MICROWAVE IMAGING OF PARALLEL PERFECTLY
CONDUCTING CYLINDERS USING REAL-CODED
GENETIC ALGORITHM COUPLED WITH
NEWTON-KANTORIVITCH METHOD

A. Qing and C. K. Lee
School of Electrical and Electronic Engineering
Nanyang Technological University
Nanyang Avenue
Singapore 639798

1. Introduction
2. Formulation of Problem
3. Real-coded Genetic Algorithm
4. Newton-Kantorivitch Method
5. RGA Coupled with NKM
6. Numerical Results
7. Conclusions
References

1. INTRODUCTION

The electromagnetic inverse scattering problem [1–4] is to recover information concerning some inaccessible region from the scattered electromagnetic fields measured outside. It has been one of the most challenging research topics in recent years due to its considerable practical importance in various areas of technology [1–14] such as non-destructive evaluation, subsurface and ground-penetrating radar, geophysical remote sensing, medical imaging, seismology and target identification etc. However, it is difficult to solve because of its ill-posedness and nonlinearity [1–4, 15–17]. It is the multiple scattering effects that make the electromagnetic inverse scattering problem inherently non-
linear while contaminated and insufficient measuring data result in the ill-posedness.

Much attention has been paid to developing the inversion algorithms and a variety of algorithms have been proposed. Generally, they fall into two classes, namely the analytic solution and the numerical solution.

Most of the analytic inversion algorithms [18–25] deal with one dimension problem. They usually make use of some approximations such as the physical optics approximation [19–20], the Born and Rytov approximation [21, 26], etc., to simplify the problem. These inversion algorithms include the layer-stripping algorithm [2, 14], the method of characteristics [2, 14, 22], the Gel’fand-Levitan integral equation method [23] and the Marchenko integral equation method [23–25], etc. However, we hardly encounter situations where a one-dimensional model can practically be used. Moreover, the simplicity of these algorithms in one dimension is not preserved in higher dimension. Consequently, they have only theoretical significance but limited practical applications.

In contrast, the numerical inversion algorithms [27–52] solve the electromagnetic inverse problem numerically and iteratively. They can deal with not only one-dimensional but higher dimensional problems.

The electromagnetic inverse problem can be transferred into an optimization problem. Correspondingly, the numerical inversion algorithms can be further classified into two categories according to the optimization algorithms adopted. The first is the local inversion algorithm and the second is the global inversion algorithm.

Most of the previous numerical inversion algorithm belong to the first category. The prominent algorithms of this category include the Born iterative method (BIM) [27, 28], the distorted Born iterative method (DBIM) [29, 30], the Newton-Kantorivitch method (NKM) [31–33], the Levenberg-Marquardt algorithm [34–36], local shape function (LSF) method [37–40], the modified gradient method (MGM) [41–43], the pseudoinverse transformation method [44], the dual space method [4] and the nonlinear parameter optimization method [45, 46], etc.

The above mentioned local inversion algorithms are gradient based. They are restricted to relatively small gradients of the object functions since the optimization methods adopted in them are local. Thus, they only converge to the true profile under certain conditions, otherwise,
they may be trapped into a local extreme or even diverge. For example, the BIM and DBIM can only give convergent solutions when the scatterer is weak while the Newton-Kantorivitch method demands a good starting point. A priori knowledge is not always available or accurate enough.

More recently, a new class of inverse scattering algorithms has emerged. They are the global inversion algorithms [47–52] based on genetic algorithms (GAs) [53–56]. GAs are a set of slowly converging probabilistic global optimization methods based on genetic recombination and evolution in nature. They operate on a randomly generated population in the search space simultaneously and perform a global optimization by the three genetic operations, i.e., selection, crossover and mutation. The GAs are less prone to converge to a local optimum than the gradient based algorithms even when the initial guess is far away from the exact one because they are stochastic and global in nature. GAs have been widely used in solving electromagnetic problems [57–61]. However, its application in solving electromagnetic inverse problem is new and incomplete.

Chiu and Liu [47] tried to reconstruct the image of a perfectly conducting cylinder by the standard genetic algorithm (SGA). However, the reported results are dubious [48, 49]. Qing and Lee [48] dealt with the same problem by RGA. RGA was also used to solve the microwave imaging problem of multiple perfectly conducting cylinders [49, 50]. Xiao et al. [51] solved the same problem by micro genetic algorithm (MGA) coupled with Powell method, while Meng et al. [52] dealt with it by SGA incorporated with local shape function (LSF) method.

The GA-based global inversion algorithms offer many advantages over the local inversion algorithms such as strong search ability, simplicity, versatility, high level of robustness and insensitiveness to ill-posedness. However, they also exhibit several disadvantages among which the unendurable long inversion time is the most notorious.

In this paper, a novel method, the RGA-NKM, is proposed for microwave imaging of parallel perfectly conducting cylinders with or without the effect of random noise. It is developed to reduce the untolerably long inversion time of RGA while keeping the merits of RGA. The main idea of the RGA-NKM is to perform a Newton-Kantorivitch type search for the local optimum after the genetic operations in each genetic evolution to improve the local search ability of RGA. It begins from an initial population which is generated randomly within the
search space. Then, the algorithm enters into evolution loop searching for the optimum solution. The evolution continues until the termination conditions are fulfilled. Numerical results and comparisons with both RGA and NKM demonstrate that although the simplicity of RGA is lost, the search ability is greatly improved and the convergence is sped up significantly while those merits of RGA such as high level of robustness, versatility and insensitivity to ill-posedness are retained.

2. FORMULATION OF PROBLEM

The problem considered here is the same as that in [49] which is depicted in Fig. 1. \( R \) stands for receivers located on the measuring circle \( \Omega \) with radius \( R_{meas} \). \( O \) is the origin. \( C_1 \) and \( C_2 \) are the contours of the two perfectly conducting cylinders. \( O_i \) is the local origin of \( C_i \) which is an arbitrary point inside \( C_i \). \( |OO_i| = d_i \). \( C_1 \) and \( C_2 \) are denoted by the shape functions \( F_1(\theta) \) and \( F_2(\theta) \) respectively. The shape functions are assumed to be trigonometric series of order \( N/2 \)

\[
F_i(\theta) = \sum_{n=0}^{N/2} A_{in} \cos(n\theta) + \sum_{n=1}^{N/2} B_{in} \sin(n\theta) \quad i = 1, 2
\]

(1)

The two cylinders are assumed to be illuminated by TM plane wave with time harmonic factor \( e^{j\omega t} \)

\[
E^{inc}(r) = E^{inc}(r) \hat{z} = e^{-jk_0(x\cos\varphi + y\sin\varphi)} \hat{z}
\]

(2)

where \( \omega = 2\pi f \) is the angular frequency, \( r = (x, y) \), \( \varphi \) is the incident angle and \( \hat{z} \) is the unit vector in the \( z \) direction.

Surface currents \( J_{sj}(r) = J_{sj}(r) \hat{z}, \quad j = 1, 2 \) are induced on the surface of the cylinders and the scattered electric field \( E^{scat}(r) = E^{scat}(r) \hat{z} \) is subsequently generated

\[
E^{scat}(r) = \sum_{j=1}^{2} \frac{-\omega\mu_0}{4} \int_{C_j} J_{sj}(r')H_0^{(2)}(k_0|r-r'|)dr' \\
= \sum_{j=1}^{2} \frac{-\omega\mu_0}{4} \int_0^{2\pi} J_{sj}(\theta')H_0^{(2)}(k_0|r-r'|)d\theta'
\]

(3)

where \( H_0^{(2)}(\cdot) \) is the second kind Hankel’s function of zero order, \( k_0 \) is the wavenumber of free space.
Figure 1. Geometry of problem in this paper.

\[
J_j(\theta') = \sqrt{F_j^2(\theta') + F_j'^2(\theta')}
\cdot J_{sj}(\theta'
\]^

where the + sign is to be employed for \( j = 1 \) and the − sign for \( j = 2 \).

At the surface of each cylinder, the electric field satisfies the boundary condition

\[
[E_{\text{scat}}(r) + E_{\text{inc}}(r)] \cdot \hat{z} = 0 \quad r \in C_i \quad (4)
\]

Thus

\[
E_{\text{inc}}(r) = 2 \sum_{j=1}^{2} \frac{\omega \mu_0}{4} \int_{0}^{2\pi} J_j(\theta') H_0^{(2)}(k_0 R_{ij}) d\theta' \quad (5)
\]

where

\[
R_{ij} = |r - r'| \quad r \in C_i, \quad r' \in C_j
\]

The distribution of surface current is obtained after solving Eq. (5) by point-matching method [62] with pulse basis function and Dirac delta test function. Consequently, the scattered electric field at receivers on \( \Omega \) is

\[
E_{\text{scat}}(r) = 2 \sum_{j=1}^{2} - \frac{\omega \mu_0}{4} \int_{0}^{2\pi} J_j(\theta') H_0^{(2)}(k_0 R_j) d\theta' \quad (6)
\]
where

\[ R_j = |r - r'| \quad r \in \Omega, \ r' \in C_j \]

For the inverse problem, the scattered electric field \( E_{\text{meas}}^{\text{scat}} \), at receivers on \( \Omega \) are measured and known while the shape functions, i.e., the coefficients of the trigonometric series are unknown.

A relative error function with respect to the coefficients of the trigonometric series is defined as

\[
fn(x) = \left\| E_{\text{meas}}^{\text{scat}} - E_n^{\text{scat}} \right\| / \left\| E_{\text{meas}}^{\text{scat}} \right\| \tag{7}
\]

where

\[
x = \begin{bmatrix}
A_{10} & A_{11} & \cdots & A_{1N/2} & B_{11} & B_{12} & \cdots & B_{1N/2} \\
A_{20} & A_{21} & \cdots & A_{2N/2} & B_{21} & B_{22} & \cdots & B_{2N/2}
\end{bmatrix}
\]

\[
\left\| E_{\text{meas}}^{\text{scat}} \right\| = \sqrt{\sum_{j=1}^{N_f \times N_a \times N_r} (E_{\text{meas}}^{\text{scat}})_j^2}
\]

\[
\left\| E_{\text{meas}}^{\text{scat}} - E_n^{\text{scat}} \right\| = \sqrt{\sum_{j=1}^{N_f \times N_a \times N_r} (E_{\text{meas}}^{\text{scat}})_j - (E_n^{\text{scat}})_j^2}
\]

\( E_{\text{meas}}^{\text{scat}} \) and \( E_n^{\text{scat}} \) are \( N_f \times N_a \times N_r \)-dimensional vectors containing the scattered electric field measured and computed after \( n \) iterations respectively. \( N_f \), \( N_a \), and \( N_r \) are the total number of frequencies, incident angles and receivers respectively.

The relative error function gives a measurement on how close the inverted results approaches the true profile.

The inverse problem can therefore be cast into an optimization problem by minimizing the relative error function with the coefficients of the trigonometric series being the parameters to be optimized.

3. REAL-CODED GENETIC ALGORITHM

The real-coded genetic algorithm has been successfully applied to solve the above inverse problem [49]. For this case, the \( i \)-th creature among the \( n \)-th population takes the form

\[
x^{n,i} = \begin{bmatrix}
A_{10}^{n,i} & A_{11}^{n,i} & \cdots & A_{1N/2}^{n,i} & B_{11}^{n,i} & B_{12}^{n,i} & \cdots & B_{1N/2}^{n,i} \\
A_{20}^{n,i} & A_{21}^{n,i} & \cdots & A_{2N/2}^{n,i} & B_{21}^{n,i} & B_{22}^{n,i} & \cdots & B_{2N/2}^{n,i}
\end{bmatrix} \tag{8}
\]
The corresponding object functions is

\[ f^n(x^{n,i}) = \frac{\|E_{scat}^{meas} - E_{scat}^{n,i}\|}{\|E_{scat}^{meas}\|} \]  \tag{9}

where \( E_{scat}^{n,i} \) is a \( N_f \times N_a \times N_r \)-dimensional vector containing the computed scattered electric field corresponding to \( x^{n,i} \).

The RGA begins from an initial population of size \( N_{pop} \) which are generated randomly within the search space. Then, the population enters the main GA loop for searching the optimum solution of the problem. Scaling, stochastic binary tournament selection, one-point crossover and mutation are involved in a GA loop. The GA loop continues until the termination conditions are fulfilled. The termination conditions for the RGA are:

1. \( \min f^n(x^{n,i}) < \varepsilon \), where \( \varepsilon \) is the required accuracy
2. Maximum evolutions are used up
3. The RGA is trapped into local minima
The flow chart of RGA for microwave imaging of two parallel perfectly conducting cylinders is depicted in Fig. 2 where the block in dotted diagram should not be included for this case.

It has been demonstrated in [49] that the RGA performs well with respect to search ability, versatility, insensitiveness to ill-posedness, robustness and simplicity. The probabilistic nature of the RGA allows random starting and requires almost no initial guesses. A false assumption on the values of $d_1, d_2$ and $\Psi$ is also acceptable. However, the inversion time is untolerably long.

4. NEWTON-KANTORIVITCH METHOD

The Newton-Kantorivitch method [31, 32] has also been applied to solve the above inverse problem.

Take variation on Eqs. (5) and (6) with respect to $J_j(\theta')$, $A_{in}$ and $B_{in}$, one obtains

\[
\delta E^{inc}(r) = \sum_{j=1}^{2} \frac{\omega\mu_0}{4} \int_{0}^{2\pi} \left[ J_j(\theta') \delta H^{(2)}_0(k_0R_{ij}) + H^{(2)}_0(k_0R_{ij}) \delta J_j(\theta') \right] d\theta' \tag{10}
\]

\[
\delta E^{scat}(r) = \sum_{j=1}^{2} \frac{\omega\mu_0}{4} \int_{0}^{2\pi} \left[ J_j(\theta') \delta H^{(2)}_0(k_0R_j) + H^{(2)}_0(k_0R_j) \delta J_j(\theta') \right] d\theta' \tag{11}
\]

where

\[
\delta E^{inc}(r) = -jk_0 \cos(\theta - \varphi) E^{inc}(r) \delta F_i(\theta) \quad r \in C_i
\]

\[
\delta H^{(2)}_0(k_0R_{ii}) = -\frac{k_0}{R_{ii}} H^{(2)}_1(k_0R_{ii}) \cdot \left\{ [F_i(\theta) - F_i(\theta') \cos(\theta - \theta')] \delta F_i(\theta) + [F_i(\theta') - F_i(\theta) \cos(\theta - \theta')] \delta F_i(\theta') \right\}
\]

\[
\delta H^{(2)}_0(k_0R_{ij}) = -\frac{k_0}{R_{ij}} H^{(2)}_1(k_0R_{ij}) \cdot \left\{ [F_i(\theta) \pm d \cos(\theta - \psi)] \delta F_i(\theta) + [F_j(\theta') \mp d \cos(\theta' - \psi)] \delta F_j(\theta') \right\} \quad i \neq j
\]

\[
\delta H^{(2)}_0(k_0R_j) = -\frac{k_0}{R_j} H^{(2)}_1(k_0R_j) \cdot [F_j(\theta') \mp d \cos(\psi - \theta')] R^{meas} \cos(\theta - \theta') \delta F_j(\theta')
\]
\[
\delta F_j(\theta) = \sum_{n=0}^{N/2} \cos(n\theta) \delta A_{jn} + \sum_{n=1}^{N/2} \sin(n\theta) \delta B_{jn}
\]

\[
\delta F_j(\theta') = \sum_{n=0}^{N/2} \cos(n\theta') \delta A_{jn} + \sum_{n=1}^{N/2} \sin(n\theta') \delta B_{jn}
\]

By applying the point-matching method with pulse basis function and Dirac delta test function on Eqs. (8) and (9), one gets

\[
0 = \mathbf{L}_{11} \cdot \delta \mathbf{J} + \mathbf{L}_{12} \cdot \delta \mathbf{x} \quad (12)
\]

\[
\delta \mathbf{E}^{\text{scat}} = \mathbf{L}_{21} \cdot \delta \mathbf{J} + \mathbf{L}_{22} \cdot \delta \mathbf{x} \quad (13)
\]

where

\[
\delta \mathbf{J} = \begin{bmatrix} \delta J_1 \\ \delta J_2 \end{bmatrix}^T
\]

The inversion equation is therefore obtained

\[
\delta \mathbf{E}^{\text{scat}} = \left( \mathbf{L}_{22} - \mathbf{L}_{21} \cdot \mathbf{L}_{11}^{-1} \cdot \mathbf{L}_{12} \right) \delta \mathbf{x} \\
\triangleq \mathbf{M} \cdot \delta \mathbf{x} \quad (14)
\]

Finally, one gets the differential increment \( \delta \mathbf{x}^n \) in the \( n \)th iteration in regularized form for multi-incidence case

\[
\delta \mathbf{x}^n = \left[ \gamma \mathbf{R}^+ \cdot \mathbf{R} + \sum_{j=1}^{N_f \times N_e \times N_r} \left( \mathbf{M}_j^o \right)^+ \cdot \mathbf{M}_j^o \right]^{-1} \]

\[
\cdot \left[ \sum_{j=1}^{N_f \times N_e \times N_r} \left( \mathbf{M}_j^o \right)^+ \cdot \delta \mathbf{E}_j^{\text{scat}} \right]
\]

where \( \mathbf{R} \) is the regularization matrix and \( \gamma \) is the regularization factor.

Consequently, the shape functions are updated according to

\[
\mathbf{x}^{n+1} = \mathbf{x}^n + \delta \mathbf{x}^n \quad (16)
\]

The flow chart of NKM for microwave imaging of two parallel perfectly conducting cylinders is depicted in Fig. 3. The searching process starts from an initial guess \( \mathbf{x}^0 \), then goes into iterations updating the shape functions by Eq. (16). The NKM-type termination conditions are:
Figure 3. Flow chart of NKM for microwave imaging of two parallel perfectly conducting cylinders.

(1) $f^n(x^n) < \varepsilon$
(2) $f^n(x^n) > f^{n-1}(x^{n-1})$
(3) Maximum iterations are used up

The numerical results in [32] show that the NKM tends to converge fast with a good starting point $x^0$ and an exact knowledge on the values of $d_1$, $d_2$ and $\Psi$. However, further study shows that it is strictly subjected to the starting point and the knowledge on the values of $d_1$, $d_2$ and $\Psi$. It is more prone to get stuck in the local minima or even diverge. A priori is crucial for ensuring the convergence of the algorithm. Unfortunately, such a priori is not always available or accurate enough.

5. RGA COUPLED WITH NKM

Obviously, RGA and NKM behave complementarily while applied to microwave imaging of parallel perfectly conducting cylinders. It is therefore expected to develop a novel algorithm which inherits the merits of both RGA and NKM. RGA coupled with NKM (RGA-NKM) is consequently proposed. NKM is hybridized with RGA to improve the performances of both NKM and RGA. The flow chart of RGA-NKM is depicted in Fig. 2. It begins from an initial population which are generated randomly within the search space. Then, the algorithm enters into evolution loop searching for the optimum solution. In each
evolution, a Newton-Kantorivitch type search for the local optimum is performed after the genetic evolution. The RGA provides the starting point of NKM. The best creature obtained by RGA after genetic evolution is used as the starting point of NKM. NKM is used to refine RGA to improve the local search ability of RGA.

6. NUMERICAL RESULTS

The RGA-NKM is used to reconstruct the first example in [49], two circular perfectly conducting cylinders with radius 0.3 m located on the x axis. \( d_1 = d_2 = 0.5 \) m, \( \psi = 0^\circ \). All the parameters are identical with those in [49] except stated otherwise. \( \varepsilon = 1\% \).

Pseudo random noise is added into the scattered electric field data to investigate the effect of noise on the reconstructed result. The noise is assumed to be additive white noise with zero mean value.

To show the effect of noise quantitatively, a relative error of shape function between the reconstructed profile and the true one is defined as

\[
DISC = \left\{ \frac{1.0}{2M} \sum_{i=1}^{2} \sum_{m=1}^{M} \left[ \frac{\rho_{im}^{\text{imag}} - \rho_{im}^{\text{true}}}{\rho_{im}^{\text{true}}} \right]^2 \right\}^{1/2}
\]

where the superscript image and true stand for the reconstructed profile and the true one respectively

\[
\rho_{im} = \left\{ \left[ F_i(\theta_m) \cos \theta_m \pm d_i \cos \psi \right]^2 + \left[ F_i(\theta_m) \sin \theta_m \pm d_i \sin \psi \right]^2 \right\}^{1/2}
\]

\[
\theta_m = \frac{2\pi}{M}m
\]

To ensure that the proposed algorithm is practically useful, a false assumption is made on the values of \( d_1, d_2 \) and \( \Psi \). To facilitate the comparison with RGA, they are set to be identical with those of example 4 in [49].

The inversion is performed on an IBM P-133 PC. It takes 64 minutes and 58 seconds to get the final results for the noise-free case as shown in Fig. 4. Seven generations of genetic evolution and 8 Newton-Kantorivitch type iterations are performed. It was demonstrated in [49] that the it took the RGA 393 generations of genetic evolution to get the final result.

The effect of random noise is shown in Figs. 5 and 6. It can be seen that the reconstructed results with signal-to-noise ratio 14 dB or
higher is very accurate. Fig. 5 also shows the effect of random noise on RGA for reconstructing this example. The typical inversion results by RGA using random noise contaminated data are shown in Fig. 7. To speed up the inversion process, the values of $d_1$, $d_2$ and $\Psi$ for this case are assumed to be known exactly in advance. This may improve the noise-tolerance level of RGA.

Obviously, the RGA converges much more slowly than the RGA-NKM. In addition, Fig. 5–7 demonstrate that the noise-tolerance level of RGA-NKM is improved slightly.

The NKM is also used to invert the above object. The values of $d_1$, $d_2$ and $\Psi$ for this case are assumed to be known exactly in advance. Four cases are considered:

Case 1: Generate the initial guess randomly within the same search range. Twenty trials were simulated. In our simulation, none of the trials succeeds

Case 2: Set the initial guess of the coefficients as their mean values of

Figure 4. Inversion results of example 1 with noise-free data (a) true profile (b) initial guess (c) starting point for NKM and (d) inversion result after one Newton-Kantorivitch iteration. The final result is exactly the same as the true profile.
Microwave imaging of parallel perfectly conducting cylinders

Figure 5. Effect of noise.

Figure 6. Inversion results of example 1 by RGA-NKM with noisy data.

the search range, i.e., $A_{i0} = 1.0$, $(A_{ij}, B_{ij}) = 0$, $i = 1 \sim 2$, $j = 1 \sim 4$. The NKM fails.

Case 3: Set the initial guess as the two extreme cases of the search range, i.e., (i) $A_{i0} = 0$, $(A_{ij}, B_{ij}) = -0.5$, $i = 1 \sim 2$, $j = 1 \sim 4$ (ii)
Case 4: Set the initial guess to be identical with that of the RGA-NKM which is the best one among the initial population. The NKM fails.

Comparing the simulation results by RGA-NKM with those by NKM, it is found that the RGA-NKM shows a much higher level of robustness than NKM.

The RGA-NKM is used to reconstruct the second and third example in [49] too. The measured(simulated) scattering data here are assumed to be noise-free. The inversion results are shown in Figs. 8 and 9. The inversion takes 50 minutes 45 seconds and 19 minutes 18 seconds respectively.

7. CONCLUSIONS

In this paper, NKM is hybridized with RGA to improve the performance of both RGA and NKM. A novel inversion algorithm, RGA-NKM, for microwave imaging of perfectly conducting cylinders with or without the effect of random noise is proposed. The main idea of the RGA-NKM is to perform a Newton-Kantorivitch type search for the local optimum after the genetic operations in each genetic
Figure 8. Inversion results of example 2 with noise-free data (a) true profile (b) initial guess (c) starting point for NKM and (d) inversion result after one Newton-Kantorivitch iteration. The final result is exactly the same as the true profile.

Figure 9. Inversion results of example 3 with noise-free data (a) true profile (b) initial guess (c) starting point for NKM and (d) inversion result after one Newton-Kantorivitch iteration. The final result is exactly the same as the true profile.
evolution. The RGA provides the starting point of NKM. The best creature obtained by RGA after genetic evolution acts as the starting point of NKM. The NKM refines RGA to improve the local search ability of RGA.

Numerical results and comparisons with both RGA and NKM demonstrate that the RGA-NKM retains the advantages of RGA such as high level of robustness, versatility and insensitiveness to ill-posedness, greatly improves the search ability and speeds up the convergence.

On the other hand, the RGA-NKM is not without drawbacks. It loses its simplicity since the NKM is hybridized.

REFERENCES


