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THEORY OF GYROELECTRIC WAVEGUIDES

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Abstract–Circular conducting waveguides filled with gyroelectric media are studied in this paper and the dyadic Green's functions in these waveguides are obtained for the first time. The electric and magnetic fields in the waveguide are expressed in terms of cylindrical vector wave function expansion. It is shown that each of the eigenmodes in the waveguide consists of two eigenwaves whose wave numbers can be determined by the characteristic equation. Dispersion relation is obtained by applying boundary conditions to the eigenmodes. Mode orthogonality is discussed before the orthogonal relations are formulated and then used to determine the expansion coefficients of electric and magnetic fields. The complete expansions of both the electric and the magnetic dyadic Green's functions are finally obtained. The calculated dispersion curves are depicted and discussed while effects of gyrotropy are shown.

1. INTRODUCTION

Gyrotropic anisotropic media, which form a subclass of general anisotropic complex media, have been a hot research topic for a long time because of the existence of natural gyrotropic anisotropic crystals and easy realization of artificial gyrotropic composites, e.g., magnetically biased plasma or ferrite. The research of gyrotropic anisotropic media can be traced back to 1950s and 1960s when most studies were conducted on the guided-wave propagation in magnetically biased plasma or ferrite [1–12]. These investigations showed that the transverse electric field patterns are fixed in an appropriately rotated coordinates system and appear to rotate about the waveguide axis as the mode propagates along the guide [11]. This Faraday effect is due to the disparity between propagation coefficients of the two counter-rotating modes [9]. In 1981, Uzunoglu examined the propagation characteristics of guided waves in dielectric cylindrical waveguides with gyroelectric cores [13] and later the excitation of guided modes along a gyroelectric cylinder and radiation patterns of dipole radiating in the proximity of the cylinder [14]. The scattering properties of a gyrotropic anisotropic cylinder were also investigated by Monzon using a spectral approach [15]. The radiation of a dipole source located near a gyrotropic layer as well as the transmission and reflection phenomena was studied analytically by Tsalamengas [16].

Recently, investigators focused their studies into two topics of gyrotropic media. One of them is again the gyrotropic waveguide. The structures include vacuum core surrounded by plasma [17] and open waveguides [18, 19]. The spectra of electromagnetic waves in plasma waveguides were analyzed [20, 21] and some numerical methods such as finite-difference method [22] and method of lines [23] were extended and applied in analyzing the characteristics of plasma waveguides. The other focused topic is the dyadic Green's functions for gyrotropic anisotropic media. Among various approaches available to the electromagnetic problems of complex media, the dyadic Green's function technique [24] plays a major role, especially in boundary value problems [25–33]. Electromagnetic fields for arbitrary sources in linear media can be obtained by integrating the dyadic Green's function as a kernel together with the source distribution. Reported dyadic Green's functions for gyrotropic anisotropic media include those for a planar stratified and arbitrarily magnetized linear plasma [25], multilayered symmetric gyroelectric media [26], bounded homogeneous gyroelectric media [27], and unbounded gyroelectric media [28–30]. However, to the authors' knowledge, the dyadic Green's functions for gyrotropic anisotropic waveguides have not been reported elsewhere yet.

In this paper, we begin with an infinite gyrotropic medium and then impose the boundary conditions of the conducting circular waveguide on electromagnetic fields. Modal analysis on the gyroelectric waveguide is provided and the dyadic Green's functions are obtained for the first time. The electric and magnetic fields in the waveguide are expressed in terms of cylindrical vector wave functions. It is shown that each of the eigenmodes in the waveguide consists of two eigenwaves whose wave numbers can be determined by the characteristic equation. Dispersion relation is obtained by applying boundary conditions to the eigenmodes. Mode orthogonality is discussed and the orthogonal relations are formulated and then used to determine the expansion coefficients of electric and magnetic fields. The complete expansions of both the electric and the magnetic dyadic Green's functions are obtained. Lastly, the calculated dispersion curves are depicted and the effects by varying the gyrotropy are discussed.

2. BASIC FORMULATION

2.1 Constitutive Relations

The constitutive relations for a gyroelectric anisotropic medium can be written in the $\{\boldsymbol{E}, \boldsymbol{H}\}$ form as

$$\boldsymbol{D} = \overline{\boldsymbol{\epsilon}} \cdot \boldsymbol{E}, \tag{1a}$$

$$\boldsymbol{B} = \mu \boldsymbol{H}, \tag{1b}$$

where the time dependence $e^{-i\omega t}$ has been assumed and will be suppressed throughout this paper. In the constitutive relations, μ is the permeability and $\overline{\boldsymbol{\epsilon}}$ is the permittivity in gyrotropic form of

$$\overline{\boldsymbol{\epsilon}} = \begin{bmatrix} \epsilon_t & -i\epsilon_g & 0 \\ i\epsilon g & \epsilon_t & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}. \tag{2}$$

This is why this kind of media is called gyroelectric (electrically gyrotropic) media. If the permittivity takes scalar form while the permeability in gyrotropic form, the media would be called gyromagnetic (magnetically gyrotropic) media, e.g., ferrite media.

Maxwell equations for gyroelectric media can be written by

$$\boldsymbol{\nabla} \times \boldsymbol{E} = i\omega\mu\boldsymbol{H}, \tag{3a}$$

$$\boldsymbol{\nabla} \times \boldsymbol{H} = -i\omega\overline{\boldsymbol{\epsilon}} \cdot \boldsymbol{E} + \boldsymbol{J}. \tag{3b}$$

Removing $\boldsymbol{H}$ from (3a) and (3b), we have, for the electric field, the following Helmholtz equation

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{E} - \omega^2\mu\overline{\boldsymbol{\epsilon}} \cdot \boldsymbol{E} = i\omega\boldsymbol{J}. \tag{4}$$

2.2 Vector Wave Functions

In general, electric field or magnetic field can be expressed by an expansion consisting of solenoidal vector wave functions and irrotational vector wave functions. In the cylindrical coordinates system (ρ, ϕ, z), the scalar potential function for defining waves in a circular waveguide is

$$\psi(\rho, \phi, z) = J_m(k_\rho \rho) e^{im\phi} e^{ik_z z}, \tag{5}$$

where k_ρ and k_z are the wavenumbers in transverse and propagating directions, respectively.

The cylindrical vector wave functions can be generated from the scalar potential function in (5) by introducing a piloting vector $\widehat{\boldsymbol{z}}$ [24]. The solenoidal vector wave functions are defined as

$$\boldsymbol{M}_m(k_z) = \boldsymbol{\nabla} \times [\psi\widehat{\boldsymbol{z}}] = \left[im\frac{J_m(k_\rho\rho)}{\rho}\widehat{\boldsymbol{\rho}} - \frac{\partial J_m(k_\rho\rho)}{\partial\rho}\widehat{\boldsymbol{\phi}} \right] e^{i(m\phi + k_z z)}, \tag{6a}$$

$$\begin{aligned} \boldsymbol{N}_m(k_z) &= \frac{1}{k}\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times [\psi\widehat{\boldsymbol{z}}] \\ &= \frac{1}{k}\left[ik_z\frac{\partial J_m(k_\rho\rho)}{\partial\rho}\widehat{\boldsymbol{\rho}} - mk_z\frac{J_m(k_\rho\rho)}{\rho}\widehat{\boldsymbol{\phi}} + k_\rho^2 J_m(k_\rho\rho)\widehat{\boldsymbol{z}} \right] e^{i(m\phi + k_z z)}, \end{aligned} \tag{6b}$$

and the irrotational vector wave function is

$$\begin{aligned}\boldsymbol{L}_m(k_z) &= \boldsymbol{\nabla}\psi \\ &= \left[\frac{\partial J_m(k_\rho \rho)}{\partial \rho}\widehat{\boldsymbol{\rho}} + im\frac{J_m(k_\rho \rho)}{\rho}\widehat{\boldsymbol{\phi}} + ik_z J_m(k_\rho \rho)\widehat{\boldsymbol{z}}\right] e^{i(m\phi + k_z z)}. \end{aligned} \tag{6c}$$

The electric field in the waveguide can be expanded completely by the above vector wave functions in the form

$$\boldsymbol{E} = \sum_m \left[\mathcal{A}(k_z)\boldsymbol{M}_m(k_z) + \mathcal{B}(k_z)\boldsymbol{N}_m(k_z) + \mathcal{C}(k_z)\boldsymbol{L}_m(k_z)\right] \tag{7}$$

where the field expansion coefficients $\mathcal{A}(k_z)$, $\mathcal{B}(k_z)$, and $\mathcal{C}(k_z)$ will be determined by the incident wave. The orthogonal properties among these cylindrical vector wave functions as well as the orthogonal relations involving gyrotropic tensor $\overline{\boldsymbol{\epsilon}}$ are given in Appendix A.

3. FIELDS IN AN UNBOUNDED GYROELECTRIC MEDIUM

In this section, we begin with an unbounded gyroelectric medium to find the combinations of the vector wave functions expressing the eigenwaves in the gyroelectric medium, and then in the next section, the boundary conditions of a circular conducting waveguide will be imposed to obtain the eigenmodes in the waveguide.

Consider a source-free gyroelectric medium. In this case, the electric field Helmholtz equation (4) should be written as

$$\boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \boldsymbol{E} - \omega^2 \mu \overline{\boldsymbol{\epsilon}} \cdot \boldsymbol{E} = 0. \tag{8}$$

3.1 Ohm-Rayleigh Method

Substituting (7) into (8), we have

$$\sum_m \boldsymbol{f}_M \cdot \boldsymbol{M}_m(k_z) + \boldsymbol{f}_N \cdot \boldsymbol{N}_m(k_z) + \boldsymbol{f}_L \cdot \boldsymbol{L}_m(k_z) = 0, \tag{9}$$

where

$$\boldsymbol{f}_M = \mathcal{A}(k_z)(k^2\overline{\boldsymbol{I}} - \omega^2\mu\overline{\boldsymbol{\epsilon}}), \tag{10a}$$

$$\boldsymbol{f}_N = \mathcal{B}(k_z)(k^2\overline{\boldsymbol{I}} - \omega^2\mu\overline{\boldsymbol{\epsilon}}), \tag{10b}$$

$$\boldsymbol{f}_L = -\mathcal{C}(k_z)\omega^2\mu\overline{\boldsymbol{\epsilon}}. \tag{10c}$$

The relationship among $\mathcal{A}(k_z)$, $\mathcal{B}(k_z)$, and $\mathcal{C}(k_z)$ can be found by applying the Ohm-Rayleigh method. Taking the anterior scalar product of (9) with $\boldsymbol{M}_{-m'}(-k_z')$, $\boldsymbol{N}_{-m'}(-k_z')$, and $\boldsymbol{L}_{-m'}(-k_z')$ in turn, performing integrations over the entire space and making use of the orthogonal relations given in Appendix A, we end up with the following equation in matrix form:

$$[\Omega][X] = 0 \tag{11}$$

where $[\Omega]$ is a 3×3 matrix given by

$$[\Omega] = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ \Omega_{21} & \Omega_{22} & \Omega_{23} \\ \Omega_{31} & \Omega_{32} & \Omega_{33} \end{bmatrix}, \tag{12}$$

with the following elements:

$$\begin{aligned} \Omega_{11} &= k^2 - \omega^2 \mu \epsilon_t, \\ \Omega_{12} &= \Omega_{21} = -\omega^2 \mu \epsilon_g \frac{k_z}{k}, \\ \Omega_{13} &= -\Omega_{31} = i\omega^2 \mu \epsilon_g, \\ \Omega_{22} &= k^2 - \omega^2 \mu \frac{\epsilon_t k_z^2 + \epsilon_z k_\rho^2}{k^2}, \\ \Omega_{23} &= -\Omega_{32} = \frac{ik_z}{k} \omega^2 \mu (\epsilon_t - \epsilon_z), \\ \Omega_{33} &= -\omega^2 \mu \frac{\epsilon_t k_\rho^2 + \epsilon_z k_z^2}{k_\rho^2}. \end{aligned} \tag{13}$$

In (11), $[X]$ is a column vector given by

$$[X] = [\mathcal{A}(k_z), \mathcal{B}(k_z), \mathcal{C}(k_z)]^T. \tag{14}$$

3.2 Characteristic Equation

In order to have nontrivial solutions for $\mathcal{A}(k_z)$, $\mathcal{B}(k_z)$, and $\mathcal{C}(k_z)$ in (11), it is required that the matrix $[\Omega]$ must be singular, i.e.,

$$\det[\Omega] = 0. \tag{15}$$

(15) results in the characteristic equation:

$$k_\rho^4 + c_2 k_\rho^2 + c_0 = 0, \tag{16}$$

where the coefficients are given by

$$c_2 = \left(1 + \frac{\epsilon_z}{\epsilon_t}\right)(k_z^2 - \omega^2\mu\epsilon_t) + \frac{\epsilon_g}{\epsilon_t}\omega^2\mu\epsilon_g, \tag{17a}$$

$$c_0 = \frac{\epsilon_z}{\epsilon_t}\left[\left(k_z^2 - \omega^2\mu\epsilon_t\right)^2 - \left(\omega^2\mu\epsilon_g\right)^2\right]. \tag{17b}$$

The solutions of (16) yield four roots of k_ρ which are designated as $\pm k_{\rho j}$ $(j = 1, 2)$. As has been discussed in [14], the characteristic equation is only dependent on k_ρ, not the azimuthal direction $\varphi = \tan^{-1}(k_{\rho y}/k_{\rho x})$ where $k_{\rho x}$ and $k_{\rho x}$ are x- and y-components of k_ρ in Cartesian coordinates. Physical insight into the wave modes shows that only two of the roots, i.e., $+k_{\rho j}$ $(j = 1, 2)$ are needed to take into account. The subscript j will be applied onwards to denote different eigenwaves.

Some features of gyroelectric media can be seen from the characteristic equation. Assume plane waves propagate in a gyroelectric medium along z-direction. In this case, $k_\rho = 0$ and (16) yields that

$$k_{z\pm} = \omega\sqrt{\mu(\epsilon_t \pm \epsilon_g)}. \tag{18}$$

The two wavenumbers discriminated by "$+$" and "$-$" represent two different directional polarizations. "$+$" is designated as the right-hand circularly polarization (RCP) while "$-$" as the left-hand circularly polarization (LCP), which are quite similar to the polarizations in a chiral medium. The difference is that the two directional polarizations of the wave in a chiral medium are due to the mutual coupling between electric and magnetic fields, while those in a gyroelectric medium are due to the coupling among the different components of the electric field.

The relations of $\mathcal{A}(k_z)$, $\mathcal{B}(k_z)$, and $\mathcal{C}(k_z)$ can be obtained from any two linear equations in (11) as long as the characteristic equation (16) is satisfied. Solving the lower two linear equations in (11), we have

$$\frac{\mathcal{A}_j(k_z)}{\mathcal{C}_j(k_z)} = \frac{ik_j^2[\epsilon_t k_{\rho j}^2 + \epsilon_z(k_z^2 - \omega^2\mu\epsilon_t)]}{\epsilon_g k_{\rho j}^2(k_j^2 - \omega^2\mu\epsilon_z)}, \tag{19a}$$

$$\frac{\mathcal{B}_j(k_z)}{\mathcal{C}_j(k_z)} = \frac{i\omega^2\mu\epsilon_z k_z k_j}{k_{\rho j}^2(k_j^2 - \omega^2\mu\epsilon_z)}. \tag{19b}$$

These two ratios are in different forms from those in [14, 27], but exactly the same if the relation of (16) is imposed. Based on the relations in (19), the values of $\mathcal{A}(k_z)$, $\mathcal{B}(k_z)$, and $\mathcal{C}(k_z)$ can be finally determined by the incident wave or its equivalent source excitation.

3.3 Eigenmodes

By following the foregoing discussions, the mth eigenmode can be expressed by the summation of the two eigenwaves, namely,

$$\boldsymbol{E}_m(k_z) = \sum_{j=1}^{2} Q_{mj}(k_z)\Big[\mathcal{A}_j(k_z)\boldsymbol{M}_{mj}(k_z) + \mathcal{B}_j(k_z)\boldsymbol{N}_{mj}(k_z) + \mathcal{C}_j(k_z)\boldsymbol{L}_{mj}(k_z)\Big], \tag{20a}$$

$$\boldsymbol{H}_m(k_z) = \frac{1}{i\omega\mu}\boldsymbol{\nabla}\times\boldsymbol{E}_m(k_z) = \sum_{j=1}^{2}\frac{Q_{mj}(k_z)}{i\omega\mu}\Big[\mathcal{A}_j(k_z)k_j\boldsymbol{N}_{mj}(k_z) + \mathcal{B}_j(k_z)k_j\boldsymbol{M}_{mj}(k_z)\Big], \tag{20b}$$

where the coefficients, $Q_{mj}(k_z)$ $(j = 1, 2)$, are to be determined by boundary conditions.

The relation between $Q_{m1}(k_z)$ and $Q_{m2}(k_z)$ in (20) can be obtained by applying the boundary conditions on the walls of the waveguide. Their values are finally determined by the incident field. The expression of the relation between $Q_{m1}(k_z)$ and $Q_{m2}(k_z)$ depends on the forms of $\mathcal{A}(k_z)$, $\mathcal{B}(k_z)$, and $\mathcal{C}(k_z)$ which can be chosen from (19) and do not affect final results. For the simplicity of the following derivations, we choose

$$\mathcal{A}_j(k_z) = \frac{\epsilon_t k_{\rho j}^2 + \epsilon_z(k_z^2 - \omega^2\mu\epsilon_t)}{\epsilon_g k_z k_{\rho j}^2}, \tag{21a}$$

$$\mathcal{B}_j(k_z) = \frac{\omega^2\mu\epsilon_z}{k_j k_{\rho j}^2}, \tag{21b}$$

$$\mathcal{C}_j(k_z) = \frac{k_j^2 - \omega^2\mu\epsilon_z}{ik_z k_j^2}. \tag{21c}$$

4. A CIRCULAR CYROELECTRIC WAVEGUIDE

In this section, we consider a circular waveguide of radius a filled with gyroelectric media as shown in Fig. 1. The walls of the waveguide is perfectly electric conducting (PEC). The wave is guided along z-axis.

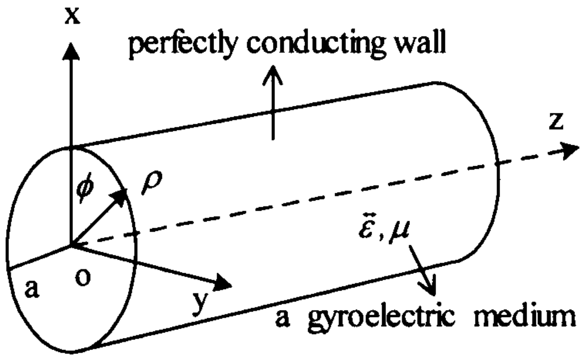

Figure 1. The geometry of a circular gyroelectric waveguide.

4.1 Boundary Conditions and Dispersion Relation

In order to match the boundary condition, $\widehat{\boldsymbol{\rho}} \times \boldsymbol{E}_m = 0$ at $\rho = a$ must be satisfied. It contains two components: one is in $\widehat{\boldsymbol{\phi}}$ direction,

$$\boldsymbol{E}_m \cdot \widehat{\boldsymbol{\phi}}\Big|_{\rho=a} = \sum_{j=1}^{2} Q_{mj}(k_z)\left[-\mathcal{A}_j(k_z)\frac{\partial J_m(k_{\rho j}a)}{\partial a} + \mathcal{D}_j(k_z)\frac{mJ_m(k_{\rho j}a)}{a}\right] = 0, \tag{22}$$

where

$$\mathcal{D}_j(k_z) = \frac{k_{\rho j}^2 - \omega^2\mu\epsilon_z}{k_z k_{\rho j}^2}; \tag{23}$$

and the other is in $\widehat{\boldsymbol{z}}$ direction

$$\boldsymbol{E}_m \cdot \widehat{\boldsymbol{z}}\Big|_{\rho=a} = \sum_{j=1}^{2} Q_{mj}(k_z) J_m(k_{\rho j}a) = 0. \tag{24}$$

These two boundary condition equations lead to the following equations in matrix form:

$$\begin{bmatrix} -\mathcal{A}_1(k_z)\frac{\partial J_m(k_{\rho 1}a)}{\partial a} + \mathcal{D}_1(k_z)\frac{mJ_m(k_{\rho 1}a)}{a} & -\mathcal{A}_2(k_z)\frac{\partial J_m(k_{\rho 2}a)}{\partial a} + \mathcal{D}_2(k_z)\frac{mJ_m(k_{\rho 2}a)}{a} \\ J_m(k_{\rho 1}a) & J_m(k_{\rho 2}a) \end{bmatrix} \begin{bmatrix} Q_{m1}(k_z) \\ Q_{m2}(k_z) \end{bmatrix} = 0. \tag{25}$$

For non-trivial solutions of (25), the determinant of the coefficient matrix must be equal to zero, which yields the dispersion relation:

$$J_m(k_{\rho 2}a)\left[-\mathcal{A}_1(k_z)\frac{\partial J_m(k_{\rho 1}a)}{\partial a}+\mathcal{D}_1(k_z)\frac{mJ_m(k_{\rho 1}a)}{a}\right] - J_m(k_{\rho 1}a)\left[-\mathcal{A}_2(k_z)\frac{\partial J_m(k_{\rho 2}a)}{\partial a}+\mathcal{D}_2(k_z)\frac{mJ_m(k_{\rho 2}a)}{a}\right]=0, \quad (26)$$

and subsequently the coefficients $Q_{mj}(k_z)$ satisfy

$$\frac{Q_{m1}(k_z)}{Q_{m2}(k_z)}=-\frac{J_m(k_{\rho 2}a)}{J_m(k_{\rho 1}a)}. \quad (27)$$

4.2 Mode Orthogonality for a Lossless Gyroelectric Waveguide

Now we examine the mode orthogonality of a gyroelectric waveguide which is assumed to be lossless and mutually bidirectional with its complementary waveguide [34]. The mode orthogonality is necessary for later obtaining the dyadic Green's functions. Consider two eigenmodes, mth mode and m'th mode. Both of the modes satisfy the Maxwell equations and the boundary conditions on the wall (at $\rho=a$) of the gyroelectric waveguide, namely,

$$\nabla\times\boldsymbol{E}_m=i\omega\mu\boldsymbol{H}_m, \quad (28a)$$

$$\nabla\times\boldsymbol{H}_m=-i\omega\overline{\boldsymbol{\epsilon}}\cdot\boldsymbol{E}_m, \quad (28b)$$

$$\widehat{\boldsymbol{\rho}}\times\boldsymbol{E}_m=0; \quad (28c)$$

and

$$\nabla\times\boldsymbol{E}_{m'}=i\omega\mu\boldsymbol{H}_{m'}, \quad (29a)$$

$$\nabla\times\boldsymbol{H}_{m'}=-i\omega\overline{\boldsymbol{\epsilon}}\cdot\boldsymbol{E}_{m'}, \quad (29b)$$

$$\widehat{\boldsymbol{\rho}}\times\boldsymbol{E}_{m'}=0. \quad (29c)$$

Hence we have the following divergence

$$\begin{aligned}\nabla\cdot[\boldsymbol{E}_m\times\boldsymbol{H}^*_{m'}+\boldsymbol{E}^*_{m'}\times\boldsymbol{H}_m] &=\boldsymbol{H}^*_{m'}\cdot\nabla\times\boldsymbol{E}_m-\boldsymbol{E}_m\cdot\nabla\times\boldsymbol{H}^*_{m'} \\ &\quad+\boldsymbol{H}_m\cdot\nabla\times\boldsymbol{E}^*_{m'}-\boldsymbol{E}^*_{m'}\cdot\nabla\times\boldsymbol{H}_m \\ &=\boldsymbol{H}^*_{m'}\cdot[i\omega\mu\boldsymbol{H}_m]-\boldsymbol{E}_m\cdot[i\omega\overline{\boldsymbol{\epsilon}}^*\cdot\boldsymbol{E}^*_{m'}] \\ &\quad+\boldsymbol{H}_m\cdot[-i\omega\mu^*\boldsymbol{H}^*_{m'}]-\boldsymbol{E}^*_{m'}\cdot[-i\omega\overline{\boldsymbol{\epsilon}}\cdot\boldsymbol{E}_m], \end{aligned} \quad (30)$$

where the asterisk denotes complex conjugation. Since the gyroelectric medium filling the waveguide has been assumed to be lossless (the values of ϵ_t, ϵ_z, ϵ_g, and μ are all real), and the following identity

$$\boldsymbol{V}_1 \cdot \overline{\boldsymbol{\mu}} \cdot \boldsymbol{V}_2 = \boldsymbol{V}_2 \cdot \overline{\boldsymbol{\mu}}^* \cdot \boldsymbol{V}_1 \tag{31}$$

holds for arbitrary vectors $\boldsymbol{V}_1$ and $\boldsymbol{V}_2$, we thus have from (30)

$$\boldsymbol{\nabla} \cdot [\boldsymbol{E}_m \times \boldsymbol{H}^*_{m'} + \boldsymbol{E}^*_{m'} \times \boldsymbol{H}_m] = 0. \tag{32}$$

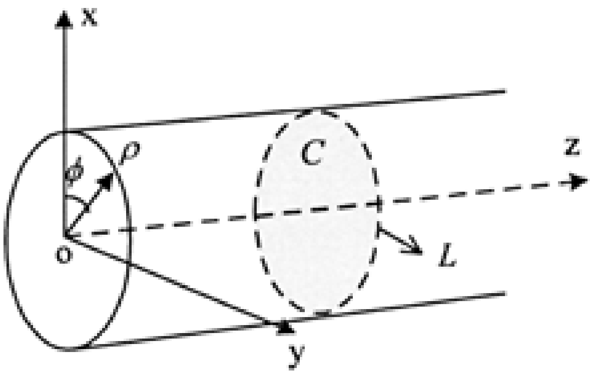

Figure 2. The cross section of the gyroelectric waveguide.

As shown in [35] where chirowaveguides were dealt with, the orthogonality relation can be obtained by further manipulating and simplifying (32). Integrating (32) over the cross section of the waveguide (surface C in Fig. 2), we have

$$\begin{aligned}\iint_C \boldsymbol{\nabla}\cdot [\boldsymbol{E}_m \times \boldsymbol{H}^*_{m'} + \boldsymbol{E}^*_{m'} \times \boldsymbol{H}_m] dS \\ = \iint_C \boldsymbol{\nabla}_t \cdot [\boldsymbol{E}_m \times \boldsymbol{H}^*_{m'} + \boldsymbol{E}^*_{m'} \times \boldsymbol{H}_m] dS \\ + \widehat{\boldsymbol{z}} \cdot \iint_C \frac{\partial}{\partial z}[\boldsymbol{E}_m \times \boldsymbol{H}^*_{m'} + \boldsymbol{E}^*_{m'} \times \boldsymbol{H}_m] dS = 0,\end{aligned} \tag{33}$$

where $\boldsymbol{\nabla}_t = \boldsymbol{\nabla} - \widehat{\boldsymbol{z}}\frac{\partial}{\partial z}$. According to Stokes' theorem, the two-dimensional integral

$$\iint_C \boldsymbol{\nabla}_t \cdot [\boldsymbol{E}_m \times \boldsymbol{H}^*_{m'} + \boldsymbol{E}^*_{m'} \times \boldsymbol{H}_m] dS$$

can be transformed into a line integral over the boundary of the surface, i.e.,

$$\int_L \widehat{\boldsymbol{n}} \cdot [\boldsymbol{E}_m \times \boldsymbol{H}^*_{m'} + \boldsymbol{E}^*_{m'} \times \boldsymbol{H}_m] dl = \int_L [\boldsymbol{H}^*_{m'} \cdot (\widehat{\boldsymbol{n}} \times \boldsymbol{E}_m) + \boldsymbol{H}_m \cdot (\widehat{\boldsymbol{n}} \times \boldsymbol{E}^*_{m'})] dl \tag{34}$$

where $\widehat{\boldsymbol{n}}$ is a unit vector normal to the waveguide wall in outward direction and dl is an infinitesimal line element along the curve L. Since the electric field on the waveguide wall must satisfy the boundary condition $\widehat{\boldsymbol{n}} \times \boldsymbol{E}|_{\rho=a} = \widehat{\boldsymbol{\rho}} \times \boldsymbol{E}|_{\rho=a} = 0$, the integral in (34) is identically zero. Thus (33) can be simplified to

$$\widehat{\boldsymbol{z}} \cdot \iint_C \frac{\partial}{\partial z} [\boldsymbol{E}_m \times \boldsymbol{H}^*_{m'} + \boldsymbol{E}^*_{m'} \times \boldsymbol{H}_m] dS = 0. \tag{35}$$

In view of the expressions of $\boldsymbol{E}_m$ and $\boldsymbol{H}_m$ in (20) and those of vector wave functions in (6), we can remove $\frac{\partial}{\partial z}$ and end up with

$$(k_z - k'_z) \iint_C [\boldsymbol{E}_m \times \boldsymbol{H}^*_{m'} + \boldsymbol{E}^*_{m'} \times \boldsymbol{H}_m] \cdot \widehat{\boldsymbol{z}}\, dS = 0. \tag{36}$$

So for any two different modes $(m \neq m'$ and $k_z \neq k'_z)$, there must have

$$\iint_C [\boldsymbol{E}_m \times \boldsymbol{H}^*_{m'} + \boldsymbol{E}^*_{m'} \times \boldsymbol{H}_m] \cdot \widehat{\boldsymbol{z}}\, dS = 0, \tag{37}$$

which is an orthogonality relation for the modes in a lossless gyroelectric waveguide.

If the two modes are the same $(m = m'$ and $k_z = k'_z)$, the integral in (37) is not necessarily zero. The value of this integral can be actually expressed by the power carried by the mode:

$$\iint_C [\boldsymbol{E}_m \times \boldsymbol{H}^*_{m'} + \boldsymbol{E}^*_{m'} \times \boldsymbol{H}_m] \cdot \widehat{\boldsymbol{z}}\, dS = 4\,\mathrm{sgn}(m) P_m \delta_{mm'}, \tag{38}$$

where P_m is the power carried by the mth mode and $\delta_{mm'}$ is a Kronecker delta, i.e.,

$$\delta_{mm'} = \begin{cases} 1, & m = m' \\ 0, & m \neq m' \end{cases} \tag{39}$$

$\mathrm{sgn}(m)$ in (38) stands for the sign of m and denotes the propagating direction of the mth mode in the waveguide.

The orthogonal relations given in (37) and (38) may be also applicable to gyroelectric waveguides of open structures. Open waveguides may not have conducting but dielectric walls. Thus the integration surface in (37) or (38) for an open waveguide should be seen as the entire transverse plane which is normal to the longitudinal axis of the guide and extended to infinity. Since open waveguides can support not only guided modes but also radiation modes. For guided modes, the wavenumbers are discrete, as discussed in our study. For radiation modes, the wavenumbers are continuous. If at least one of mth mode and m'th mode is a guided mode, (38) still holds. If both of them are radiation modes, the Kronecker delta function $\delta_{mm'}$ should be replaced by Dirac delta function $\delta(m-m')$.

Since the eigenmodes have been proved to be mutually orthogonal, the electric and magnetic fields outside the source region can be expanded as follows:

$$\boldsymbol{E} = \sum_m \sum_{k_{\rho 1,2}} \Gamma_m(\pm k_z) \boldsymbol{E}_m(\pm k_z), \qquad z \gtrless z', \tag{40a}$$

$$\boldsymbol{H} = \sum_m \sum_{k_{\rho 1,2}} \Gamma_m(\pm k_z) \boldsymbol{H}_m(\pm k_z), \qquad z \gtrless z', \tag{40b}$$

where the coefficient Γ_m is known for a given excitation. The upper lines in the equations (40) are for modes propagating in the positive z-direction while the lower lines are for those propagating in the negative z-direction.

5. DYADIC GREEN'S FUNCTIONS

It is well-known that the electric and magnetic fields can be related to the source directly through

$$\boldsymbol{E}(\boldsymbol{r}) = i\omega\mu \iiint_V \overline{\boldsymbol{G}}_{EJ}\left(\boldsymbol{r}, \boldsymbol{r}'\right) \cdot \boldsymbol{J}\left(\boldsymbol{r}'\right) dV', \tag{41a}$$

$$\boldsymbol{H}(\boldsymbol{r}) = \iiint_V \overline{\boldsymbol{G}}_{MJ}\left(\boldsymbol{r}, \boldsymbol{r}'\right) \cdot \boldsymbol{J}\left(\boldsymbol{r}'\right) dV'. \tag{41b}$$

$\overline{\boldsymbol{G}}_{EJ}(\boldsymbol{r}, \boldsymbol{r}')$ is the dyadic Green's function of electric type and $\overline{\boldsymbol{G}}_{MJ}(\boldsymbol{r}, \boldsymbol{r}')$ is the dyadic Green's function of magnetic type. They are both associated with the arbitrary electric source $\boldsymbol{J}(\boldsymbol{r}')$. $\boldsymbol{r}$ and $\boldsymbol{r}'$ are the field point and the source point, respectively. Once the dyadic Green's functions are known, the electric and magnetic fields can be readily calculated.

Expressing the source $\boldsymbol{J}(\boldsymbol{r}')$ by the unit dyad and the Dirac delta function as

$$\boldsymbol{J}(\boldsymbol{r}) = \iiint_V \overline{\boldsymbol{I}}\delta\left(\boldsymbol{r}-\boldsymbol{r}'\right)\cdot\boldsymbol{J}\left(\boldsymbol{r}'\right)dV', \tag{42}$$

and substituting (41) and (42) into (3), we can obtain the Maxwell equations for gyroelectric media in dyadic form

$$\nabla\times\overline{\boldsymbol{G}}_{EJ} = \overline{\boldsymbol{G}}_{MJ}, \tag{43a}$$
$$\nabla\times\overline{\boldsymbol{G}}_{MJ} = \omega^2\mu\overline{\boldsymbol{\epsilon}}\cdot\overline{\boldsymbol{G}}_{EJ} + \overline{\boldsymbol{I}}\delta\left(\boldsymbol{r}-\boldsymbol{r}'\right). \tag{43b}$$

In this section, we derive the complete expansions of dyadic Green's functions for a circular gyroelectric waveguide.

5.1 Determination of Γ_m

To obtain the expressions of the dyadic Green's functions, firstly we must determine the expansion coefficient $\Gamma_m(k_z)$ in (40). We conduct a procedure in the following similar to that in [36] except that the conjugation forms are used here.

Let $(\boldsymbol{E}_t,\ \boldsymbol{H}_t)$ be a set of source-free "test" fields within the waveguide, i.e.,

$$\nabla\times\boldsymbol{E}_t = i\omega\mu\cdot\boldsymbol{H}_t, \tag{44a}$$
$$\nabla\times\boldsymbol{H}_t = -i\omega\overline{\boldsymbol{\epsilon}}\cdot\boldsymbol{E}_t. \tag{44b}$$

Applying complex conjugation to (44), we can readily have

$$\nabla\times\boldsymbol{E}_t^* = -i\omega\mu^*\boldsymbol{H}_t^*, \tag{45a}$$
$$\nabla\times\boldsymbol{H}_t^* = i\omega\overline{\boldsymbol{\epsilon}}^*\cdot\boldsymbol{E}_t^*, \tag{45b}$$

where the parameters ϵ_t, ϵ_z, ϵ_g, and μ have been assumed to be real quantities.

Consider the following divergence

$$\begin{aligned}\nabla\cdot\left[\boldsymbol{E}_t^*\times\boldsymbol{H}+\boldsymbol{E}\times\boldsymbol{H}_t^*\right] = \boldsymbol{H}\cdot\left[\omega\xi_c\overline{\boldsymbol{\mu}}^*\cdot\boldsymbol{E}_t^* - i\omega\overline{\boldsymbol{\mu}}^*\cdot\boldsymbol{H}_t^*\right]\\ -\boldsymbol{H}_t^*\cdot\nabla\times\boldsymbol{E}+\boldsymbol{E}\cdot\nabla\times\boldsymbol{H}_t^*.\end{aligned} \tag{46}$$

Replacing the terms involving the curl in (46) by their equivalents as given in (3), (44), and (45) and making use of the identity (31), we can finally arrive at

$$\nabla \cdot [\boldsymbol{E}_t^* \times \boldsymbol{H} + \boldsymbol{E} \times \boldsymbol{H}_t^*] = -\boldsymbol{E}_t^* \cdot \boldsymbol{J}. \tag{47}$$

Integrating over a volume containing the source $\boldsymbol{J}$ and bounded by a closed surface S, and converting the volume integral of the divergence to a surface integral, we have

$$\iint_S [\boldsymbol{E}_t^* \times \boldsymbol{H} + \boldsymbol{E} \times \boldsymbol{H}_t^*] \cdot \widehat{\boldsymbol{n}} dS = -\iiint \boldsymbol{E}_t^* \cdot \boldsymbol{J} dV \tag{48}$$

where $\widehat{\boldsymbol{n}}$ is again a unit vector normal to S in outward direction, and S consists of the waveguide wall and two cross-sectional planes, i.e., $S = S_+ + S_c + S_-$, as shown in Fig. 3.

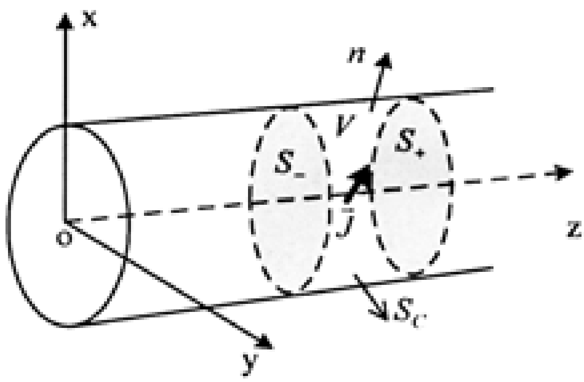

Figure 3. The volume V containing a current source.

There is no contribution to the surface integral arising from the guide wall since $\widehat{\boldsymbol{n}} \times \boldsymbol{E}$ and $\widehat{\boldsymbol{n}} \times \boldsymbol{E}_t^*$ are zero on the guide wall. So (48) can be rewritten as

$$\iint_{S_+} [\boldsymbol{E}_t^* \times \boldsymbol{H} + \boldsymbol{E} \times \boldsymbol{H}_t^*] \cdot \widehat{\boldsymbol{z}}\, dS + \iint_{S_-} [\boldsymbol{E}_t^* \times \boldsymbol{H} + \boldsymbol{E} \times \boldsymbol{H}_t^*] \cdot (-\widehat{\boldsymbol{z}}) dS$$
$$= -\iiint \boldsymbol{E}_t^* \cdot \boldsymbol{J} dV \tag{49}$$

Applying the mode orthogonality (38), we have

$$\iint_{S_-} \Big[\boldsymbol{E}_t^*(k_z) \times \sum \Gamma_m(-k_z)\boldsymbol{H}_m(-k_z) + \sum \Gamma_m(-k_z)\boldsymbol{E}_m(-k_z) \times \boldsymbol{H}_t^*(k_z)\Big] \cdot (-\widehat{\boldsymbol{z}})dS = 0, \tag{50a}$$

and

$$\Gamma_t(k_z) \iint_{S_+} [\boldsymbol{E}_t^*(k_z) \times \boldsymbol{H}_t(k_z) + \boldsymbol{E}_t(k_z) \times \boldsymbol{H}_t^*(k_z)] \cdot \widehat{\boldsymbol{z}}\, dS = -\iiint \boldsymbol{E}_t^*(k_z) \cdot \boldsymbol{J} dV, \tag{50b}$$

for the mode functions $\boldsymbol{E}_t^*(k_z)$ and $\boldsymbol{H}_t^*(k_z)$.

Similarly, for the mode functions $\boldsymbol{E}_t^*(-k_z)$ and $\boldsymbol{H}_t^*(-k_z)$, we have

$$\iint_{S_+} \Big[\boldsymbol{E}_t^*(-k_z) \times \sum \Gamma_m(k_z)\boldsymbol{H}_m(k_z) + \sum \Gamma_m(k_z)\boldsymbol{E}_m(k_z) \times \boldsymbol{H}_t^*(-k_z)\Big] \cdot \widehat{\boldsymbol{z}}\, dS = 0, \tag{51a}$$

and

$$\Gamma_t(-k_z) \iint_{S_-} [\boldsymbol{E}_t^*(-k_z) \times \boldsymbol{H}_t(-k_z) + \boldsymbol{E}_t(-k_z) \times \boldsymbol{H}_t^*(-k_z)] \cdot (-\widehat{\boldsymbol{z}})dS = -\iiint \boldsymbol{E}_t^*(-k_z) \cdot \boldsymbol{J} dV. \tag{51b}$$

The surface integrals in (50b) and (51b) are rewritten as

$$I_m(\pm k_z) = \iint_{S_\pm} [\boldsymbol{E}_m^*(\pm k_z) \times \boldsymbol{H}_m(\pm k_z) + \boldsymbol{E}_m(\pm k_z) \times \boldsymbol{H}_m^*(\pm k_z)] \cdot (\pm\widehat{\boldsymbol{z}})dS, \tag{52}$$

where subscript m has been used in place of subscript t. They can be evaluated by substituting the expressions of $\boldsymbol{E}_m$ and $\boldsymbol{H}_m$ and their conjugates into (52). The expressions of $I_m(\pm k_z)$ are given in Appendix B.

Thus, $\Gamma_m(k_z)$, where subscript t is also replaced by m, can be expressed by $I_m(k_z)$ as

$$\Gamma_m(\pm k_z) = \frac{-\iiint \boldsymbol{E}_m^*(\pm k_z)\cdot \boldsymbol{J}dV}{I_m(\pm k_z)} = \frac{-\iiint \boldsymbol{E}^{*\prime}_m(\pm k_z)\cdot \boldsymbol{J}'dV'}{I_m(\pm k_z)}. \tag{53}$$

5.2 Magnetic Dyadic Green's Function

Since the magnetic field is solenoidal, i.e., $\boldsymbol{\nabla}\cdot\boldsymbol{H} = 0$, the expansion of $\boldsymbol{H}$ in (40b) is complete. Therefore, the magnetic dyadic Green's function can be obtained by simply equating the right-hand side of (41b) and the right-hand side of (40b) and extracting the corresponding part for $\overline{\boldsymbol{G}}_{MJ}(\boldsymbol{r},\boldsymbol{r}')$, namely, from

$$\iiint \overline{\boldsymbol{G}}_{MJ}\left(\boldsymbol{r},\boldsymbol{r}'\right)\cdot \boldsymbol{J}(\boldsymbol{r}')dV = \sum_m\sum_{k_{\rho 1,2}}\Gamma_m(\pm k_z)\boldsymbol{H}_m(\pm k_z)$$

$$= \sum_m\sum_{k_{\rho 1,2}}\boldsymbol{H}_m(\pm k_z)\frac{-\iiint \boldsymbol{E}^{*\prime}_m(\pm k_z)\cdot \boldsymbol{J}'dV'}{I_m(\pm k_z)}, \qquad z \gtrless z' \tag{54}$$

to

$$\overline{\boldsymbol{G}}_{MJ}\left(\boldsymbol{r},\boldsymbol{r}'\right) = \sum_m\sum_{k_{\rho 1,2}}\frac{-1}{I_m(\pm k_z)}\boldsymbol{H}_m(\pm k_z)\boldsymbol{E}^{*\prime}_m(\pm k_z), \quad z \gtrless z'. \tag{55}$$

Taking note of

$$\boldsymbol{E}_m^*(\pm k_z) = (-1)^m\boldsymbol{E}_{-m}(\mp k_z), \tag{56}$$

we can rewrite (55) as

$$\overline{\boldsymbol{G}}_{MJ}(\boldsymbol{r},\boldsymbol{r}') = \sum_m\sum_{k_{\rho 1,2}}\frac{(-1)^{m+1}}{I_m(\pm k_z)}\boldsymbol{H}_m(\pm k_z)\boldsymbol{E}'_{-m}(\mp k_z), \quad z \gtrless z'. \tag{57}$$

5.3 Electric Dyadic Green's Function

Unlike the magnetic field, the expansion (40a) is only valid in the region outside the source. The complete expansion of the electric field should include an irrotational term related to the source.

In the same manner as above for deriving the magnetic dyadic Green's function, we can obtain the solenoidal part of the electric dyadic Green's function as follows:

$$\begin{aligned}\overline{\boldsymbol{G}}_{EJ}^{sol}(\boldsymbol{r},\boldsymbol{r}') &= \frac{1}{i\omega\mu}\sum_m\sum_{k_{\rho1,2}}\frac{-1}{I_m(\pm k_z)}\boldsymbol{E}_m(\pm k_z)\boldsymbol{E}^{*\prime}_m(\pm k_z)\\ &= \frac{1}{i\omega\mu}\sum_m\sum_{k_{\rho1,2}}\frac{(-1)^{m+1}}{I_m(\pm k_z)}\boldsymbol{E}_m(\pm k_z)\boldsymbol{E}'_{-m}(\mp k_z),\quad z \gtrless z'. \quad (58)\end{aligned}$$

The irrotational term of the electric dyadic Green's function can be found by means of the method of $\overline{\boldsymbol{G}}_{MJ}$ [24]. Detailed derivation of the irrotational term is given in Appendix C. The final results of the complete electric dyadic Green's function are

$$\overline{\boldsymbol{G}}_{EJ}^{sin}(\boldsymbol{r},\boldsymbol{r}') = -\frac{1}{\omega^2\mu\epsilon_z}\widehat{\boldsymbol{z}}\,\widehat{\boldsymbol{z}}\delta(\boldsymbol{r}-\boldsymbol{r}'), \quad (59)$$

and thus

$$\begin{aligned}\overline{G}_{EJ}(\boldsymbol{r},\boldsymbol{r}') = &-\frac{1}{\omega^2\mu\epsilon_z}\widehat{\boldsymbol{z}}\,\widehat{\boldsymbol{z}}\,\delta(\boldsymbol{r}-\boldsymbol{r}')\\ &+\frac{1}{i\omega\mu}\sum_m\sum_{k_{\rho1,2}}\frac{(-1)^{m+1}}{I_m(\pm k_z)}\boldsymbol{E}_m(\pm k_z)\boldsymbol{E}'_{-m}(\mp k_z),\quad z \gtrless z'. \quad (60)\end{aligned}$$

5.4 Reduction to Non-Gyrotropic Cases

The inspection of (19a) or (21a) tells us that the expressions here cannot be directly reduced to non-gyrotropic cases. When the gyrotropy vanishes, i.e., $\epsilon_g = 0$, we should go back to the matrix-form equation (11) which can be reduced to

$$\Omega_{11}\mathcal{A}(k_z) = 0, \quad (61a)$$

$$\Omega_{22}\mathcal{B}(k_z) + \Omega_{23}\mathcal{C}(k_z) = 0, \quad (61b)$$

$$\Omega_{32}\mathcal{B}(k_z) + \Omega_{33}\mathcal{C}(k_z) = 0, \quad (61c)$$

For (61) to have nontrivial solutions, there must be

$$\Omega_{11} = 0, \quad (62a)$$

$$\Omega_{22}\Omega_{33} - \Omega_{23}\Omega_{32} = 0. \quad (62b)$$

The solution of (62a) corresponds to ordinary waves, while that of (62b) corresponds to extraordinary waves in uniaxial anisotropic media.

It is observed that in (61), $\mathcal{B}(k_z)$ is still related to $\mathcal{C}(k_z)$, while $\mathcal{A}(k_z)$ is shown of no more relation with $\mathcal{C}(k_z)$. This indicates that in (7) $\mathcal{A}(k_z)\boldsymbol{M}_m(k_z)$ and $[\mathcal{B}(k_z)\boldsymbol{N}_m(k_z) + \mathcal{C}(k_z)\boldsymbol{L}_m(k_z)]$ should satisfy boundary conditions individually. Hence, the vector wave functions in the expansion can be straightforwardly determined to be $\boldsymbol{M}_m(k_z)|_{k_\rho=\eta}$, $\boldsymbol{N}_m(k_z)|_{k_\rho=\lambda}$, and $\boldsymbol{L}_m(k_z)|_{k_\rho=\lambda}$ where η satisfies $\frac{\partial J_m(\eta a)}{\partial a} = 0$ and λ, $J_m(\lambda a) = 0$.

Applying Ohm-Rayleigh method, the electric dyadic Green's function for a circular conducting waveguide filled with a uniaxial medium can be obtained as [32]

$$
\begin{aligned}
&\overline{\boldsymbol{G}}_{EJ}(\boldsymbol{r}, \boldsymbol{r}') \\
&= -\frac{1}{\omega^2\mu\epsilon_z}\widehat{\boldsymbol{z}}\widehat{\boldsymbol{z}}\,\delta(\boldsymbol{r}-\boldsymbol{r}') + \frac{i}{4\pi}\sum_m \left\{ \frac{1}{\eta^2 k_{z\eta} I_\eta}\boldsymbol{M}_{m,\eta}(\pm k_{z\eta})\boldsymbol{M}'_{-m,\eta}(\mp k_{z\eta}) \right. \\
&\quad + \frac{\lambda^2 + k_{z\lambda}^2}{\omega^2\mu\epsilon_t\lambda^2 k_{z\lambda} I_\lambda}\left[\boldsymbol{N}_{m,\lambda}(\pm k_{z\lambda}) + \frac{\epsilon_t - \epsilon_z}{\epsilon_z}\boldsymbol{N}_{m,\lambda}(\pm k_{z\lambda})\cdot\widehat{\boldsymbol{z}}\widehat{\boldsymbol{z}}\right] \\
&\quad \left. \left[\boldsymbol{N}'_{-m,\lambda}(\mp k_{z\lambda}) + \frac{\epsilon_t - \epsilon_z}{\epsilon_z}\boldsymbol{N}'_{-m,\lambda}(\mp k_{z\lambda})\cdot\widehat{\boldsymbol{z}}\widehat{\boldsymbol{z}}\right]\right\}, \quad z \gtrless z'
\end{aligned}
\tag{63}
$$

where

$$
k_{z\eta}^2 = \omega^2\mu\epsilon_t - \eta^2, \tag{64a}
$$

$$
k_{z\lambda}^2 = \omega^2\mu\epsilon_t - \frac{\epsilon_t}{\epsilon_z}\lambda^2, \tag{64b}
$$

$$
I_\eta = \int_0^a J_m^2(\eta\rho)\rho d\rho = \frac{1}{2}a^2\left(1 - \frac{m^2}{\eta^2 a^2}\right)J_m^2(\eta a), \tag{64c}
$$

$$
I_\lambda = \int_0^a J_m^2(\lambda\rho)\rho d\rho = \frac{1}{2}a^2 J_{m-1}^2(\lambda a). \tag{64d}
$$

This Green's function is obviously reducible to the isotropic case by letting $\epsilon_t = \epsilon_z = \epsilon$. The resultant expression is in agreement with that given by Tai [24].

Readers may refer to the published papers [31–33] to find more details of our previous studies on waveguides filled with uniaxial anisotropic media.

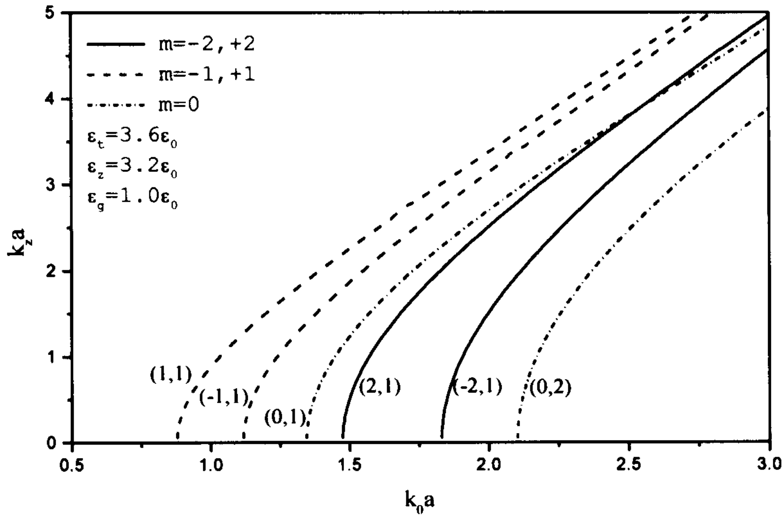

Figure 4. Dispersion curves for a circular gyroelectric waveguide with ideally conducting boundary.

6. NUMERICAL RESULTS

The dispersion equation given in (26) can be used to draw the dispersion curve for any mode once the medium parameters are given. The dispersion curves for $m = 0, \pm 1, \pm 2$ and $\epsilon_t = 3.6\epsilon_0$, $\epsilon_z = 3.2\epsilon_0$, $\epsilon_g = 1.0\epsilon_0$ are shown in Fig. 4 where $k_z a$ is plotted versus $k_0 a$. ϵ_0 and μ_0 are the permittivity and permeability in free space and $k_0 = \omega\sqrt{\mu_0\epsilon_0}$. Unlike those in the circular chirowaveguide, each pair of modes with $+m$ and $-m$ in the gyroelectric waveguide starts with different cutoff frequencies. In chirowaveguides, $+m$ mode and $-m$ mode are degenerated at cutoff frequency [37]. Therefore, in chirowaveguides, $+m$ mode is normally generated in pair with $-m$ mode. While in gyroelectric waveguides, $+m$ mode and $-m$ mode are not degenerated at any frequency except that as the frequency increases, $+m$ mode and $-m$ mode approach to each other. So in gyroelectric waveguides, the RCP (designated by " + ") wave of any eigenmode can exist individually at specific frequency where the LCP (designated by " − ") wave is cut off.

Another numerical result is the dispersion curves for various values of the gyrotropy ϵ_g which are shown in Fig. 5 for $\epsilon_g = 0.5\epsilon_0$, $1.0\epsilon_0$, and

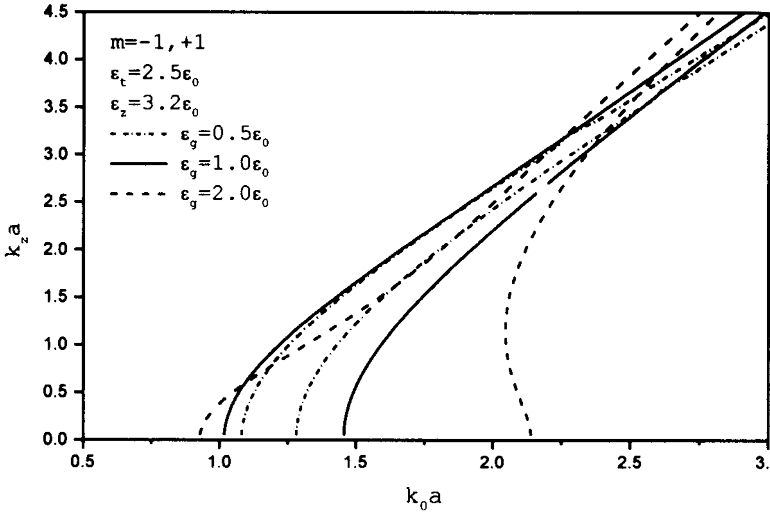

Figure 5. Dispersion curves for $m = \pm 1$ modes with $\epsilon_g = 0.5\epsilon_0, 1.0\epsilon_0, 2.0\epsilon_0$.

$2.0\epsilon_0$. It can be seen that with the increase of the gyrotropy ϵ_g , the cutoff frequency of $+m$ mode decreases while that of $-m$ mode increases. The larger the gyrotropy ϵ_g is, the more widely apart the start points of $+m$ mode and $-m$ mode are. This finding implies that increase of ϵ_g could be helpful to select expected RCP waves.

7. CONCLUSIONS

In conclusion, circular conducting gyroelectric waveguides were studied and the dyadic Green's functions were obtained for the first time in this paper. An unbounded gyroelectric medium was dealt with first and then boundary conditions of the circular conducting waveguide were imposed. The electric and magnetic fields in the waveguide are expressed in terms of cylindrical vector wave functions. It was shown that each of the eigenmodes in the waveguide consists of two eigenwaves whose wave numbers can be determined by the characteristic equation. Dispersion relation was obtained by applying boundary conditions to the eigenmodes. Mode orthogonality was discussed and the orthogonal relations were formulated and used to determine the expansion coefficients of the electric and magnetic fields. The complete expansions of both the electric and the magnetic dyadic Green's functions were made. In the end, some numerical results were obtained, and the calculated dispersion curves were plotted and the effects of the gyrotropy were also discussed.

APPENDIX A. THE ORTHOGONAL RELATIONS OF CYLINDRICAL VECTOR WAVE FUNCTIONS

$$\iiint \boldsymbol{M}_{-m'}\left(-k_z'\right)\cdot\boldsymbol{M}_m(k_z)dV=\Delta, \tag{A1}$$

$$\iiint \boldsymbol{N}_{-m'}\left(-k_z'\right)\cdot\boldsymbol{N}_m(k_z)dV=\Delta, \tag{A2}$$

$$\iiint \boldsymbol{L}_{-m'}\left(-k_z'\right)\cdot\boldsymbol{L}_m(k_z)dV=\frac{k^2}{k_\rho^2}\Delta, \tag{A3}$$

$$\iiint \boldsymbol{M}_{-m'}\left(-k_z'\right)\cdot\boldsymbol{N}_m(k_z)dV=0, \tag{A4}$$

$$\iiint \boldsymbol{M}_{-m'}\left(-k_z'\right)\cdot\boldsymbol{L}_m(k_z)dV=0, \tag{A5}$$

$$\iiint \boldsymbol{N}_{-m'}\left(-k_z'\right)\cdot\boldsymbol{L}_m(k_z)dV=0, \tag{A6}$$

$$\iiint \boldsymbol{M}_{-m'}\left(-k_z'\right)\cdot\overline{\boldsymbol{\epsilon}}\cdot\boldsymbol{M}_m(k_z)dV=\epsilon_t\Delta, \tag{A7}$$

$$\iiint \boldsymbol{M}_{-m'}\left(-k_z'\right)\cdot\overline{\boldsymbol{\epsilon}}\cdot\boldsymbol{N}_m(k_z)dV=\epsilon_g\frac{k_z}{k}\Delta, \tag{A8}$$

$$\iiint \boldsymbol{M}_{-m'}\left(-k_z'\right)\cdot\overline{\boldsymbol{\epsilon}}\cdot\boldsymbol{L}_m(k_z)dV=-i\epsilon_g\Delta, \tag{A9}$$

$$\iiint \boldsymbol{N}_{-m'}\left(-k_z'\right)\cdot\overline{\boldsymbol{\epsilon}}\cdot\boldsymbol{M}_m(k_z)dV=\epsilon_g\frac{k_z}{k}\Delta, \tag{A10}$$

$$\iiint \boldsymbol{N}_{-m'}\left(-k_z'\right)\cdot\overline{\boldsymbol{\epsilon}}\cdot\boldsymbol{N}_m(k_z)=\frac{\epsilon_t k_z^2+\epsilon_z k_\rho^2}{k^2}\Delta, \tag{A11}$$

$$\iiint \boldsymbol{N}_{-m'}\left(-k_z'\right)\cdot\overline{\boldsymbol{\epsilon}}\cdot\boldsymbol{L}_m(k_z)dV=-i(\epsilon_t-\epsilon_z)\frac{k_z}{k}\Delta, \tag{A12}$$

$$\iiint \boldsymbol{L}_{-m'}\left(-k_z'\right)\cdot\overline{\boldsymbol{\epsilon}}\cdot\boldsymbol{M}_m(k_z)dV=i\epsilon_g\Delta, \tag{A13}$$

$$\iiint \boldsymbol{L}_{-m'}\left(-k_z'\right)\cdot\overline{\boldsymbol{\epsilon}}\cdot\boldsymbol{N}_m(k_z)dV=i(\epsilon_t-\epsilon_z)\frac{k_z}{k}\Delta, \tag{A14}$$

$$\iiint \boldsymbol{L}_{-m'}\left(-k_z'\right)\cdot\overline{\boldsymbol{\epsilon}}\cdot\boldsymbol{L}_m(k_z)dV=\frac{\epsilon_t k_\rho^2+\epsilon_z k_z^2}{k_\rho^2}\Delta, \tag{A15}$$

where

$$\Delta = 4\pi^2(-1)^m k_\rho^2 \delta_{m,m'}\delta_{k_\rho,k'_\rho}\delta(k_z - k'_z). \tag{A16}$$

APPENDIX B. THE EXPRESSION OF $I_m(\pm k_z)$

$$\begin{aligned}
&I_m(\pm k_z)\\
&= \pm\frac{4\pi}{i\omega\mu}\Bigg\{Q_{m1}(\pm k_z)Q_{m2}(\pm k_z)\times\Bigg\{\Bigg[i\mathcal{A}_1(\pm k_z)\mathcal{B}_2(\pm k_z)k_2\\
&\quad + i\mathcal{A}_2(\pm k_z)\mathcal{B}_1(\pm k_z)\frac{k_z^2}{k_1} \pm \mathcal{A}_2(\pm k_z)\mathcal{C}_1(\pm k_z)k_z + i\mathcal{A}_2(\pm k_z)\mathcal{B}_1(\pm k_z)k_1\\
&\quad + i\mathcal{A}_1(\pm k_z)\mathcal{B}_2(\pm k_z)\frac{k_z^2}{k_2} \pm \mathcal{A}_1(\pm k_z)\mathcal{C}_2(\pm k_z)k_z\Bigg]\Phi_{12}\\
&\quad + \Bigg[\pm 2i\mathcal{A}_1(\pm k_z)\mathcal{A}_2(\pm k_z)k_z \pm i\mathcal{B}_1(\pm k_z)\mathcal{B}_2(\pm k_z)k_z\left(\frac{k_2}{k_1}+\frac{k_1}{k_2}\right)\\
&\quad \cdot\mathcal{B}_2(\pm k_z)\mathcal{C}_1(\pm k_z)k_2 + \mathcal{B}_1(\pm k_z)\mathcal{C}_2(\pm k_z)k_1\Bigg]\Psi_{12}\Bigg\}\\
&\quad + \sum_{j=1}^{2} Q_{mj}^2(\pm k_z)\Bigg\{\Bigg[i\mathcal{A}_j(\pm k_z)\mathcal{B}_j(\pm k_z)k_j\\
&\quad + i\mathcal{A}_j(\pm k_z)\mathcal{B}_j(\pm k_z)\frac{k_z^2}{k_j} \pm \mathcal{A}_j(\pm k_z)\mathcal{C}_j(\pm k_z)k_z\Bigg]\Phi_j\\
&\quad + \Big[\pm i\mathcal{A}_j^2(\pm k_z) \pm i\mathcal{B}_j^2(\pm k_z)k_z + \mathcal{B}_j(\pm k_z)\mathcal{C}_j(\pm k_z)k_j\Big]\Psi_j\Bigg\}\Bigg\},
\end{aligned} \tag{B1}$$

where Φ_j $(j = 1, 2)$, Φ_{12}, Ψ_j $(j = 1, 2)$, and Ψ_{12} are integrals of Bessel functions given by

$$\Phi_j = 2m\int_0^a \rho d\rho \frac{J_m(k_{\rho j}\rho)}{\rho}\frac{\partial J_m(k_{\rho j}\rho)}{\partial\rho} = mJ_m^2(k_{\rho j}a), \tag{B2}$$

$$\begin{aligned}
\Phi_{12} &= m\int_0^a \rho d\rho\left[\frac{J_m(k_{\rho 1}\rho)}{\rho}\frac{\partial J_m(k_{\rho 2}\rho)}{\partial\rho} + \frac{J_m(k_{\rho 2}\rho)}{\rho}\frac{\partial J_m(k_{\rho 1}\rho)}{\partial\rho}\right]\\
&= mJ_m(k_{\rho 1}a)J_m(k_{\rho 2}a),
\end{aligned} \tag{B3}$$

$$\begin{aligned}
\Psi_j &= \int_0^a \rho d\rho\left\{\left[\frac{mJ_m(k_{\rho j}\rho)}{\rho}\right]^2 + \left[\frac{\partial J_m(k_{\rho j}\rho)}{\partial\rho}\right]^2\right\}\\
&= \frac{1}{2}(k_{\rho j}a)^2\left[J_{m-1}^2(k_{\rho j}a) - J_{m-2}(k_{\rho j}a)J_m(k_{\rho j}a)\right] - mJ_m^2(k_{\rho j}a),
\end{aligned} \tag{B4}$$

$$\Psi_{12} = \int_0^a \rho d\rho \left[\frac{mJ_m(k_{\rho 1}\rho)}{\rho} \frac{mJ_m(k_{\rho 2}\rho)}{\rho} + \frac{\partial J_m(k_{\rho 1}\rho)}{\partial \rho} \frac{\partial J_m(k_{\rho 2}\rho)}{\partial \rho} \right]$$
$$= \frac{k_{\rho 1}k_{\rho 2}}{k_{\rho 1}^2 - k_{\rho 2}^2} \left[k_{\rho 1} a J_m(k_{\rho 1}a) J_{m-1}(k_{\rho 2}a) - k_{\rho 2} a J_{m-1}(k_{\rho 1}a) J_m(k_{\rho 2}a) \right]$$
$$- nJ_m(k_{\rho 1}a) J_m(k_{\rho 2}a). \tag{B5}$$

APPENDIX C. METHOD OF $\overline{G}_{MJ}$

At $z = z'$, the magnetic dyadic Green's function $\overline{\boldsymbol{G}}_{MJ}$ is discontinuous. For the present problem, the boundary condition for the tangential magnetic field at $z = z'$ surface can be written in the form

$$\hat{\boldsymbol{z}} \times \left(H^+ - H^- \right) = \boldsymbol{J} \cdot \hat{\boldsymbol{\rho}}, \tag{C1a}$$

or in a dyadic form,

$$\hat{\boldsymbol{z}} \times \left(\overline{\boldsymbol{G}}_{MJ}^+ - \overline{\boldsymbol{G}}_{MJ}^- \right) = \left(\overline{I} - \hat{\boldsymbol{z}}\hat{\boldsymbol{z}} \right) \delta \left(\rho - \rho' \right), \tag{C1b}$$

where the superscript $+$ stands for $z > z'$ and $-$ for $z < z'$.

Since $\overline{\boldsymbol{G}}_{MJ}$ is discontinuous at $z = z'$, we can write

$$\overline{\boldsymbol{G}}_{MJ} \left(\boldsymbol{r}, \boldsymbol{r}' \right) = \overline{\boldsymbol{G}}_{MJ}^+ \left(\boldsymbol{r}, \boldsymbol{r}' \right) U \left(z - z' \right) + \overline{\boldsymbol{G}}_{MJ}^- \left(\boldsymbol{r}, \boldsymbol{r}' \right) U \left(z' - z \right), \tag{C2}$$

where U is the unit step function. Thus, by making use of (C1b), we can derive

$$\begin{aligned}
\boldsymbol{\nabla} \times \overline{\boldsymbol{G}}_{MJ} &= \boldsymbol{\nabla} \times \left[\overline{\boldsymbol{G}}_{MJ}^+ \left(\boldsymbol{r}, \boldsymbol{r}' \right) U \left(z - z' \right) + \overline{\boldsymbol{G}}_{MJ}^- \left(\boldsymbol{r}, \boldsymbol{r}' \right) U \left(z' - z \right) \right] \\
&= \left(\boldsymbol{\nabla} \times \overline{\boldsymbol{G}}_{MJ}^+ \right) U \left(z - z' \right) + \boldsymbol{\nabla} U \left(z - z' \right) \times \overline{\boldsymbol{G}}_{MJ}^+ \\
&\quad + \left(\boldsymbol{\nabla} \times \overline{\boldsymbol{G}}_{MJ}^- \right) U \left(z' - z \right) + \boldsymbol{\nabla} U \left(z' - z \right) \times \overline{\boldsymbol{G}}_{MJ}^- \\
&= \left(\boldsymbol{\nabla} \times \overline{\boldsymbol{G}}_{MJ}^+ \right) U \left(z - z' \right) + \left(\boldsymbol{\nabla} \times \overline{\boldsymbol{G}}_{MJ}^- \right) U \left(z' - z \right) \\
&\quad + \hat{\boldsymbol{z}} \delta \left(z - z' \right) \times \left(\overline{\boldsymbol{G}}_{MJ}^+ - \overline{\boldsymbol{G}}_{MJ}^- \right), \\
&= \left(\boldsymbol{\nabla} \times \overline{\boldsymbol{G}}_{MJ}^+ \right) U \left(z - z' \right) + \left(\boldsymbol{\nabla} \times \overline{\boldsymbol{G}}_{MJ}^- \right) U \left(z' - z \right) \\
&\quad + (\overline{\boldsymbol{I}} - \hat{\boldsymbol{z}}\hat{\boldsymbol{z}}) \delta \left(\boldsymbol{r} - \boldsymbol{r}' \right).
\end{aligned} \tag{C3}$$

Calling the relation (43b) and noting that

$$\overline{\epsilon}^{-1}\cdot\boldsymbol{M}=\frac{\epsilon_t}{\epsilon_t^2-\epsilon_g^2}\boldsymbol{M}+\frac{i\epsilon_g}{\epsilon_t^2-\epsilon_g^2}\frac{ik}{k_z}\boldsymbol{N}_t, \tag{C4a}$$

$$\overline{\epsilon}^{-1}\cdot\boldsymbol{N}=\frac{\epsilon_t}{\epsilon_t^2-\epsilon_g^2}\boldsymbol{N}_t+\frac{1}{\epsilon_z}\boldsymbol{N}_z+\frac{i\epsilon_g}{\epsilon_t^2-\epsilon_g^2}\frac{ik_z}{k}\boldsymbol{M}, \tag{C4b}$$

with

$$\boldsymbol{N}_t=\boldsymbol{N}\cdot\left(\widehat{\boldsymbol{\rho}}\,\widehat{\boldsymbol{\rho}}+\widehat{\boldsymbol{\phi}}\,\widehat{\boldsymbol{\phi}}\right), \tag{C4c}$$

$$\boldsymbol{N}_z=\boldsymbol{N}\cdot\widehat{\boldsymbol{z}}\,\widehat{\boldsymbol{z}}, \tag{C4d}$$

we have

$$\begin{aligned}\overline{\boldsymbol{G}}_{EJ}(\boldsymbol{r},\boldsymbol{r}')=&\frac{1}{\omega^2\mu\overline{\epsilon}}\cdot\Big[\left(\boldsymbol{\nabla}\times\overline{\boldsymbol{G}}_{MJ}^{+}\right)U\left(z-z'\right)\\&+(\boldsymbol{\nabla}\times\overline{\boldsymbol{G}}_{MJ}^{-})U\left(z-z'\right)-\widehat{\boldsymbol{z}}\,\widehat{\boldsymbol{z}}\delta\left(\boldsymbol{r}-\boldsymbol{r}'\right)\Big]\\=&-\frac{1}{\omega^2\mu\epsilon_z}\widehat{\boldsymbol{z}}\,\widehat{\boldsymbol{z}}\delta\left(\boldsymbol{r}-\boldsymbol{r}'\right)+\sum_{k_{\rho1,2}}\sum_m\frac{(-1)^{m+1}}{I_m(\pm k_z)}\\&\times\left\{\sum_{j=1}^{2}\left[\frac{\mathcal{A}_j(\pm k_z)\epsilon_t k_j^2\mp\mathcal{B}_j(\pm k_z)\epsilon_g k_j k_z}{\omega^2\mu(\epsilon_t^2-\epsilon_g^2)}\boldsymbol{M}_{mj}(\pm k_z)\right.\right.\\&+\frac{\mp\mathcal{A}_j(\pm k_z)\epsilon_g k_j^3+\mathcal{B}_j(\pm k_z)\epsilon_t k_j^2 k_z}{\omega^2\mu(\epsilon_t^2-\epsilon_g^2)k_z}\boldsymbol{N}_{mjt}(\pm k_z)\\&\left.\left.+\frac{\mathcal{B}_j(\pm k_z)k_j^2}{\omega^2\mu\epsilon_z}\boldsymbol{N}_{mjz}(\pm k_z)\right]\right\}\boldsymbol{E}'_{-m}(\mp k_z),\quad z\gtrless z'.\end{aligned} \tag{C5}$$

Taking note of

$$\frac{\mathcal{A}_j(\pm k_z)\epsilon_t k_j^2\mp\mathcal{B}_j(\pm k_z)\epsilon_g k_j k_z}{\omega^2\mu(\epsilon_t^2-\epsilon_g^2)}=\mathcal{A}_j(\pm k_z), \tag{C6a}$$

$$\frac{\mp\mathcal{A}_j(\pm k_z)\epsilon_g k_j^3+\mathcal{B}_j(\pm k_z)\epsilon_t k_j^2 k_z}{\omega^2\mu(\epsilon_t^2-\epsilon_g^2)k_z}=\mathcal{B}_j(\pm k_z), \tag{C6b}$$

$$\frac{\mathcal{B}_j(\pm k_z)k_j^2}{\omega^2\mu\epsilon_z}=\mathcal{B}_j(\pm k_z)\pm\mathcal{C}_j(\pm k_z)\frac{ik_zk_j}{k_{\rho j}^2}, \tag{C6c}$$

we can find that the combination

$$\sum_{j=1}^{2}\left[\frac{\mathcal{A}_j(\pm k_z)\epsilon_t k_j^2 \mp \mathcal{B}_j(\pm k_z)\epsilon_g k_j k_z}{\omega^2\mu(\epsilon_t^2-\epsilon_g^2)}\boldsymbol{M}_{mj}(\pm k_z) + \frac{\mp\mathcal{A}_j(\pm k_z)\epsilon_g k_j^3 + \mathcal{B}_j(\pm k_z)\epsilon_t k_j^2 k_z}{\omega^2\mu(\epsilon_t^2-\epsilon_g^2)k_z}\boldsymbol{N}_{mjt}(\pm k_z) + \frac{\mathcal{B}_j(\pm k_z)k_j^2}{\omega^2\mu\epsilon_z}\boldsymbol{N}_{mjz}(\pm k_z)\right] \tag{C7}$$

in (C5) is actually $\boldsymbol{E}_m(\pm k_z)$. Hence we have the results given in (59) and (60).

REFERENCES

1. Suhl, H., and L. R. Walker, "Topics in guided-wave propagation through gyromagnetic media: Part I — The completely filled cylindrical guide," *Bell Syst. Tech. Journal,* Vol. 33, 579–659, May 1954.
2. Allis, W. P., S. J. Buchsbaum, and A. Bers, *Waves in Anisotropic Plasmas*, MIT Press, Cambridge, Mass., 1963.
3. Toda, M., "Propagation in a solid state plasma waveguide in a transverse magnetic field," *J. Phys. Soc. Japan*, Vol. 19, 1126–1130, 1964.
4. Hirota, R., "Theory of a solid state plasma waveguide in a transverse magnetic field," *J. Phys. Soc. Japan,* Vol. 19, 1130–1134, 1964.
5. Collin, R. E., "Electromagnetic potentials and field expansions for plasma radiation in waveguide," *IEEE Trans. Microwave Theory Tech.,* Vol. MTT-12, 413–420, 1965.
6. Gabriel, G. J., and M. E. Brodwin, "The solution of guided waves in inhomogeneous anisotropic media by perturbation and variational methods," *IEEE Trans. Microwave Theory and Techniques,* Vol. MTT-13, 364–370, May 1965.
7. Gabriel, G. J., and M. E. Brodwin, "Distinctions between gyroelectric and gyromagnetic media in rectangular waveguide," *IEEE Trans. Microwave Theory and Techniques,* Vol. MTT-14, 292–293, June 1966.
8. Hirota, R., and K. Suzuki, "Propagation of waves in a bounded solid state plasma in transverse magnetic fields," *J. Phys. Soc. Japan,* Vol. 21, 1112–1118, 1996.
9. Kuno, H. J., and W. D. Hershberger, "Microwave faraday effect and propagation in a circular solid-state plasma waveguide,"

IEEE Trans. Microwave Theory Tech., Vol. MTT-15, 661-668, Dec. 1967.

10. Champlin, K. S., G. H. Glover, and D. E. O'Connor, "Plasma-filled waveguide with axial magnetization: I — Variational determination of normal modes," *Journal of Applied Physics,* Vol. 40, 3532–3538, Aug. 1969.
11. Champlin, K. S., and G. H. Glover, ""twist" modes in magneto-plasma-filled circular waveguides," *IEEE Trans. Microwave Theory Tech.,* Vol. MTT-18, No. 9, 566–570, Sept. 1970.
12. Hirota, R., and K. Suzuki, "Field distribution in a magneto-plasma-loaded waveguide at room temperature," *IEEE Trans. Microwave Theory Tech.,* Vol. MTT-18, No. 4, 188–195, Apr. 1970.
13. Uzunoglu, N. K., "Propagation in cylindrical dielectric waveguides with gyroelectric cores," *IEE Proceedings: Part H*, Vol. 128, No. 3, 164–167, June 1981.
14. Uzunoglu, N. K., P. G. Cottis, and J. G. Fikioris, "Excitation of electromagnetic waves in a gyroelectric cylinder," *IEEE Trans. Ant. Prop.,* Vol. AP-33, No. 1, 90–95, Jan. 1995.
15. Monzon, J. C., "Three-dimensional scattering by an infinite homogeneous anisotropic circular cylinder: A spectral approach," *IEEE Transactions on Antennas & Propagation,* No. 6, 670–682, June 1987.
16. Tsalamengas, J. L., and N. K. Uzunoglu, "Radiation from a dipole near a general anisotropic layer," *IEEE Transactions on Antennas & Propagation,* Vol. 38, No. 1, 9–16, Jan. 1990.
17. Shen, H.-M., "Plasma waveguide," *Proceedings of SPIE - The International Society for Optical Engineering,* Vol. 1226, 302–313, Int. Soc. for Optical Engineering, Bellingham, WA, USA, 1989.
18. Hu, B. J., and C. L. Ruan, "Plasma circular cylindrical waveguide in lossy material," *International Journal of infrared & Millimeter Waves,* Vol. 18, No. 5, 1015–1022, May 1997.
19. Hu, B. J., and C. L. Ruan, "Propagation properties of a plasma waveguide in an external magnetic field," *Journal of Physics D — Applied Physics,* Vol. 31, No. 17, 2151–2154, Sept. 7, 1998.
20. Takeda, S., "Propagation of waves through magnetized plasmas in waveguides," *Japanese Journal of Applied Physics Part 2 — Letters,* Vol. 33, No. 5B, L757–L759, May 15, 1994.
21. Ivanov, S. T., and N. I. Nikolaev, "Spectrum of electromagnetic waves in a planar gyrotropic plasma waveguide," *Japanese Journal of Applied Physics Part 1 — Regular Papers Short Notes & Review Papers,* Vol. 37, No. 9A, 5033–5038, Sept. 1998.

22. Erdmann, A., M. Shamonin, P. Hertel, and H. Dotsch, "Finite difference analysis of gyrotropic waveguides," *Optics Communications,* Vol. 102, No. 1–2, 25–30, Sept. 15, 1993.
23. Pregla, R., "Method of lines for the analysis of multilayered gyrotropic waveguide structures," *IEE Proceedings-H Microwaves Antennas & Propagation,* Vol. 140, No. 3, 183–192, June 1993.
24. Tai, C. T., *Dyadic Green's Functions in Electromagnetic Theory,* 2nd edition, IEEE Press, Piscataway, New Jersey, 1994.
25. Habashy, T. M., S. M. Ali, J. A. Kong, and M. D. Grossi, "Dyadic Green's functions in a planar stratified, arbitrarily magnetized linear plasma," *Radio Science,* Vol. 26, No. 3, 701–715, May–June 1991.
26. Barkeshli, S., "Electromagnetic dyadic Green's function for multilayered symmetric gyroelectric media," *Radio Science,* Vol. 28, No. 1, 23–36, Jan.–Feb. 1993.
27. Ren, W., "Contributions to the electromagnetic wave theory of bounded homogeneous anisotropic media," *Physical Rev. E,* Vol. 47, No. 1, 664–673, Jan. 1993.
28. Weiglhofer, W. S., "Dyadic Green's function representation in electrically gyrotropic media," *AEU, Archiv fur Elektronik und Ubertragungstechnik: Electronics and Communication,* Vol. 47, No. 3, 125–130, May 1993.
29. Olyslager, F., "Time-harmonic two- and three-dimensional closed form Green's dyadics for gyrotropic, bianisotropic, and anisotropic media," *Electromagnetics,* Vol. 17, No. 4, 369–386, July–Aug. 1997.
30. Cottis, P. G., "Radiation in an unbounded gyroelectric medium," *Journal of Electromagnetic Waves and Applications,* Vol. 12, No. 2, 249–268, 1998.
31. Liu, S., L. W. Li, M. S. Leong, and T. S. Yeo, "Dyadic Green's function in a rectangular conducting waveguide filled with uniaxial anisotropic materials," *Proceedings of Progress in Electromagnetics Research Symposium, PIERS 1999,* Vol. 1 of 2, 421, National Central University, Chung-Li, Taiwan, Taipei, Taiwan, Mar. 22–26, 1999.
32. Liu, S., L. W. Li, T. S. Yeo, and M. S. Leong, "Arbitrary cross-sectional cylindrical conducting waveguide filled with uniaxial anisotropic media: Electromagnetic fields and dyadic Green's functions," *Proceedings of The 1999 Asia-Pacific Microwave Conference (APMC'99),* Vol. 2 of 3, 279–282, Singapore, Nov. 30–Dec. 3, 1999,
33. Liu, S., L. W. Li, M. S. Leong, and T. S. Yeo, "Rectangular conducting waveguide filled with uniaxial anisotropic media: A

modal analysis and dyadic Green's functions," *Progress in Electromagnetics Research,* Vol. PIER 25, 111–130, 2000.

34. McIsaac, P. R., "Mode orthogonality in reciprocal and nonreciprocal waveguides," *IEEE Transactions on Microwave Theory & Techniques,* Vol. 39, No. 11, 1808–1816, Nov. 1991.
35. Engheta, N. and P. Pelet, "Orthogonality relations in chirowaveguides," *IEEE Trans. Microwave Theory Tech.,* Vol. MTT-38, No. 11, 1631–1634, 1990.
36. Pathak, P. H., "On the eigenfunction expansion of the electric and magnetic field dyadic Green's functions," *IEEE Trans. Antennas Propagat.,* Vol. AP-31, No. 6, 837–846, Sept. 1983.
37. Liu, S., L. W. Li, M. S. Leong, and T. S. Yeo, "Electromagnetic field and dispersion curves in circular conducting chirowaveguide," *Proc. of the 1999 (2nd) International Conference on Computational Electromagnetics and Its Applications,* 12–15, Beijing, China, Nov. 1–4, 1999.