymbol{r}')\delta(\boldsymbol{r}-\boldsymbol{r}'),&\end{aligned} \tag{38a}$$

$$\begin{aligned}\boldsymbol{\nabla}\times\boldsymbol{\nabla}\times\boldsymbol{H}(\boldsymbol{r}) - 2\xi_f\boldsymbol{\nabla}\times\boldsymbol{H}(\boldsymbol{r}) - k^2\boldsymbol{H}(\boldsymbol{r})&\\ = \left[\boldsymbol{\nabla}\times\boldsymbol{J}(\boldsymbol{r}')\delta(\boldsymbol{r}-\boldsymbol{r}')\right],&\end{aligned} \tag{38b}$$

where $k^2 = \omega\mu(\omega\epsilon - i\sigma)$, and μ, ϵ and σ are the permeability, the permittivity and the conductivity, respectively. In free space, we have $k_0^2 = \omega^2\mu_0\epsilon_0$.

The solution to Eq. (38) may be represented in terms of the integral involving the electric type of dyadic Green's function as follows [23]:

$$\boldsymbol{E}(\boldsymbol{r}) = i\omega\mu\iiint_V \overline{\boldsymbol{G}}_e\left(\boldsymbol{r},\boldsymbol{r}'\right)\cdot\boldsymbol{J}(\boldsymbol{r}')dV', \tag{39}$$

where $\boldsymbol{E}(\boldsymbol{r})$ represents the total electric field and $\overline{\boldsymbol{G}}_e(\boldsymbol{r}, \boldsymbol{r}')$ stands for the electric type of dyadic Green's functions.

To solve for the unknown scattered field, the following well-known integrals of scattered fields excited by the current distribution $\boldsymbol{J}(\boldsymbol{r}')$ in free space are recommended:

$$\boldsymbol{E}^s(\boldsymbol{r}) = i\omega\mu \iiint_V \overline{\boldsymbol{G}}_{es}(\boldsymbol{r}, \boldsymbol{r}') \cdot \boldsymbol{J}(\boldsymbol{r}') dV', \tag{40a}$$

$$\boldsymbol{H}^s(\boldsymbol{r}) = \iiint_V \nabla \times \overline{\boldsymbol{G}}_{es}(\boldsymbol{r}, \boldsymbol{r}') \cdot \boldsymbol{J}(\boldsymbol{r}') dV'. \tag{40b}$$

Different from that in (39), the Green's function $\overline{\boldsymbol{G}}_{es}(\boldsymbol{r}, \boldsymbol{r}')$ used in (40) identifies the scattering dyadic Green's function exclusive of contributions due to direct waves.

3.2.1 Dyadic Green's Function for Multilayered Spherically Chiral Media

A general expression of spatial-domain dyadic Green's function for defining the electromagnetic radiation fields in spherically arbitrary multilayered and chiral media is available [45]. For the case of scattering in the present paper, an equivalent current source is assumed to be located at infinity and to be in the first region where the field is concerned. According to Part A of Section V in [45], the scattering dyadic Green's function is formulated as follows:

$$\begin{aligned}
&\overline{\boldsymbol{G}}_{es}^{(fs)}(\boldsymbol{r}, \boldsymbol{r}') \\
&\quad = \frac{i}{2\pi(k_s^{(r)} + k_s^{(l)})} \sum_{m,n} (2 - \delta_0) \frac{(2n+1)(n-m)!}{n(n+1)(n+m)!} \\
&\qquad \cdot \Big\{ \mathcal{C}_{12}^{fs} \boldsymbol{V}^{(3)}_{\substack{e\\o}mn}(k_f^{(r)}) \boldsymbol{V}'^{(3)}_{\substack{e\\o}mn}(k_s^{(r)}) + \mathcal{C}_{22}^{fs} \boldsymbol{W}^{(3)}_{\substack{e\\o}mn}(k_f^{(l)}) \boldsymbol{V}'^{(3)}_{\substack{e\\o}mn}(k_s^{(r)}) \\
&\qquad + \mathcal{C}_{14}^{fs} \boldsymbol{V}^{(3)}_{\substack{e\\o}mn}(k_f^{(r)}) \boldsymbol{W}'^{(3)}_{\substack{e\\o}mn}(k_s^{(l)}) + \mathcal{C}_{24}^{fs} \boldsymbol{W}^{(3)}_{\substack{e\\o}mn}(k_f^{(l)}) \boldsymbol{W}'^{(3)}_{\substack{e\\o}mn}(k_s^{(l)}) \Big\},
\end{aligned} \tag{41}$$

where the normalized spherical vector wave functions are defined as

$$\boldsymbol{V}_{\substack{e\\o}mn}(k) = \frac{\boldsymbol{M}_{\substack{e\\o}mn}(k) + \boldsymbol{N}_{\substack{e\\o}mn}(k)}{\sqrt{2}}, \tag{42a}$$

$$\boldsymbol{W}_{\substack{e\\o}mn}(k) = \frac{\boldsymbol{M}_{\substack{e\\o}mn}(k) - \boldsymbol{N}_{\substack{e\\o}mn}(k)}{\sqrt{2}}. \tag{42b}$$

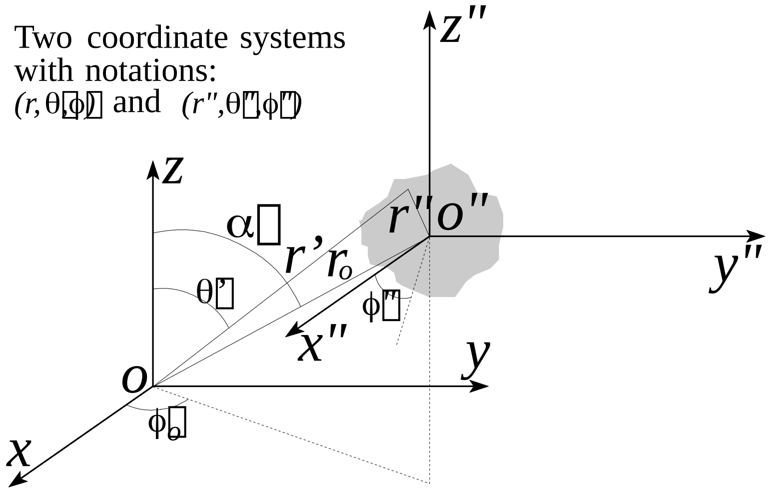

Figure 1. Coordinate translation.

The outmost layer is assumed to be free space, so the scattering dyadic Green's function is simplified as follows:

$$\begin{aligned}
&\overline{\boldsymbol{G}}_{es}^{(11)}(\boldsymbol{r},\boldsymbol{r}') \\
&\quad= \frac{i}{4\pi k_0}\sum_{m,n}(2-\delta_0)\frac{(2n+1)(n-m)!}{n(n+1)(n+m)!} \\
&\qquad \cdot\Big\{\mathcal{C}_{12}^{11}\boldsymbol{V}^{(3)}_{\substack{e\\o}mn}(k_0)\boldsymbol{V}'^{(3)}_{\substack{e\\o}mn}(k_0)+\mathcal{C}_{22}^{11}\boldsymbol{W}^{(3)}_{\substack{e\\o}mn}(k_0)\boldsymbol{V}'^{(3)}_{\substack{e\\o}mn}(k_0) \\
&\qquad +\mathcal{C}_{14}^{11}\boldsymbol{V}^{(3)}_{\substack{e\\o}mn}(k_0)\boldsymbol{W}'^{(3)}_{\substack{e\\o}mn}(k_0)+\mathcal{C}_{24}^{11}\boldsymbol{W}^{(3)}_{\substack{e\\o}mn}(k_0)\boldsymbol{W}'^{(3)}_{\substack{e\\o}mn}(k_0)\Big\}, \quad (43)
\end{aligned}$$

3.2.2 Source Which Generates Plane Waves at Infinity

The coordinates translation is shown in Fig. 1. The fields are observed under the coordinates system (r,θ,ϕ) and the point source at infinity is located in the coordinates system (r'',θ'',ϕ''). r_0 denotes the position of the point source with the distance $r_0\to\infty$, at the polar angle $\theta=\alpha$, and at the azimuth angle $\phi=0$ (where the sphere is symmetrical with respect to the z-axis so that a simple azimuth angle can be chosen). r' represents the position of the differential element in the source region. It can be seen from the figure that the position $\boldsymbol{r}_0$ of the point-source in the field coordinates, the position $\boldsymbol{r}'$ of a differential element in the field coordinates, and the position $\boldsymbol{r}''$ of the differential element in the source coordinates are related by

$$\boldsymbol{r}' = \boldsymbol{r}_0 + \boldsymbol{r}''. \tag{44}$$

The source at infinity can be regarded as a point source, therefore a delta function can be used to describe the source. For the different polarizations, the source can be expressed by

$$\boldsymbol{J}^I(\boldsymbol{r}') = E_I f(r_0)\delta(\boldsymbol{r}' - \boldsymbol{r}_0)\widehat{\boldsymbol{\theta}}, \tag{45a}$$

$$\boldsymbol{J}^{II}(\boldsymbol{r}') = E_{II} f(r_0)\delta(\boldsymbol{r}' - \boldsymbol{r}_0)\widehat{\boldsymbol{\phi}}. \tag{45b}$$

where $f(r_0)$ to be determined is a function of the distance of the point source. The plane wave can be considered as the one excited by the point source that is located at infinity (i.e., $r_0 \to \infty$). Since $\boldsymbol{r}_0$ is a constant vector, the vectors $\boldsymbol{r}'$ and $\boldsymbol{r}''$ and their derivatives in the two coordinates systems are related by

$$d\boldsymbol{r}' = d\boldsymbol{r}'', \tag{46a}$$

$$\boldsymbol{r}' - \boldsymbol{r}_0 = \boldsymbol{r}'', \tag{46b}$$

so that Eq. (45) can be rewritten as

$$\begin{aligned}\boldsymbol{J}^I(\boldsymbol{r}') &= E_I f(r_0)\delta(\boldsymbol{r}'')\widehat{\boldsymbol{\theta}} \\ &= E_I f(r_0)\frac{\delta(r'')\delta(\theta'')\delta(\phi'')}{[r'']^2 \sin\theta''}\widehat{\boldsymbol{\theta}}, \end{aligned} \tag{47a}$$

$$\begin{aligned}\boldsymbol{J}^{II}(\boldsymbol{r}') &= E_{II} f(r_0)\delta(\boldsymbol{r}'')\widehat{\boldsymbol{\phi}} \\ &= E_{II} f(r_0)\frac{\delta(r'')\delta(\theta'')\delta(\phi'')}{[r'']^2 \sin\theta''}\widehat{\boldsymbol{\phi}}. \end{aligned} \tag{47b}$$

Furthermore, we have the equivalent current distributions as follows:

$$\begin{bmatrix} \boldsymbol{J}^I(\boldsymbol{r}') \\ \boldsymbol{J}^{II}(\boldsymbol{r}') \end{bmatrix} dr' = f(r_0) \begin{bmatrix} E_I\widehat{\boldsymbol{\theta}} \\ E_{II}\widehat{\boldsymbol{\phi}} \end{bmatrix} \delta(r'')\delta(\theta'')\delta(\phi'')dr''d\theta''d\phi''. \tag{48}$$

3.2.3 Radiated Field

For simplicity, we define the following intermediates resulted in from substituting Eq. (41) and Eq. (48) into Eq. (39):

$$H^{I,II}_{\substack{e\\o}} = \iiint_v \boldsymbol{V}'_{\substack{e\\o}mn}(k_0) \cdot \boldsymbol{J}^{I,II}(r')dr', \tag{49a}$$

$$K_{\substack{e\\o}}^{I,II} = \iiint_v \boldsymbol{W}'_{\substack{e\\o}mn}(k_0) \cdot \boldsymbol{J}^{I,II}(r')dr'. \tag{49b}$$

Substitution of Eq. (48) into Eq. (39) leads to $r' \to r_0$, $\theta' \to \alpha$, and $\phi' \to 0$ in the integral containing the vector eigenfunctions, $\boldsymbol{V'}_{\substack{e\\o}mn}(k_0)$ and $\boldsymbol{W'}_{\substack{e\\o}mn}(k_0)$. Thus, we finally obtain the following expressions:

$$H_{\substack{e\\o}}^{I} = \frac{f(r_0)}{\sqrt{2}} E_I \cdot \begin{bmatrix} \dfrac{\partial P_n^m(\cos\alpha)}{\partial \alpha} \dfrac{d[(k_0r_0)h_n^{(1)}(k_0r_0)]}{(k_0r_0)d(k_0r_0)} \\ \dfrac{mP_n^m(\cos\alpha)}{\sin\alpha} h_n^{(1)}(k_0r_0) \end{bmatrix}, \tag{50a}$$

$$H_{\substack{e\\o}}^{II} = \frac{f(r_0)}{\sqrt{2}} E_{II} \cdot \begin{bmatrix} -\dfrac{\partial P_n^m(\cos\alpha)}{\partial \alpha} h_n^{(1)}(k_0r_0) \\ \dfrac{mP_n^m(\cos\alpha)}{\sin\alpha} \dfrac{d[(k_0r_0)h_n^{(1)}(k_0r_0)]}{(k_0r_0)d(k_0r_0)} \end{bmatrix}, \tag{50b}$$

$$K_{\substack{e\\o}}^{I} = \frac{f(r_0)}{\sqrt{2}} E_I \cdot \begin{bmatrix} -\dfrac{\partial P_n^m(\cos\alpha)}{\partial \alpha} \dfrac{d[(k_0r_0)h_n^{(1)}(k_0r_0)]}{(k_0r_0)d(k_0r_0)} \\ \dfrac{mP_n^m(\cos\alpha)}{\sin\alpha} h_n^{(1)}(k_0r_0) \end{bmatrix}, \tag{50c}$$

$$K_{\substack{e\\o}}^{II} = \frac{f(r_0)}{\sqrt{2}} E_{II} \cdot \begin{bmatrix} -\dfrac{\partial P_n^m(\cos\alpha)}{\partial \alpha} h_n^{(1)}(k_0r_0) \\ -\dfrac{mP_n^m(\cos\alpha)}{\sin\alpha} \dfrac{d[(k_0r_0)h_n^{(1)}(k_0r_0)]}{(k_0r_0)d(k_0r_0)} \end{bmatrix}, \tag{50d}$$

where the function f(r_0) is constructed as in [48]:

$$f(r_0) = i\frac{4\pi}{\omega\mu_0} r_0 e^{-ik_0r_0}. \tag{51}$$

According to Eq. (39), the scattered field of the multilayered chiral sphere is written as follows:

$$\begin{aligned} \boldsymbol{E}_{I,II}^{s} &= i\omega\mu \iiint_A \overline{\boldsymbol{G}}_{es}(\boldsymbol{r}, \boldsymbol{r}') \cdot \boldsymbol{J}^{I,II}(\boldsymbol{r}')dS', \\ &= -\frac{\omega\mu}{8\pi k_0} f(r_0) \sum_{n=1}^{\infty}\sum_{m=0}^{n} (2-\delta_{m0})\mathcal{N}_{mn} \left\{ \left[\mathcal{C}_{12}^{11} \boldsymbol{V}_{\substack{e\\o}mn}^{(3)}(k_0) \right.\right. \\ &\quad \left. + \mathcal{C}_{22}^{11} \boldsymbol{W}_{\substack{e\\o}mn}^{(3)}(k_0) \right] H_{\substack{e\\o}}^{I,II} + \left[\mathcal{C}_{14}^{11} \boldsymbol{V}_{\substack{e\\o}mn}^{(3)}(k_0) \right. \\ &\quad \left.\left. + \mathcal{C}_{24}^{11} \boldsymbol{W}_{\substack{e\\o}mn}^{(3)}(k_0) \right] K_{\substack{e\\o}}^{I,II} \right\}. \end{aligned} \tag{52}$$

By substituting the expression of $f(r_0)$ into Eq. (52) and using the asymptotic form of large-argument spherical Hankel function given as follows:

$$h_n^{(1)}(k_0 r_0) = (-i)^{n+1} \frac{e^{ik_0 r_0}}{k_0 r_0}, \tag{53a}$$

$$\frac{d[(k_0 r_0) h_n^{(1)}(k_0 r_0)]}{(k_0 r_0) d(k_0 r_0)} = (-i)^n \frac{e^{ik_0 r_0}}{k_0 r_0}; \tag{53b}$$

Eq. (52) can thus be further reduced to

$$\begin{aligned} \boldsymbol{E}^s_{\substack{I\\II}} = & \frac{1}{2k_0^2} \begin{bmatrix} E_I \\ E_{II} \end{bmatrix} \sum_{n=1}^{\infty} \sum_{m=0}^{n} (-i)^n (2-\delta_{m0}) \mathcal{N}_{mn} \\ & \cdot \left\{ \boldsymbol{M}^{(3)}_{\substack{o\\e}mn}(k_0) \left[(\mathcal{C}^{11}_{12} + \mathcal{C}^{11}_{22} + \mathcal{C}^{11}_{14} + \mathcal{C}^{11}_{24}) \right] \begin{bmatrix} -a \\ b \end{bmatrix} \right. \\ & + \boldsymbol{M}^{(3)}_{\substack{e\\o}mn}(k_0) \left[(\mathcal{C}^{11}_{12} + \mathcal{C}^{11}_{22} - \mathcal{C}^{11}_{14} - \mathcal{C}^{11}_{24}) \right] \begin{bmatrix} -ib \\ -ia \end{bmatrix} \\ & + \boldsymbol{N}^{(3)}_{\substack{o\\e}mn}(k_0) \left[(\mathcal{C}^{11}_{12} - \mathcal{C}^{11}_{22} + \mathcal{C}^{11}_{14} - \mathcal{C}^{11}_{24}) \right] \begin{bmatrix} -a \\ b \end{bmatrix} \\ & \left. + \boldsymbol{N}^{(3)}_{\substack{e\\o}mn}(k_0) \left[(\mathcal{C}^{11}_{12} - \mathcal{C}^{11}_{22} - \mathcal{C}^{11}_{14} + \mathcal{C}^{11}_{24}) \right] \begin{bmatrix} -ib \\ -ia \end{bmatrix} \right\} \end{aligned} \tag{54}$$

Substituting the expression for scattering coefficients [45] into the equation, we can easily obtain:

$$\begin{aligned} \boldsymbol{E}^s_{\substack{I\\II}} = & \frac{\left[\mathcal{P}^i_{\substack{o\\e}mn} \boldsymbol{M}^{(3)}_{\substack{o\\e}mn}(k_0) + \mathcal{Q}^i_{\substack{e\\o}mn} \boldsymbol{M}^{(3)}_{\substack{e\\o}mn}(k_0) \right]}{2\left[\mathcal{T}^{(1)}_{12}\mathcal{T}^{(1)}_{21} - \mathcal{T}^{(1)}_{11}\mathcal{T}^{(1)}_{22} \right]} \\ & \times \left[\mathcal{T}^{(1)}_{13}\mathcal{T}^{(1)}_{22} - \mathcal{T}^{(1)}_{23}\mathcal{T}^{(1)}_{12} + \mathcal{T}^{(1)}_{11}\mathcal{T}^{(1)}_{23} - \mathcal{T}^{(1)}_{13}\mathcal{T}^{(1)}_{21} \right] \\ & + \frac{\left[\mathcal{P}^i_{\substack{o\\e}mn} \boldsymbol{M}^{(3)}_{\substack{o\\e}mn}(k_0) - \mathcal{Q}^i_{\substack{e\\o}mn} \boldsymbol{M}^{(3)}_{\substack{e\\o}mn}(k_0) \right]}{2\left[\mathcal{T}^{(1)}_{12}\mathcal{T}^{(1)}_{21} - \mathcal{T}^{(1)}_{11}\mathcal{T}^{(1)}_{22} \right]} \\ & \times \left[\mathcal{T}^{(1)}_{14}\mathcal{T}^{(1)}_{22} - \mathcal{T}^{(1)}_{24}\mathcal{T}^{(1)}_{12} + \mathcal{T}^{(1)}_{11}\mathcal{T}^{(1)}_{24} - \mathcal{T}^{(1)}_{14}\mathcal{T}^{(1)}_{21} \right] \\ & + \frac{\left[\mathcal{P}^i_{\substack{o\\e}mn} \boldsymbol{N}^{(3)}_{\substack{o\\e}mn}(k_0) + \mathcal{Q}^i_{\substack{e\\o}mn} \boldsymbol{N}^{(3)}_{\substack{e\\o}mn}(k_0) \right]}{2\left[\mathcal{T}^{(1)}_{12}\mathcal{T}^{(1)}_{21} - \mathcal{T}^{(1)}_{11}\mathcal{T}^{(1)}_{22} \right]} \end{aligned}$$

$$\times\left[\mathcal{T}_{13}^{(1)}\mathcal{T}_{22}^{(1)}-\mathcal{T}_{23}^{(1)}\mathcal{T}_{12}^{(1)}-\mathcal{T}_{11}^{(1)}\mathcal{T}_{23}^{(1)}+\mathcal{T}_{13}^{(1)}\mathcal{T}_{21}^{(1)}\right]$$

$$+\frac{\left[\mathcal{P}^{i}_{\substack{o\\e}mn}\boldsymbol{N}^{(3)}_{\substack{o\\e}mn}(k_0)-\mathcal{Q}^{i}_{\substack{e\\o}mn}\boldsymbol{N}^{(3)}_{\substack{e\\o}mn}(k_0)\right]}{2\left[\mathcal{T}_{12}^{(1)}\mathcal{T}_{21}^{(1)}-\mathcal{T}_{11}^{(1)}\mathcal{T}_{22}^{(1)}\right]}$$

$$\times\left[\mathcal{T}_{14}^{(1)}\mathcal{T}_{22}^{(1)}-\mathcal{T}_{24}^{(1)}\mathcal{T}_{12}^{(1)}-\mathcal{T}_{11}^{(1)}\mathcal{T}_{24}^{(1)}+\mathcal{T}_{14}^{(1)}\mathcal{T}_{21}^{(1)}\right] \tag{55}$$

with

$$\mathcal{P}_{\substack{o\\e}mn}=-(-i)^n(2-\delta_{m0})\mathcal{N}_{mn}\left\{\begin{array}{c}\dfrac{mP_n^m(\cos\alpha)}{\sin\alpha}E_I\\ -\dfrac{\partial P_n^m(\cos\alpha)}{\partial\alpha}E_{II}\end{array}\right\}, \tag{56a}$$

$$\mathcal{Q}_{\substack{e\\o}mn}=(-i)^{n+1}(2-\delta_{m0})\mathcal{N}_{mn}\left\{\begin{array}{c}\dfrac{\partial P_n^m(\cos\alpha)}{\partial\alpha}E_I\\ \dfrac{mP_n^m(\cos\alpha)}{\sin\alpha}E_{II}\end{array}\right\}, \tag{56b}$$

We note, however, that the direction of propagation of the plane wave created here is opposite to the one considered by eigenfunction expansion [23]. In order to interpret this problem, we refer to Mie scattering.

According to Stratton [51], the incident wave of Mie scattering is in positive $\hat{\boldsymbol{z}}$-axis direction. Under the time dependence $e^{-i\omega t}$, the incident electric field is expanded as:

$$\boldsymbol{E}_I^i=\sum_{n=0}^{\infty}\left[a_n^I\boldsymbol{M}_{o1n}^{(1)}(k_0)+b_n^I\boldsymbol{N}_{e1n}^{(1)}(k_0)\right], \tag{57a}$$

$$\boldsymbol{E}_{II}^i=\sum_{n=0}^{\infty}\left[a_n^{II}\boldsymbol{M}_{e1n}^{(1)}(k_0)+b_n^{II}\boldsymbol{N}_{o1n}^{(1)}(k_0)\right]. \tag{57b}$$

with

$$\left\{\begin{array}{c}a_n^I\\a_n^{II}\end{array}\right\}=i^n\frac{2n+1}{n(n+1)}\left\{\begin{array}{c}1\\-1\end{array}\right\}, \tag{58a}$$

$$\left\{\begin{array}{c}b_n^I\\b_n^{II}\end{array}\right\}=i^{n+1}\frac{2n+1}{n(n+1)}\left\{\begin{array}{c}-1\\-1\end{array}\right\}, \tag{58b}$$

In Eq. (15), we let $m=1$ and $\alpha=0$. Following Ferrer's definition of associate Legendre function in [51], we easily obtain:

$$\left.\frac{P_n^1(\cos\alpha)}{\sin\alpha}\right|_{\alpha=0}=\frac{n(n+1)}{2}, \tag{59a}$$

$$\left.\frac{\partial P_n^1(\cos\alpha)}{\partial\alpha}\right|_{\alpha=0}=\frac{n(n+1)}{2}. \tag{59b}$$

By substituting the above equation, Eq. (15) can be reduced to Eq. (57).

In the method of dyadic Green's function, we also can obtain the expansion of incident wave by applying the following expressions:

$$\boldsymbol{E}(\boldsymbol{r}) = i\omega\mu \iiint_V \overline{\boldsymbol{G}}_0\left(\boldsymbol{r}, \boldsymbol{r}'\right) \cdot \boldsymbol{J}(\boldsymbol{r}')dV', \tag{60}$$

$\overline{\boldsymbol{G}}_0$ expanded by vector spherical wave function is available in [51]. In order to match the wave propagation direction, substitution Eq. (48) into the above equation leads to $r' \to r_0$, $\theta' \to \pi$, and $\phi' \to 0$ in the integral containing the vector eigenfunctions. When $\alpha = \pi$, we borrow the following equations:

$$\left.\frac{P_n^1(\cos\alpha)}{\sin\alpha}\right|_{\alpha=\pi} = -(-1)^n \frac{n(n+1)}{2}, \tag{61a}$$

$$\left.\frac{\partial P_n^1(\cos\alpha)}{\partial\alpha}\right|_{\alpha=\pi} = (-1)^n \frac{n(n+1)}{2}. \tag{61b}$$

Apply the expression for source and $f(r_0)$ and after integration, the same expression for coefficients as Eq. (57) is obtained.

As to the scattered field of Mie scattering, Stratton obtained the result by applying method of eigenfunction expansion in [51] (p. 564). Tai also found the result by using the method of dyadic Green's function in [23] (p. 217). The difference between them is different wave propagation direction. To calculate Mie scattering using the method of dyadic Green's function, if we let $\theta' \to 0$, we can obtain the same result as in [23]. That means the incident wave is in negative $\hat{\boldsymbol{z}}$-axis direction which is different from that considered by Stratton [51]. This can be easily matched by let $\theta' \to \pi$ and applying Eq. (61) in calculation [52] (p. 1466).

In the present paper, when applying the method of dyadic Green's function, we let $\theta' \to \alpha$ which is the angle between the incident wave direction and positive $\hat{\boldsymbol{z}}$-axis.(see Fig. 1). While in the method of eigenfunction expansion, we still follow the procedure of Stratton [51] in which the incident angle is between the incident wave direction and negative $\hat{\boldsymbol{z}}$-axis. The different wave propagation direction results in the difference between two methods. To solve this problem, we only need to let $\theta' \to \pi + \alpha$ and apply the following relation:

$$\frac{P_n^m(-\cos\alpha)}{\sin(\pi+\alpha)} = -(-1)^{n-m} \frac{P_n^m(\cos\alpha)}{\sin\alpha}, \tag{62a}$$

$$\frac{\partial P_n^m(-\cos\alpha)}{\partial\alpha} = (-1)^{n-m}\frac{\partial P_n^m(\cos\alpha)}{\partial\alpha}. \tag{62b}$$

It should be point out there are two different definition of Associated Legenfre functions. We may write such relation

$$P_n^{m(H)}(\cos\theta) = (-1)^m P_n^{m(F)}(\cos\theta) \tag{63}$$

with $P_n^{m(H)}$ represents Hobson's definition of associated Legendre functions which can be found in some books such as [53] (p. 951) and in Matheatica software. While $P_n^{m(F)}$ denotes associated Legendre functions without the factor $(-1)^m$ defined by Ferrer which is utilized by Stratton [51] (p. 401). Hence, the associate Legendre functions in Eq. (15) is referring to the definition with superscript (F).

After the manipulation, it is easily verified that the coefficients of the spherical vector wave functions $\boldsymbol{M}^{(3)}_{omn}(k_0)$, $\boldsymbol{M}^{(3)}_{emn}(k_0)$, $\boldsymbol{N}^{(3)}_{omn}(k_0)$ and $\boldsymbol{N}^{(3)}_{emn}(k_0)$ are the same as the $C^{I,II}_{\substack{e\\o}M}$ and $C^{I,II}_{\substack{e\\o}N}$ in the Eq. (37). Hence, the scattered fields due to the multilayer chiral sphere obtained by the two methods are identical.

From the above procedure, it is seen that both the scattering theory and the specific radiation theory give the identical results of the scattered electromagnetic fields. In other words, the scattering problem can be transformed into the radiation problem for which the dyadic Green's functions have been derived and the current distribution at infinity has been known.

4. SCATTERING BY A MULTILAYERED CHIRAL CYLINDER

Assume a plane electromagnetic wave is obliquely incident upon the cylinder which is infinitely long. Also two pairs of incident electromagnetic waves, i.e., parallel (I) (TM) and perpendicular (II) (TE) polarizations incident at an arbitrary angle on a cylinder whose center O is at the origin of the Cartesian coordinates system, are considered. The plane wave is specified by its propagation vector $\boldsymbol{k}$ with associated spherical components k, γ, and α. The angle γ is the angle between the propagation $\boldsymbol{k}$ and the axis of the cylinder and α is the angle between the projection of the vector $\boldsymbol{k}$ onto the transverse plane and the x-axis. Let Θ_i be the angle between the vectors $\boldsymbol{r}$ and $\boldsymbol{k}$.

Then we have

$$\cos\Theta_i = \cos\theta\cos\gamma + \sin\theta\sin\gamma\cos(\phi-\alpha). \tag{64}$$

The incident waves of the two polarizations can be expressed by:

$$\boldsymbol{E}_I^i = \frac{i}{k}\left(\frac{\mu_0}{\epsilon_0}\right)^{\frac{1}{2}}\boldsymbol{\nabla}\times\boldsymbol{H}_I^i, \tag{65a}$$

$$\boldsymbol{H}_I^i = \frac{kE_I}{\omega\mu_0}(\sin\alpha\widehat{\boldsymbol{x}} - \cos\alpha\widehat{\boldsymbol{y}})e^{ik_0 r\cos\Theta_i}; \tag{65b}$$

and

$$\boldsymbol{E}_{II}^i = E_{II}(\sin\alpha\widehat{\boldsymbol{x}} - \cos\alpha\widehat{\boldsymbol{y}})e^{ik_0 r\cos\Theta_i}, \tag{66a}$$

$$\boldsymbol{H}_{II}^i = \frac{1}{ik}\left(\frac{\epsilon_0}{\mu_0}\right)^{\frac{1}{2}}\boldsymbol{\nabla}\times\boldsymbol{E}_{II}^i; \tag{66b}$$

where E_I and E_{II} are the amplitude of the incident electric fields with parallel and vertical polarizations.

4.1 The Method of Eigenfunction Expansion

4.1.1 The Eigenfunction Expansion

Since the problem defined is now changed into cylindrical coordinates, the circular cylindrical coordinates system is the most convenient one.

The incident electromagnetic fields can be expanded in terms of cylindrical vector wave eigenfunctions defined in the cylindrical coordinates system as follows:

$$\boldsymbol{M}_{\substack{e\\o}n\eta}(h) = \left[\mp\frac{nZ_n(\eta\rho)}{\rho}\begin{matrix}\sin\\\cos\end{matrix}n\phi\widehat{\boldsymbol{\rho}} - \frac{dZ_n(\eta\rho)}{d\rho}\begin{matrix}\cos\\\sin\end{matrix}n\phi\widehat{\boldsymbol{\phi}}\right]e^{ihz}, \tag{67a}$$

$$\begin{aligned}\boldsymbol{N}_{\substack{e\\o}n\eta}(h) = \frac{1}{k}\Bigg[&ih\frac{dZ_n(\eta\rho)}{d\rho}\begin{matrix}\cos\\\sin\end{matrix}n\phi\widehat{\boldsymbol{\rho}} \mp \frac{inh}{\rho}Z_n(\eta\rho)\begin{matrix}\sin\\\cos\end{matrix}n\phi\widehat{\boldsymbol{\phi}}\\&+\eta^2 Z_n(\eta\rho)\begin{matrix}\cos\\\sin\end{matrix}n\phi\widehat{\boldsymbol{z}}\Bigg]e^{ihz},\end{aligned} \tag{67b}$$

where $Z_n(k\rho)$ represents the cylindrical Bessel functions of order n and h is the longitudinal wave number. Also, $\boldsymbol{M}^{(3)}_{\substack{e\\o}n\eta}(h)$ and

$\boldsymbol{N}^{(3)}_{\substack{e\\o}n\eta}(h)$ are defined in terms of the cylindrical Hankel function of the first kind, $H_n^{(1)}(\eta\rho)$.

The incident waves under the two polarizations have, as introduced by Rao and Barakat [54, 55], the following forms:

$$\begin{aligned}\boldsymbol{E}^i_I = & \frac{E_I}{k\sin\gamma}\sum_{n=0}^{\infty}(2-\delta_{n0})i^n\Big[\cos(n\alpha)\boldsymbol{N}^{(1)}_{en\eta}(h_0,\gamma)\\ & +\sin(n\alpha)\boldsymbol{N}^{(1)}_{on\eta}(h_0,\gamma)\Big],\end{aligned} \tag{68a}$$

$$\begin{aligned}\boldsymbol{H}^i_I = & (\frac{\epsilon_0}{\mu_0})^{\frac{1}{2}}\frac{E_I}{ik\sin\gamma}\sum_{n=0}^{\infty}(2-\delta_{n0})i^n\Big[\cos(n\alpha)\boldsymbol{M}^{(1)}_{en\eta}(h_0,\gamma)\\ & +\sin(n\alpha)\boldsymbol{M}^{(1)}_{on\eta}(h_0,\gamma)\Big],\end{aligned} \tag{68b}$$

and

$$\begin{aligned}\boldsymbol{E}^i_{II} = & \frac{E_{II}}{ik\sin\gamma}\sum_{n=0}^{\infty}(2-\delta_{n0})i^n\Big[\cos(n\alpha)\boldsymbol{M}^{(1)}_{em\eta}(h_0,\gamma)\\ & +\sin(n\alpha)\boldsymbol{M}^{(1)}_{on\eta}(h_0,\gamma)\Big],\end{aligned} \tag{69a}$$

$$\begin{aligned}\boldsymbol{H}^i_{II} = & -(\frac{\epsilon_0}{\mu_0})^{\frac{1}{2}}\frac{E_{II}}{k\sin\gamma}\sum_{n=0}^{\infty}(2-\delta_{n0})i^n\Big[\cos(n\alpha)\boldsymbol{N}^{(1)}_{en\eta}(h_0,\gamma)\\ & +\sin(n\alpha)\boldsymbol{N}^{(1)}_{on\eta}(h_0,\gamma)\Big],\end{aligned} \tag{69b}$$

and where the cylindrical Bessel functions of the first kind, i.e., $Z_n(x) = J_n(x)$, are used in the above cylindrical vector wave functions. The tangential wave number $\eta = k\sin\gamma$ and the longitudinal one is $h_0 = k\cos\gamma$. For generality, combining the above equations, we rewrite them as the follows:

$$\boldsymbol{E}^i_{\substack{I\\II}} = \sum_{n=0}^{\infty}\begin{bmatrix} Q^i_{en}\boldsymbol{N}^{(1)}_{en\eta}(h_0,\gamma)+Q^i_{on}\boldsymbol{N}^{(1)}_{on\eta}(h_0,\gamma)\\ P^i_{en}\boldsymbol{M}^{(1)}_{en\eta}(h_0,\gamma)+P^i_{on}\boldsymbol{M}^{(1)}_{on\eta}(h_0,\gamma)\end{bmatrix}, \tag{70a}$$

$$\boldsymbol{H}^i_{\substack{I\\II}} = -\frac{i}{\eta_0}\sum_{n=0}^{\infty}\begin{bmatrix} Q^i_{en}\boldsymbol{M}^{(1)}_{en\eta}(h_0,\gamma)+Q^i_{on}\boldsymbol{M}^{(1)}_{on\eta}(h_0,\gamma)\\ P^i_{en}\boldsymbol{N}^{(1)}_{en\eta}(h_0,\gamma)+P^i_{on}\boldsymbol{N}^{(1)}_{on\eta}(h_0,\gamma)\end{bmatrix}, \tag{70b}$$

Rewrite the coefficients of the expanded incident electromagnetic fields, $P^i_{\substack{e\\o}n}$ and $Q^i_{\substack{e\\o}n}$, as follows:

$$P^i_{\substack{e\\o}n} = -(2-\delta_{n0})i^{n+1}\frac{E_I}{k\sin\gamma}\begin{bmatrix}\cos(n\alpha)\\ \sin(n\alpha)\end{bmatrix}, \tag{71a}$$

$$Q^{i}_{\substack{e\\o}n} = (2-\delta_{n0}) i^n \frac{E_{II}}{k\sin\gamma}\begin{bmatrix}\cos(n\alpha)\\ \sin(n\alpha)\end{bmatrix}. \tag{71b}$$

Again, δ_{mn} ($=1$ for $m=n$; and 0 for $m\neq n$) here denotes the Kronecker symbol. It should be noted that the form of the expansion of incident wave for cylindrical systems is different from that for spherical systems. The polarization-I and polarization-II incident waves correspond to $\boldsymbol{N}$ and $\boldsymbol{M}$ terms respectively. Furthermore, the notation ${}^{o}_{e}n$ and ${}^{e}_{o}n$ here have a different meaning from those in Eq. (14) where the upper (or lower) notation denotes the parallel (or the perpendicular) polarization. While in the above equation, it means the summation of both upper and lower modes should be taken into account.

The vector wave eigenfunction expansion in multilayered chiral cylinder can be written as a superposition of right-handed and left-handed circularly polarized fields. Since the coupling between the even and odd modes exit in vector cylindrical wave functions, both the even and odd modes should be taken into account. The right-handed and left-handed circularly polarized fields can be expressed using wave functions as follows:

$$\boldsymbol{E}^{(p)}_{Rn\eta} = \boldsymbol{M}^{(p)}_{\substack{e\\o}n\eta}(h^{(R)}) + \boldsymbol{N}^{(p)}_{\substack{e\\o}n\eta}(h^{(R)}) + \boldsymbol{M}^{(p)}_{\substack{o\\e}n\eta}(h^{(R)}) + \boldsymbol{N}^{(p)}_{\substack{o\\e}n\eta}(h^{(R)}), \tag{72a}$$

$$\boldsymbol{E}^{(p)}_{Ln\eta} = \boldsymbol{M}^{(p)}_{\substack{e\\o}n\eta}(h^{(L)}) - \boldsymbol{N}^{(p)}_{\substack{e\\o}n\eta}(h^{(L)}) + \boldsymbol{M}^{(p)}_{\substack{o\\e}n\eta}(h^{(L)}) - \boldsymbol{N}^{(p)}_{\substack{o\\e}n\eta}(h^{(L)}), \tag{72b}$$

where p equals 1 or 3. The superscript (1) represents the first type of cylindrical Bessel function and the superscript (3) denotes the first kind of cylindrical Hankel function used in the vector wave functions. Following the procedure in the scattering by multilayered chiral sphere, we can expand the electric field in the layer from 2nd to $(N-1)$th layer of the cylinder as follows:

$$\begin{aligned}\boldsymbol{E}_f = \sum_{n=0}^{\infty}\Big\{ &\mathcal{A}_{\substack{e\\o}f}\left[\boldsymbol{M}^{(3)}_{\substack{e\\o}n\eta}(h_f^{(R)}) + \boldsymbol{N}^{(3)}_{\substack{e\\o}n\eta}(h_f^{(R)})\right]\\ &+\mathcal{B}_{\substack{o\\e}f}\left[\boldsymbol{M}^{(3)}_{\substack{o\\e}n\eta}(h_f^{(R)}) + \boldsymbol{N}^{(3)}_{\substack{o\\e}n\eta}(h_f^{(R)})\right]\\ &+\mathcal{C}_{\substack{e\\o}f}\left[\boldsymbol{M}^{(3)}_{\substack{e\\o}n\eta}(h_f^{(L)}) - \boldsymbol{N}^{(3)}_{\substack{e\\o}n\eta}(h_f^{(L)})\right]\\ &+\mathcal{D}_{\substack{o\\e}f}\left[\boldsymbol{M}^{(3)}_{\substack{o\\e}n\eta}(h_f^{(L)}) - \boldsymbol{N}^{(3)}_{\substack{o\\e}n\eta}(h_f^{(L)})\right]\\ &+\mathcal{A}'_{\substack{e\\o}f}\left[\boldsymbol{M}^{(1)}_{\substack{e\\o}n\eta}(h_f^{(R)}) + \boldsymbol{N}^{(1)}_{\substack{e\\o}n\eta}(h_f^{(R)})\right]\end{aligned}$$

$$
\begin{aligned}
&+\mathcal{B}'_{\substack{o\\e}f}\left[\boldsymbol{M}^{(1)}_{\substack{o\\e}n\eta}(h_f^{(R)})+\boldsymbol{N}^{(1)}_{\substack{o\\e}n\eta}(h_f^{(R)})\right]\\
&+\mathcal{C}'_{\substack{e\\o}f}\left[\boldsymbol{M}^{(1)}_{\substack{e\\o}n\eta}(h_f^{(L)})-\boldsymbol{N}^{(1)}_{\substack{e\\o}n\eta}(h_f^{(L)})\right]\\
&+\mathcal{D}'_{\substack{o\\e}f}\left[\boldsymbol{M}^{(1)}_{\substack{o\\e}n\eta}(h_f^{(L)})-\boldsymbol{N}^{(1)}_{\substack{o\\e}n\eta}(h_f^{(L)})\right]\Bigg\}, \qquad (73a)
\end{aligned}
$$

$$
\boldsymbol{H}_f = -\frac{i}{\eta_f}\left[\boldsymbol{E}_{f,R}-\boldsymbol{E}_{f,L}\right]. \qquad (73b)
$$

The electric fields in the out-most and inner-most regions are written as follows, respectively:

$$
\begin{aligned}
\boldsymbol{E}_1 &= \boldsymbol{E}^i+\boldsymbol{E}^s\\
&= \boldsymbol{E}^i+\sum_{n=0}^{\infty}\Bigg\{\mathcal{A}_{\substack{e\\o}1}\left[\boldsymbol{M}^{(3)}_{\substack{e\\o}n\eta}(h_0,\gamma)+\boldsymbol{N}^{(3)}_{\substack{e\\o}n\eta}(h_0,\gamma)\right]\\
&\quad+\mathcal{B}_{\substack{o\\e}1}\left[\boldsymbol{M}^{(3)}_{\substack{o\\e}n\eta}(h_0,\gamma)+\boldsymbol{N}^{(3)}_{\substack{o\\e}n\eta}(h_0,\gamma)\right]\\
&\quad+\mathcal{C}_{\substack{e\\o}1}\left[\boldsymbol{M}^{(3)}_{\substack{e\\o}n\eta}(h_0,\gamma)-\boldsymbol{N}^{(3)}_{\substack{e\\o}n\eta}(h_0,\gamma)\right]\\
&\quad+\mathcal{D}_{\substack{o\\e}1}\left[\boldsymbol{M}^{(3)}_{\substack{o\\e}n\eta}(h_0,\gamma)-\boldsymbol{N}^{(3)}_{\substack{o\\e}n\eta}(h_0,\gamma)\right]\Bigg\}, \qquad (74a)\\
\boldsymbol{E}_N &= \sum_{n=0}^{\infty}\Bigg\{\mathcal{A}'_{\substack{e\\o}N}\left[\boldsymbol{M}^{(1)}_{\substack{e\\o}n\eta}(h_0,\gamma)+\boldsymbol{N}^{(1)}_{\substack{e\\o}n\eta}(h_0,\gamma)\right]\\
&\quad+\mathcal{B}'_{\substack{o\\e}N}\left[\boldsymbol{M}^{(1)}_{\substack{o\\e}n\eta}(h_0,\gamma)+\boldsymbol{N}^{(1)}_{\substack{o\\e}n\eta}(h_0,\gamma)\right]\\
&\quad+\mathcal{C}'_{\substack{e\\o}N}\left[\boldsymbol{M}^{(1)}_{\substack{e\\o}n\eta}(h_0,\gamma)-\boldsymbol{N}^{(1)}_{\substack{e\\o}n\eta}(h_0,\gamma)\right]\\
&\quad+\mathcal{D}'_{\substack{o\\e}N}\left[\boldsymbol{M}^{(1)}_{\substack{o\\e}n\eta}(h_0,\gamma)-\boldsymbol{N}^{(1)}_{\substack{o\\e}n\eta}(h_0,\gamma)\right]\Bigg\}, \qquad (74b)
\end{aligned}
$$

4.1.2 Determination of Scattering Coefficients by Boundary Conditions

The electric and magnetic field satisfies the following boundary conditions at the cylindrical interfaces $\rho = a_j$ (where $j = 1, 2, \cdots, N-1$):

$$
\widehat{\boldsymbol{\rho}}\times\boldsymbol{E_f} = \widehat{\boldsymbol{\rho}}\times\boldsymbol{E}_{(f+1)}, \qquad (75a)
$$

$$
\widehat{\boldsymbol{\rho}}\times\boldsymbol{H_f} = \widehat{\boldsymbol{\rho}}\times\boldsymbol{H}_{(f+1)}. \qquad (75b)
$$

Without any loss of generality of the problem, we extend Eq. (75) into a linear equation system and replace it by coefficient matrix equation system, we have the following equation:

$$\mathbf{F}_f C_f = \mathbf{F}_{f+1} C_{f+1}, \tag{76}$$

where

$$[\boldsymbol{F}_{jm}] = \begin{bmatrix} \left[\boldsymbol{U}^{\hbar}_{jm}\right] & \left[\boldsymbol{U}^{\Im}_{jm}\right] \\ \left[\boldsymbol{D}^{\hbar}_{jm}\right] & \left[\boldsymbol{D}^{\Im}_{jm}\right] \end{bmatrix}, \tag{77a}$$

$$[\boldsymbol{C}_f] = \begin{bmatrix} \mathcal{A}_{\substack{e\\o}f} \\ \mathcal{B}_{\substack{o\\e}f} \\ \mathcal{C}_{\substack{e\\o}f} \\ \mathcal{D}_{\substack{o\\e}f} \\ \mathcal{A}'_{\substack{e\\o}f} \\ \mathcal{B}'_{\substack{o\\e}f} \\ \mathcal{C}'_{\substack{e\\o}f} \\ \mathcal{D}'_{\substack{o\\e}f} \end{bmatrix}; \tag{77b}$$

with

$$\left[\boldsymbol{U}^{\Xi}_{jm}\right] = \begin{bmatrix} \left[\boldsymbol{P}^{\Xi}_{jm}\right] & \left[\boldsymbol{Q}^{\Xi}_{jm}\right] \\ \left[\boldsymbol{R}^{\Xi,r}_{jm}\right] & -\left[\boldsymbol{R}^{\Xi,l}_{jm}\right] \end{bmatrix}, \tag{78a}$$

$$\left[\boldsymbol{D}^{\Xi}_{jm}\right] = \frac{1}{\mu_f} \begin{bmatrix} \left[\boldsymbol{S}^{\Xi}_{jm}\right] & \left[\boldsymbol{L}^{\Xi}_{jm}\right] \\ \left[-\lambda^{(r)}_{f+}\boldsymbol{R}^{\Xi,r}_{jm}\right] & \left[\lambda^{(l)}_{f+}\boldsymbol{R}^{\Xi,l}_{jm}\right] \end{bmatrix}. \tag{78b}$$

To simplify the complicated and tedious representation, the following operators are introduced:

$$\Im^{(r,l)}_{im} = \left.\frac{J_n(\rho)}{\rho}\right|_{\rho=\eta^{(r,l)}_j a_m}, \tag{79a}$$

$$\hbar^{(r,l)}_{im} = \left.\frac{H^{(1)}_n(\rho)}{\rho}\right|_{\rho=\eta^{(r,l)}_j a_m}, \tag{79b}$$

$$\partial\Im^{(r,l)}_{im} = \left.\frac{d[J_n(\rho)]}{\rho}\right|_{\rho=\eta^{(r,l)}_j a_m}, \tag{79c}$$

$$\partial\hbar^{(r,l)}_{im} = \left.\frac{d[H^{(1)}_n(\rho)]}{\rho}\right|_{\rho=\eta^{(r,l)}_j a_m}, \tag{79d}$$

$$\zeta_j^{(r,l)} = \frac{ih\eta_j^{(r,l)}}{k_j^{(r,l)}}, \tag{79e}$$

$$\varrho_{jm}^{(r,l)} = \frac{a_m}{k_j^{(r,l)}} \left(\eta_j^{(r,l)}\right)^3, \tag{79f}$$

$$\lambda_{f\pm}^{(r,l)} = \left(k_f^{(r,l)} \pm \xi_f\right), \tag{79g}$$

$$\Xi = \hbar, or \Im, \tag{79h}$$

$$\left[\boldsymbol{P}_{jm}^{\Xi}\right] = \begin{bmatrix} \mp n\xi_j^{(r)}\Xi_{jm}(r) & -\eta_j^{(r)}\partial\Xi_{jm}^{(r)} \\ -\eta_j^{(r)}\partial\Xi_{jm}^{(r)} & \pm n\xi_j^{(r)}\Xi_{jm}(r) \end{bmatrix}, \tag{79i}$$

$$\left[\boldsymbol{Q}_{jm}^{\Xi}\right] = \begin{bmatrix} \pm n\xi_j^{(l)}\Xi_{jm}(l) & -\eta_j^{(l)}\partial\Xi_{jm}^{(l)} \\ -\eta_j^{(l)}\partial\Xi_{jm}^{(l)} & \mp n\xi_j^{(l)}\Xi_{jm}(l) \end{bmatrix}, \tag{79j}$$

$$\left[\boldsymbol{R}_{jm}^{\Xi,(r,l)}\right] = \begin{bmatrix} \varrho_{jm}^{(r,l)}\Xi_{jm}(l) & 0 \\ 0 & -\varrho_{jm}^{(r,l)}\Xi_{jm}(l) \end{bmatrix}, \tag{79k}$$

$$\left[\boldsymbol{S}_{jm}^{\Xi}\right] = \begin{bmatrix} \pm n\xi_j^{(r)}\Xi_{jm}^{(r)}\lambda_{f-}^{(r)} & -\eta_j^{(r)}\partial\Xi_{jm}^{(r)}\lambda_{f-}^{(r)} \\ -\eta_j^{(r)}\partial\Xi_{jm}^{(r)}\lambda_{f-}^{(r)} & \mp n\xi_j^{(r)}\Xi_{jm}^{(r)}\lambda_{f-}^{(r)} \end{bmatrix}, \tag{79l}$$

$$\left[\boldsymbol{L}_{jm}^{\Xi}\right] = \begin{bmatrix} \mp n\xi_j^{(l)}\Xi_{jm}^{(l)}\lambda_{f+}^{(l)} & -\eta_j^{(l)}\partial\Xi_{jm}^{(l)}\lambda_{f+}^{(l)} \\ -\eta_j^{(l)}\partial\Xi_{jm}^{(l)}\lambda_{f+}^{(l)} & \pm n\xi_j^{(l)}\Xi_{jm}^{(l)}\lambda_{f+}^{(l)} \end{bmatrix}, \tag{79m}$$

$$j = 1, 2, \cdots, N,$$
$$m = j - 1 \text{ or } j.$$

It should be pointed out that the symbols in (79a)–(79d) look identical to, but are defined differently from, those in (24a)–(24d). Following the same procedure in the problem of scattering by a multilayered chiral sphere, we obtain:

$$\mathbf{C}_N = \left[\mathbf{T}_{N-1}\mathbf{T}_{N-2}\cdots\mathbf{T}_2\mathbf{T}_1\right]\mathbf{C}_1 \tag{80}$$

which can be written as:

$$\mathbf{C}_N = \mathbf{T}^{(1)}\mathbf{C}_1 \tag{81}$$

where

$$\mathbf{T}^{(k)} = \left[\mathcal{T}_{j\ell}^{(k)}\right]_{8\times 8} = [\mathbf{T}_{N-1}][\mathbf{T}_{N-2}]\cdots[\mathbf{T}_{k+1}][\mathbf{T}_k], \tag{82}$$

with the transmission matrix in the eigen-expansion domain is defined by:

$$\mathbf{T}_f = \mathbf{F}_{f+1}^{-1}\mathbf{F}_f = \left[T_{j\ell}^f\right]_{8\times 8}. \tag{83}$$

We now rewrite Eq. (81) as:

$$\mathbf{C}_N = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \mathcal{A}'_{\substack{e\\o}N} \\ \mathcal{B}'_{\substack{o\\e}N} \\ \mathcal{C}'_{\substack{e\\o}N} \\ \mathcal{D}'_{\substack{e\\o}N} \end{bmatrix}, \tag{84a}$$

$$\begin{aligned} \mathbf{T}^{(1)} &= [\mathbf{T}_{N-1}][\mathbf{T}_{N-2}]\cdots[\mathbf{T}_2][\mathbf{T}_1] \\ &= \begin{bmatrix} \left[\mathbf{T}^{(\mathbf{1})}_{(\mathbf{11})}\right]_{4\times4} & \left[\mathbf{T}^{(\mathbf{1})}_{(\mathbf{12})}\right]_{4\times4} \\ \left[\mathbf{T}^{(\mathbf{1})}_{(\mathbf{21})}\right]_{4\times4} & \left[\mathbf{T}^{(\mathbf{1})}_{(\mathbf{22})}\right]_{4\times4} \end{bmatrix}, \end{aligned} \tag{84b}$$

$$\mathbf{C}_1 = \begin{bmatrix} \mathcal{A}_{\substack{e\\o}1} \\ \mathcal{B}_{\substack{o\\e}1} \\ \mathcal{C}_{\substack{e\\o}1} \\ \mathcal{D}_{\substack{o\\e}1} \\ \mathcal{A}'_{\substack{e\\o}1} \\ \mathcal{B}'_{\substack{o\\e}1} \\ \mathcal{C}'_{\substack{e\\o}1} \\ \mathcal{D}'_{\substack{o\\e}1} \end{bmatrix}. \tag{84c}$$

After some simple matrix algebraic manipulations, we easily obtain the following matrix equation:

$$\begin{bmatrix} \mathcal{A}_{\substack{e\\o}1} \\ \mathcal{B}_{\substack{o\\e}1} \\ \mathcal{C}_{\substack{e\\o}1} \\ \mathcal{D}_{\substack{o\\e}1} \end{bmatrix} = -\left[\mathbf{T}^{(\mathbf{1})}_{\mathbf{11}}\right]^{-1}_{4\times4} \cdot \left[\mathbf{T}^{(\mathbf{1})}_{\mathbf{11}}\right]_{4\times4} \cdot \begin{bmatrix} \mathcal{A}'_{\substack{e\\o}1} \\ \mathcal{B}'_{\substack{o\\e}1} \\ \mathcal{C}'_{\substack{e\\o}1} \\ \mathcal{D}'_{\substack{o\\e}1} \end{bmatrix}. \tag{85}$$

According to Eq. (73a), we obtain that $\mathcal{B}'_{\substack{o\\e}1}$ and $\mathcal{D}'_{\substack{o\\e}1}$ both equal 0. Thus, we further assume:

$$\left[\mathbf{X}^{(\mathbf{1})}\right]_{4\times4} = -\left[\mathbf{T}^{(\mathbf{1})}_{\mathbf{11}}\right]^{-1}_{4\times4} \cdot \left[\mathbf{T}^{(\mathbf{1})}_{\mathbf{11}}\right]_{4\times4}$$

$$= \begin{bmatrix} \mathcal{X}_{11}^{(1)} & \mathcal{X}_{12}^{(1)} & \mathcal{X}_{13}^{(1)} & \mathcal{X}_{14}^{(1)} \\ \mathcal{X}_{21}^{(1)} & \mathcal{X}_{22}^{(1)} & \mathcal{X}_{23}^{(1)} & \mathcal{X}_{24}^{(1)} \\ \mathcal{X}_{31}^{(1)} & \mathcal{X}_{32}^{(1)} & \mathcal{X}_{33}^{(1)} & \mathcal{X}_{34}^{(1)} \\ \mathcal{X}_{41}^{(1)} & \mathcal{X}_{42}^{(1)} & \mathcal{X}_{43}^{(1)} & \mathcal{X}_{44}^{(1)} \end{bmatrix}. \tag{86}$$

Finally we obtain:

$$\begin{bmatrix} \mathcal{A}^{I}_{\substack{e\\o}1} \\ \mathcal{B}^{I}_{\substack{o\\e}1} \\ \mathcal{C}^{I}_{\substack{e\\o}1} \\ \mathcal{D}^{I}_{\substack{o\\e}1} \end{bmatrix} = \begin{bmatrix} \mathcal{X}_{11}^{(1)} & \mathcal{X}_{12}^{(1)} & \mathcal{X}_{13}^{(1)} & \mathcal{X}_{14}^{(1)} \\ \mathcal{X}_{21}^{(1)} & \mathcal{X}_{22}^{(1)} & \mathcal{X}_{23}^{(1)} & \mathcal{X}_{24}^{(1)} \\ \mathcal{X}_{31}^{(1)} & \mathcal{X}_{32}^{(1)} & \mathcal{X}_{33}^{(1)} & \mathcal{X}_{34}^{(1)} \\ \mathcal{X}_{41}^{(1)} & \mathcal{X}_{42}^{(1)} & \mathcal{X}_{43}^{(1)} & \mathcal{X}_{44}^{(1)} \end{bmatrix} \begin{bmatrix} \frac{Q^{i}_{\substack{e\\o}n}}{2} \\ 0 \\ -\frac{Q^{i}_{\substack{e\\o}n}}{2} \\ 0 \end{bmatrix}, \tag{87a}$$

$$\begin{bmatrix} \mathcal{A}^{II}_{\substack{e\\o}1} \\ \mathcal{B}^{II}_{\substack{o\\e}1} \\ \mathcal{C}^{II}_{\substack{e\\o}1} \\ \mathcal{D}^{II}_{\substack{o\\e}1} \end{bmatrix} = \begin{bmatrix} \mathcal{X}_{11}^{(1)} & \mathcal{X}_{12}^{(1)} & \mathcal{X}_{13}^{(1)} & \mathcal{X}_{14}^{(1)} \\ \mathcal{X}_{21}^{(1)} & \mathcal{X}_{22}^{(1)} & \mathcal{X}_{23}^{(1)} & \mathcal{X}_{24}^{(1)} \\ \mathcal{X}_{31}^{(1)} & \mathcal{X}_{32}^{(1)} & \mathcal{X}_{33}^{(1)} & \mathcal{X}_{34}^{(1)} \\ \mathcal{X}_{41}^{(1)} & \mathcal{X}_{42}^{(1)} & \mathcal{X}_{43}^{(1)} & \mathcal{X}_{44}^{(1)} \end{bmatrix} \begin{bmatrix} \frac{P^{i}_{\substack{e\\o}n}}{2} \\ 0 \\ \frac{P^{i}_{\substack{e\\o}n}}{2} \\ 0 \end{bmatrix}. \tag{87b}$$

Therefore, the scattering coefficients are derived. The scattered field can then be written as follows:

$$\begin{aligned} \boldsymbol{E}^{s}_{I} = \sum_{n=0}^{\infty} \Bigg\{ & \left[\left(\mathcal{X}_{11}^{(1)} - \mathcal{X}_{13}^{(1)} + \mathcal{X}_{31}^{(1)} - \mathcal{X}_{33}^{(1)} \right) \frac{Q^{i}_{\substack{e\\o}n}}{2} \right] \boldsymbol{M}^{(3)}_{\substack{e\\o}n}(h_0, \gamma) \\ & + \left[\left(\mathcal{X}_{21}^{(1)} - \mathcal{X}_{23}^{(1)} + \mathcal{X}_{41}^{(1)} - \mathcal{X}_{43}^{(1)} \right) \frac{Q^{i}_{\substack{e\\o}n}}{2} \right] \boldsymbol{M}^{(3)}_{\substack{o\\e}n}(h_0, \gamma) \\ & + \left[\left(\mathcal{X}_{11}^{(1)} - \mathcal{X}_{13}^{(1)} - \mathcal{X}_{31}^{(1)} + \mathcal{X}_{33}^{(1)} \right) \frac{Q^{i}_{\substack{e\\o}n}}{2} \right] \boldsymbol{N}^{(3)}_{\substack{e\\o}n}(h_0, \gamma) \\ & + \left[\left(\mathcal{X}_{21}^{(1)} - \mathcal{X}_{23}^{(1)} - \mathcal{X}_{41}^{(1)} + \mathcal{X}_{43}^{(1)} \right) \frac{Q^{i}_{\substack{e\\o}n}}{2} \right] \boldsymbol{N}^{(3)}_{\substack{o\\e}n}(h_0, \gamma) \Bigg\}, \end{aligned} \tag{88a}$$

$$\boldsymbol{E}^{s}_{II} = \sum_{n=0}^{\infty} \Bigg\{ \left[\left(\mathcal{X}_{11}^{(1)} + \mathcal{X}_{13}^{(1)} + \mathcal{X}_{31}^{(1)} + \mathcal{X}_{33}^{(1)} \right) \frac{P^{i}_{\substack{e\\o}n}}{2} \right] \boldsymbol{M}^{(3)}_{\substack{e\\o}n}(h_0, \gamma)$$

$$+\left[\left(\mathcal{X}_{21}^{(1)}+\mathcal{X}_{23}^{(1)}+\mathcal{X}_{41}^{(1)}+\mathcal{X}_{43}^{(1)}\right)\frac{P_{\substack{e\\o}n}^{i}}{2}\right]\boldsymbol{M}_{\substack{o\\e}n}^{(3)}(h_0,\gamma)$$

$$+\left[\left(\mathcal{X}_{11}^{(1)}+\mathcal{X}_{13}^{(1)}-\mathcal{X}_{31}^{(1)}-\mathcal{X}_{33}^{(1)}\right)\frac{P_{\substack{e\\o}n}^{i}}{2}\right]\boldsymbol{N}_{\substack{e\\o}n}^{(3)}(h_0,\gamma)$$

$$\left.+\left[\left(\mathcal{X}_{21}^{(1)}+\mathcal{X}_{23}^{(1)}-\mathcal{X}_{41}^{(1)}-\mathcal{X}_{43}^{(1)}\right)\frac{P_{\substack{e\\o}n}^{i}}{2}\right]\boldsymbol{N}_{\substack{o\\e}n}^{(3)}(h_0,\gamma)\right\}. \tag{88b}$$

4.2 The Dyadic Green's Function Method

According to the dyadic Green's function method in Section 3.2, the scattered field can be obtained by solving the following well-known integrals consisting of the dyadic Green's function $\overline{\boldsymbol{G}}_{es}(\boldsymbol{r},\boldsymbol{r}')$ and the current distribution $\boldsymbol{J}(\boldsymbol{r}')$ in free space:

$$\boldsymbol{E}^s(\boldsymbol{r})=i\omega\mu\iiint_V\overline{\boldsymbol{G}}_{es}(\boldsymbol{r},\boldsymbol{r}')\cdot\boldsymbol{J}(\boldsymbol{r}')dV', \tag{89a}$$

$$\boldsymbol{H}^s(\boldsymbol{r})=\iiint_V\nabla\times\overline{\boldsymbol{G}}_{es}(\boldsymbol{r},\boldsymbol{r}')\cdot\boldsymbol{J}(\boldsymbol{r}')dV'. \tag{89b}$$

4.2.1 Dyadic Green's Function for Multilayered Cylindrically Chiral Media

The dyadic Green's function for a cylindrically arbitrary multilayered medium was constructed by Yin and Wang [46] and the generalized coefficients of the dyadic Green's function were provided by Li et al. [47]. For the present case, a current source is assumed to be located at infinity and to be in the first region where the field is of our interest. According to Part A of Section IV in [47], the scattering dyadic Green's function is formulated as follows:

$$\begin{aligned}&\overline{\boldsymbol{G}}_{es}^{(11)}(\boldsymbol{r},\boldsymbol{r}')\\&\quad=\frac{i}{4\pi(k_s^{(r)}+k_s^{(l)})}\int_{-\infty}^{\infty}dh(2-\delta_{0n})\\&\qquad\cdot\left\{\boldsymbol{V}_{\substack{e\\o}n\eta_f^{(r)}}^{(3)}(h)\left[\mathcal{C}_{12}^{11}\boldsymbol{V}_{\substack{e\\o}n\eta_s^{(r)}}^{\prime(3)}(-h)+\mathcal{C}_{14}^{11}\boldsymbol{W}_{\substack{e\\o}n\eta_s^{(l)}}^{\prime(3)}(-h)\right]\right.\end{aligned}$$

$$
\begin{aligned}
&+ \boldsymbol{V}^{(3)}_{\substack{o\\e}n\eta_f^{(r)}}(h)\left[\mathcal{C}^{11\prime}_{12}\boldsymbol{V}'^{(3)}_{\substack{e\\o}n\eta_s^{(r)}}(-h)+\mathcal{C}^{11\prime}_{14}\boldsymbol{W}'^{(3)}_{\substack{e\\o}n\eta_s^{(l)}}(-h)\right]\\
&+ \boldsymbol{W}^{(3)}_{\substack{e\\o}n\eta_f^{(l)}}(h)\left[\mathcal{C}^{11}_{22}\boldsymbol{V}'^{(3)}_{\substack{e\\o}n\eta_s^{(r)}}(-h)+\mathcal{C}^{11}_{24}\boldsymbol{W}'^{(3)}_{\substack{e\\o}n\eta_s^{(l)}}(-h)\right]\\
&+ \boldsymbol{W}^{(3)}_{\substack{o\\e}n\eta_f^{(l)}}(h)\left[\mathcal{C}^{11\prime}_{22}\boldsymbol{V}'^{(3)}_{\substack{e\\o}n\eta_s^{(r)}}(-h)+\mathcal{C}^{11\prime}_{24}\boldsymbol{W}'^{(3)}_{\substack{e\\o}n\eta_s^{(l)}}(-h)\right]\Bigg\},
\end{aligned}
\tag{90}
$$

where the same definitions as in (42) but for different (cylindrical) vector wave functions are given below:

$$
\boldsymbol{V}_{\substack{e\\o}n\eta}(h)=\frac{\boldsymbol{M}_{\substack{e\\o}n\eta}(h)+\boldsymbol{N}_{\substack{e\\o}n\eta}(h)}{\sqrt{2}},
\tag{91a}
$$

$$
\boldsymbol{W}_{\substack{e\\o}n\eta}(h)=\frac{\boldsymbol{M}_{\substack{e\\o}n\eta}(h)-\boldsymbol{N}_{\substack{e\\o}n\eta}(h)}{\sqrt{2}}.
\tag{91b}
$$

The outmost region is assumed to be free space, the scattering dyadic Green's function is simplified as:

$$
\begin{aligned}
&\overline{\boldsymbol{G}}^{(11)}_{es}(\boldsymbol{r},\boldsymbol{r}')\\
&\quad=\frac{i}{8\pi k_0}\int_{-\infty}^{\infty}dh(2-\delta_{0n})\\
&\qquad\cdot\Bigg\{\boldsymbol{V}^{(3)}_{\substack{e\\o}n\eta_0}(h_0)\left[\mathcal{C}^{11}_{12}\boldsymbol{V}'^{(3)}_{\substack{e\\o}n\eta_0}(-h_0)+\mathcal{C}^{11}_{14}\boldsymbol{W}'^{(3)}_{\substack{e\\o}n\eta_0}(-h_0)\right]\\
&\qquad+\boldsymbol{V}^{(3)}_{\substack{o\\e}n\eta_0}(h_0)\left[\mathcal{C}^{11\prime}_{12}\boldsymbol{V}'^{(3)}_{\substack{e\\o}n\eta_0}(-h_0)+\mathcal{C}^{11\prime}_{14}\boldsymbol{W}'^{(3)}_{\substack{e\\o}n\eta_0}(-h_0)\right]\\
&\qquad+\boldsymbol{W}^{(3)}_{\substack{e\\o}n\eta_0}(h_0)\left[\mathcal{C}^{11}_{22}\boldsymbol{V}'^{(3)}_{\substack{e\\o}n\eta_0}(-h_0)+\mathcal{C}^{11}_{24}\boldsymbol{W}'^{(3)}_{\substack{e\\o}n\eta_0}(-h_0)\right]\\
&\qquad+\boldsymbol{W}^{(3)}_{\substack{o\\e}n\eta_0}(h_0)\left[\mathcal{C}^{11\prime}_{22}\boldsymbol{V}'^{(3)}_{\substack{e\\o}n\eta_0}(-h_0)+\mathcal{C}^{11\prime}_{24}\boldsymbol{W}'^{(3)}_{\substack{e\\o}n\eta_0}(-h_0)\right]\Bigg\}.
\end{aligned}
\tag{92}
$$

4.2.2 Source Which Generates a Plane Wave at Infinity

Following the procedure in [48], we can derive the current distribution for the source at infinity by applying the cylindrical systems. It has the same form as in Eq. (47). For different polarizations, the sources can be expressed by:

$$
\begin{bmatrix}\boldsymbol{J}^{I}(r')\\ \boldsymbol{J}^{II}(r')\end{bmatrix}=f(r_0)\begin{bmatrix}E_I\widehat{\boldsymbol{\theta}}\\ E_{II}\widehat{\boldsymbol{\phi}}\end{bmatrix}\delta(r'')\delta(\theta'')\delta(\phi'').
\tag{93}
$$

4.2.3 Radiated Field

Substituting Eq. (92) and Eq. (93) into Eq. (39), we can find the radiated field. First, we introduce the approximation for large argument for the Hankel function in vector cylindrical wave function, that is:

$$H_n^{(1)}(\eta r) = \left(\frac{2}{\pi\eta r}\right)^{\frac{1}{2}} (-i)^{n+\frac{1}{2}} e^{i\eta r}. \tag{94}$$

The functions $\boldsymbol{M}'^{(3)}_{\substack{e\\o}n\eta}(-h)$ and $\boldsymbol{N}'^{(3)}_{\substack{e\\o}n\eta}(-h)$, therefore, become:

$$\boldsymbol{M}'^{(3)}_{\substack{e\\o}n\eta}(-h) = (-i)^{n+\frac{3}{2}}\eta(\frac{2}{\pi\eta r})^{\frac{1}{2}} e^{i(\eta r'-hz')} \begin{matrix}\cos\\ \sin\end{matrix}(n\phi)\widehat{\boldsymbol{\phi}}, \tag{95a}$$

$$\boldsymbol{N}'^{(3)}_{\substack{e\\o}n\eta}(-h) = (-i)^{n+\frac{1}{2}}\frac{\eta}{k}(\frac{2}{\pi\eta r})^{\frac{1}{2}} e^{i(\eta r'-hz')} \begin{matrix}\cos\\ \sin\end{matrix}(n\phi)(h\widehat{\boldsymbol{r}}+\eta\widehat{\boldsymbol{z}}). \tag{95b}$$

We now change the cylindrical variables into spherical variables. In order to obtain the same propagation direction wave, we let

$$\eta = -k\sin(\pi+\beta), \tag{96a}$$

$$h = -k\cos(\pi+\beta), \tag{96b}$$

$$r' = R\sin\theta, \tag{96c}$$

$$z' = -R\cos\theta. \tag{96d}$$

Hence, we have

$$\eta r' - hz' = kR'\cos[\theta-(\pi+\beta)], \tag{97a}$$

$$h\widehat{\boldsymbol{r}}+\eta\widehat{\boldsymbol{z}} = k(\widehat{\boldsymbol{r}}\cos(\pi+\beta)-\widehat{\boldsymbol{z}}\sin(\pi+\beta)) = k\widehat{\boldsymbol{\theta}}, \tag{97b}$$

Substitution of Eq. (95) into Eq. (92) and Eq. (39) leads to $R'\to r_0$, $\theta'\to\pi+\gamma$, and $\phi'\to\pi+\alpha$ in the integral containing the vector wave eigenfunctions $\boldsymbol{V}'_{\substack{e\\o}n\eta}(-h)$ and $\boldsymbol{W}'_{\substack{e\\o}n\eta}(-h)$. According to [47], the coefficients can be expressed as:

$$\mathcal{C}_{12}^{11} = \frac{k_0}{[\eta_0]^2}\mathcal{X}_{11}^{(1)}, \tag{98a}$$

$$\mathcal{C}_{14}^{11} = \frac{k_0}{[\eta_0]^2}\mathcal{X}_{13}^{(1)}, \tag{98b}$$

$$\mathcal{C}_{12}^{11\prime} = \frac{k_0}{[\eta_0]^2} \mathcal{X}_{21}^{(1)}, \tag{98c}$$

$$\mathcal{C}_{14}^{11\prime} = \frac{k_0}{[\eta_0]^2} \mathcal{X}_{23}^{(1)}, \tag{98d}$$

$$\mathcal{C}_{22}^{11} = \frac{k_0}{[\eta_0]^2} \mathcal{X}_{31}^{(1)}, \tag{98e}$$

$$\mathcal{C}_{24}^{11} = \frac{k_0}{[\eta_0]^2} \mathcal{X}_{33}^{(1)}, \tag{98f}$$

$$\mathcal{C}_{22}^{11\prime} = \frac{k_0}{[\eta_0]^2} \mathcal{X}_{41}^{(1)}, \tag{98g}$$

$$\mathcal{C}_{24}^{11\prime} = \frac{k_0}{[\eta_0]^2} \mathcal{X}_{43}^{(1)}. \tag{98h}$$

Substituting the coefficients and $f(r_0)$ into Eq. (88). Meanwhile notice that:

$$\cos(n(\pi+\alpha)) = (-1)^n \cos(n\alpha), \tag{99a}$$

$$\sin(n(\pi+\alpha)) = (-1)^n \sin(n\alpha). \tag{99b}$$

By using the method of saddle-point integration [23], we obtain the scattered fields as follows:

$$\begin{aligned}
\boldsymbol{E}_I^s = & \frac{E_I}{2k_0 \sin\gamma} \sum_{n=0}^{\infty} \begin{bmatrix} \cos(n\alpha) \\ \sin(n\alpha) \end{bmatrix} (2-\delta_{0n}) i^n \\
& \cdot \Big\{ \left[\mathcal{X}_{11}^{(1)} - \mathcal{X}_{13}^{(1)} - \mathcal{X}_{31}^{(1)} + \mathcal{X}_{33}^{(1)} \right] \boldsymbol{N}_{\substack{e \\ o}n}^{(3)}(k_0\cos\theta) \\
& + \left[\mathcal{X}_{11}^{(1)} - \mathcal{X}_{13}^{(1)} + \mathcal{X}_{31}^{(1)} - \mathcal{X}_{33}^{(1)} \right] \boldsymbol{M}_{\substack{e \\ o}n}^{(3)}(k_0\cos\theta) \\
& + \left[\mathcal{X}_{21}^{(1)} - \mathcal{X}_{23}^{(1)} - \mathcal{X}_{41}^{(1)} + \mathcal{X}_{43}^{(1)} \right] \boldsymbol{N}_{\substack{o \\ e}n}^{(3)}(k_0\cos\theta) \\
& + \left[\mathcal{X}_{21}^{(1)} - \mathcal{X}_{23}^{(1)} + \mathcal{X}_{41}^{(1)} - \mathcal{X}_{33}^{(1)} \right] \boldsymbol{M}_{\substack{o \\ e}n}^{(3)}(k_0\cos\theta) \Big\},
\end{aligned} \tag{100a}$$

$$\begin{aligned}
\boldsymbol{E}_{II}^s = & -\frac{E_{II}}{2k_0 \sin\gamma} \sum_{n=0}^{\infty} \begin{bmatrix} \cos(n\alpha) \\ \sin(n\alpha) \end{bmatrix} (2-\delta_{0n}) i^{n+1} \\
& \cdot \Big\{ \left[\mathcal{X}_{11}^{(1)} + \mathcal{X}_{13}^{(1)} + \mathcal{X}_{31}^{(1)} + \mathcal{X}_{33}^{(1)} \right] \boldsymbol{M}_{\substack{e \\ o}n}^{(3)}(k_0\cos\theta) \\
& + \left[\mathcal{X}_{11}^{(1)} + \mathcal{X}_{13}^{(1)} - \mathcal{X}_{31}^{(1)} - \mathcal{X}_{33}^{(1)} \right] \boldsymbol{N}_{\substack{e \\ o}n}^{(3)}(k_0\cos\theta)
\end{aligned}$$

$$+ \left[\mathcal{X}_{21}^{(1)} + \mathcal{X}_{23}^{(1)} + \mathcal{X}_{41}^{(1)} + \mathcal{X}_{43}^{(1)}\right] \boldsymbol{M}^{(3)}_{\substack{o\\e}n}(k_0\cos\theta)$$
$$+ \left[\mathcal{X}_{21}^{(1)} + \mathcal{X}_{23}^{(1)} - \mathcal{X}_{41}^{(1)} - \mathcal{X}_{43}^{(1)}\right] \boldsymbol{N}^{(3)}_{\substack{o\\e}n}(k_0\cos\theta)\bigg\}. \quad (100b)$$

It is observed that the above result is the same as that obtained by the method of eigenfunction expansion in Eq. (88).

5. SCATTERING BY PLANARLY STRATIFIED CHIRAL MEDIA

5.1 The Method of Eigenfunction Expansion

Spectral-domain dyadic Green's function in layered chiral media has been well documented by Ali [38]. For simplicity, the main procedure and expressions are omitted here. Following the notations of Ali [38], the field in region n can be written as:

$$\begin{aligned}\boldsymbol{E}_n(z_n) &= f_n^{(r)}\boldsymbol{e}_r[\gamma_n^{(r)}]\exp[i\gamma_n^{(r)}z_n] + f_n^{(l)}\boldsymbol{e}_l[\gamma_n^{(l)}]\exp[i\gamma_n^{(l)}z_n]\\ &\quad + g_n^{(r)}\boldsymbol{e}_r[-\gamma_n^{(r)}]exp[-i\gamma_n^{(r)}z_n] + g_n^{(l)}\boldsymbol{e}_l[-\gamma_n^{(l)}]exp[-i\gamma_n^{(l)}z_n] \quad (101)\end{aligned}$$

which can be written in a matrix form as:

$$\begin{aligned}\boldsymbol{E}_n(z_n) = &\left[\boldsymbol{e}_r(\gamma_n^{(r)}) \qquad \boldsymbol{e}_l(\gamma_n^{(l)})\right]\cdot\boldsymbol{A}_n(z_n)\\ &+\left[\boldsymbol{e}_r(-\gamma_n^{(r)}) \qquad \boldsymbol{e}_l[-\gamma_n^{(l)}]\right]\cdot\boldsymbol{B}_n(z_n) \quad (102)\end{aligned}$$

where

$$\boldsymbol{e}_r(\pm\gamma_n^{(r)}) = i\widehat{\boldsymbol{h}}(\pm\gamma_n^{(r)}) + \widehat{\boldsymbol{v}}(\pm\gamma_n^{(r)}), \quad (103a)$$
$$\boldsymbol{e}_l(\pm\gamma_n^{(l)}) = i\widehat{\boldsymbol{h}}(\pm\gamma_n^{(l)}) - \widehat{\boldsymbol{v}}(\pm\gamma_n^{(l)}); \quad (103b)$$

with

$$\widehat{\boldsymbol{h}}(\gamma_n^{(r,l)}) = \frac{1}{k_s}(\widehat{\boldsymbol{x}}k_y - \widehat{\boldsymbol{y}}k_s), \quad (104a)$$
$$\widehat{\boldsymbol{v}}(\gamma_n^{(r,l)}) = \frac{1}{k_n^{r,l}}[-\gamma_n^{r,l}\widehat{\boldsymbol{K}_s} + k_s\widehat{\boldsymbol{z}}], \quad (104b)$$
$$\widehat{\boldsymbol{K}_s} = \frac{(\widehat{\boldsymbol{x}}k_x + \widehat{\boldsymbol{y}}k_y)}{k_s}; \quad (104c)$$

and $\boldsymbol{A}_n(z_n)$ and $\boldsymbol{B}_n(z_n)$ are, respectively, the column matrices representing the amplitudes of the upgoing and the downgoing waves defined by

$$\boldsymbol{A}_n(z_n) = \begin{bmatrix} \mathcal{A}_n^{(r)} \\ \mathcal{A}_n^{(l)} \end{bmatrix} = \bar{\boldsymbol{G}}_n(+z_n) \cdot \boldsymbol{f}_n, \tag{105a}$$

$$\boldsymbol{B}_n(z_n) = \begin{bmatrix} \mathcal{B}_n^{(r)} \\ \mathcal{B}_n^{(l)} \end{bmatrix} = \bar{\boldsymbol{G}}_n(-z_n) \cdot \boldsymbol{g}_n; \tag{105b}$$

with

$$\bar{\boldsymbol{G}}_n(\pm z_n) = \begin{bmatrix} exp[\pm i\gamma_n^{(r)} z_n] & 0 \\ 0 & exp[\pm i\gamma_n^{(l)} z_n] \end{bmatrix}, \tag{105c}$$

$$\boldsymbol{f}_n = \begin{bmatrix} f_n^{(r)} \\ f_n^{(l)} \end{bmatrix}, \tag{105d}$$

$$\boldsymbol{g}_n = \begin{bmatrix} g_n^{(r)} \\ g_n^{(l)} \end{bmatrix}. \tag{105e}$$

In Eq. (105a), $\mathcal{A}_n^{(r)}$ and $\mathcal{A}_n^{(l)}$ are, respectively, the amplitudes of the upgoing waves of the right- and the left-handed polarizations while $\mathcal{B}_n^{(r)}$ and $\mathcal{B}_n^{(l)}$ are those of the down-going waves of the right- and the left-handed polarizations. In present case, the incident waves exist in the upmost layer which is assumed to be free space, thus only the downward global reflection and the downward global transmission are present. Utilizing the notation $\bar{\boldsymbol{R}}_{\cap n}$ in [38], we easily obtain:

$$\boldsymbol{A}_1 = \bar{\boldsymbol{R}}_{\cap n} \cdot \boldsymbol{B}_1 = \bar{\boldsymbol{G}}_1(+z_1) \cdot \boldsymbol{f}_1, \tag{106}$$

Thus, the scattered field is:

$$\boldsymbol{E}^s = [\boldsymbol{e}_r(\gamma_0) \qquad \boldsymbol{e}_l(\gamma_0)] \cdot \bar{\boldsymbol{R}}_{\cap n} \bar{\boldsymbol{G}}_1(+z_1) \cdot \boldsymbol{g}_1 \tag{107}$$

where γ_0 is the longitudinal wave number in the upmost layer. We can easily find:

$$\boldsymbol{g}_1 = \begin{bmatrix} E_0 \\ E_0 \end{bmatrix} \tag{108}$$

for TE waves; and

$$\boldsymbol{g}_1 = \begin{bmatrix} E_0 \\ -E_0 \end{bmatrix} \tag{109}$$

for TM waves with E_0 as the amplitude of the indent wave.

According to [38],

$$\bar{\boldsymbol{u}}_{2\times 2}^{(-)} = \bar{\boldsymbol{R}}_{\cap n} = \begin{bmatrix} u_{11}^{(-)} & u_{12}^{(-)} \\ u_{21}^{(-)} & u_{22}^{(-)} \end{bmatrix} \tag{110}$$

We can therefore write the scattered field as:

$$\begin{aligned} \boldsymbol{E}_I^s = & \left(u_{11}^{(-)} - u_{12}^{(-)}\right) E_0 \boldsymbol{e}_r(\gamma_0) \exp(i\gamma_0 z) \\ & + \left(u_{21}^{(-)} - u_{22}^{(-)}\right) E_0 \boldsymbol{e}_l(\gamma_0) \exp(i\gamma_0 z) \end{aligned} \tag{111}$$

for the first polarized (TM) incident waves; and

$$\begin{aligned} \boldsymbol{E}_{II}^s = & \left(u_{11}^{(-)} + u_{12}^{(-)}\right) E_0 \boldsymbol{e}_r(\gamma_0) \exp(i\gamma_0 z) \\ & + \left(u_{21}^{(-)} + u_{22}^{(-)}\right) E_0 \boldsymbol{e}_l(\gamma_0) \exp(i\gamma_0 z) \end{aligned} \tag{112}$$

for the second polarized (TE) incident waves.

5.2 The Dyadic Green's Function Method

The technique of fast Fourier transform (FTT) [56] suggests that an alternative representation of the Green functions for planar stratified media is to cast the eigenfunction expansion in the form of two-dimensional Fourier transform. Tai [23] has used the free-space dyadic Green's function to illustrate the formulation. The desirable vector wave functions to used to represent the free-space dyadic Green functions are defined by:

$$\begin{aligned} \boldsymbol{M}(\boldsymbol{k}) &= \boldsymbol{\nabla} \times \left[e^{i\boldsymbol{k}\cdot\boldsymbol{r}} \widehat{\boldsymbol{z}}\right] \\ &= i(k_2 \widehat{\boldsymbol{x}} - k_1 \widehat{\boldsymbol{y}}) e^{i\boldsymbol{k}\cdot\boldsymbol{r}}, \end{aligned} \tag{113a}$$

$$\begin{aligned} \boldsymbol{N}(\boldsymbol{k}) &= \frac{1}{k} \boldsymbol{\nabla} \times \boldsymbol{M}(\boldsymbol{k}) \\ &= \frac{1}{k} \left[-k_1 k_3 \widehat{\boldsymbol{x}} - k_2 k_3 \widehat{\boldsymbol{y}} + (k_1^2 + k_2^2) \widehat{\boldsymbol{z}}\right] e^{i\boldsymbol{k}\cdot\boldsymbol{r}}; \end{aligned} \tag{113b}$$

where

$$\boldsymbol{k} = k_1 \widehat{\boldsymbol{x}} + k_2 \widehat{\boldsymbol{y}} + k_3 \widehat{\boldsymbol{z}}, \tag{114a}$$

$$k = \sqrt{k_1^2 + k_2^2 + k_3^2}, \tag{114b}$$

$$\boldsymbol{r} = x \widehat{\boldsymbol{x}} + y \widehat{\boldsymbol{y}} + z \widehat{\boldsymbol{z}}. \tag{114c}$$

Ali [38] has derived the dyadic Green's function for a chiral medium and a planarly layered chiral media in Cartesian coordinates. Ren [57] also obtained the dyadic Green's function for a chiral medium in Cartesian coordinates written by aforementioned wave function. By using the integral representations of wave functions [51], the dyadic Green's function expanded by vector cylindrical wave function for planarly stratified chiral medium is obtained [57]. In our case, however, the scattering dyadic Green's functions can be further reduced to:

$$\begin{aligned}\overline{\boldsymbol{G}}_{es}(\boldsymbol{r},\boldsymbol{r}')&\\ =\frac{i}{2\pi}\int_0^\infty\frac{d\lambda}{\lambda}\sum_{n=0}^{\infty}(2-\delta_{n0})&\\ \cdot\Bigg\{\frac{1}{\gamma^{(0)}}\boldsymbol{R}'_{\substack{e\\o}n\lambda}(\gamma^{(0)})\Big[u_{11}^{(-)}\boldsymbol{R}_{\substack{e\\o}n\lambda}(\gamma^{(0)})+u_{21}^{(-)}\boldsymbol{L}_{\substack{e\\o}n\lambda}(\gamma^{(0)})\Big]&\\ +\frac{1}{\gamma^{(0)}}\boldsymbol{L}'_{\substack{e\\o}n\lambda}(\gamma^{(0)})\Big[u_{12}^{(-)}\boldsymbol{R}_{\substack{e\\o}n\lambda}(\gamma^{(0)})+u_{22}^{(-)}\boldsymbol{L}_{\substack{e\\o}n\lambda}(\gamma^{(0)})\Big]\Bigg\}.&\quad(115)\end{aligned}$$

The same two kinds of sources located at infinity used in the problem of electromagnetic scattering by cylindrical multilayered chiral are employed here to calculate the radiated field. By applying the saddle point method and following the same procedure in [23] (p. 213), the radiated field can be written as:

$$\begin{aligned}\boldsymbol{E}_I^s=&\frac{E_0}{k_0\sin\gamma}\sum_{n=0}^{\infty}\begin{bmatrix}\cos(n\alpha)\\ \sin(n\alpha)\end{bmatrix}(2-\delta_{0n})i^n\\ &\cdot\Big\{\Big[u_{11}^{(-)}+u_{21}^{(-)}-u_{12}^{(-)}-u_{22}^{(-)}\Big]\boldsymbol{M}^{(3)}_{\substack{e\\o}n}(k\cos\gamma)\\ &+\Big[u_{11}^{(-)}-u_{21}^{(-)}-u_{12}^{(-)}+u_{22}^{(-)}\Big]\boldsymbol{N}^{(3)}_{\substack{e\\o}n}(k\cos\gamma)\Big\},\quad(116a)\\ \boldsymbol{E}_{II}^s=&-\frac{E_0}{k_0\sin\gamma}\sum_{n=0}^{\infty}\begin{bmatrix}\cos(n\alpha)\\ \sin(n\alpha)\end{bmatrix}(2-\delta_{0n})i^{n+1}\\ &\cdot\Big\{\Big[u_{11}^{(-)}+u_{21}^{(-)}+u_{12}^{(-)}+u_{22}^{(-)}\Big]\boldsymbol{M}^{(3)}_{\substack{e\\o}n}(k\cos\gamma)\\ &+\Big[u_{11}^{(-)}-u_{21}^{(-)}+u_{12}^{(-)}-u_{22}^{(-)}\Big]\boldsymbol{N}^{(3)}_{\substack{e\\o}n}(k\cos\gamma)\Big\}.\quad(116b)\end{aligned}$$

We can easily find Eq. (104) satisfy the following relation:

$$i\widehat{\boldsymbol{h}}e^{i\gamma z}=\frac{\boldsymbol{M}(k)}{k_s},\tag{117a}$$

$$\widehat{\boldsymbol{v}}e^{i\gamma z} = \frac{\boldsymbol{N}(k)}{k_s}, \tag{117b}$$

where

$$k_s = k_0 \sin\gamma \tag{118}$$

with the $\boldsymbol{M}(k)$ and $\boldsymbol{N}(k)$ defined in Eq. (113). It should be noted that the scattering coefficients obtained by the two methods are same. In 1981, Kong [58] expressed the electric field in each layer of multilayered isotropic media in term of an integration of Hankel functions. In 1994, Li et al. [42] developed the dyadic Green functions for planarly stratified media expanded in cylindrical wave function. In the present paper, the vector cylindrical wave function is used to present the problem of scattering by plannarly stratified chiral medium. According to the expansion [54]:

$$e^{i\boldsymbol{k}\cdot\boldsymbol{r}} = \sum_{n=0}^{\infty}(2-\delta_{n0})i^n J_n(k\rho\sin\gamma)\cos[n(\phi-\alpha)]e^{ikz\sin\gamma} \tag{119}$$

and the definition of vector cylindrical wave function and vector Cartesian wave function, the relation between them can be easily found. Thus, the results obtained from the two methods are identical.

6. CONCLUSIONS

In this paper, the electromagnetic wave scattering by a multilayered chiral sphere, a multilayered chiral cylinder and a planarly stratified chiral structure have been studied comparatively. Vector wave function expansion technique is applied in the comparative analysis. Boundary conditions are matched and a series of recursive transmission matrix equations is obtained and so are the scattering coefficients. Cartesian coordinates system, cylindrical coordinates system and spherical coordinates system are applied respectively to study the scattering problem; and correspondingly different vector wave functions are employed. Furthermore, the dyadic Green's functions for multilayered chiral media of different structures are utilized to calculate the radiated fields. The plane waves of perpendicular and parallel polarizations are equivalent to two different point sources located at infinity. The electromagnetic fields are formulated in terms of integrals consisting a volumetric current distribution and a dyadic Green's function. It is thus found from the comparative studies that the scattering problem can be considered as a specific radiation problem where the point radiated source is located at infinity. The principle used behind is very straightforward, elegant, and quite standard especially when the dyadic Green's functions are known.

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