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SCATTERING FROM A CYLINDRICAL OBJECT BURIED IN A GEOMETRY WITH PARALLEL PLANE INTERFACES

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1. INTRODUCTION

Scattering of electromagnetic waves from buried objects is complicated due to interfaces which contains the object. Due to the presence of interfaces, a process of multiple interaction of fields between object and interfaces takes place. Multiple interaction process leads to complicated induced current distribution on the buried object. As far as the induced current distribution on the buried objects is concerned, as yet no analytic solution is available even for simple shapes such as circular cylinder. Complexity in the problem increases, if the shape of the buried object is not symmetric.

Buried object problem can be simplified by certain assumptions and approximations. One assumption is that object is far away from interfaces containing the object. In such situations, interaction terms may be evaluated using an asymptotic approximation for large separation of the object from the interfaces[1–5] and the analysis becomes relatively simple. Even with above assumption and approximation, situation remains complicated if number of interfaces are increased. This is because, increase in the number of interfaces increases the interaction terms. It is discussed elsewhere [5], when separation of the object from the interfaces is large, the interaction effects can be neglected. An object under this approximation is termed as deeply buried object and the actual induced current distribution can be approximated by a current distribution due to primary excitation. Primary excitation is defined as the field distribution that exists when no inhomogeneity is present and corresponding induced current distribution is termed as primary induced current distribution. Deeply buried assumption decouples the original scattering problem into two relatively simple parts. One which deals with the geometry of the scattering problem. In this part of the problem field radiated by a line source in the given geometry is derived. Other part of the problem deals with the cylindrical object as if it is in a homogeneous medium. In this part the primary currents excited on the cylindrical object are calculated. Now the scattered field due to the buried object can be obtained by considering the primary induced current on the object as source and fields due to the line sources are summed up to yield the scattered fields from the buried object.

2. GREEN'S FUNCTION

Consider the geometry shown in Fig. 1. This geometry contains a system of plane stratified dielectric layers. It is assumed that $k_{i+1} > k_i$, $i = 1, 2, \cdots, n-1$. These dielectric layers are assumed to be lossless, homogeneous and isotropic. A line source carrying a unit current is located at a point (x', y') in these dielectric layers. Different regions of interest in the geometry are mentioned in the Fig. 1. Region I is $y' \leq y \leq d_1$ and region II is $-d_2 \leq y \leq y'$. The region above the stratified dielectric layers is region III while region IV is below the stratified dielectric layers. It is desired to derive a general form for far-zone fields radiated from this buried line source. Therefore it is appropriate to use cylindrical coordinate system with (ρ, ϕ) as the coordinates of the point of observation and (ρ', ϕ') as the coordinates of the source. Radiated fields from a line source in each region can be expressed as superposition of plane waves. Regions outside the dielectric layers, i.e., region III and region IV, are unbounded and fields radiated from buried line source as a spectrum of plane waves in direction θ must travel away from parallel dielectric layers as [6]

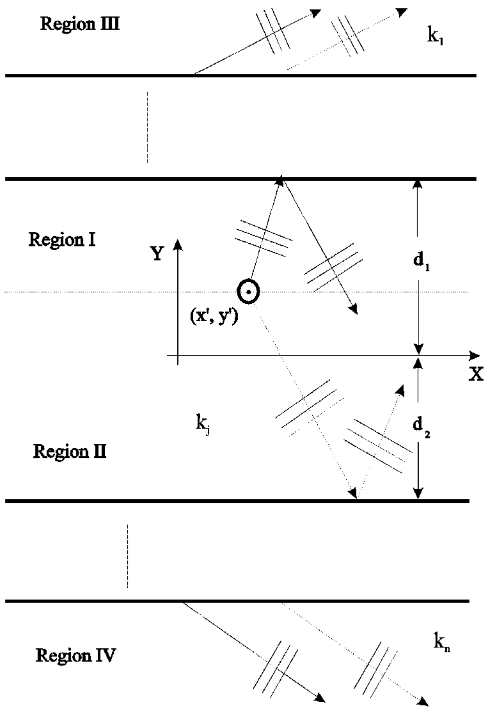

Figure 1. A line source is placed in a system of parallel dielectric layers.

$$G(\rho,\phi;\rho',\phi') = \int_C A(\theta,\rho',\phi')\exp\{ik\rho\cos(\theta\pm\phi)\}d\theta$$

where C is the contour in the complex θ-plane. k is the propagation constant of an unbounded medium in which point of observation lies. In the above expression plus sign corresponds to the field traveling in region IV while negtive sign corresponds to the field traveling in region III.

Fig. 1 partially shows corresponding diagram of the plane wave fields contributing to the total field in regions outside the dielectric layers. The total field in both outside regions is a combination of two types of plane wave contributions. One type of contributions take into account the fields which are radiated from the line source in region I as if no interface is present in the vicinity of the line source. This field propagates in both regions outside the dielectric layers after suffering reflections and refractions at the dielectric interfaces. Similarly the other type of contributions are the fields which are radiated from the line source in region II as if no interface is present in the vicinity of line source. This field also contributes to the fields propagating in regions outside the dielectric layers, after suffering reflections and refractions at the dielectric interfaces. This means that the total radiated field at any point outside these parallel stratified dielectric layers can be written as the combination of two terms as

$$\begin{aligned} G(\rho,\phi;\rho',\phi') = &\int_C A_a(\theta)\exp\{ik\rho'\psi_a(\theta,\phi')\}\exp\{ik\rho\cos(\theta\pm\phi)\}d\theta + \\ &\int_C A_b(\theta)\exp\{ik\rho'\psi_b(\theta,\phi')\}\exp\{ik\rho\cos(\theta\pm\phi)\}d\theta \end{aligned} \tag{2.1}$$

where $A_a(\theta)$ corresponds to the amplitude of field which is primarily radiated from buried line source in region I in direction θ as if no dielectric interface is present. In the second term of (2.1), $A_b(\theta)$ corresponds to the amplitude the field which is primarily radiated from the line source in a region II in direction $-\theta$ as if no dielectric interface is present. Since a change in location of the source only changes the phase of the corresponding spectrum function therefore the amplitude term is independent of ρ' and ϕ'. Position of the source is described by the phase term. For any fixed direction of observation ϕ, there corresponds only one value of θ which contributes to the far-zone fields. This particular direction of observation is called stationary point. The dominant part of the far-zone field is obtained by applying the method of stationary phase on the integrals in (2.1) [7–8]. This method yields the following form of far-zone radiated fields

$$\begin{aligned} G(\rho,\phi;\rho',\phi') \sim \frac{\exp(ik\rho)}{\sqrt{k\rho}} &\left[f_a(\phi)\exp\{ik\rho'\psi_a(\phi,\phi')\}\right. \\ &\left. + f_b(\phi)\exp\{ik\rho'\psi_b(\phi,\phi')\}\right]. \end{aligned} \tag{2.2}$$

This means that each term in the expression (2.2) can be written as the product of two factors. One factor which is independent of the position (ρ',ϕ') of the line source in the geometry and other factor which takes into account the position of the line source in the geometry as a phase term. Considering above arguments the representation of far-zone fields in regions outside the dielectric layers takes the following

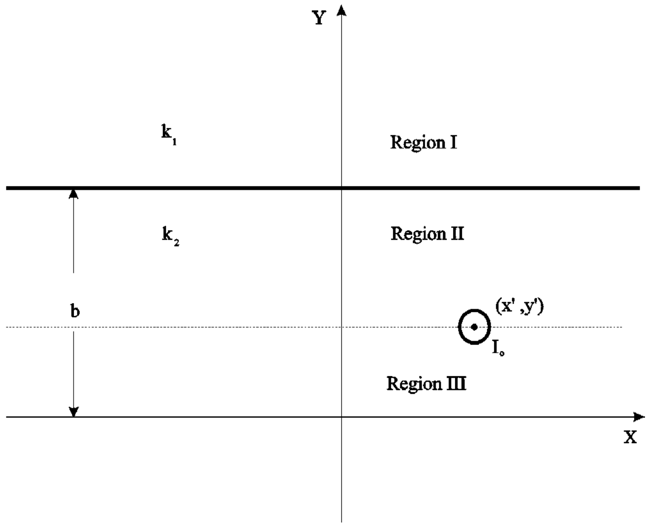

Figure 2. A line source buried in a dielectric half-space.

form

$$G(\rho,\phi;\rho',\phi') \sim G_a(\rho,\phi;0,0)\exp\{ik\rho'\psi_a(\phi,\phi')\} + G_b(\rho,\phi;0,0)\exp\{ik\rho'\psi_b(\phi,\phi')\} \quad (2.3)$$

where

$$G_i(\rho,\phi;0,0) = f_i(\phi)\frac{\exp(ik\rho)}{\sqrt{k\rho}} \qquad i = a, b.$$

It may be noted that G_a is the contribution of a line source located at origin and radiating primarily in region I while G_b is the contribution of a line source located at origin and radiating primarily in region II and the total far-zone field for the same location at origin is

$$G(\rho,\phi;0,0) \sim G_a(\rho,\phi;0,0) + G_b(\rho,\phi;0,0).$$

In order to develop better understanding of the generalized form of the expression (2.3) for Green's function, two examples are considered. First example deals with the far-zone fields radiated from a unit line source buried in the dielectric half-space geometry as shown in Fig. 2. It is of interest to compare the far-zone field in region I and region II of this geometry with the expression (2.3). From this comparison corresponding factors in each term will be identified. The dominant term of radiated field $G_1(\rho,\phi;\rho',\phi')$ in region I is given as [4]

$$G_1(\rho,\phi;\rho',\phi') \sim G_a(\rho,\phi;0,0)\exp\Big(-ik_1\rho'\cos\phi\cos\phi' - i\rho'\sin\phi'\sqrt{k_2^2-k_1^2\cos^2\phi}\Big) \quad (2.4)$$

where

$$G_a(\rho,\phi;0,0) = \frac{-\omega\mu k_1}{\sqrt{2\pi}(k_1^2-k_2^2)}\left\{k_1\sin^2\phi - \sin\phi\sqrt{k_2^2-k_1^2\cos^2\phi}\right\} \exp\Big(-ik_1 b\sin\phi + ib\sqrt{k_2^2-k_1^2\cos^2\phi}\Big)\frac{\exp(ik_1\rho - i\pi/4)}{\sqrt{k_1\rho}}.$$

Dominant term of radiated field $G_3(\rho,\phi;\rho',\phi')$ in region III is given as [4]

$$G_3(\rho,\phi;\rho',\phi') \sim G_a(\rho,\phi;0,0)\exp\{-ik_2\rho'\cos(\phi-\phi')\} + G_b(\rho,\phi;0,0)\exp\{-ik_2\rho'\cos(\phi+\phi')\} \quad (2.5)$$

where

$$G_a(\rho,\phi;0,0) = \frac{-\omega\mu\exp(ik_2\rho - i\pi/4)}{2\sqrt{2\pi}\sqrt{k_2\rho}}$$

$$G_b(\rho,\phi;0,0) = \frac{-\omega\mu\exp(ik_2\rho - i\pi/4)}{2\sqrt{2\pi}}R(-\phi)\exp(-i2k_2 b\sin\phi).$$

The factor $R(\phi)$ is recognized as the Fresnel reflection coefficient and is given as

$$R(\phi) = \frac{k_2\sin\phi - \sqrt{k_1^2-k_2^2\cos^2\phi}}{k_2\sin\phi + \sqrt{k_1^2-k_2^2\cos^2\phi}}.$$

It is obvious that far-zone field expressions for line source buried in dielectric half-space geometry can be written in a form given by (2.3). It may be noted that expression (2.4) does not contain term which corresponds to the second term in the general form given in (2.3). This is because region below the line source is unbounded therefore field which is primarily radiated to region below the line source will not contribute to the total far-zone fields in region above the line source.

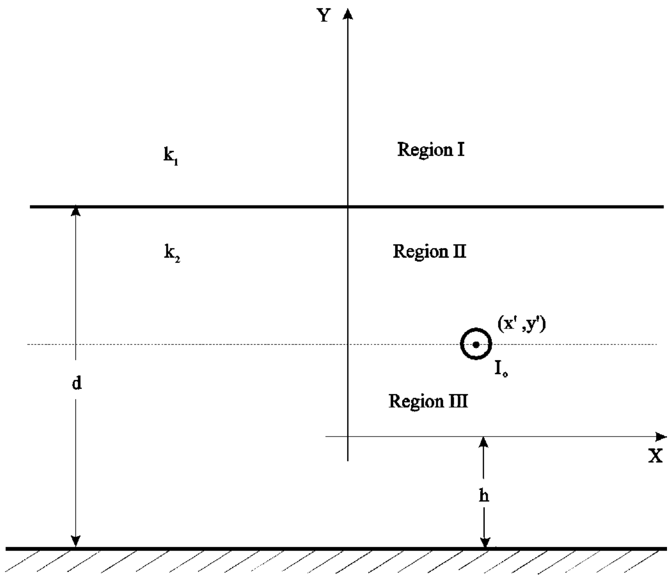

Figure 3. A line source buried in a grounded dielectric layer.

The second example deals with a unit line source buried in the grounded dielectric layer geometry as shown in Fig. 3. Dominant term of radiated field $G_1(\rho, \phi; \rho', \phi')$ in region I is given as [4, 9]

$$\begin{aligned} &G_1(\rho, \phi; \rho', \phi') \\ &\sim G_a(\rho, \phi; 0, 0) \exp\left\{-ik_1\rho' \cos\phi' \cos\phi + i\rho' \sin\phi' \sqrt{k_2^2 - k_1^2 \cos^2\phi}\right\} \\ &\quad G_b(\rho, \phi; 0, 0) \exp\left\{-ik_1\rho' \cos\phi' \cos\phi - i\rho' \sin\phi' \sqrt{k_2^2 - k_1^2 \cos^2\phi}\right\} \end{aligned} \tag{2.6}$$

where

$$\begin{aligned} G_a(\rho, \phi; 0, 0) = \frac{\omega\mu}{2\sqrt{2\pi}} \frac{\exp(ik_1\rho - i\pi/4)}{\sqrt{k_1\rho}} \times \\ \frac{k_1 \sin\phi \;\; \exp\left\{ih\sqrt{k_2^2 - k_1^2 \cos^2\phi}) - ik_1(d-h)\sin\phi\right\}}{\left(\begin{aligned}&\sqrt{k_2^2 - k_1^2\cos^2\phi}\cos\left(d\sqrt{k_2^2 - k_1^2\cos^2\phi}\right) \\ &\qquad -ik_1 \sin\phi \sin\left(d\sqrt{k_2^2 - k_1^2\cos^2\phi}\right)\end{aligned}\right)} \end{aligned}$$

$$G_b(\rho,\phi;0,0) = \frac{-\omega\mu}{2\sqrt{2\pi}} \frac{\exp(ik_1\rho - i\pi/4)}{\sqrt{k_1\rho}} \times \frac{k_1 \sin\phi \ \exp\left\{-ih\sqrt{k_2^2 - k_1^2\cos^2\phi}) - ik_1(d-h)\sin\phi\right\}}{\left(\begin{array}{l}\sqrt{k_2^2 - k_1^2\cos^2\phi}\cos\left(d\sqrt{k_2^2 - k_1^2\cos^2\phi}\right) \\ \qquad -ik_1\sin\phi\sin\left(d\sqrt{k_2^2 - k_1^2\cos^2\phi}\right)\end{array}\right)}.$$

The generalized form of the Green's function, which is given in (2.3), will be utilized to study the scattering from a buried object.

3. BURIED CYLINDRICAL OBJECT

The scattered field from the arbitrary shaped object which is buried in parallel layered geometry can be obtained as

$$E^s = \int_{s'} J_{sz}(\rho', \phi') G(\rho, \phi; \rho', \phi') ds'$$

where $J_{sz}(\rho', \phi')$ is the corresponding induced current on the buried object. If the point of observation lies in far-zone then the above expression with (2.3) yields the following scattered fields

$$\begin{aligned} E^s &\sim G_a(\rho,\phi;0,0) \int_{s'} J_{sz}(\rho',\phi') \exp\{i\rho'\psi_a(\phi,\phi')\} ds' \\ &\quad + G_b(\rho,\phi;0,0) \int_{s'} J_{sz}(\rho',\phi') \exp\{ik\rho'\psi_b(\phi,\phi')\} ds' \\ &\sim G_a(\rho,\phi;0,0) N_a + G_b(\rho,\phi;0,0) N_b. \end{aligned} \tag{3.1}$$

Induced current distribution J_{sz} may be written as combination of two terms as

$$J_{sz} = J_{sz}^p + J_{sz}^i.$$

Term J_{sz}^p takes into account the effects due to primary excitation. Primary excitation is defined as the field that exist as if no scatterer is present. Term J_{sz}^i takes into account interaction effects of the reflected scattered field. Quantities N_a and N_b in above expression depend upon primary excitation, shape of buried object and geometry containing the object. These two quantities can be identified from the induced current distribution on the buried object. Exact analytic solution to the induced current distribution on the buried object is generally not available so it is not possible to give a general form of these two required quantities.

The expressions for quantities N_a and N_b can be simplified using certain assumptions and approximations. A useful assumption in this regard may be that the object is deeply buried. Assumption of deeply buried states that the object is so far-away from the the interfaces containing the cylinder that the reflected scattered field has no interaction with the buried object. In this way, one can approximate actual induced current distribution on deeply buried object by the induced current distribution as if object is in a homogeneous medium, i.e., $J_{sz} \sim J_{sz}^{p}$. Under deeply buried assumption the quantities N_a and N_b in the above expression becomes independent of the geometry containing the object and these quantities depends only on primary excitation and the shape of the buried object. In this situation quantities N_a and N_b are called shape functions of the object for a specific excitation. Now it is relatively easier to calculate these quantities.

Let E^a and E^b denote far-zone fields scattered in regions above and below the buried object respectively. These far-zone scattered fields can be written in terms of Green's function and shape functions as given by the following set of equations

$$\begin{aligned} E^a &\sim G_a^a N_a^a + G_b^a N_b^a \\ E^b &\sim G_a^b N_a^b + G_b^b N_b^b. \end{aligned} \tag{3.2}$$

The subscripts in the above set of expressions stand for a region in which fields are primarily radiated from object in the geometry. The superscripts stand for a region in which point of observation lies in the far-zone. For example $G_b^a N_b^a$ is the field radiated primarily by the object in region II and observed in region III. It may be noted that if there is no interface below the object, e.g., a dielectric half-space geometry, then $G_b^a N_b^a = 0$. In this situation (3.2) reduces to the following form

$$\begin{aligned} E^a &\sim G_a^a N_a^a \\ E^b &\sim G_a^b N_a^b + G_b^b N_b^b. \end{aligned}$$

On the other hand if there is no interface above the object then $G_a^b N_a^b = 0$, so the (3.2) takes the following form

$$E^a \sim G_a^a N_a^a + G_b^a N_b^a$$
$$E^b \sim G_b^b N_b^b.$$

If there are no interfaces around the object, i.e., homogeneous space case, then $G_a^b N_a^b = G_b^a N_b^a = 0$ and $G_a^a = G_b^b = G_h$. Thus (3.2) simplifies to the following

$$E^a \sim G_h N_a^a$$
$$E^b \sim G_h N_b^b.$$

where G_h is the Green's function of a line source in a homogeneous medium.

4. SHAPE FUNCTIONS

In this section, few simple scattering geometries will be considered. Each scattering geometry contains a perfectly conducting circular cylinder with axis of the cylinder is coincident with z-axis. The far-zone scattered field will be evaluated, in the unbounded regions for each geometry, in conjunction with the deeply buried assumption. The scattered field will be compared with the expression (3.2) and the corresponding factors in each term, e.g., shape function, will be identified. The form of the general expression (3.2) can be simplified for the case of a circular cylinder.

First consider the simplest case of a circular cylinder of radius a which is placed in a homogeneous space with propagation constant k_2. The cylinder is excited by a normally incident, TE polarized plane wave

$$E_z^i = T_{12} \exp(-ik_2 y)$$

where T_{12} is amplitude of plane wave. Far-zone scattered field from this circular cylinder [10] can be written as a product of two factors, i.e.,

$$E_z^s \sim G(\rho, \phi; 0, 0) N_1$$

where

$$G(\rho, \phi; 0, 0) = \frac{-\omega\mu}{2\sqrt{2\pi}} \frac{\exp(ik_2\rho - \pi/4)}{\sqrt{k_2\rho}}$$

is far-zone Green's function for the line source located at the origin of the coordinate system in the homogeneous medium having propagation constant k_2 [10] and

$$N_1(\phi) = \frac{4T_{12}}{\omega\mu} \sum_{n=-\infty}^{n=\infty} (i)^n \frac{J_n(k_2 a)}{H_n^{(1)}(k_2 a)} \exp(in\phi) \tag{4.1}$$

is shape function of circular cylinder of radius a.

Consider the scattering geometry, in which a perfectly conducting circular cylinder of radius a is deeply buried in a dielectric half-space. This geometry of this case is shown in Fig. 4. The cylinder is excited by a normal incident, TE plane wave as shown in Fig. 4. The electric field associated with the incident plane wave is given by $E_z^i = \exp(-ik_1 y)$. Different regions for the geometry have been mentioned in the figure. Region I in this geometry is above the dielectric interface, while region II is between the dielectric interface and the buried cylinder. Region III is below the cylinder. Far-zone scattered field expressions due to primary excitation, i.e.,

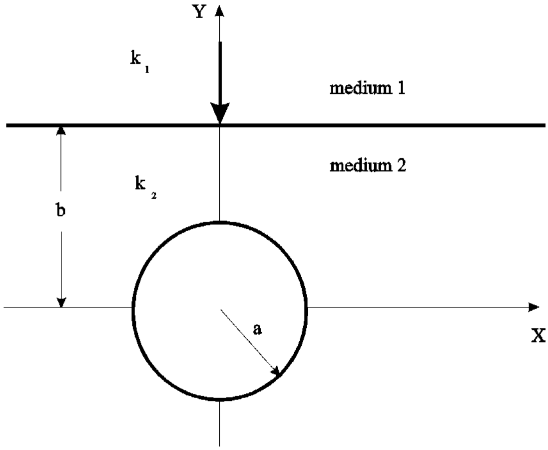

Figure 4. A perfectly conducting circular cylinder deeply buried in a dielectric half-space.

$$E_z = T_{12} \exp(-ik_2 y)$$

are considered in regions I and III one by one. T_{12} is Fresnel transmission coefficient. First consider the field expression in region I of this geometry [4].

$$
\begin{aligned}
E_z^s \\
\sim & \frac{-8k_1^2}{\sqrt{2\pi}(k_1^2-k_2^2)(k_1+k_2)} \exp(ik_2 b - ik_1 b\sin\phi + ib\sqrt{k_2^2-k_1^2\cos^2\phi}) \\
& \times \left\{k_1\sin^2\phi - \sin\phi\sqrt{k_2^2-k_1^2\cos^2\phi}\right\} \sum_{n=-\infty}^{n=\infty}\left\{\frac{(i)^n J_n(k_2a)\exp(in\phi_0)}{H_n^{(1)}(k_2a)}\right\} \\
& \times \frac{\exp\{ik_1\rho - i\pi/4\}}{\sqrt{k_1\rho}} \qquad (4.2)
\end{aligned}
$$

where

$$\phi_0 = \tan^{-1}\sqrt{(\frac{k_2}{k_1\cos\phi})^2 - 1}.$$

It is obvious after observing shape function N_1 of the cylinder and Green's function (2.4) that expression (4.2) can be written as a product of two factors as given below

$$E_z^a \sim G_a^a(\rho,\phi;0,0)N_a^a \qquad (4.3)$$

where $G_a^a(\rho,\phi;0,0)$ is far-zone Green's function for the line source located at origin of the coordinate system in the dielectric half-space geometry and is given in (2.4). The other factor N_a^a is similar to shape function of a cylinder in homogeneous medium (4.1) because the excitation is a normally incident plane wave. The only modification is introduction of ϕ_0 in place of ϕ in N_1. Shape function $N_1(\phi)$ of cylinder in homogeneous medium is modified in order to account the refraction effects of the field traveling between the two different media. The modified shape function is given by the following expression

$$N_a^a = \frac{4T_{12}}{\omega\mu}\sum_{n=-\infty}^{n=\infty}\left\{(i)^n\frac{J_n(k_2a)}{H_n^{(1)}(k_2a)}\exp(in\phi_0)\right\} = N_1(\phi_0). \qquad (4.3a)$$

It may be noted that $G_b^a = 0$ because there is no interface below the cylinder.

Consider now far-zone scattered field expression in the region III of dielectric half-space geometry [4].

$$E_z^s \sim \frac{-4k_1 \exp(ik_2 b)\exp(ik_2\rho - i\pi/4)}{\sqrt{2\pi}(k_2+k_1)\sqrt{k_2\rho}} \times \sum_{n=-\infty}^{n=\infty} \frac{(i)^n J_n(k_2 a)}{H_n^{(1)}(k_2 a)}$$

$$\cdot \{\exp(in\phi) + \exp(-in\phi)R(-\phi)\exp(-i2k_2 b\sin\phi)\}\,. \quad (4.4)$$

It is obvious after comparing the (4.4) with (4.3a) and (2.5) that (4.4) can also be expanded in terms of Green's function of the geometry and shape functions of the cylinder in a homogeneous medium as

$$E_z^b \sim G_a^b(\rho,\phi;0,0)N_a^b + G_b^b(\rho,\phi;0,0)N_b^b$$

where $G_a^b(\rho,\phi;0,0) + G_b^b(\rho,\phi;0,0)$ is far-zone field radiated in region III from the line source buried in the dielectric half-space geometry and located at the origin of the coordinate system and is given in (2.5). Shape functions N_a^b and N_b^b are related to shape function of the circular cylinder in a homogeneous medium by the following relations

$$N_a^b = N_1(-\phi)$$
$$N_b^b = N_1(\phi).$$

From the above relations, the total far-zone scattered field from the deeply buried cylinder in the region III of dielectric half-space geometry can be written as

$$E_z^b \sim G_a^b(\rho,\phi;0,0)N_1(-\phi) + G_b^b(\rho,\phi;0,0)N_1(\phi). \quad (4.5)$$

Now consider the case of a perfectly conducting circular cylinder deeply buried in the grounded dielectric layer. The geometry of the problem is shown in Fig. 5. The source of excitation is TE plane wave and is given by the following expression

$$E_z = \exp[-ik_1\{(x\cos\theta_i + (y-d+h)\sin\theta_i\}].$$

It may be noted that source of incidence is a plane wave that is external to this geometry is a plane wave. Due to the presence of perfectly conducting sheet below the cylinder the source of excitation for the induced currents on the buried cylinder becomes a standing wave in y direction. The corresponding field expression for the standing wave is given below

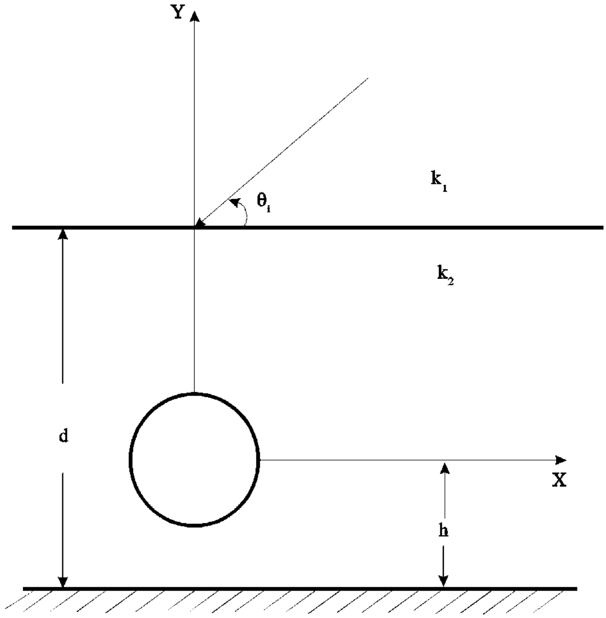

Figure 5. A perfectly conducting circular cylinder deeply buried in a grounded dielectric layer.

$$E_{2z} = \frac{-2ik_1 \sin\theta_i \sin\{k_2(y+h)\sin\theta_1\}}{k_2 \sin\theta_1 \cos(k_2 d \sin\theta_1) - ik_1 \sin\theta_i \sin(k_2 d \sin\theta_1)} \cdot \exp(-ik_2 x \cos\theta_1). \tag{4.6}$$

To be able to identify terms in the expression (3.2), it is required to calculate the far-zone scattered field from a circular cylinder in a homogeneous medium and excited with a standing wave as a primary excitation. Consider a perfectly conducting circular cylinder of radius a placed in a homogeneous medium having propagation constant k_2 which is excited with a standing wave as a source of excitation. Shape function N_2 of the circular cylinder in a homogeneous medium with standing wave field given in (4.6) as a source of excitation may be easily calculated by an eigenfunction expansion and is given by the following expression

$$N_2(\phi) = \frac{-8iT}{\omega\mu} \sum_{n=-\infty}^{n=\infty} (-i)^n \frac{J_n(k_2 a)}{H_n^{(1)}(k_2 a)} \sin(k_2 h \sin\theta_1 + n\theta_1) \exp(in\phi)$$

where the value of T is given below

$$T = \frac{k_1 \sin\theta_i}{k_2 \sin\theta_1 \cos(k_2 d \sin\theta_1) - ik_1 \sin\theta_i \sin(k_2 d \sin\theta_1)}.$$

Far-zone scattered field from a circular cylinder deeply buried in the grounded dielectric layer is [4]

$$E_z^s \sim \frac{\sin\phi \exp\{-ik_1(d-h)\sin\phi\}}{\sqrt{2\pi}(\beta\cos\beta d - ik_1\sin\phi\sin\beta d)} \frac{8k_1^2 \sin\theta_i \exp(ik_1\rho - i\pi/4)}{(\beta_1\cos\beta_1 d \; - ik_1\sin\theta_i \sin\beta_1 d)\sqrt{k_1\rho}}$$
$$\times \sum_{n=-\infty}^{\infty} \frac{J_n(k_2 a)}{H_n^{(1)}(k_2 a)} \exp(-in\pi)\sin(k_2 h\sin\theta_1 + n\theta_1)\sin(h\beta - n\phi_0). \tag{4.7}$$

The above scattered field expression can also be written in terms of the Green's function (2.6) and modified shape functions N_2, i.e.,

$$E_z^a \sim G_a^a(\rho,\phi;0,0)N_a^a + G_b^a(\rho,\phi;0,0)N_b^a$$

where $G_a^a(\rho,\phi;0,0)+G_b^a(\rho,\phi;0,0)$ is far-zone scattered field due to the line source buried in the grounded dielectric layer and located at the origin of the coordinate system. The factors N_a^a and N_b^a are related to the homogeneous medium shape function N_2 by the following relation

$$N_a^a = N_2(\phi_0)$$
$$N_b^a = N_2(-\phi_0).$$

The parameter ϕ_0 in place of ϕ in shape function N_2 is used to account for the refraction of fields, which are propagating from medium 1 to the medium 2. In medium 1 the total far-zone scattered field from the cylinder deeply buried in the grounded dielectric layer can be written in a more appropriate form as given below

$$E_z^a \sim G_a^a(\rho,\phi;0,0)N_2(\phi_0) + G_b^a(\rho,\phi;0,0)N_2(-\phi_0). \tag{4.8}$$

It is deduced from above discussion in this section that far-zone scattered fields from the deeply buried circular cylinder in two scattering geometries which are considered above can be written in a form given in (3.2). It is also evident from this discussion that in the case of deeply buried circular cylinder, the general representation for far-zone scattered fields (3.2) becomes even simpler and can be written in a more appropriate form as given below

$$E_z^a \sim G_a^a N(\phi) + G_b^a N(-\phi)$$
$$E_z^b \sim G_a^b N(-\phi) + G_b^b N(\phi). \tag{4.9}$$

If the point of observation lies in a medium other than the medium in which scatterer is located, the observation angle ϕ in the shape function is modified to a new observation angle ϕ_0 to account for the refraction effects of the field propagating between two media.

From the above discussion following important conclusion is made. Suppose that someone obtains the far-zone scattered fields from an object which is deeply buried in a particular geometry. This scattered field expression can be utilized to calculate scattered field from other object of arbitrary cross-section. This can be achieved by replacing shape function in the derived expression with shape function of desired object.

REFERENCES

1. Hongo, K. and A. Hamamura, "Asymptotic solutions for the scattered field of plane wave by a cylindrical obstacle buried in a dielectric half-space," *IEEE Trans. Antennas Propagat.,* Vol. 34, 1306–1312, Nov. 1986.
2. Rizvi, A. A. and Q. A. Naqvi, "An approximate solution to scattering from a deeply buried circular cylinder," *Communications Lab.,* Technical report 9, Nov. 1997.
3. Naqvi, Q. A., A. A. Rizvi, and M. A. Ashraf, "Asymptotic solutions for the scattered field of plane wave by a cylindrical obstacle buried in a grounded dielectric layer," *Progress in Electromagnetics Research, PIER 20,* 247–260, 1998.
4. Naqvi, Q. A., "Scattering of electromagnetic waves from a buried cylinder," Ph.D. Thesis, Quaid-i-Azam University, Islamabad, Pakistan, 1997.
5. Naqvi, Q. A. and A. A. Rizvi, "An approximation to induced currents on the deeply buried cylinder," *IEEE National Multitopic Conference,* 172–175, April 1997.
6. Clemmow, P. C., *The Plane Wave Spectrum Representation of Electromagnetic Fields,* Pergamon Press, 1996.
7. Bender, C. M. and S. A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers,* McGraw-Hill, New York, 1978.

8. Engheta, N. and C. Elachi, "Radiation characteristics of a source in a thin substrate mounted over a dielectric medium," *IEEE Trans. Antennas Propagat.,* Vol. 36, 322–330, March 1988.
9. Naqvi, Q. A. and A. A. Rizvi, "Low contrast circular cylinder buried in a grounded dielectric layer," *Journal of Electromagnetic Waves and Applications,* Vol. 12, No. 11, 1527–1536, 1998.
10. Balanis, C. A., *Advanced Engineering Electromagnetics,* John Wiley & Sons, 1989.