Progress In Electromagnetics Research, PIER 27, 37–59, 2000

SCATTERING OF ELECTROMAGNETIC WAVES FROM A DEEPLY BURIED CIRCULAR CYLINDER

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1. INTRODUCTION

The scattering of electromagnetic waves by buried objects has wide applications in civil and military environments. For example, the scattering of electromagnetic waves from buried cylindrical objects has application in remote sensing of buried pipes and cables and in the understanding of mutual interaction between the buried object and surrounding media. This type of problems have been treated mainly by integral equation combined with numerical techniques such as moment method. Butler *et al.* [1] solved an integral equation using a numerical method for the induced currents on the, perfectly conducting circular cylinder buried near the planer interface of the two homogeneous media. They used these currents to calculate the far-zone scattered fields from coupled and partially buried cylinders at the interface between two media [2, 3].

The exact analytic formulations of buried object are available in some particular cases. The reason why such analytic results are few and far between is that the determination of electromagnetic fields scattered by inhomogeneities reduces to the solution of a Helmholtz's equation in a complicated configuration. The classical method of solution of Helmholtz's equation, i.e., method of separation of variables, yields analytic results only for those inhomogeneities whose surfaces coincide with the coordinate surfaces of orthogonal coordinate systems in which Helmholtz's equation is separable. The exact analytic analysis of a buried circular cylinder was first undertaken by D'Yakonov [4] but the results of his analytic work are not suitable for numerical evaluation. This has been pointed out by Howard [5] and Ogunade [6]. Ogunade [6] extended D'Yakanov's exact analytic solution to obtain solution for the scattered field of an embedded circular cylinder using conventional eigenfunction expansion. This treatment required much numerical calculation to get the desired physical quantities as has been pointed out by Hongo and Hamamura [7]. Ogunade [6] considers conducting circular cylinder inside a dielectric cylinder and takes the limit that the radius of the dielectric cylinder goes to infinity. Therefore his results cannot be extended to arbitrary configurations and other cross-sections. Hongo and Hamamura [7] calculated the far-zone scattered field expressions for a strip buried in a dielectric half-space. They have also given the numerical results for the perfectly conducting circular cylinder buried in a dielectric half-space. They assumed in their analysis that the obstacle is so deeply buried that only the first order reflection from the dielectric interface towards the buried obstacle is important. Higher order multiple reflections between the dielectric interface and the buried object were ignored to calculate the total far-zone scattered fields from the buried object. They evaluated the far-zone scattered fields directly without using the corresponding induced current distribution on the buried obstacle.

The scattered fields from a cylinder can be calculated by integrating the Green's function of the configuration over the corresponding induced current distribution. Techniques for computing the radiated fields due to a line source located at the interface of the two media or in one of the two half-spaces are available in the literature [1, 8–9]. The only complexity involved in the evaluation of scattered fields from a buried cylinder is the determination of unknown induced current distribution on the buried cylinder. Although it is not possible to give a general, exact and analytic solution for the induced current distribution on the buried cylinder, but there exists a situation in which one can approximate the actual induced current distribution on the buried cylinder. In this situation it is assumed that the cylinder is deeply buried. The assumption of deeply buried states that the cylinder is so deeply buried that the scattered fields due to primary induced currents on the cylinder, after reflection from the interface containing the cylinder, have no interaction with the buried cylinder. Thus actual induced current distribution on the deeply buried cylinder can be approximated with induced current distribution as if the cylinder is in a homogeneous medium [10]. This approximate induced current may be utilized to calculate the scattered fields from a cylinder deeply buried in a certain configuration.

This assumption of deeply buried cylinder decouples the original scattering problem into two relatively simple parts. One part deals with the configuration of the scattering problem. In this part of the problem, field radiated by a line source in the given configuration is derived. The other part of the problem deals with the cylinder as if it is in a homogeneous medium. In this part the currents excited on the cylinder are calculated. Now the scattered field due to the buried cylinder can be obtained by considering the primary induced current on the buried cylinder as source and fields due to the line sources are summed up to yield the scattered fields from the buried cylinder.

In this paper, approximation to the induced currents is utilized on a circular cylinder which is deeply buried in a dielectric half-space configuration. Circular cylinder is selected for buried object, because it can be a model for various practical cylindrical objects, e.g., pipes and cables. Far-zone scattered fields and other scattering parameters of deeply buried circular cylinder are calculated. It is found that after this approximation to the induced currents the problem becomes relatively easy and the resulting expression is quite simple to give a physical interpretation. Different scattering parameters can be studied in terms of corresponding scattering parameters for the case of homogeneous medium. In this way one can develop better understanding of

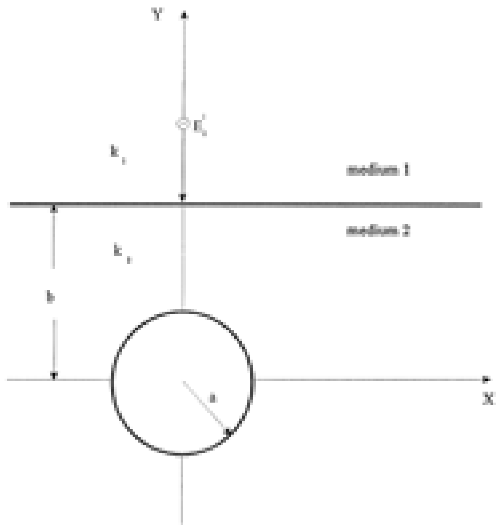

Figure 1. A perfectly conducting circular cylinder deeply buried in a dielectric half-space.

interaction between the buried object and surrounding media. It may be noted that this approximation may be extended to other parallel layer configurations and objects of arbitrary shapes.

2. EXCITATION OF CURRENTS

Geometry of the scattering problem is shown in Fig. 1. A perfectly conducting circular cylinder of radius a is deeply buried in a dielectric half-space. The depth of the circular cylinder from the dielectric interface is b. The cylinder is of infinite extent and its axis is coincident with z coordinate axis. Medium 1 has propagation constant k_1 and fills space $y > b$, while medium 2 has propagation constant k_2 and is placed in space $y < b$.

2.1. TE Case

A perpendicularly polarized plane wave is normally incident from medium 1 whose electric field is given as

$$E_z^i = \exp\{-ik_1(y-b)\}.$$

The incident field varies harmonically with time as $\exp(-i\omega t)$ which has been suppressed. The field transmitted in medium 2 is given as

$$E_z^t = T_{12}\exp(-ik_2 y) \tag{2.1}$$

where T_{12} is the Fresnel transmission coefficient for the fields traveling from medium 1 to medium 2 and is given as

$$T_{12} = \frac{2k_1}{k_1 + k_2}\exp(ik_2 b).$$

Since the cylinder is deeply buried, therefore transmitted field (2.1) is considered as a field incident on the embedded cylinder. The electric field given by (2.1) is now converted in a series of cylindrical waves to facilitate the application of the boundary conditions in cylindrical coordinates (ρ, ϕ). Using the plane wave expansion, the transmitted field can be written as

$$E_z^t = T_{12}\sum_{n=-\infty}^{n=\infty}(-1)^n J_n(k_2\rho)\exp(in\phi).$$

The scattered field due to the cylinder can be represented as

$$E_z^{sh} = \sum_{n=-\infty}^{n=\infty} A_n H_n^{(1)}(k_2\rho)\exp(in\phi) \tag{2.2}$$

where A_n are constants which are easily determined by applying boundary condition, that total tangential electric field must be zero on the cylinder. The constants A_n are given as

$$A_n = -\frac{(-1)^n T_{12} J_n(k_2 a)}{H_n^{(1)}(k_2 a)}.$$

Since the problem is independent of z coordinate so total tangential magnetic field H_ϕ^{tot} from the Maxwell's equation can be calculated as

$$H_\phi^{\text{tot}} = \frac{i}{\omega\mu}\frac{\partial E_z^{\text{tot}}}{\partial\rho} \tag{2.3}$$

where $E_z^{\text{tot}} = E_z^{sh} + E_z^t$ is the total electric field in the medium 2. The surface current density on the perfect conductor is given as

$$\mathbf{e}_\rho \times H_\phi \mathbf{e}_\phi = J_{sz}(\rho,\phi)\mathbf{e}_z, \quad \text{at } \rho = a \tag{2.4}$$

where $\mathbf{e}_\rho$ is outward normal to the conductor. Therefore,

$$J_{sz}(a,\phi) = \frac{2T_{12}}{\pi\omega\mu a}\sum_{n=-\infty}^{n=\infty}(-1)^n\frac{\exp(in\phi)}{H_n^{(1)}(k_2a)} \tag{2.5}$$

where the Wronskian for the Hankel function has been used to simplify the expression.

2.2. TM Case

A parallel polarized plane wave is normally incident from medium 1 whose magnetic field is given as

$$H_z^i = \exp\{-ik_1(y-b)\}.$$

The corresponding induced current distribution on the circular cylinder is

$$J_{s\phi}(a,\phi) = \frac{-i2T_{12}}{\pi k_2 a}\sum_{n=-\infty}^{n=\infty}(-1)^n\frac{\exp(in\phi)}{H_n^{(1)'}(k_2a)}. \tag{2.6}$$

The surface current distributions in (2.5) and (2.6) will now be used to calculate the scattered field taking into account the effect of interface between the two dielectric media.

3. FAR-ZONE SCATTERED FIELDS

The scattered electric field above the dielectric interface due to the perfectly conducting cylinder can be obtained by calculating the scattered field for each element of current on the conducting cylinder and summing over all such elements of the surface current.

3.1. TE Case

The total scattered electric field due to the perfectly conducting cylinder can be written as

$$E_z^s = a\int_0^{2\pi} J_{sz}(a,\phi')G_e(\rho,\phi;a,\phi')d\phi' \tag{3.1}$$

where

$$G_e(\rho,\phi;a,\phi') = \begin{cases} G_{e1}(\rho,\phi;a,\phi'), & y \geq b \\ G_{e2}(\rho,\phi;a,\phi'), & y \leq 0. \end{cases}$$

Substituting the value of Green's function $G_{e1}(\rho,\phi;a,\phi')$ from [11] and current density (2.5), expression for the scattered field is obtained

$$
\begin{aligned}
E_z^s = {} & \frac{-2T_{12}k_1}{\pi\sqrt{2\pi}(k_1^2-k_2^2)}\frac{\exp(ik_1\rho - i\pi/4)}{\sqrt{k_1\rho}} \\
& \left\{k_1\sin^2\phi - \sin\phi\sqrt{k_2^2-k_1^2\cos^2\phi}\right\} \\
& \times \exp(-ik_1 b\sin\phi + ib\sqrt{k_2^2-k_1^2\cos^2\phi})\sum_{n=-\infty}^{n=\infty}\frac{(-1)^n}{H_n^{(1)}(k_2 a)} \\
& \times \int_0^{2\pi}\exp(-ik_1 a\cos\phi\cos\phi' \\
& - ia\sqrt{k_2^2-k_1^2\cos^2\phi}\sin\phi' + in\phi')\,d\phi'.
\end{aligned}
\tag{3.2}
$$

Simplifying the integral in above expression by substituting

$$
\phi_0 = \tan^{-1}\sqrt{\left(\frac{k_2}{k_1\cos\phi}\right)^2 - 1}
$$

and converting resulting exponential term to corresponding plane wave representation, it can be easily integrated as follows

$$
\begin{aligned}
& \sum_{k=-\infty}^{k=\infty}\int_0^{2\pi}(-i)^k J_k(k_2 a)\exp\{i(n+k)\phi' - ik\phi_0\}d\phi' \\
& \qquad = 2\pi(-i)^n J_n(k_2 a)\exp(in\phi_0).
\end{aligned}
\tag{3.3}
$$

Substituting these results in (3.2) yields

$$
\begin{aligned}
E_z^s = {} & \frac{-8k_1^2}{\sqrt{2\pi}(k_1^2-k_2^2)(k_1+k_2)}\exp(ik_2 b - ik_1 b\sin\phi + ib\sqrt{k_2^2-k_1^2\cos^2\phi}) \\
& \times\left\{k_1\sin^2\phi - \sin\phi\sqrt{k_2^2-k_1^2\cos^2\phi}\right\}\sum_{n=-\infty}^{n=\infty}\left\{\frac{(i)^n J_n(k_2 a)\exp(in\phi_0)}{H_n^{(1)}(k_2 a)}\right\} \\
& \times\frac{\exp\{ik_1\rho - i\pi/4\}}{\sqrt{k_1\rho}}, \qquad (0 \le \phi \le \pi).
\end{aligned}
\tag{3.4}
$$

This is the total scattered field in medium 1 due to the buried perfectly conducting cylinder of radius a.

Substituting the value of Green's function $G_{e2}(\rho, \phi; a, \phi')$ from [11] and current density (2.5), the far-zone field scattered from a buried perfectly conducting circular cylinder is obtained as

$$E_z^s = \frac{-T_{12}\exp(i\rho - i\pi/4)}{\pi\sqrt{2\pi}\sqrt{k_2\rho}} \sum_{n=-\infty}^{n=\infty} (-1)^n \frac{I_- + R(-\phi) I_+ \exp(-i2k_2 b\sin\phi)}{H_n^{(1)}(k_2 a)} \tag{3.5}$$

where I_- and I_+ are defined as

$$I_\pm = \int_0^{2\pi} \exp\{-ik_2 a\cos(\phi \pm \phi') + in\phi'\} d\phi'.$$

The integral I_- and I_+ can be solved in a same way as the integral involved in the scattered field expression for medium 1. Thus (3.5) simplifies to

$$\begin{aligned} E_z^s = & \frac{-4k_1\exp(ik_2 b)\exp(ik_2\rho - i\pi/4)}{\sqrt{2\pi}(k_2 + k_1)\sqrt{k_2\rho}} \times \sum_{n=-\infty}^{n=\infty} \frac{(i)^n J_n(k_2 a)}{H_n^{(1)}(k_2 a)} \\ & \{\exp(in\phi) + \exp(-in\phi) R(-\phi)\exp(-i2k_2 b\sin\phi)\}, \\ & (-\pi \le \phi \le 0) \end{aligned} \tag{3.6}$$

where

$$R(-\phi) = \frac{n\sin\phi - \sqrt{1 - n^2\cos^2\phi}}{n\sin\phi + \sqrt{1 - n^2\cos^2\phi}}.$$

The above expression gives the total far-zone scattered field due to the perfectly conducting cylinder of radius a in the region whose propagation constant is k_2.

3.2. TM Case

The scattered magnetic field by the cylinder can be obtained using following relation

$$H_z^s = a\int_0^{2\pi} J_{s\phi}(a, \phi') G_h(\rho, \phi; a, \phi') d\phi' \tag{3.7}$$

where

$$G_h(\rho,\phi;a,\phi') = \begin{cases} G_{h1}(\rho,\phi;a,\phi'), & y \geq b \\ G_{h2}(\rho,\phi;a,\phi'), & y \leq 0. \end{cases}$$

$G_h(\rho,\phi;a,\phi')$ is the Green's function due to an arbitrary oriented electric dipole placed at point (ρ',ϕ') in medium 2 [3,12]. Substituting the expressions G_{h1}, the corresponding Green's function in medium 1 and the current distribution $J_{s\phi}$ in Eq. (3.7), following is obtained

$$H_z^s = \frac{-8\epsilon_1 k_1 k_2}{\sqrt{2\pi}(k_1+k_2)}\left\{\frac{\exp(ik_2 b - ik_1 b\sin\phi + ib\sqrt{k_2^2 - k_1^2\cos^2\phi})}{\epsilon_1\sqrt{k_2^2 - k_1^2\cos^2\phi} + \epsilon_2 k_1\sin\phi}\right\}\sin\phi$$
$$\times\frac{\exp(ik_1\rho - i\pi/4)}{\sqrt{k_1\rho}}\sum_{n=-\infty}^{n=\infty}(i)^n\frac{J_n'(k_2 a)\exp(in\phi_0)}{H_n^{(1)'}(k_2 a)},\ (0\leq\phi\leq\pi). \quad (3.8)$$

The far zone scattered magnetic field in medium 2, below the perfectly conducting buried cylinder is given as

$$H_z^s = \frac{-2T_{12}}{\sqrt{2\pi}}\frac{\exp(ik_2\rho - i\pi/4)}{\sqrt{k_2\rho}}\times\sum_{n=-\infty}^{n=\infty}\frac{(i)^n J_n'(k_2 a)}{H_n^{(1)'}(k_2 a)}$$
$$\{\exp(in\phi) + R_h(-\phi)\exp(-i2k_2 b\sin\phi)\exp(-in\phi)\},$$
$$(-\pi\leq\phi\leq 0) \quad (3.9)$$

where

$$R_h(-\phi) = \frac{\sin\phi + n\sqrt{1-n^2\cos^2\phi}}{\sin\phi - n\sqrt{1-n^2\cos^2\phi}}.$$

4. THE SCATTERED FIELDS

The complicated looking expression for the total scattered electric field obtained in the previous section can be explained quite simply. A combination of wave and ray optics will be used for explaining the form of the scattered field. For this purpose it is expedient to introduce some new variables. The relative index of refraction is defined as $n = k_2/k_1$. Two angles ψ_1 and ψ_2 are defined, which are angles of refraction and incidence respectively, for a ray traveling from medium 2 to medium 1. Analytically these angles can be written as

$$\psi_1 = \frac{\pi}{2} - \phi, \qquad \psi_2 = \sin^{-1}\left(\frac{\sin\psi_1}{n}\right), \qquad 0 \leq \phi \leq \pi.$$

Using these definitions, expression (3.4) can be rewritten after some manipulation as a product of various factors which are defined below

$$E_z^s = PQSWTUV \tag{4.1}$$

where

$$\begin{aligned}
P &= \frac{2}{1+n} \\
Q &= \exp(ik_2 b) \\
S &= -\sqrt{\frac{2}{\pi}} \exp\left(\frac{-i\pi}{4}\right) \sum_{m=-\infty}^{\infty} (i)^m \frac{J_m(k_2 a)}{H_m^{(1)}(k_2 a)} \exp\left\{im\left(\frac{\pi}{2} - \psi_2\right)\right\} \\
W &= \frac{\cos\psi_1}{n\cos\psi_2} \\
T &= \frac{2n\cos\psi_2}{\cos\psi_1 + n\cos\psi_2} \\
U &= \exp\{ik_1 b(n\cos\psi_2 - \cos\psi_1)\} \\
V &= \frac{\exp(ik_1\rho)}{\sqrt{k_1\rho}}.
\end{aligned}$$

Consider a unit amplitude TE wave normally incident on a plane dielectric interface. In medium 2 a transmitted wave is excited with an amplitude given by the Fresnel transmission coefficient. This contributes the factor P in (4.1). As this wave travels towards the cylinder it undergoes a phase change $ik_2 b$. This is indicated by the factor Q. The scattering of a plane wave by a circular cylinder in a homogeneous medium is a well known problem. The scattered field due to the cylinder for large distance from the cylinder can be written as $E_0 S \exp(ik_2\rho)/\sqrt{k_2\rho}$, where E_0 is the amplitude of the incident plane wave. In the present case this amplitude is given as PQ. The factor S describes the angular distribution of the field scattered from the cylinder buried in a homogeneous medium. Therefore the field scattered by the cylinder in medium 2 is a cylindrical wave whose amplitude in any given direction is PQS. Consider Fig. 2 in which a ray of the scattered

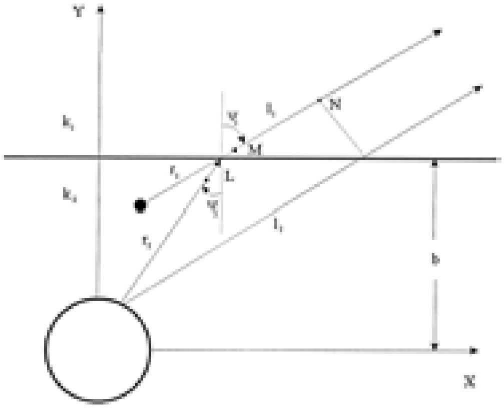

Figure 2. Ray diagram of scattered fields from buried cylinder for above the dielectric interface.

field in direction ψ_2 strikes the plane interface at point L after covering a distance r_2. The field at L is given as, $PQS\exp(ik_2r_2)/\sqrt{k_2r_2}$. This ray is now diffracted by the plane interface and the amplitude of this ray is modified by the Fresnel transmission coefficient T from medium 2 to medium 1 for the angle at which the ray is incident. Therefore the field at point M, just above the interface, on this ray is given as, $PQST\exp(ik_2r_2)/\sqrt{k_2r_2}$. The radius of curvature of the incident wavefront at point L is r_2. The radius of curvature of the diffracted wavefront is denoted by r_1. The relation between the two radii of curvature following Born and Wolf [13] may be easily calculated as

$$r_1 = r_2 \frac{\cos^2\psi_1}{n\cos^2\psi_2}. \tag{4.2}$$

As the diffracted ray travels to point N, at a distance l_1 from M in medium 1, the field amplitude on the ray according to the inverse squar law becomes

$$PQST\frac{\exp(ik_2r_2)}{\sqrt{k_2r_2}}\sqrt{\frac{r_1}{r_1+l_1}}\exp(ik_1l_1).$$

Finally as the ray travels to a point ρ in the far-zone from N it travels an additional distance $\rho - l_2$, where l_2 is the distance from the origin to the interface on a ray parallel to the emergent ray. The amplitude at ρ will be

$$E_z^s = PQST\frac{\exp(ik_2r_2)}{\sqrt{k_2r_2}}\sqrt{\frac{r_1}{r_1+l_1}}\exp(ik_1l_1)$$
$$\sqrt{\frac{r_1+l_1}{\rho-l_2+l_1+r_1}}\exp\{ik_1(\rho-l_2)\}.$$

As ρ is much greater than r_1, l_1 and l_2, therefore $\rho \gg l_1 - l_2 + r_1$. Using (4.2) the expression for the scattered field reduces to

$$E_z^s = PQSWT\exp(ik_2r_2 + ik_1l_1 - ik_1l_2)\frac{\exp(ik_1\rho)}{\sqrt{k_1\rho}}.$$

From Fig. 2, it can be easily shown that the phase factor $k_2r_2 + k_1l_1 - k_1l_2$ can be written as $k_1b(n\cos\psi_2 - \cos\psi_1)$. Thus the last two terms in the above expression can be identified as the factors U and V.

The scattered field below the interface can be explained in a similar manner, by rewriting them as a sum of two terms containing various factors which are defined below. Let

$$E_z^s = PQ(S_+)V + PQ(S_-)RUV \tag{4.3}$$

where

$$\begin{aligned}
P &= \frac{2}{1+n} \\
Q &= \exp(ik_2b) \\
S_\pm &= -\sqrt{\frac{2}{\pi}}\exp\left(\frac{-i\pi}{4}\right)\sum_{m=-\infty}^{\infty}(i)^m\frac{J_m(k_2a)}{H_m^{(1)}(k_2a)}\exp(\pm im\phi) \\
V &= \frac{\exp(ik_2\rho)}{k_2\rho} \\
U &= \exp(-2ik_2b\sin\phi) \\
R &= \frac{-k_2\sin\phi - \sqrt{k_1^2 - k_2^2\cos^2\phi}}{-k_2\sin\phi + \sqrt{k_1^2 - k_2^2\cos^2\phi}}.
\end{aligned}$$

Consider Fig. 3, where a plane wave with amplitude PQ is incident on the cylinder. The field scattered by the cylinder in the two regions

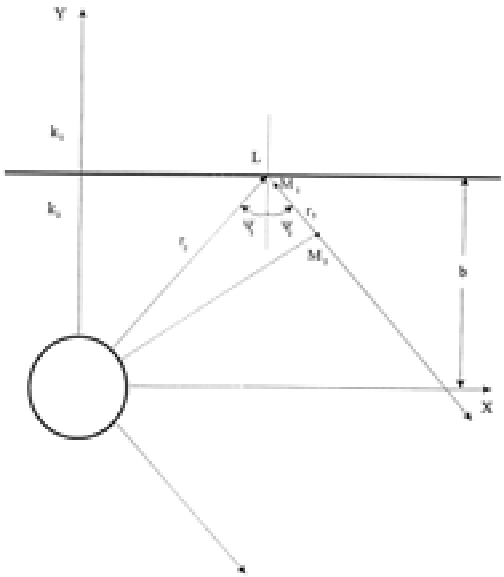

Figure 3. Ray diagram of scattered fields from buried cylinder for below the dielectric interface.

$0 < \phi < \pi$ and $\pi < \phi < 2\pi$ is separated and corresponding angular distributions are denoted by S_- and S_+ respectively. The scattered field in the direction ϕ is contributed by two rays. One ray gives the directly scattered field and its contribution is $PQ(S_+)V$. The amplitude on a ray traveling towards the interface is $PQ(S_-)$. This ray is now reflected by the plane interface and the amplitude of this ray is modified by the Fresnel reflection coefficient R from medium 2 to medium 1 for the angle at which the ray is incident. Therefore the field at point M_1, just below the interface, on this ray is given as $PQ(S_-)R\exp(ik_2r_2)/\sqrt{k_2r_2}$. Finally as the ray travels to a point ρ in the far-zone from M_1 it travels an additional distance $\rho + r_3$ than the direct scattered ray, where r_3 is the distance from the dielectric interface to the point M_2. The amplitude at ρ will be

$$PQ(S_-)R\frac{\exp(ik_2r_2)}{\sqrt{k_2r_2}}\sqrt{\frac{r_2}{\rho + r_3}}\exp\{ik_2(\rho + r_3)\}.$$

The expression for the scattered field contributed by both rays is

$$E_z^s = PQ(S_+)V \;+\; PQ(S_-)R\exp\{ik_2(r_2+r_3)\}\frac{\exp(ik_2\rho)}{\sqrt{k_2\rho}}.$$

From Fig. 3 it can be easily shown that the phase factor $2k_2(r_2+r_3)$ can be written as $-2k_2 b\sin\phi$. Thus the last two terms in the above expression can be identified as the factors U and V.

5. FIELD PATTERNS

The normalized field is obtained by dividing the field component $|E_z^s|$ by its value at $\phi=\pi/2$, i.e.,

$$E_n = \frac{|E_z^s|}{|E_z^s(\pi/2)|}. \tag{5.1}$$

The normalized field expression for buried circular cylinder contains slowly convergent series. The coefficients of the series are $J_m(k_2a)/H_m^{(1)}(k_2a)$. These coefficients for fixed argument k_2a, taking m to be large behave as [14]

$$\frac{1}{1-2i\{k_2a\exp(1)/2m\}^{-2m}}.$$

In order to truncate the series, the value of m corresponding to the coefficient value 10^{-30} of the series is selected. The values of m which give the coefficients less than 10^{-30} are neglected.

The normalized field patterns E_n of the buried circular cylinder are plotted in Fig. 4. It is seen that at the dielectric interface field pattern has a null. Above the dielectric interface field pattern contains a single lobe. The lobe has its maximum value in the direction $\phi=\pi/2$. Below the dielectric interface the field patterns is divided into three sectors. In sectors $2\pi-\phi_c\le\phi\le 2\pi$ and $\pi\le\phi\le\pi+\phi_c$ field pattern contains many lobes. The number of lobes in the field pattern increases with the increase in the depth. The increase in the side lobes is due to the increase in the interference between the scattered field and the scattered field reflected by the dielectric interface. In the middle sector the field pattern contains one lobe with ripples. The variation in the field pattern in the sector $\pi+\phi_c\le\phi\le 2\pi-\phi_c$ is low.

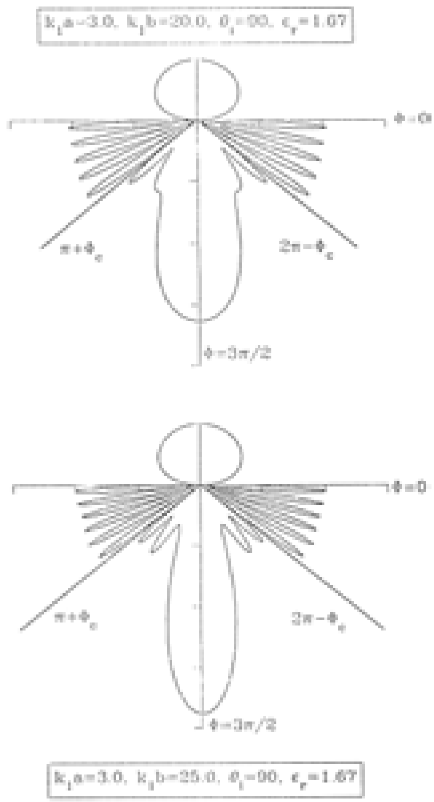

Figure 4. The normalized field pattern of buried cylinder.

In the radar scattering problems, the scattered field above the dielectric interface is of interest. The field pattern above the dielectric interface contains single lobe regardless of radius of the circular cylinder and dielectric in which the circular cylinder is buried. Three dB beamwidth of this lobe above the dielectric interface is $\pi - 2\phi^*$. Where ϕ^* is the angle at which E_n decreases to $1/\sqrt{2}$. Three dB beamwidth of the lobe as function of radius a/λ_1 of the buried cylinder and dielectric constant n is shown in Fig. 5, where λ_1 is the wavelength in medium 1.

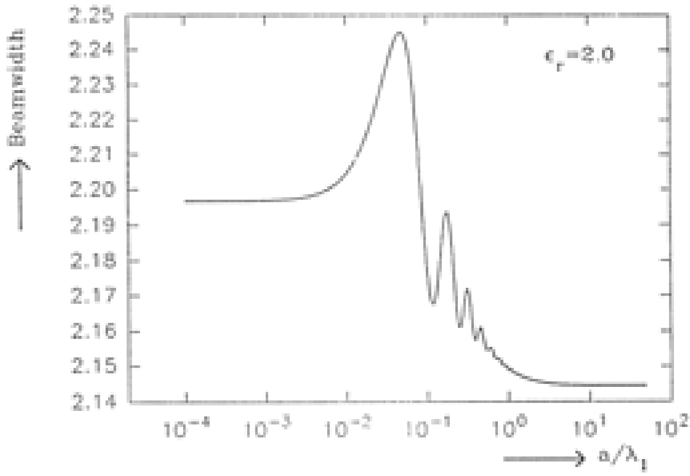

Figure 5.a The effect of radius a/λ_1 on the beamwidth.

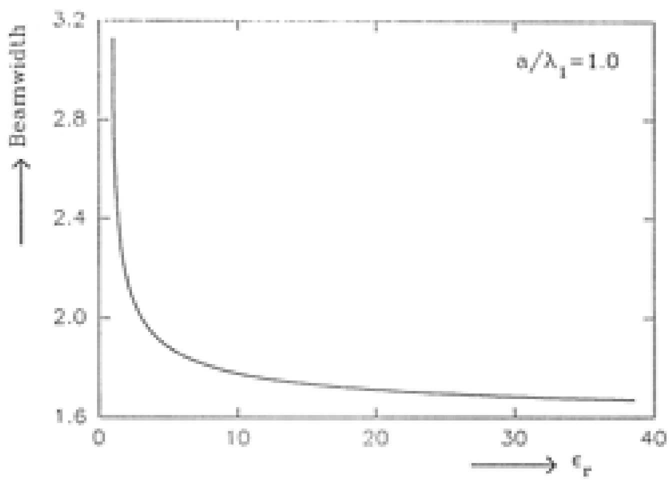

Figure 5.b The effect of relative dielectric constant ϵ_r on the beamwidth.

6. SCATTERING WIDTH AND SCATTERED POWER

6.1. TE Case

From the geometry in Fig. 1, it is clear that field is independent of z and has only the components E_z^s, H_ϕ^s and H_ρ^s. By the use of (2.3), H_ϕ^s can be calculated from a knowledge of E_z^s. The Poynting vector in the far-zone has only the ρ component and is given as

$$P^{\pm} = \frac{1}{2}\sqrt{\frac{\epsilon_{\pm}}{\mu}}|E_z^s|^2, \qquad \epsilon_+ = \epsilon_1, \quad \epsilon_- = \epsilon_2 \tag{6.1}$$

where P^+ is the far-zone Poynting vector in the region above the dielectric interface of two homogeneous media and P^- for below the dielectric interface.

One of the important parameters in two dimensional scattering problems is the scattering width. When the transmitter and receiver are at same location the scattering width is referred to as monostatic scattering width. The monostatic scattering width σ for the buried circular cylinder is [15]

$$\sigma = \lim_{\rho\to\infty} 2\pi\rho \frac{P^+(\pi/2)}{P^{\text{inc}}} \tag{6.2}$$

which is obtained by taking the ratio of Poynting vector in the backward direction to the incident power. The incident power P^{inc} is $\sqrt{\epsilon_1/4\mu}$. Using (6.2) yields the following expression for the scattering width

$$\sigma = \frac{64}{(1+n)^4 k_1}\left|\sum_{m=-\infty}^{\infty}(-1)^m \frac{J_m(k_2 a)}{H_m(k_2 a)}\right|^2.$$

The scattering width of the buried cylinder in terms of scattering width of the cylinder in homogeneous medium 2 is

$$\sigma = nT_{12}^4\sigma_1$$

where σ_1 is the scattering width of the cylinder placed in a homogeneous whose propagation is k_2. This expression gives the relation between the scattering widths of the circular cylinder in homogeneous medium and when buried in dielectric half-space.

Another quantity of interest is the total scattered power from the buried cylinder. For this purpose the Poynting vector in the far-zone both above and below the dielectric interface is calculated. Substituting the values of E_z^s from (3.4) and (3.6) in (6.1), the following expression for scattered power above and below the dielectric interface are obtained

$$\begin{aligned} P^+ &= \frac{G}{\rho}\left|T(\phi)\right|^2\left|Q(\psi)\right|^2 \\ P^- &= \frac{G}{\rho}\left|Q(\phi)\ +\ R(\phi)\,F(\phi)\,Q(-\phi)\right|^2 \end{aligned} \tag{6.3}$$

where

$$G = \frac{4}{\pi\omega\mu(1+n)^2}$$

$$Q(\phi) = \sum_{m=-\infty}^{\infty}(i)^m\frac{J_m(k_2a)}{H_m^{(1)}(k_2a)}\exp(im\phi)$$

$$\psi = \tan^{-1}\sqrt{(n/\cos\phi)^2-1}$$

$$F(\phi) = \exp\{-i2nk_1b\sin\phi\}$$

$$R(\phi) = \frac{-n\sin\phi-\sqrt{1-n^2\cos^2\phi}}{-n\sin\phi+\sqrt{1-n^2\cos^2\phi}}$$

$$T(\phi) = \frac{2\sin\phi}{1-n^2}\left\{\sin\phi-\sqrt{n^2-\cos^2\phi}\right\}.$$

The total scattered power from the buried perfectly conducting cylinder in the far-zone is

$$P = \int_0^{\pi}P^+\rho d\phi + \int_{\pi}^{2\pi}P^-\rho d\phi.$$

Substituting the values of P^+ and P^- in the above equation yields

$$\begin{aligned}P =&G\left\{\int_0^{\pi} |T(\phi)|^2\,|Q(\psi)|^2 d\phi + \int_{\pi}^{2\pi} |Q(\phi)|^2 d\phi\right.\\&+\int_{\pi}^{2\pi} |R(\phi)|^2 |Q(-\phi)|^2 d\phi + \int_{\pi}^{2\pi} Q^*(\phi)\,R(\phi)\,F(\phi) Q(-\phi) d\phi\\&\left.+\int_{\pi}^{2\pi} Q(\phi)\,R^*(\phi)\,F^*(\phi) Q^*(-\phi) d\phi\right\}. \qquad (6.4)\end{aligned}$$

It can be easily shown that sum of first three integrals in the (6.4) is $\int_0^{2\pi} |Q(\phi)|^2 d\phi$.

$$\begin{aligned}P = G\int_0^{2\pi} |Q(\phi)|^2 d\phi + G&\left\{\int_{\pi}^{2\pi} W(\phi)\exp(-i2k_2 b\sin\phi) d\phi\right.\\&\left.+\int_{\pi}^{2\pi} W^*(\phi)\exp(i2k_2 b\sin\phi) d\phi\right\} \qquad (6.5)\end{aligned}$$

where

$$W(\phi) = Q^*(\phi) R(\phi) Q(-\phi).$$

Since the cylinder is deeply buried, the last two integrals of (6.5) are solved asymptotically for large $k_2 b$. The method of stationary phase [16] is used. The stationary point of integrand for the both integrals is at $\phi = -\pi/2$. It is found that (6.5) yields the following expression for the scattered power from buried circular cylinder in the far-zone

$$\begin{aligned}P = G\int_0^{2\pi} |Q(\phi)|^2 d\phi + G\sqrt{\pi/k_2 b}\Big[&W(-\pi/2)\exp\{-i(2k_2 b - \pi/4)\}\\&+ W^*(-\pi/2)\exp\{i(2k_2 b - \pi/4)\}\Big]\end{aligned}$$

where

$$W(-\pi/2) = \left(\frac{n-1}{n+1}\right) Q^*(-\pi/2) Q(\pi/2).$$

The first term of above expression is the total scattered power from the cylinder in homogeneous medium with propagation constant k_2. The incident power on the cylinder is $\sqrt{\epsilon_2/4\mu}\,T_{12}^2$. The second term gives the effect of the interaction between cylinder and the dielectric interface. Using a result obtained by Papas [17] the total scattered power from the circular cylinder in homogeneous medium can be written as

$$\begin{aligned} P &= G2\pi\Re[Q(-\pi/2)] \\ &\quad\times\left\{1+\sqrt{\pi/k_2b}\frac{n-1}{\pi(n+1)}\frac{|Q^*(-\pi/2)Q(\pi/2)|}{\Re[Q(-\pi/2)]}\cos(2k_2b-\pi/4)\right\} \\ &= C_1 + C_2\sqrt{1/k_2b}\,\cos(2k_2b-\pi/4) \end{aligned} \tag{6.6a}$$

where $\Re$ stands for the real part. The first term C_1 and factor C_2 of the second term in (6.6a) are functions of radius a. It is clear from above expression that the behaviour of the total scattered power from the buried cylinder is decaying oscillatory around a mean value C_1 as the depth b of buried cylinder is increased. The period of these decaying oscillations with depth is $\lambda_2/4$, where λ_2 is the wavelength in medium 2. It is numerically observed that when a is large, the ratio

$$\lim_{a\to\infty}\frac{|Q^*(-\pi/2)Q(\pi/2)|}{\Re[Q(-\pi/2)]}$$

tends to $\pi\sqrt{a/2\lambda_2}$. Using the numerical results (6.6a) simplifies to

$$P = G2\pi\Re[Q(-\pi/2)]\left\{1+\sqrt{\frac{a}{b}}\frac{n-1}{2(n+1)}\cos(2k_2b-\pi/4)\right\}. \tag{6.6b}$$

Above expression gives the total scattered power from circular cylinders of large radii in the far-zone. This expression shows that the behaviour of total scattered power with the increase in depth is decaying oscillatory with period $\lambda_2/4$ around a mean value which is the power scattered from the cylinder in homogeneous medium having propagation constant k_2.

6.2. TM Case

The far zone poynting vector for TM case is given as

$$P_h^{\pm} = \frac{1}{2}\sqrt{\frac{\mu}{\epsilon_\pm}}|H_z^s|^2, \qquad \epsilon_+ = \epsilon_1, \quad \epsilon_- = \epsilon_2.$$

The corresponding expression for scattering width therefore becomes

$$\sigma_h = \frac{64}{(1+n)^4 k_1}\left|\sum_{n=-\infty}^{n=\infty}(-1)^n \frac{J_n'(k_2 a)}{H_n^{(1)'}(k_2 a)}\right| = nT_{12}^4\sigma_1. \tag{6.7}$$

The poynting vectors in the far zone, both above and below the interface have been calculated as

$$P_h^+ = \frac{G_h}{\rho}|T_h(\phi)|^2|Q_h(\psi)|^2 \tag{6.8}$$

$$P_h^- = \frac{G_h}{\rho}|Q_h(\phi) + R_h(-\phi)F(\phi)|Q_h(-\phi)|^2 \tag{6.9}$$

where

$$G_h = \frac{4}{\pi\omega\epsilon_1(1+n)^2}$$

$$Q_h(\phi) = \sum_{m=-\infty}^{m=\infty}(i)^m \frac{J_m'(k_2 a)\exp(im\phi)}{H_m^{(1)'}(k_2 a)}$$

$$\psi = \tan^{-1}\sqrt{(n/\cos\phi)^2 - 1}$$

$$F(\phi) = \exp(-i2nk_1 b\sin\phi)$$

$$T_h(\phi) = \frac{2\sin\phi}{\sqrt{1-(\cos\phi/n)^2} + n\sin\phi}.$$

Hence for TM case, the total power scattered from the buried conducting cylinder in the far zone is

$$P_h = \int_0^\pi P_h^+\rho d\phi + \int_\pi^{2\pi} P_h^-\rho d\phi\,.$$

Substituting P^+ and P^- in the above equation and after manipulation following results are obtained

$$\begin{aligned} P_h &= G_h 2\pi \Re[Q_h(-\pi/2)] \\ &\quad \times \left\{ 1 + \sqrt{\pi/k_2 b} \frac{n-1}{\pi(n+1)} \frac{|Q_h^*(-\pi/2) Q_h(\pi/2)|}{\Re[Q_h(-\pi/2)]} \cos(2k_2 b - \pi/4) \right\} \\ &= C_1' + C_2' \sqrt{1/k_2 b} \cos(2k_2 b - \pi/4). \end{aligned} \tag{6.10}$$

The behaviour of the total scattered power from the buried cylinder is decaying oscillatory around a mean value C_1' as the depth b of buried cylinder is increased.

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