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PROPAGATION OF ELECTROMAGNETIC WAVES ON THE LATERAL SURFACE OF A FERRITE/SEMICONDUCTOR SUPERLATTICE AT QUANTUM HALL-EFFECT CONDITIONS

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Abstract–The principal result of this work is the existence of coupled magnon-plasmon surface polaritons, i.e., nonradiative spin-electromagnetic surface waves on the lateral surface of a superlattice that consists of the alternating layers of GaAs-AlGaAs quantum well system and ferromagnetic insulator. Dispersion relations for the waves are considered at quantum Hall-effect conditions when a static quantizing magnetic field is perpendicular to the quantum well plane.

1. INTRODUCTION

During the past several years there have been quite a number of studies on the collective excitations in the superlattices (SL) that consist of alternating layers of two or three different materials [e.g., 1, 2]. The coupled magnon-plasmon surface polaritons in the infrared region on the interface between antiferrodielectric and semiconductor have been investigated in [3, 4] for the isotropic and uniaxial media. The purpose of this work is the investigation of the coupled magnon-plasmon polaritons on the lateral surface of SL that consists of the alternating layers of GaAs-AlGaAs quantum well system and ferromagnet insulators. This work continues our study of the coupled spin-electromagnetic waves in a bigyrotropic media. Previous investigation was performed for the waves localized at the boundary of vacuum with the SL that consist of the n-InSb type semiconductor and ferromagnet layers in the presence of a non-quantizing static magnetic field parallel to the lateral surface of the SL [5]. In this work the particular attention is paid to those collective modes which arise from the coupling between the two-dimensional (2D) magnetoplasmons, magnons and fluctuating electromagnetic fields in the quantum Hall-effect conditions. The dispersion relations and the localization conditions for the waves are considered when a quantizing magnetic field is perpendicular to the quantum well planes, at the arbitrary orientation of the wave vector with respect to the magnetic field. The principal new result of this work is the existence of the nondamping spin-electromagnetic surface waves with quantizing frequencies propagating along the lateral surface of the SL. The waves are solutions of Maxwell equations in their full form. The effective anisotropic medium method is used to express the dielectric and magnetic permeability tensors for the SL in the terms of the permeabilities for the individual layers. This method is based on using of the continuity of some macroscopic field components at the interfaces of the layers. It is valid only if the period of the SL is less than the wavelength of the considered waves.

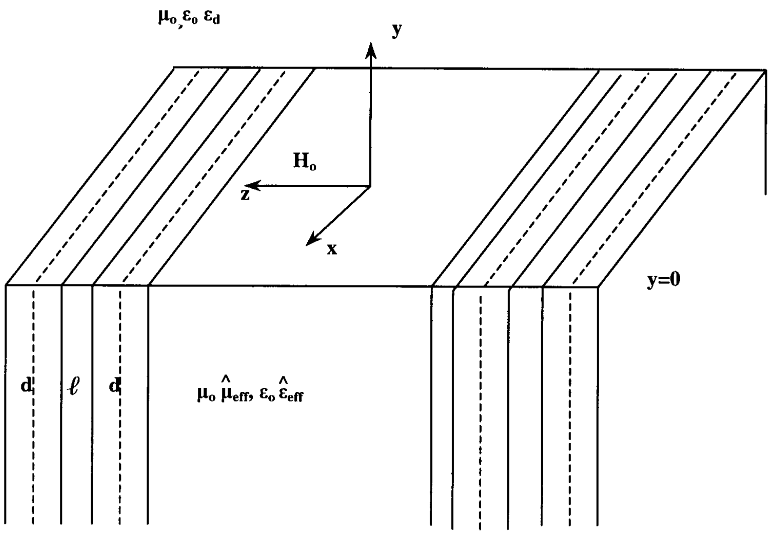

Figure 1. Geometry of the ferrite/semiconductor superlattice with quantum wells in the externally applied static magnetic field. Dashed lines note 2D electron gas planes.

2. GENERAL THEORY

2.1. Effective Permeability Tensors

The ferrite/semiconductor SL is considered as being infinite in the x and z directions and filling the space $y < 0$ (Fig. 1). The space $y > 0$ is taken to be an isotropic insulator of dielectric constant ε_d (for vacuum $\varepsilon_d = 1$). The layers of the SL are parallel to the xy plane and are perpendicular to the quantizing static magnetic field $\mathbf{H}_0\mathbf{IIoz}$. A 2D electron gas plane is described by a local conductivity tensor

$$\hat{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix}. \tag{1}$$

In the quantum Hall-effect conditions [6]

$$\sigma_{xx} = 0, \qquad \sigma_{xy} = -e^2 s/h, \tag{2}$$

where the integer s is the number of the filling Landau levels. An individual semiconductor layer of thickness d representable as GaAs-

AlGaAs quantum well system can be described by the gyrotropic dielectric permeability tensor

$$\hat{\varepsilon}(\omega) = \begin{pmatrix} \varepsilon_L & -i\varepsilon_a & 0 \\ i\varepsilon_a & \varepsilon_L & 0 \\ 0 & 0 & \varepsilon_L \end{pmatrix}, \tag{3}$$

where ε_L is the average static dielectric constant and

$$\varepsilon_a = -\sigma_{xy}/\varepsilon_0\omega d. \tag{4}$$

An individual ferrite layer of thickness l is described by the static dielectric constant ε_F and magnetic permeability tensor [5, 7]

$$\hat{\mu}(\omega) = \begin{pmatrix} \mu & i\mu_a & 0 \\ -i\mu_a & \mu & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{5}$$

where

$$\mu = \left[\omega^2 - \omega_H(\omega_H + \omega_M)\right] / \left(\omega^2 - \omega_H^2\right), \quad \mu_a = \omega\omega_M / \left(\omega^2 - \omega_H^2\right), \tag{6}$$

$\omega_H = \gamma H_0$, $\omega_M = \gamma M_0$, γ is the gyromagnetic ratio and M_0 is the saturation magnetization. In Eqs. (6) is assumed that the dissipation of the waves can be neglected. In the case when the period of the SL $L = d + l$ is less than the wavelength of the waves, one may average (3) and (5) and obtain the effective permeability tensors in the form

$$\hat{\varepsilon}_{\text{eff}} = \overline{\varepsilon} \begin{pmatrix} 1 & -i\nu & 0 \\ i\nu & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{7}$$

$$\hat{\mu}_{\text{eff}} = \overline{\mu} \begin{pmatrix} 1 & i\zeta & 0 \\ -i\zeta & 1 & 0 \\ 0 & 0 & \overline{\mu}^{-1} \end{pmatrix}, \tag{8}$$

where

$$\overline{\varepsilon} = (\varepsilon_L d + \varepsilon_F l)/L, \quad \overline{\mu} = (\mu l + d)/L = 1 - \omega_H\omega_1 / \left(\omega^2 - \omega_H^2\right), \tag{9a}$$

$$\nu = \varepsilon_a d/\overline{\varepsilon}L = s\nu_0/\omega, \quad \nu_0 = e^2/\varepsilon_0\overline{\varepsilon}hL, \tag{9b}$$

$$\zeta = \mu_a l/\overline{\mu}L = \omega\omega_1 / \left[\omega^2 - \omega_H(\omega_H + \omega_1)\right], \quad \omega_1 = \omega_M l/L. \tag{9c}$$

Thus, the ferrite/semiconductor SL under considering conditions is really a bigyrotropic media described by the dielectric and magnetic permeability tensors (7) and (8).

2.2. Partial Waves and Electromagnetic Field Structure

To obtain the dispersion relations for the nonradiative surface waves propagating along the lateral surface of the SL one must find the solutions of Maxwell equations

$$rot\mathbf{H} = -i\omega\varepsilon_0\hat{\varepsilon}_{\text{eff}}\mathbf{E}, \qquad rot\mathbf{E} = i\omega\mu_0\hat{\mu}_{\text{eff}}\mathbf{H}, \tag{10}$$

for which the fields are exponentially damped as one leaves the surface $y = 0$. For the plane waves $\mathbf{E}$, $\mathbf{H} \sim \exp[i(\mathbf{kr} - \omega t)]$ Eq. (10) give

$$\hat{a}\mathbf{E} = 0, \tag{11}$$

where

$$\hat{a} = \begin{pmatrix} \alpha - \mu_v\chi^2 & -i(\alpha\nu + \delta n_z^2 - \mu_v\chi n_x) & n_z(n_x - \chi\zeta) \\ i(\alpha\nu + \delta n_z^2 + \mu_v\chi n_x) & \alpha + \mu_v\chi^2 & in_z(\chi - \zeta n_x) \\ n_z(n_x + \chi\zeta) & -in_z(\chi + \zeta n_x) & \chi^2 - n_x^2 + \overline{\varepsilon}\mu_v \end{pmatrix}, \tag{12}$$

$$\alpha = n_z^2 - \overline{\varepsilon}\mu_v, \quad \delta = \zeta - \nu, \qquad \mu_v = \overline{\mu}(1 - \zeta^2), \tag{13a}$$

$$n_x = k_x/k_0, \qquad \chi = ik_y/k_0, \quad n_z = k_z/k_0. \tag{13b}$$

$$k_0 = \omega/c, \qquad c = (\varepsilon_0\mu_0)^{-1/2}. \tag{13c}$$

By setting $\text{Det}\,\hat{a} = 0$ one may obtain two partial waves with different values for χ at the given values for the tangential components of the wave vector k_x and k_z:

$$\chi_{1,2}^2 = n_x^2 + \frac{1}{2}\left\{\alpha + \beta \mp \left[(\alpha - \beta)^2 + 4\overline{\varepsilon}\delta^2 n_z^2\right]^{1/2}\right\}, \tag{14}$$

where

$$\beta = n_z^2/\overline{\mu} - \varepsilon_v, \quad \varepsilon_v = \overline{\varepsilon}\left(1 - \nu^2\right). \tag{15}$$

Thus, the electromagnetic field in the space region $y < 0$ can be written as a superposition of the partial waves with the different polarizations:

$$\mathbf{E}(\mathbf{r}, t) = \{\mathbf{E}_1 \exp(k_0\chi_1 y) + \mathbf{E}_2 \exp(k_0\chi_2 y)\} \exp[i(k_x x + k_z z - \omega t)] \tag{16}$$

and an analogical expression for the magnetic field vector $\mathbf{H}(\mathbf{r}, t)$.

To find the polarization of the partial waves one can write Eq. (10) in the form

$$\hat{b}\mathbf{H} = 0, \tag{17}$$

where

$$\hat{b} = \begin{pmatrix} \overline{\mu}(\varepsilon_v \chi^2/\overline{\varepsilon\mu} - \beta) & -i\overline{\mu}(\beta\nu - \delta\varepsilon_v + \varepsilon_v \chi n_x/\overline{\varepsilon\mu}) & n_z(n_x + \chi\nu) \\ i\overline{\mu}(\delta\varepsilon_v - \beta v + \varepsilon_v \chi n_x/\overline{\varepsilon\mu}) & \overline{\mu}(\varepsilon_v n_x^2/\overline{\varepsilon\mu} + \beta) & in_z(\chi + vn_x) \\ n_z(n_x - \chi\nu) & -in_z(\chi - \nu n_x) & \chi^2 - n_x^2 + \varepsilon_v \end{pmatrix} \tag{18}$$

It is not difficult to see that the equation $\operatorname{Det} \hat{b} = 0$ gives the values for χ^2 which are identical with (14). Using (17) and (18), the field structure of the partial wave with χ_1 can be described as

$$\mathbf{H}_1 = (h_{1x}, ih_{1y}, 1)H_{1z}, \tag{19}$$

$$\mathbf{E}_1 = (ie_{1x}, e_{1y}, -ie_{1z})H_{1z}/\varepsilon_v, \tag{20}$$

where

$$\begin{pmatrix} e_{1x} \\ e_{1y} \\ e_{1z} \end{pmatrix} = \begin{pmatrix} -\nu n_z & n_z & \chi_1 + \nu n_x \\ -n_z & \nu n_z & n_x + \nu\chi_1 \\ \varepsilon_v \chi_1/\overline{\varepsilon} & \varepsilon_v n_x/\overline{\varepsilon} & 0 \end{pmatrix} \begin{pmatrix} h_{1x} \\ h_{1y} \\ 1 \end{pmatrix}, \tag{21}$$

$$h_{1x} = B_{31}/B_{33} = B_{11}/B_{13}, \quad h_{1y} = -iB_{32}/B_{33} = -iB_{12}/B_{13}, \tag{22}$$

$B_{ij} = (-1)^{i+j}$ — minor of the tensor $\hat{b}$ corresponding to the element b_{ij}, e.g.,

$$\begin{aligned} B_{11} &= b_{22}b_{33} - b_{23}b_{32} \\ &= \varepsilon_v \left[n_x^2 n_z^2 \left(1 - \overline{\mu}^{-1}\right) + \left(n_x^2 - \overline{\varepsilon}\,\overline{\mu}\right)\left(\beta + n_x^2 - \chi_1^2\right)\right], \end{aligned} \tag{23a}$$

$$\begin{aligned} B_{12} &= b_{23}b_{31} - b_{21}b_{33} \\ &= -i\varepsilon_v \left[\left(\overline{\varepsilon}\,\overline{\mu}\zeta + \chi_1 n_x\right)\left(\beta + n_x^2 - \chi_1^2\right) \right. \\ &\quad \left. - \overline{\varepsilon}\delta n_z^2 + \chi_1 n_x n_z^2 \left(1 - \overline{\mu}^{-1}\right)\right], \end{aligned} \tag{23b}$$

$$\begin{aligned} B_{13} &= b_{21}b_{32} - b_{22}b_{31} \\ &= \varepsilon_v n_z \left[n_x \left(\alpha + n_x^2 - \chi_1^2\right) - \delta\overline{\varepsilon}\,\overline{\mu}(\chi_1 + \zeta n_x)\right]. \end{aligned} \tag{23c}$$

Analogically, using (11), the field structure of the partial wave for which $\chi = \chi_2$ can be described as

$$\mathbf{E}_2 = (e_{2x}, ie_{2y}, 1)E_{2z}, \qquad \mathbf{H}_2 = (-ih_{2x}, h_{2y}, ih_{2z})E_{2z}/\mu_v, \tag{24}$$

where

$$\begin{pmatrix} h_{2x} \\ h_{2y} \\ h_{2z} \end{pmatrix} = \begin{pmatrix} \zeta n_z & n_z & \chi_2 - \zeta n_x \\ n_z & \zeta n_z & \chi_2\zeta - n_x \\ \mu_v\chi_2 & \mu_v n_x & 0 \end{pmatrix} \begin{pmatrix} e_{2x} \\ e_{2y} \\ 1 \end{pmatrix}, \tag{25}$$

$$e_{2x} = A_{31}/A_{33} = A_{11}/A_{13}, \quad e_{2y} = -iA_{32}/A_{33} = -iA_{12}/A_{13} \tag{26}$$

and $A_{ij} = (-1)^{i+j}$ — minor of the tensor $\hat{a}$ for the element a_{ij}. Note that the polarization parameters e_{1i}, h_{2i}, $i = x, y, z$ and h_{1j}, e_{2j}, $j = x, y$ in Eqs. (20)–(26) are real quantities.

To simplify all the above expressions let us consider two special cases for which the wave given by (16) splits in two.

a) $k_z = 0$ (Voigt configuration). In this case there are two independent waves propagating in the direction perpendicular to the static magnetic field: TH-polarized wave describing by the relations

$$\chi_1^2 = n_x^2 - \varepsilon_v, \tag{27}$$

$$\mathbf{H}_1 = (0, 0, 1)H_{1z}, \quad \mathbf{E}_1 = \{i(\chi_1 + \nu n_x)/\varepsilon_v, (n_x + \chi_1\nu)/\varepsilon_v, 0\}H_{1z} \tag{28}$$

and TE-polarized wave for which

$$\chi_2^2 = n_x^2 - \overline{\varepsilon}\mu_v, \tag{29}$$

$$\mathbf{E}_2 = (0, 0, 1)E_{2z}, \quad \mathbf{H}_2 = \{i(\zeta n_x - \chi_2)/\mu_v, (\chi_2\zeta - n_x)/\mu_v, 0\}E_{2z}. \tag{30}$$

b) $\delta = 0$. In this special case one has $\alpha = \beta\overline{\mu}$, $\chi_2^2 - \chi_1^2 = \alpha - \beta$ and

$$\omega^2 = s\nu_0\omega_H(\omega_H + \omega_1)/(s\nu_0 - \omega_1). \tag{31}$$

There are two independent waves with discrete frequencies (31): an extraordinary wave for which

$$\chi_1^2 = n_x^2 - \varepsilon_v + n_z^2\overline{\mu}^{-1}, \tag{32}$$

$$\begin{aligned} \mathbf{H}_1 &= (n_x n_z/\alpha, -i\chi_1 n_z/\alpha, 1)H_{1z}, \\ \mathbf{E}_1 &= -\{i(\chi_1 + \nu n_x)/\beta, (n_x + \chi_1\nu)/\beta, 0)\}H_{1z} \end{aligned} \tag{33}$$

and an ordinary wave describing by the relations

$$\chi_2^2 = n_x^2 + n_z^2 - \overline{\varepsilon}\mu_v, \tag{34}$$

$$\begin{aligned} \mathbf{E}_2 &= (n_x n_z/\alpha, -i\chi_2 n_z/\alpha, 1)E_{2z}, \\ \mathbf{H}_2 &= \{i(\chi_2 - \nu n_x)\overline{\varepsilon}/\alpha, (n_x - \chi_2\nu)\overline{\varepsilon}/\alpha, 0\}E_{2z}. \end{aligned} \tag{35}$$

Note that the parameters $\overline{\mu}$, μ_v, and ε_v in Eqs. (32)–(35) have discrete values:

$$\overline{\mu} = (\omega_H+\omega_1)/(\omega_H+s\nu_0),\ \mu_v = (\omega_H+\omega_1-s\nu_0)/\omega_H,\ \varepsilon_v = \overline{\varepsilon}\mu_v/\overline{\mu}. \tag{36}$$

Both ordinary and extraordinary waves are elliptically polarized. The waves can propagate independently from each other in the volume of the SL with refraction indices

$$n_0^2 = \overline{\varepsilon}\mu_v \tag{37a}$$

and

$$n_e^2 = \overline{\varepsilon}\mu_v / \left[1 + (\overline{\mu} - 1)\sin^2\vartheta\right], \tag{37b}$$

where ϑ is the angle between the wave vector $\mathbf{k}$ and the axis of the SL. The existence of the volume waves with discrete frequencies and discrete refraction indices first has been mentioned in [8]. The waves exist only if the condition

$$\omega_1 \prec s\nu_0 \prec \omega_1 + \omega_H \tag{38}$$

is fulfilled. For the existence of the extraordinary wave the additional condition

$$\sin^2\theta \prec (s\nu_0 + \omega_H)/(s\nu_0 - \omega_1) \tag{39}$$

must be fulfilled too.

2.3 Dispersion Relations and Localization Conditions for the Surface Waves

In the previous section the solutions of Eq. (10) have been considered in the space region $y < 0$ filled by the SL. In the region $y > 0$ filled by an insulator with dielectric constant ε_d the plane electromagnetic wave can be written as

$$\mathbf{E}_0,\ \mathbf{H}_0 \sim \exp[i(k_x x + k_z z) - k_0\chi_0 y], \tag{40}$$

where

$$\chi_0^2 = n_x^2 + n_z^2 - \varepsilon_d. \tag{41}$$

Excluding normal components of the field vectors E_{0y} and H_{0y} from Maxwell equations, for the tangential components of the vector $\mathbf{E}_0$ one can obtain

$$\begin{pmatrix} E_{0x} \\ E_{0z} \end{pmatrix} = \frac{i}{\chi_0 \varepsilon_d} \begin{pmatrix} n_x n_z & \varepsilon_d - n_x^2 \\ n_z^2 - \varepsilon_d & -n_x n_z \end{pmatrix} \begin{pmatrix} H_{0x} \\ H_{0z} \end{pmatrix}. \tag{42}$$

Using the standard continuity conditions for the tangential components at the plane $y = 0$ as well as Eqs. (16), (20), (24), (40), and (42), one can obtain a set of four linear homogeneous algebraical equations for the four unknowns H_{1z}, E_{2z}, H_{0x}, and H_{0z}. Nontrivial solutions of the set exist only if the determinant of coefficients at the unknowns is equal to zero. This condition gives the dispersion relation for the surface waves in the form

$$F_1 G_2 - n_z^2 F_2 G_1 = 0, \tag{43}$$

where

$$F_1 = (\alpha + \gamma_1) f_1 + \delta n_x n_z^2 g_1, \quad F_2 = \delta f_2 + n_x g_2 (\beta + \gamma_2), \tag{43a}$$

$$G_1 = \delta \overline{\varepsilon} p_1 - n_x (\alpha + \gamma_1) q_1, \quad G_2 = (\beta + \gamma_2) p_2 - \delta n_x n_z^2 q_2, \tag{43b}$$

$$\begin{aligned} f_j &= n_x^2 n_z^2 \overline{\mu}^{-1} + (n_x^2 - \varepsilon_d)\gamma_j + \chi_0 \varepsilon_d (\nu n_x + \chi_j), \\ g_j &= \chi_j - \zeta n_x + \rho \chi_0. \end{aligned} \tag{43c}$$

$$p_j = \gamma_0 (\chi_j - \zeta n_x) + \rho \chi_0 \gamma_j, \quad q_j = \gamma_j - \gamma_0 \overline{\mu}^{-1}. \tag{43d}$$

$$\gamma_j = n_x^2 - \chi_j^2, \qquad j = 0, 1, 2, \quad \rho = \varepsilon_d / \overline{\varepsilon} \prec 1. \tag{43e}$$

Using (14), after some algebra Eq. (43) can be written as

$$\begin{aligned} &\chi_0 \left[(1 + \rho) u_0 + (\beta + \alpha\rho) \left(n_x^2 + \chi_1 \chi_2 \right) \right] \\ &+ (\chi_1 + \chi_2) \left[\rho u_0 + n_x^2 \left(\zeta \nu n_x^2 - \chi_1 \chi_2 \right) u_1 \right] \\ &+ n_x \left[-\gamma_0 u_2 + \left(n_x^2 + \chi_1 \chi_2 \right) u_3 + \chi_0 (\chi_1 + \chi_2) u_4 \right] = 0, \end{aligned} \tag{44}$$

where

$$u_0 = \alpha\beta - \overline{\varepsilon}\delta^2 n_z^2, \quad u_1 = \rho n_z^2 \overline{\mu}^{-1} - \varepsilon_v \mu_v, \quad u_3 = \delta \left(n_z^2 + \varepsilon_d \right), \tag{44a}$$

$$u_2 = \alpha\zeta - \beta\nu + \delta(\varepsilon_v + \overline{\varepsilon}\mu_v), \quad u_4 = \alpha\nu\rho - \beta\zeta + \delta n_z^2 \left(\rho + \overline{\mu}^{-1} \right). \tag{44b}$$

Nonradiative surface waves exist only if all the χ_j, $j = 0, 1, 2$ are real and positive quantities. Using (14) and (41) one can obtain the conditions of the existence in the form

$$n_x^2 + n_z^2 \succ \varepsilon_d,\ n_x^2 + \alpha \succ 0,\ n_x^2 + \beta \succ 0,\ \left(n_x^2 + \alpha\right)\left(n_x^2 + \beta\right) - \overline{\varepsilon}\delta^2 n_z^2 \succ 0. \tag{45}$$

3. RESULTS AND DISCUSSSION

The solutions of Eq. (43) or (44) give the frequency of the surface wave as nonexplicit function of the tangential wave numbers k_x and k_z at the given value of the static quantizing magnetic field. However it is impossible to find an analytical expression for $\omega(k_x, k_z)$ and one must proceed by an approximation. But above all let us try to make some physical conclusions on the base of general dispersion relation (44) and localization conditions (45). At first it is obviously that the solutions of Eq. (44) correspond to the mixed spin-electromagnetic surface waves with quantizing frequencies due to the terms containing $\nu \sim s$. Besides, we conclude that there is nonreciprocity between the left $(k_x < 0)$ and right $(k_x > 0)$ directions of propagating due to the terms $\sim n_x$. Reciprocity would be realized only if $k_x = 0$, i.e., in the case of Faraday configuration. Then Eqs. (45), (14), and (44) give

$$\alpha \succ 0, \quad \beta \succ 0, \quad u_0 = \chi_1^2 \chi_2^2 \tag{46}$$

and

$$(\chi_1 + \chi_2)(\chi_0^2 + \rho\chi_1\chi_2) + \chi_0[\beta + \alpha\rho + (1 + \rho)\chi_1\chi_2] = 0. \tag{47}$$

One may easy see that all the terms in the left-hand side of Eq. (47) are real and positive quantities. That means Eq. (47) has no solutions, i.e., there is no nonradiative surface wave in the case of Faraday configuration. In other words, the surface waves exist only when $n_x \neq 0$. Thus, we conclude that the nonreciprocity is one of the most characteristical properties of the surface electromagnetic waves in the presence of a static magnetic field.

As it has been mentioned above both Eqs. (43) and (44) are quite complicated and to analyze the behavior of the dispersion curves at $n_x \neq 0$ some simplifying approximations must be used.

a) $k_z = 0$ (Voigt configuration). In this case Eq. (44) splits in two:

$$\rho(\chi_1 + \nu n_x) + \chi_0\left(1 - \nu^2\right) = 0, \tag{48}$$

$$\chi_2 - \zeta n_x + \chi_0 \mu_v = 0, \tag{49}$$

where χ_0, χ_1, and χ_2 are given by (41), (27), and (29). Eqs. (48) and (49) correspond to the TM- and TE-polarized surface waves, respectively. It is obviously that TM-waves are only sensitive to the parameters of the semiconductor layers and correspond to the surface plasmon-polaritons with discrete spectrum, while TE-waves are only sensitive to the parameters of the ferrite layers and correspond to the surface magnon-polaritons with classical spectrum. We conclude that the mixed surface waves arising from the coupling between magnetoplasmons, magnons and pure electromagnetic waves can exist only for the directions of propagation for which both k_x and k_z are not equal to zero. As to the TM-waves in the case $k_z = 0$, which are sensitive to the quantum Hall conditions, one can describe them by the quantized refractive indices $(n = ck_x/\omega)$

$$n_{\pm}^2 = \frac{\varepsilon_d}{\omega^2 - \Omega_+^2}\left[\frac{\omega^2}{1+\rho} - \Omega_+^2 \pm \frac{2\omega^2\Omega_-\rho^{3/2}}{(1+\rho)^2(\Omega_-^2 - \omega^2)^{1/2}}\right], \tag{50}$$

where

$$\Omega_{\pm} = s\nu_0/(1 \pm \rho). \tag{50a}$$

Thus, there are two TM-waves, and one of them has a resonance $(n_- \to \infty)$ at $\omega = \Omega_+$ while the second one $n_+ \to \infty$ at $\omega = \Omega_-$. In Fig. 2 the region of existence is shown schematically for the nondamping surface TM-waves in the plane $(\omega^2, c^2k_x^2/\varepsilon_d)$ for the given value of the integer s. On the left-hand side the region of existence is bounded by the light-line $\omega^2 = c^2k_x^2/\varepsilon_d$ (line 1) and straight line (2) described by the equation

$$\omega^2 = s^2\nu_0^2 + \rho c^2k_x^2/\varepsilon_d. \tag{51}$$

The value of the frequency

$$\omega = s\nu_0(1-\rho)^{-1/2}, \tag{52}$$

at which the lines are intersected correspond to the solution of Eq. $\varepsilon_v = \varepsilon_d$. This value of ω as well as the upper boundary of the transparency region $\omega = \Omega_-$ jump up with increasing of the integer s. The curve 3 in Fig. 2 for which $\chi_1 = \chi_{01}\varepsilon_{v1}/\varepsilon_d$ can be described by the equation

$$c^2k_x^2/\varepsilon_d = \omega^2\left(s^2\nu_0^2 - \omega^2\right)/\left[s^2\nu_0^2 - (1+\rho)\omega^2\right]. \tag{53}$$

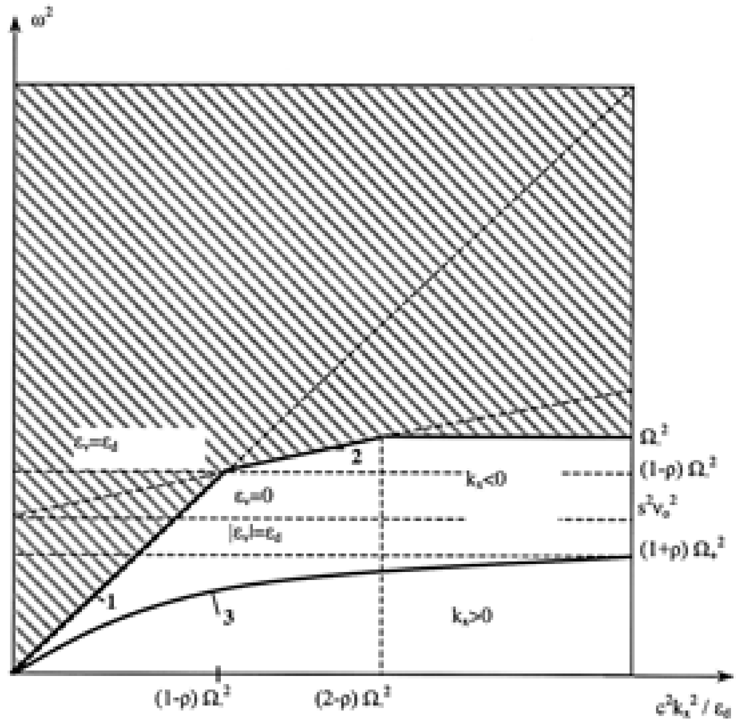

Figure 2. Region of existence of the surface TM-waves. The shaded region is forbidden for the propagation.

This curve separates the regions of propagation in the opposite directions perpendicular to the external magnetic field $\mathbf{H}_0$: $k_x < 0$ above and $k_x > 0$ under the curve 3.

The spectrum of the TM-modes is given schematically in Fig. 3 for the first three values of the integer s. The low-frequency modes describing by the refractive index n_- can propagate only to the right $(k_x > 0)$ while the high-frequency modes with $n = n_+$ propagate only to the left. The frequency of the both TM-modes increases with increasing $|k_x|$. At the given value of the integer s the frequency of the low mode rises from $\omega = 0$ at $k_x = 0$ to the asymptotic value $\omega = \Omega_+(s)$ in the non-retardation limit $ck_x \to \infty$ Each of the high-frequency branches exists in the frequency region

$$\Omega_+(s) \prec \omega \prec \Omega_-(s) \tag{54}$$

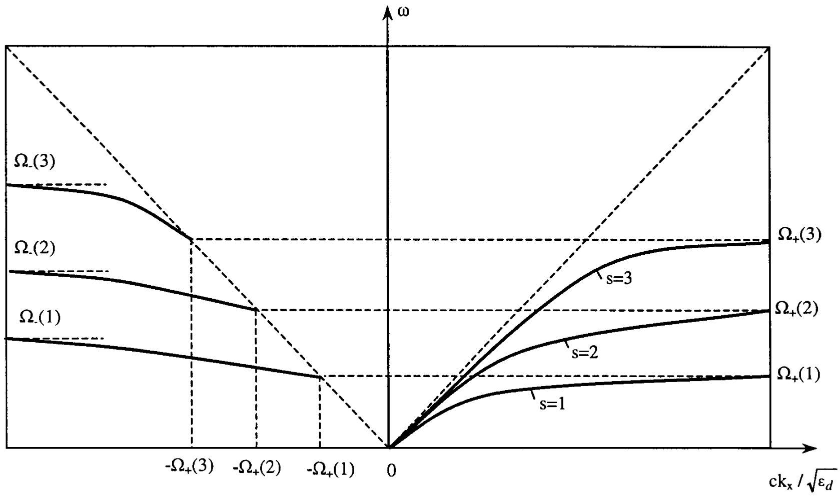

Figure 3. Spectrum of the surface TM-modes (dispersion curves) for s=1,2,3 at $\rho \prec 0.2$.

and in the wave number region where

$$c|k_x|\varepsilon_d^{-1/2} \succ \Omega_+(s). \tag{55}$$

b) At the simplifying condition $\delta = 0$ Eq. (43) gives

$$f_1 p_2 + n_x^2 n_z^2 q_1 g_2 = 0. \tag{56}$$

This dispersion relation corresponds to the mixed spin-electromagnetic surface waves with discrete frequencies given by (31). Unlike the independent partial waves described by Eqs. (31)–(38) the surface wave (56) really is a superposition of the ordinary and extraordinary waves in the region $y < 0$. It is interesting to mention that there is some restriction on the value of the angle θ between $\mathbf{H}_0$ and the tangential wave vector $\mathbf{k}_{\|}$. To see that, let us consider the spatial case when the additional condition

$$q_1 = \overline{\varepsilon}(\mu_v - \rho)/\overline{\mu} = 0 \tag{57}$$

is fulfilled. This is possible only if

$$s\nu_0 = \omega_1 + (1 - \rho)\omega_H \tag{57a}$$

or

$$\omega^2 = (\omega_H + \omega_1)\left[\omega_H + \omega_1(1-\rho)^{-1}\right]. \tag{57b}$$

Then $\chi_2 = \chi_0$ and Eq. (56) splits in two:

$$\chi_0(1+\rho) = \nu n_x \tag{58}$$

and

$$\chi_1 + \chi_0\overline{\mu}^{-1} = -\nu n_x. \tag{59}$$

That means there are two independent mixed surface waves with the same frequency (57b) but with different wave numbers. One of them described by (58) exists only if $k_x > 0$ while the second one can propagate only to the left $(k_x < 0)$. Both (58) and (59) can be written as

$$n_{\|}^2 = \varepsilon_d/(1 - C_i \sin^2\vartheta), \tag{60}$$

where

$$C_1 = s\nu_0(1-\rho)/(\omega_H + \omega_1)(1+\rho)^2 \tag{60a}$$

for the wave (58) and

$$C_2 = (\omega_H + \omega_1)(s\nu_0 + \omega_1 + 2\sqrt{s\nu_0\omega_1})/(1-\rho)\omega_H^2 \tag{60b}$$

for the wave (59). Using (57a) one can see that $C_1 < 1$ and there is no restriction for the angle θ while $C_2 > 1$ and the wave described by (59) propagates $(n_{\|}^2 > 0)$ only in the directions for which $\sin\theta < C_2^{-1/2}$.

4. SUMMARY

In conclusion we have derived effective dielectric and magnetic permeability tensors for the superlattice in quantum Hall-effect conditions. Using these tensors we have obtained the dispersion relation for the surface spin-electromagnetic waves propagating along the lateral surface of SL. The existence of the nondamping mixed surface waves with quantizing frequencies is shown. The field structure in the waves is examined. The nonreciprocity between the opposite propagation directions perpendicular to the external quantizing magnetic field is found. The region of existence as well as the spectrum of the TM-polarized surface waves in the case of Voigt configuration are considered. It is shown that there is no nonradiative surface wave in the case of Faraday configuration. Also some new special waves with quantizing frequencies are found in the case of arbitrary direction of propagation different from the mentioned above configurations.

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