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ANALYSIS OF SCATTERING AND COUPLING PROBLEM OF DIRECTIONAL COUPLER FOR RECTANGULAR DIELECTRIC WAVEGUIDES

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Abstract–In this paper, scattering and coupling problems of the directional coupler for the dielectric rectangular waveguides are analyzed by the mode-matching method in the sense of least squares for the fundamental mode incidence. This directional coupler is composed of three parallel dielectric rectangular waveguides cores which are placed at equal space in the dielectric medium. Namely, respective cores are core regions of three respective rectangular waveguides. The central rectangular core among them has periodic groove structures of finite extent on its two surfaces which face each other and other two waveguide cores are perfect. In the central waveguide, the fundamental mode is incident from perfect part toward the periodic structure of this waveguide. The power of the incident mode to the central waveguide is coupled to other two waveguides through periodic groove structure. The coupled mode propagates in the other waveguides to the same or opposite direction for the direction of the incident mode when the Bragg condition is selected appropriately. The method of this paper results in the integral equations of Fredholm type of the second kind for the unknown spectra of scattered fields. The results of the first order approximate solutions of the integral equations are presented in this paper.

1. INTRODUCTION

In optical waveguide used in integrated optics and light transmission circuit and so on, it is often desired to transfer optical power from one to another waveguide. Therefore dielectric rectangular waveguides [1–7] and directional couplers [3, 5] have been presented and studied enthusiastically. In this paper, scattering and coupling problems of directional couplers for dielectric rectangular waveguides are analyzed by the mode-matching method in the sense of least squares for the fundamental mode incidence. This directional coupler is composed of three parallel waveguide cores with rectangular cross section which are placed at equal space in the dielectric medium. Respective cores form core regions of three respective waveguides. The central rectangular core among them has periodic groove structures on two surfaces which face each other and other two waveguide cores are perfect. In the central waveguide, the fundamental mode is incident from the perfect part toward the periodic structure of this waveguide. The power of the incident fundamental mode to the central waveguide is coupled to other two waveguides through periodic groove structure. When the Bragg condition is selected appropriately, the mode which is coupled to other two waveguides propagates to the same or opposite direction for the direction of the incident mode of the central waveguide. In this analysis, we shall apply a mode-matching method in the sense of least squares [8–11] for analyzing the electromagnetic fields of above-mentioned directional coupler when the fundamental mode is incident to the perfect part of the waveguide having a periodic structure of finite extent. Approximate wave functions of scattered fields in each region of the coupler are described by the Fourier transform with band-limited spectra. These integral transforms can be regarded as modal expansions and the expansion theorem of mode-matching method in the sense of least squares can be applied [12]. These approximate wave functions are determined in such way that the mean-square boundary residual is minimized. This method results in simultaneous integral equations of Fredholm type of the second kind for the unknown spectra [8–11]. The results of analyses for coupling efficiency and scattered fields are presented on the basis of the first order approximate solutions of the integral equations in this paper.

2. FORMULATION OF THE PROBLEM

The discussion is developed about the directional couplers for dielectric rectangular waveguides. This coupler is composed of three parallel waveguide cores which are placed at equal space in the dielectric medium. The central waveguide core among them has sinusoidal groove structures of finite extent on the both surfaces of y direction as shown by Fig. 1. In this discussion, the fundamental mode is incident from the perfect part toward the periodic structure of the central waveguide. Fig. 1(a) shows the overhead view of the coupler, Fig. 1(b) shows the plane figure of it and Fig. 1(c) shows the cross figure. Three parallel waveguide cores with rectangular cross section are placed at equal space H in the dielectric medium. Each core region is denoted as the waveguide hereinafter. This coupler has a symmetrical structure about x-z and y-z plane which contain origin o and incident wave is assumed to be the fundamental symmetric mode with regard to x and y direction in the square cross section of the perfect part the central waveguide. Therefore scattered fields are considered that they are symmetrical about x-z and y-z plane. Then the scattering and coupling problems of this coupler can be analyzed with regard to the region of $x \geq 0$ and $y \geq 0$ and formulation of only that region is given in this paper. Fig. 2 shows cross sections of this coupler at $z = 0$. In this figure, waveguide, regions and surfaces of this coupler are denoted by two superscripts. The first superscript denotes those of $x \geq 0$ and $x < 0$ by u and l, respectively. The second superscript denotes those of $y \geq 0$ and $y < 0$ by r and l, respectively. The cross section of the perfect part of waveguide-1 is square with width $2a$ and that of waveguide-2^r is rectangle and its width is $2a$, $2b$ in x, y direction, respectively. The waveguide-1 has a periodic groove structure in the finite region $(|x| \leq a,\ |z| \leq t)$ on surface of y direction of the core. The boundary surface $S_{1\xi}^{u,r}$ with sinusoidal grooves is given as follows:

$$y = \xi(x, z) = \begin{cases} a + \delta a \eta(z), & 0 \leq x \leq a, \quad |z| \leq t \\ a, & 0 \leq x \leq a, \quad |z| > t \end{cases}, \tag{1}$$

where

$$\eta(z) = \cos(Kz), \tag{2}$$

$$t = \left(N + \frac{1}{4}\right) D, \tag{3}$$

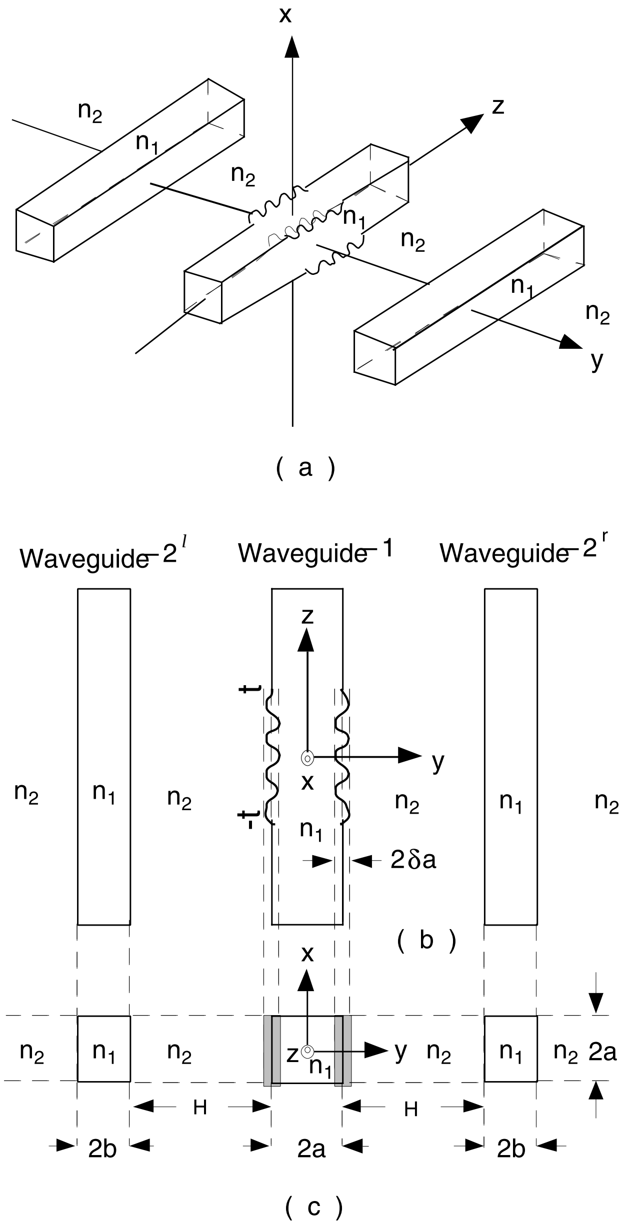

Figure 1. Geometry of directional coupler for rectangular dielectric waveguides. (a) Overhead view of the coupler. (b) Plane figure. (c) Cross figure.

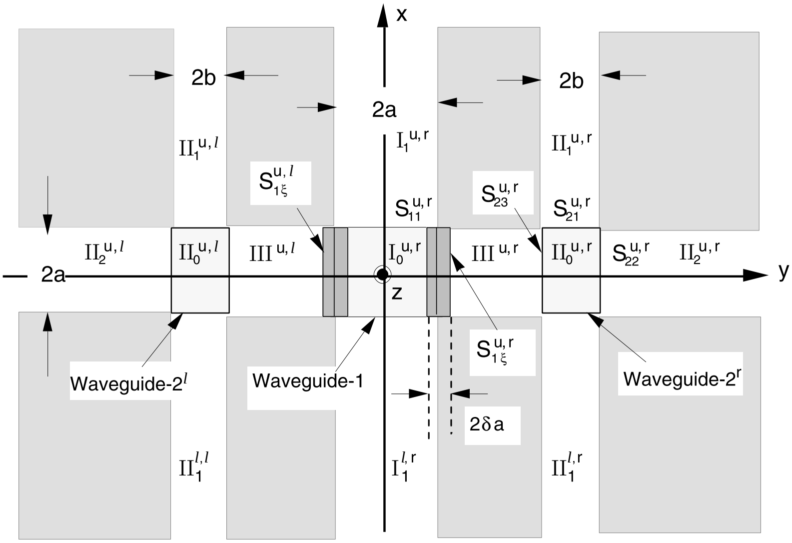

Figure 2. Cross section of directional coupler composed of three rectangular waveguides cores and its field distribution.

$$K = \frac{2\pi}{D}, \tag{4}$$

and D is the spatial period in the z direction. N is an integer and it denotes the groove number. $2\delta a$ is the depth of the groove and δ is a perturbation parameter. $2t$ is the length of the periodic groove structure in the z direction. The boundary surface $S_{1,\xi}^{u,l}$ is given by

$$y = -\xi(x, z). \tag{5}$$

The regions occupied by respective media $(x \geq 0,\ y \geq 0,\ -\infty \leq z \leq \infty)$ are denoted as follows:

$$\begin{cases} \mathrm{I}_0^{u,r}; & (0 \leq x \leq a,\ 0 \leq y \leq \xi(x,z),\ |z| \leq \infty) \\ \mathrm{I}_1^{u,r}; & (x > a,\ 0 \leq y \leq \xi(x,z),\ |z| \leq \infty) \\ \mathrm{III}^{u,r}; & (0 \leq x \leq a,\ \xi(x,z) < y < a + H,\ |z| \leq \infty) \\ \mathrm{II}_0^{u,r}; & (0 \leq x \leq a,\ a + H \leq y \leq a + H + 2b,\ |z| \leq \infty) \\ \mathrm{II}_1^{u,r}; & (x > a,\ a + H \leq y \leq a + H + 2b,\ |z| \leq \infty) \\ \mathrm{II}_2^{u,r}; & (0 \leq x \leq a,\ y > a + H + 2b,\ |z| \leq \infty) \end{cases}. \tag{6}$$

Also boundary surfaces of core of waveguide-2^r; $S_{23}^{u,r}$, $S_{22}^{u,r}$ are described as follows:

$$y = H + a, \qquad 0 \leq x \leq a, \qquad |z| \leq \infty \tag{7}$$

$$y = H + a + 2b, \qquad 0 \leq x \leq a, \qquad |z| \leq \infty \tag{8}$$

Each region has refractive index, n_1, n_2, n_2, n_1, n_2 and n_2, respectively. Refractive indices are such that

$$n_1 > n_2, \tag{9}$$

and

$$\frac{n_1 - n_2}{n_1} \ll 1. \tag{10}$$

The wavenumber in free space is k_0 and wave length is λ_0. The implied time factor is $\exp(-j\omega t)$.

Superscripts u, r of regions and surface are neglected such as I_0, I_1, II_0, II_1, II_2, III and $S_{1,\xi}$ hereinafter because of symmetrical structure.

In this paper, Marcatili's field expression and approach is applied [3–5]. The incident wave to the perfect part of the waveguide-1 is assumed to be the fundamental mode. It is polarized in x direction and has only one maximum in x and y direction in the rectangular cross section with regard to the electric and magnetic field distribution. In Marcatili's approach, this fundamental mode has mainly E_x, E_z, H_y, and H_z components and H_x, E_y components are neglected. Marcatili's field expression will be valid only if the effect of the shaded areas of Fig. 2 may be ignored. Moreover it is assumed that E_x component of this mode and its derivatives in x and y direction are continuous over the entire cross Section [5] in this paper. When these conditions are satisfied, E_x, E_z, and H_z components satisfy the fundamental boundary condition on each surface of waveguide-1. Then H_y component satisfies the fundamental boundary condition approximately under the weakly guiding approximation given by Eq. (10). It is defined that this mode is E_{11}^x mode and above-mentioned continuos conditions with regard to E_x component are boundary conditions in this paper. The x component of the electric field of this E_{11}^x mode which is incident to the perfect part of waveguide-1 is denoted as $E_x^{(1)}$ hereinafter and it is given by

$$E_x^{(1)}(x, y, z) = \widetilde{E}_x^{(1)}(x, y) e^{j\beta_1 z}, \tag{11}$$

$$\widetilde{E}_x^{(1)}(x, y) = X^{(1)}(x) \cdot Y^{(1)}(y), \tag{12}$$

where

$$X^{(1)}(x) = \begin{cases} \cos(\kappa_{10}x); & |x| \le a \\ \cos(\kappa_{10}a)\exp\{j\kappa_{11}(|x|-a)\}; & |x| > a \end{cases}, \tag{13}$$

$$Y^{(1)}(y) = \begin{cases} \cos(\zeta_{10}y); & |y| \le a \\ \cos(\zeta_{10}a)\exp\{j\zeta_{13}(|y|-a)\}; & |y| > a \end{cases}. \tag{14}$$

In Eq. (11), β_1 is a propagation constant in the z direction of waveguide-1 and κ_{10}, κ_{11}, ζ_{10} and ζ_{13} are wavenumbers of this waveguide in x and y directions, respectively. Wavenumbers, κ_{10} and ζ_{10} are positive real numbers and κ_{11}, ζ_{13} are pure positive imaginary numbers. Those wavenumbers satisfy following relations from Hermholtz equation in respective regions:

$$(\kappa_{10})^2 + (\zeta_{10})^2 + (\beta_1)^2 = {n_1}^2{k_0}^2, \tag{15}$$
$$(\kappa_{11})^2 + (\zeta_{10})^2 + (\beta_1)^2 = {n_2}^2{k_0}^2, \tag{16}$$
$$(\kappa_{10})^2 + (\zeta_{13})^2 + (\beta_1)^2 = {n_2}^2{k_0}^2, \tag{17}$$

where k_0 is the wavenumber in vacuum.

Eigenvalue equations for wavenumbers of x and y directions are obtained from the boundary conditions in which $E_x^{(1)}$ and its derivatives in x and y directions are continuous over the entire cross section. From boundary condition at $x = \pm a$ and $y = \pm a$, eigenvalue equations of x and y directions are obtained, respectively as follows [13]:

$$I_{ex}^{(1)}(\kappa_{10}, \kappa_{11}) = j\kappa_{11}\cos(\kappa_{10}a) + \kappa_{10}\sin(\kappa_{10}a) = 0, \tag{18}$$
$$I_{ey}^{(1)}(\zeta_{10}, \zeta_{13}) = j\zeta_{13}\cos(\zeta_{10}a) + \zeta_{10}\sin(\zeta_{10}a) = 0. \tag{19}$$

When the waveguide-2 is isolated and x, y coordinates of the center of its cross section is $(0, H+a+b)$, the x component of the electric field of E_{11}^x mode of this waveguide is given similarly to one of waveguide-1 and denoted as $E_x^{(2)}$. Then it is given as follows:

$$E_x^{(2)}(x,y,z) = \widetilde{E}_x^{(2)}(x,y)e^{j\beta_2 z}, \tag{20}$$
$$\widetilde{E}_x^{(2)}(x,y) = X^{(2)}(x) \cdot Y^{(2)}(y) \tag{21}$$

where

$$X^{(2)}(x) = \begin{cases} \cos(\kappa_{20}x); & |x| \le a \\ \cos(\kappa_{20}a)\exp\{j\kappa_{21}(|x|-a)\}; & |x| > a \end{cases}, \tag{22}$$

and

$$Y^{(2)}(y) = \begin{cases} \cos\{\zeta_{20}(y-H-a-b)\}\,; & H+a \le y \le H+a+2b \\ \cos(\zeta_{20}b)\exp\{-j\zeta_{23}(y-H-a)\}\,; & y < H+a \\ \cos(\zeta_{20}b)\exp\{j\zeta_{23}(y-H-a-2b)\}\,; & y > H+a+2b \end{cases}. \tag{23}$$

In Eq. (20), β_2 is a propagation constant of the lowest order even mode of waveguide-2 when it is isolated. Wavenumbers κ_{20}, ζ_{20} are positive real numbers and κ_{21}, ζ_{23} are positive pure imaginary numbers. The propagation mode of this waveguide has same wavenumbers of the y direction in the region III and II_2. Therefore they are denoted as ζ_{23}. Wavenumbers of above equations satisfy following equations;

$$(\kappa_{20})^2 + (\zeta_{20})^2 + (\beta_2)^2 = {n_1}^2{k_0}^2, \tag{24}$$
$$(\kappa_{21})^2 + (\zeta_{20})^2 + (\beta_2)^2 = {n_2}^2{k_0}^2, \tag{25}$$
$$(\kappa_{20})^2 + (\zeta_{23})^2 + (\beta_2)^2 = {n_2}^2{k_0}^2. \tag{26}$$

From boundary conditions on S_{21} and S_{23} and S_{22}, eigenvalue equation of x and y directions of waveguide-2 are given as follows:

$$I_{ex}^{(2)}(\kappa_{20},\kappa_{21}) = j\kappa_{21}\cos(\kappa_{20}a) + \kappa_{20}\sin(\kappa_{20}a) = 0, \tag{27}$$
$$I_{ey}^{(2)}(\zeta_{20},\zeta_{23}) = j\zeta_{23}\cos(\zeta_{20}b) + \zeta_{20}\sin(\zeta_{20}b) = 0. \tag{28}$$

Eigenvalue equations Eqs. (18) and (27) are identical and wavenumbers of x direction of two waveguides satisfy following equations:

$$(\kappa_{10})^2 - (\kappa_{11})^2 = ({n_1}^2 - {n_2}^2){k_0}^2, \tag{29}$$
$$(\kappa_{20})^2 - (\kappa_{21})^2 = ({n_1}^2 - {n_2}^2){k_0}^2. \tag{30}$$

Consequently, from Eqs. (18), (27), (29), and (30), the relations are given as follows:

$$\begin{cases} \kappa_{10} = \kappa_{20} \\ \kappa_{11} = \kappa_{21} \end{cases}. \tag{31}$$

In the case that three parallel rectangular waveguides cores are perfect, namely, the central waveguide has not periodic structure and the central waveguide core and other two cores are different and the space between waveguides H is sufficiently large, the power of incident mode to the central waveguide is coupled to other two waveguides scarcely

[14, 15]. In this paper, it is postulated that the distance between waveguides H is large and the central waveguide core is different from other two ones with regard to width in the y direction. Therefore the incident mode to the central waveguide is considered that this mode is coupled scarcely to other two waveguides when these cores of waveguides are perfect.

When E_{11}^{x} mode is incident to the central waveguide having periodic groove structure, total fields of E_x component in each region $(x \geq 0,\ y \geq 0)$ of this coupler are given as follows:

$$E^{(t)}_{\mathrm{I}_m,x}(x,y,z) = E^{(i)}_{\mathrm{I}_m,x}(x,y,z) + \Psi_{\mathrm{I}_m}(x,y,z), \quad (m=0,1), \tag{32}$$

$$E^{(t)}_{\mathrm{II}_m,x}(x,y,z) = \Psi_{\mathrm{II}_m}(x,y,z), \quad (m=0,1,2), \tag{33}$$

$$E^{(t)}_{\mathrm{III},x}(x,y,z) = E^{(i)}_{\mathrm{III},x}(x,y,z) + \Psi_{\mathrm{III}}(x,y,z). \tag{34}$$

Scattered field of Eqs.(32)–(34) are given by

$$\Psi_V(x,y,z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \psi_V(h)\phi_V(h,x,y)\exp(jhz)dh,$$
$$(V = \mathrm{I}_0, \mathrm{I}_1, \mathrm{II}_0, \mathrm{II}_1, \mathrm{II}_2, \mathrm{III};\ x,y,z \in V). \tag{35}$$

Both faces of the x direction of the central waveguide-1 and four faces of waveguide-2 are perfect. From boundary conditions on those faces, relations of spectra and $\phi_V(h,x,y)$ are derived to satisfy the boundary conditions, namely, Ψ_V, $\dfrac{\partial \Psi_V}{\partial x}$, and $\dfrac{\partial \Psi_V}{\partial y}$ are continuous through surfaces, S_{11}, S_{21}, S_{22}, and S_{23} and they are given by

$$\psi_{\mathrm{I}_0}(h) = \psi_{\mathrm{I}_1}(h), \tag{36}$$

$$\psi_{\mathrm{II}_0}(h) = \psi_{\mathrm{II}_1}(h) = \psi_{\mathrm{II}_2}(h), \tag{37}$$

$$\psi_{\mathrm{III}}(h) = \frac{\exp\left\{\left(-j\widetilde{\zeta}_{23}(h)H\right)\right\}}{j\widetilde{\zeta}_{23}(h)}\psi_{\mathrm{II}_0}(h). \tag{38}$$

and $\phi_V(h,x,y)$, $(V = \mathrm{I}_0, \mathrm{I}_1, \mathrm{II}_0, \mathrm{II}_1, \mathrm{II}_2, \mathrm{III};\ x,y \in V)$ are given as follows:

$$\phi_{\mathrm{I}_0}(h,x,y) = \cos(\kappa_{10}x)\cos\left\{\widetilde{\zeta}_{10}(h)y\right\}, \tag{39}$$

$$\phi_{\mathrm{I}_1}(h,x,y) = \cos(\kappa_{10}a)\cos\left\{\widetilde{\zeta}_{10}(h)y\right\}\exp\{j\kappa_{11}(x-a)\}, \tag{40}$$

$$\phi_{\mathrm{III}}(h,x,y) = \cos(\kappa_{10}x)\left\{\varphi^{(1)}_{\mathrm{III}}(h,y) + \varphi^{(2)}_{\mathrm{III}}(h,y)\right\}, \tag{41}$$

$$\phi_{\mathrm{II}_0}(h,x,y) = \cos(\kappa_{10}x)\left\{\varphi^{(1)}_{\mathrm{II}_0}(h,y) + \varphi^{(2)}_{\mathrm{II}_0}(h,y)\right\}, \tag{42}$$

$$\phi_{\mathrm{II}_1}(h,x,y) = \cos(\kappa_{10}a)\exp\{j\kappa_{11}(x-a)\}\left\{\varphi^{(1)}_{\mathrm{II}_0}(h,y)+\varphi^{(2)}_{\mathrm{II}_0}(h,y)\right\}, \tag{43}$$

$$\phi_{\mathrm{II}_2}(h,x,y) = \widetilde{\zeta}_{20}(h)\cos(\kappa_{10}x)\exp\left\{j\widetilde{\zeta}_{23}(h)(y-H-a-2b)\right\}, \tag{44}$$

where

$$\varphi^{(1)}_{\mathrm{III}}(h,y) = \tilde{I}^{(2)}_{ey}(h,b)\tilde{I}^{(2)}_{oy}(h,b)\exp\left\{j\widetilde{\zeta}_{13}(h)(y-a)\right\}, \tag{45a}$$

$$\begin{aligned}\varphi^{(2)}_{\mathrm{III}}(h,y) = &-\frac{1}{2}\exp\left(j\widetilde{\zeta}_{23}(h)H\right)\left(\widetilde{\zeta}^2_{20}(h)-\widetilde{\zeta}^2_{23}(h)\right)\\ &\cdot\sin\left\{2\widetilde{\zeta}_{20}(h)b\right\}\exp\left\{-j\widetilde{\zeta}_{23}(h)(y-H-a)\right\},\end{aligned} \tag{45b}$$

and

$$\varphi^{(1)}_{\mathrm{II}_0}(h,y) = \tilde{I}^{(2)}_{oy}(h,b)\cos\left[\widetilde{\zeta}_{20}(h)\{y-(H+a+b)\}\right], \tag{46a}$$

$$\varphi^{(2)}_{\mathrm{II}_0}(h,y) = \tilde{I}^{(2)}_{ey}(h,b)\sin\left[\widetilde{\zeta}_{20}(h)\{y-(H+a+b)\}\right], \tag{46b}$$

and

$$\tilde{I}^{(2)}_{ey}(h,b) = j\widetilde{\zeta}_{23}(h)\cos\left\{\widetilde{\zeta}_{20}(h)b\right\} + \widetilde{\zeta}_{20}(h)\sin\left\{\widetilde{\zeta}_{20}(h)b\right\}, \tag{47}$$

$$\tilde{I}^{(2)}_{oy}(h,b) = \widetilde{\zeta}_{20}(h)\cos\left\{\widetilde{\zeta}_{20}(h)b\right\} - j\widetilde{\zeta}_{23}(h)\sin\left\{\widetilde{\zeta}_{20}(h)b\right\}, \tag{48}$$

and

$$\widetilde{\zeta}^2_{10}(h) = {n_1}^2{k_0}^2 - h^2 - {\kappa_{10}}^2, \tag{49a}$$

$$\widetilde{\zeta}_{13}(h) = {n_2}^2{k_0}^2 - h^2 - {\kappa_{10}}^2, \tag{49b}$$

$$\widetilde{\zeta}^2_{20}(h) = {n_1}^2{k_0}^2 - h^2 - {\kappa_{20}}^2, \tag{49c}$$

$$\widetilde{\zeta}^2_{23}(h) = {n_2}^2{k_0}^2 - h^2 - {\kappa_{20}}^2. \tag{49d}$$

When the right hand sides of Eqs. (49b) and (49d) is negative, $\widetilde{\zeta}_{13}(h)$ and $\widetilde{\zeta}_{23}(h)$ are defined as positive pure imaginary numbers.

The boundary condition on the surface $S_{1\xi}$ with the periodic groove structure of finite extent is as follows:

$$\Psi_{\mathrm{I}_0}(x,y,z) - \Psi_{\mathrm{III}}(x,y,z) = \begin{cases} e^{j\beta_1 z}f(x,y), & |z|\le t\\ 0, & |z|>t\end{cases}, \tag{50}$$

$$\frac{\partial}{\partial y}\Psi_{\mathrm{I}_0}(x,y,z) - \frac{\partial}{\partial y}\Psi_{\mathrm{III}}(x,y,z) = \begin{cases} e^{j\beta_1 z}g(x,y), & |z|\le t\\ 0, & |z|>t\end{cases}. \tag{51}$$

where

$$f(x,y) = \cos(\kappa_{10}x)\cos(\zeta_{10}a)\exp\{j\zeta_{13}(y-a)\} \\ - \cos(\kappa_{10}x)\cos(\zeta_{10}y), \tag{52}$$

$$\tilde{g}(x,y) = \frac{\partial}{\partial y}\tilde{f}(x,y). \tag{53}$$

The continuos condition of $\frac{\partial}{\partial x}\Psi_{\mathrm{I}_0}$ and $\frac{\partial}{\partial x}\Psi_{\mathrm{III}}$ on the surface, $S_{1\xi}$ is satisfied naturally when Eq. (50) is satisfied. The above-mentioned conditions are equivalent to the fundamental boundary conditions on $S_{1\xi}$, namely, the continuous conditions of tangential component of the electric and magnetic field on $S_{1\xi}$. If the scattered fields of the regions I_0, I_1, II_0, II_1, II_2, and III are described by the Fourier transforms with plane waves whose integral domain are not limited, they generally diverge [16]. Therefore band-limited superpositions of plane waves are used generally as approximate wave functions for scattered fields in each region as follows:

$$\Psi_{VW}(x,y,z) = \frac{1}{2\pi}\int_{-w}^{w}\psi_{VW}(h)\phi_V(h,x,y)\exp(jhz)dh, \\ (V = \mathrm{I}_0, \mathrm{I}_1, \mathrm{II}_0, \mathrm{II}_1, \mathrm{II}_2, \mathrm{III};\ x,y,z \in V), \tag{54}$$

where $\phi_V(h,x,y)$, $(V = \mathrm{I}_0, \mathrm{I}_1, \mathrm{II}_0, \mathrm{II}_1, \mathrm{II}_2, \mathrm{III})$ is given by Eqs. (39)–(49). In Eq. (54), $\psi_{VW}(h)$, $(V = \mathrm{I}_0, \mathrm{I}_1, \mathrm{II}_0, \mathrm{II}_1, \mathrm{II}_2, \mathrm{III})$ is band-limited spectra of each region.

From Eqs. (35), (39), (41), (50)–(53), the boundary conditions on $S_{1\xi}$ are independent to the x direction and they are given by applying Eq. (54) as follows:

$$\widetilde{\Psi}_{\mathrm{I}_0 W}(y,z) - \widetilde{\Psi}_{\mathrm{III}W}(y,z) = \begin{cases} e^{j\beta_1 z}\tilde{f}(y), & |z| \le t \\ 0, & |z| > t \end{cases}, \tag{55}$$

$$\frac{\partial}{\partial y}\widetilde{\Psi}_{\mathrm{I}_0 W}(y,z) - \frac{\partial}{\partial y}\widetilde{\Psi}_{\mathrm{III}W}(y,z) = \begin{cases} e^{j\beta_1 z}\tilde{g}(y), & |z| \le t \\ 0, & |z| > t \end{cases}, \tag{56}$$

where

$$\widetilde{\Psi}_{\mathrm{I}_0 W}(y,z) = \frac{1}{2\pi}\int_{-W}^{W}\psi_{\mathrm{I}_0 W}(h)\tilde{\phi}_{\mathrm{I}_0}(h,y)\exp(jhz)dh, \tag{57}$$

$$\widetilde{\Psi}_{\mathrm{III}W}(y,z) = \frac{1}{2\pi}\int_{-W}^{W}\psi_{\mathrm{III}W}(h)\tilde{\phi}_{\mathrm{III}}(h,y)\exp(jhz)dh, \tag{58}$$

and

$$\tilde{\phi}_{\mathrm{I}_0}(h,y) = \cos\left\{\widetilde{\zeta}_{10}(h)y\right\}, \tag{59}$$

$$\tilde{\phi}_{\mathrm{III}}(h,y) = \left\{\varphi^{(1)}_{\mathrm{III}}(h,y) + \varphi^{(2)}_{\mathrm{III}}(h,y)\right\}, \tag{60}$$

and

$$\tilde{f}(y) = \cos(\zeta_{10}a)\exp\{j\zeta_{13}(y-a)\} - \cos(\zeta_{10}y), \tag{61}$$

$$\tilde{g}(y) = \frac{\partial}{\partial y}\tilde{f}(y). \tag{62}$$

3. ALGORITHM

From concept of the method of least squares, the following mean-square error is defined on $S_{1,\xi}$ to obtain the wave functions $\widetilde{\Psi}_{\mathrm{I}_0 W}(y,z)$ and $\widetilde{\Psi}_{\mathrm{III}W}(y,z)$ that satisfy the boundary conditions stated in Eqs. (55)–(62) approximately:

$$\Omega_W = \frac{\displaystyle\int_{-\infty}^{\infty}\left|\widetilde{\Psi}_W(z,y) - e^{j\beta_1 z}\tilde{f}(y)\right|^2 dz}{\displaystyle\int_{-\infty}^{\infty}\left|\tilde{f}(y)\right|^2 dz} + \frac{\displaystyle\int_{-\infty}^{\infty}\left|\partial_y\widetilde{\Psi}_W(z,y) - e^{j\beta_1 z}\tilde{g}(y)\right|^2 dz}{\displaystyle\int_{-\infty}^{\infty}\left|\tilde{g}(y)\right|^2 dz}, \tag{63}$$

where

$$\widetilde{\Psi}_W(z,y) = \widetilde{\Psi}_{\mathrm{I}_0 W}(y,z) - \widetilde{\Psi}_{\mathrm{III}W}(y,z). \tag{64}$$

in which y is function of z. Since $\widetilde{\Psi}_{VW}(y,z)$, $(V = \mathrm{I}_0, \mathrm{III})$ are given by Eqs. (57) and (58), Ω_W is functional of spectra $\psi_{VW}(h)$, $(V = \mathrm{I}_0, \mathrm{III})$. Therefore, if the first variations of Ω_W with respect to the complex conjugate spectra of $\psi_{VW}(h)$, $(V = \mathrm{I}_0, \mathrm{III})$ are set equal to zero, the simultaneous integral equations with two unknown spectra can be derived. These equations are in the form of the Fredholm integral equation of the second kind for the vector whose components are two unknown spectra. The integral equation [8, 10] is given by

$$\left(\boldsymbol{A}+\alpha^2\boldsymbol{B}\right)\cdot\boldsymbol{\psi}_{_W}(h)=\int_{-W}^{W}\left\{\boldsymbol{K}(h,h',\delta)+\alpha^2\boldsymbol{L}(h,h',\delta)\right\}\cdot\boldsymbol{\psi}_{_W}(h')dh'+\boldsymbol{F}(h,\delta)+\alpha^2\boldsymbol{G}(h,\delta) \tag{65}$$

in which "$\cdot$" denotes an inner product. In Eq. (65) $\boldsymbol{\psi}_{_W}(h)$, $\boldsymbol{F}(h,\delta)$, $\boldsymbol{G}(h,\delta)$ are vectors, and $\boldsymbol{A}$, $\boldsymbol{B}$ and the integral kernels $\boldsymbol{K}(h,h',\delta)$, $\boldsymbol{L}(h,h',\delta)$ are dyadics and they and α^2 are shown in Refs. 8 and 10. The vector $\boldsymbol{\psi}_{_W}(h)$ is given by

$$\boldsymbol{\psi}_{_W}(h)=\begin{bmatrix}\psi_{_{\mathrm{I}_0W}}(h)\\ \psi_{_{\mathrm{III}W}}(h)\end{bmatrix} \tag{66}$$

4. FIRST ORDER APPROXIMATE SOLUTIONS

In this paper, the couplers with very shallow grooves are analyzed and the parameter δ is assumed to be very small in comparison with unity. Therefore the perturbation method [17] is applied to solve the integral equation. In Eq. (65), the solution $\boldsymbol{\psi}_{_W}(h)$ is expressed by the perturbation expansion of the form

$$\boldsymbol{\psi}_{_W}(h)=\delta\boldsymbol{\psi}_{_W}^{(1)}(h)+\delta^2\boldsymbol{\psi}_{_W}^{(2)}(h)+\delta^3\boldsymbol{\psi}_{_W}^{(3)}(h)+\cdots. \tag{67}$$

Also terms such as $\boldsymbol{K}(h,h',\delta)$, $\boldsymbol{L}(h,h',\delta)$, $\boldsymbol{F}(h,\delta)$, $\boldsymbol{G}(h,\delta)$ and α^2 can be expanded into the Taylor series about $\delta=0$. When terms of equal powers of δ are equated and the condition for the existence of the solution is taken into account, equations for the first order expansion coefficients $\boldsymbol{\psi}_{_W}^{(1)}(h)$ is obtained as follows:

$$\boldsymbol{A}\cdot\boldsymbol{\psi}_{_W}^{(1)}(h)=0, \tag{68}$$

$$\boldsymbol{B}\cdot\boldsymbol{\psi}_{_W}^{(1)}(h)=\boldsymbol{G}^{(1)}(h). \tag{69}$$

where $\boldsymbol{G}^{(1)}(h)$ is the first order expansion coefficient of $\boldsymbol{G}(h,\delta)$. The first order approximate solution of Eq. (65) is defined as the first term of the perturbation expansion of Eq. (67). Eqs. (68) and (69) can be solved analytically and $\boldsymbol{\psi}_{_W}^{(1)}(h)$ is given by

$$\boldsymbol{\psi}_{_W}^{(1)}(h)=\begin{bmatrix}\psi_{_{\mathrm{I}_0W}}(h)\\ \psi_{_{\mathrm{III}W}}(h)\end{bmatrix}=-\frac{G^{(1)}(h)}{I(h,H,a,b)}\begin{bmatrix}\tilde{\phi}_{\mathrm{III}}(h,a)\\ \tilde{\phi}_{\mathrm{I}_0}(h,a)\end{bmatrix}, \tag{70}$$

where

$$G^{(1)}(h) = \delta a \left(\zeta_{10}^2 - \zeta_{13}^2\right) \cos(\kappa_{10} a) F^{(1)}\{\beta_1, h, \eta(z)\}, \tag{71}$$

$$\begin{aligned} F^{(1)}\{\beta_1, h, \eta(z)\} &= \int_{-t}^{t} \cos(Kz) \exp\{j(\beta_1 - h)z\} dz \\ &= \left[\frac{\sin\{(K+\beta_1-h)t\}}{K+\beta_1-h} + \frac{\sin\{(K-\beta_1+h)t\}}{K-\beta_1+h}\right], \end{aligned} \tag{72}$$

$$\begin{aligned} I(h, H, a, b) &= -\tilde{I}_{ey}^{(1)}(h, a)\tilde{I}_{ey}^{(2)}(h, b)\tilde{I}_{oy}^{(2)}(h, b) \\ &\quad + \exp\left\{2j\widetilde{\zeta}_{23}(h)H\right\} I_1(h, b) I_2(h, a) \end{aligned} \tag{73}$$

and

$$\tilde{I}_{ey}^{(1)}(h, a) = j\widetilde{\zeta}_{13}(h) \cos\left\{\widetilde{\zeta}_{10}(h)a\right\} + \widetilde{\zeta}_{10}(h) \sin\left\{\widetilde{\zeta}_{10}(h)a\right\}, \tag{74}$$

$$I_1(h, b) = \frac{1}{2}\left\{\widetilde{\zeta}_{20}^2(h) - \widetilde{\zeta}_{23}^2(h)\right\} \sin\left\{2\widetilde{\zeta}_{20}(h)b\right\}, \tag{75}$$

$$I_2(h, a) = \left[\widetilde{\zeta}_{10}(h) \sin\left\{\widetilde{\zeta}_{10}(h)a\right\} - j\widetilde{\zeta}_{13}(h) \cos\left\{\widetilde{\zeta}_{10}(h)a\right\}\right]. \tag{76}$$

From Eq. (73), the eigenvalue equation of the compound structure composed of three parallel perfect waveguide cores without a periodic groove structure [11, 15] is given by

$$I(\beta, H, a, b) = 0, \tag{77}$$

where β is the eigenvalue of the compound perfect waveguide. The first order approximate scattered fields of the regions I_0, I_1, II_0, II_1, II_2 and III are given by

$$\begin{aligned} \Psi_{VW}^{(1)}(x, y, z) &= \frac{1}{2\pi} \int_{-W}^{W} \left[\frac{G^{(1)}(h)}{I(h, H, a, b)} f_V(x, y) \exp(jhz)\right] dh, \\ &\quad (V = \mathrm{I}_0, \mathrm{I}_1, \mathrm{II}_0, \mathrm{II}_1, \mathrm{II}_2, \mathrm{III}), \end{aligned} \tag{78}$$

where

$$\begin{aligned} f_{\mathrm{I}_0}(h, x, y) &= \left[\tilde{I}_{ey}^{(2)}(h, b)\tilde{I}_{oy}^{(2)}(h, b) - \exp\left\{2j\widetilde{\zeta}_{23}(h)H\right\} I_1(h, b)\right] \\ &\quad \cdot \cos(\kappa_{10} x) \cos\left\{\widetilde{\zeta}_{10}(h)y\right\}, \end{aligned} \tag{79}$$

$$f_{\mathrm{I}_1}(h, x, y) = \left[\tilde{I}_{ey}^{(2)}(h, b)\tilde{I}_{oy}^{(2)}(h, b) - \exp\left\{2j\widetilde{\zeta}_{23}(h)H\right\} I_1(h, b)\right]$$

$$\cdot \cos(\kappa_{10}a)\exp\{j\kappa_{11}(x-a)\}\cos\left\{\widetilde{\zeta}_{10}(h)y\right\}, \tag{80}$$

$$f_{\text{II}_0}(x,y) = j\widetilde{\zeta}_{23}(h)\exp\left\{j\widetilde{\zeta}_{23}(h)H\right\}\cos\left\{\widetilde{\zeta}_{10}(h)a\right\} \cdot \cos(\kappa_{10}x)I_3(h,y), \tag{81}$$

$$f_{\text{II}_1}(x,y) = j\widetilde{\zeta}_{23}(h)\exp\left\{j\widetilde{\zeta}_{23}(h)H\right\}\cos\left\{\widetilde{\zeta}_{10}(h)a\right\}\cos(\kappa_{10}a) \cdot \exp\{j\kappa_{11}(x-a)\}I_3(h,y), \tag{82}$$

$$f_{\text{II}_2}(x,y) = j\widetilde{\zeta}_{20}(h)\widetilde{\zeta}_{23}(h)\exp\left\{j\widetilde{\zeta}_{23}(h)H\right\}\cos\left\{\widetilde{\zeta}_{10}(h)a\right\}\cos(\kappa_{10}x) \cdot \exp\left\{j\widetilde{\zeta}_{23}(h)(y-H-a-2b)\right\}, \tag{83}$$

$$f_{\text{III}}(h,x,y) = \cos\left\{\widetilde{\zeta}_{10}(h)a\right\}\cos(\kappa_{10}x)\left[\tilde{I}_{ey}^{(2)}(h,b)\tilde{I}_{oy}^{(2)}(h,b) \cdot \exp\left\{j\widetilde{\zeta}_{13}(h)(y-a)\right\} - \exp\left\{j\widetilde{\zeta}_{23}(h)H\right\}I_1(h,b) \cdot \exp\left\{-j\widetilde{\zeta}_{23}(h)(y-H-a)\right\}\right], \tag{84}$$

and

$$I_3(h,y) = \left[\!\left[\tilde{I}_{oy}^{(2)}(h,b)\cos\left[\widetilde{\zeta}_{20}(h)\{y-(H+a+b)\}\right] + \tilde{I}_{ey}^{(2)}(h,b)\sin\left[\widetilde{\zeta}_{20}(h)\{y-(H+a+b)\}\right]\right]\!\right]. \tag{85}$$

The amplitudes of modes which are excited by the periodic groove structure of finite extent are derived from Eqs. (78)–(85) by the calculation of residues at the propagation constant [18, 19], namely, at $h = \pm\beta$. In this case, following equation is satisfied:

$$I(\pm\beta,H,a,b) = -\tilde{I}_{ey}^{(1)}(\pm\beta,a)\tilde{I}_{ey}^{(2)}(\pm\beta,b)\tilde{I}_{oy}^{(2)}(\pm\beta,b) + \exp\left\{2j\widetilde{\zeta}_{23}(\pm\beta)H\right\}I_1(\pm\beta,b)I_2(\pm\beta,a) = 0. \tag{86}$$

Because the mode fields in regions with refractive index n_2 are the evanescent fields, they decay in proportion to $\exp\{j\widetilde{\zeta}_{23}(\beta)(y-H-a-2b)\}$ for example. Therefore $\widetilde{\zeta}_{23}(\beta)$ is a pure positive imaginary number and $\widetilde{\zeta}_{23}(\beta)$ is given by

$$j\widetilde{\zeta}_{23}(\beta) = -\sqrt{\beta^2+{\kappa_{10}}^2-{n_2}^2{k_0}^2}. \tag{87}$$

In eigenvalue equation,

$$\exp\left\{2j\widetilde{\zeta}_{23}(\pm\beta)H\right\} \ll 1, \tag{88}$$

is satisfied when H is large space between waveguides. Therefore eigenvalue equation can be approximated as follows:

$$I(\pm\beta, H, a, b) \cong -\tilde{I}_{ey}^{(1)}(\pm\beta, a)\tilde{I}_{ey}^{(2)}(\pm\beta, b)\tilde{I}_{oy}^{(2)}(\pm\beta, b). \tag{89}$$

Therefore the mode field of the compound waveguide, which consists of three parallel perfect waveguide cores placing at equal space each other, can be considered approximately as superposition of fields of modes of isolated waveguide-2^r and waveguide-2^l and the mode of isolated central waveguide-1 when cores of waveguides are placed at large space [15].

From above discussion and Eqs. (78)–(85) and (89), it is considered approximately that modes of waveguide-2^r and waveguide-2^l and modes of the waveguide-1 are coupled with incident mode through the periodic groove structure of finite extent and propagate in those waveguides.

In this paper, scattering and coupling problems are analyzed in the region of normalized frequency where only the fundamental E_{11}^x mode can exist in the waveguide-1 and waveguide-2^r. It is given by

$$0 \leq V_2 < V_1 \leq \frac{\pi}{2}, \tag{90a}$$

$$V_1 = \sqrt{{n_1}^2 - {n_2}^2}k_0 a, \tag{90b}$$

$$V_2 = \sqrt{{n_1}^2 - {n_2}^2}k_0 b, \tag{90c}$$

$$\frac{V_2}{V_1} = \frac{b}{a} < 1. \tag{90d}$$

In waveguide-2^r, the fundamental E_{11}^x modes of this waveguide are coupled with the incident mode by the periodic groove structure of waveguide-1 and propagate to positive(forward) and negative (backward) z direction. Then E_x component of these coupled fundamental modes having the first order approximate amplitudes are denoted as $E_{2,x}^{(1)(f,b)}(x, y, z)$. Wavenumbers, κ_{20}, κ_{21} and ζ_{20}, ζ_{23} are derived by Eqs. (27) and (28) in the region of the normalized frequency, V_1 and V_2 given by Eqs. (90a)–(90d), respectively. The propagation constant β_2

of the waveguide-2^r is given by using one of Eqs. (24)–(26) after obtaining those wavenumbers. Propagation constants $\pm\beta_2$ satisfy the equation given by Eqs. (47), (49c), and (49d) as follows:

$$\tilde{I}_{ey}^{(2)}(\pm\beta_2, b) = j\widetilde{\zeta}_{23}(\pm\beta_2)\cos\left\{\widetilde{\zeta}_{20}(\pm\beta_2)b\right\} + \widetilde{\zeta}_{20}(h)\sin\left\{\widetilde{\zeta}_{20}(\pm\beta_2)b\right\} = 0. \tag{91}$$

Then $E_{2,x}^{(1)(f,b)}(x,y,z)$ is derived from the calculation of the residues at $h = \pm\beta_2$ [18, 19]. Consequently, $E_{2,x}^{(1)(f,b)}(x,y,z)$ is given as follows:

$$E_{2,x}^{(1)(f,b)}(x,y,z) = -\frac{j}{2}A_2 G^{(1)}(\pm\beta_2)\exp\left\{j\widetilde{\zeta}_{23}(\beta_2)H\right\}\exp(\pm j\beta_2 z)$$
$$\cdot\begin{bmatrix} \cos(\kappa_{10}x)\cos\left\{\widetilde{\zeta}_{20}(\beta_2)b\right\}\exp\left\{-j\widetilde{\zeta}_{23}(\beta_2)(y-H-a)\right\}; \\ \hfill x,y,z\in \mathrm{III} \\ \cos(\kappa_{10}x)\cos\left[\widetilde{\zeta}_{20}(\beta_2)\left\{y-(H+a+b)\right\}\right];\ x,y,z\in \mathrm{II}_0 \\ \cos(\kappa_{10}a)\exp\{j\kappa_{21}(x-a)\}\cos\left[\widetilde{\zeta}_{20}(\beta_2)\left\{y-(H+a+b)\right\}\right]; \\ \hfill x,y,z\in \mathrm{II}_1 \\ \cos(\kappa_{10}x)\cos\left\{\widetilde{\zeta}_{20}(\beta_2)b\right\}\exp\left\{j\widetilde{\zeta}_{23}(\beta_2)(y-H-a-2b)\right\}; \\ \hfill x,y,z\in \mathrm{II}_2 \end{bmatrix} \tag{92}$$

where

$$G^{(1)}(\pm\beta_2) = \delta a\left(\zeta_{10}^2 - \zeta_{13}^2\right)\cos(\kappa_{10}a)\cdot\left[\frac{\sin\left\{(K+\beta_1-(\pm\beta_2))\,t\right\}}{K+\beta_1-(\pm\beta_2)} + \frac{\sin\left\{(K-\beta_1+(\pm\beta_2))\,t\right\}}{K-\beta_1+(\pm\beta_2)}\right] \tag{93}$$

A_2 is given by

$$A_2 = \overline{A}_2\left\{j\widetilde{\zeta}_{23}(\beta_2)\right\}\frac{\cos\left\{\widetilde{\zeta}_{10}(\beta_2)a\right\}}{I_{ey}^{(1)}(\beta_2, a)}, \tag{94}$$

where

$$\overline{A}_2 = \frac{j\widetilde{\zeta}_{23}(\beta_2)\cos\left\{\widetilde{\zeta}_{20}(\beta_2)b\right\}}{\beta_2\left[1 - j\widetilde{\zeta}_{23}(\beta_2)b\right]}. \tag{95}$$

In above equations, $\widetilde{\zeta}_{20}(\beta_2)$ and $\widetilde{\zeta}_{23}(\beta_2)$ satisfy naturally following relations:

$$\begin{cases} \widetilde{\zeta}_{20}(\beta_2) = \zeta_{20} \\ \widetilde{\zeta}_{23}(\beta_2) = \zeta_{23} \end{cases}. \tag{96}$$

In the waveguide-1, the fundamental E_{11}^x modes of this waveguide are exited by the periodic groove structure and propagate to forward and backward in this waveguide. Then E_x component of these fundamental E_{11}^x modes with the first order approximate amplitudes are denoted as $E_{1,x}^{(1)(f,b)}(x, y, z)$. It is derived in the same manner as $E_{2,x}^{(1)(f,b)}(x, y, z)$ and given as follows:

$$E_{1,x}^{(1)(f,b)}(x, y, z) = -\frac{j}{2} A_1 G^{(1)}(\pm\beta_1)\exp(\pm j\beta_1 z) \cdot \begin{bmatrix} \cos(\kappa_{10}x)\cos\left\{\widetilde{\zeta}_{10}(\beta_1)a\right\}\exp\left\{j\widetilde{\zeta}_{13}(\beta_1)(y-a)\right\};\ x, y, z \in \mathrm{III} \\ \cos(\kappa_{10}x)\cos\left\{\widetilde{\zeta}_{10}(\beta_1)y\right\};\ x, y, z \in \mathrm{I}_0 \\ \cos(\kappa_{10}a)\exp\{j\kappa_{11}(x-a)\}\cos\left\{\widetilde{\zeta}_{10}(\beta_1)y\right\};\ x, y, z \in \mathrm{I}_1 \end{bmatrix}, \tag{97}$$

where

$$A_1 = \frac{j\widetilde{\zeta}_{13}(\beta_1)\cos\left\{\widetilde{\zeta}_{10}(\beta_1)a\right\}}{\beta_1\left[1 - j\widetilde{\zeta}_{13}(\beta_1)a\right]}. \tag{98}$$

In Eqs. (97) and (98), $\widetilde{\zeta}_{10}(\beta_1)$ and $\widetilde{\zeta}_{13}(\beta_1)$ satisfy naturally following equations:

$$\begin{cases} \widetilde{\zeta}_{10}(\beta_1) = \zeta_{10} \\ \widetilde{\zeta}_{13}(\beta_1) = \zeta_{13} \end{cases}. \tag{99}$$

From Eqs. (92) and (93), it is considered that E_{11}^x mode which propagates positive z direction (forward) in the waveguide-2^r is coupled with the incident mode strongly when the Bragg condition is satisfied as follows:

$$K = \frac{2\pi}{D} = \pm(\beta_1 - \beta_2). \tag{100}$$

Signs of right hand side of this equation are defined in such way that K takes a positive number. Similarly, E_{11}^{x} mode which propagates the negative z direction (backward) in the waveguide-2^r is coupled with the incident mode strongly when the Bragg condition is satisfied as follows:

$$K = \frac{2\pi}{D} = \beta_1 + \beta_2. \tag{101}$$

In this paper, scattering and coupling problems of the directional coupler is analyzed when the Bragg condition of Eq. (101) is satisfied.

Applying the method of steepest descent to scattered fields represented by the integral transform, the first order approximation of the far-field of the scattered wave in the region II_2^r, $\widetilde{E}_{\mathrm{II}_2}^{(1)}(x, \rho, \theta)$ is obtained as follows:

$$\widetilde{E}_{\mathrm{II}_2}^{(1)}(x, \rho, \theta) = -\sqrt{\frac{\gamma_0 k_0}{2\pi}} \cdot \exp\left(-j\frac{\pi}{4}\right) \frac{\exp(j\gamma_0 k_0 \rho)}{\rho^{1/2}} \cdot \sin\theta \cos(\kappa_{10} x) \psi_{\mathrm{II}_2}^{(1)}(h), \tag{102}$$

$$h^2 + \widetilde{\zeta}_{23}^2(h) = n_2^2 k_0^2 - \kappa_{\mathrm{I}_0}^2 = \gamma_0^2 k_0^2, \tag{103}$$

$$\begin{cases} h = \gamma_0 k_0 \cos\theta \\ \widetilde{\zeta}_{23}(h) = \gamma_0 k_0 \sin\theta \end{cases}. \tag{104}$$

where $\psi_{\mathrm{II}_2}^{(1)}(h)$ is given from Eqs. (78) and (83) as follows:

$$\psi_{22}^{(1)}(h) = \frac{G^{(1)}(h)}{I(h, H, a, b)} j\widetilde{\zeta}_{10}(h)\widetilde{\zeta}_{23}(h) \exp\left\{j\widetilde{\zeta}_{23}(h)H\right\} \cos\left\{\widetilde{\zeta}_{10}(h)a\right\}. \tag{105}$$

and θ is measured counterclockwise from the z axis. The far-field of the scattered field given by above equations depends on the x coordinate. However the far-field pattern can be obtained independently on the x coordinate on the y-z plane with any x coordinate in the region II_2^r because it is drawn by normalizing with the maximum amplitude of the scattered fields on any y-z plane in that region. Therefore the profile of the far-field pattern is independent on the x coordinate in that region. The profile of the far-field pattern of the region II_2^l is symmetry to one of II_2^r with regard to x-z plane.

The incident power to waveguide-1 is denoted as Pi and the reflected power derived from the first order approximate amplitude is denoted as $PW1r$. The radiation power is the sum of ones in the region II_2^r and II_2^l and it is denoted as $Prad$. Moreover powers which are coupled with the incident mode and propagate forward and backward in the waveguide-2^r are denoted as $PW2f$ and $PW2b$, respectively. The transmitted power of the waveguide-1 is denoted as $PW1t$. The same amount of powers are coupled with waveguide-2^l as ones of waveguide-2^r. Then $PW1t$ is given by

$$PW1t = Pi - PW1r - Prad - 2 \cdot (PW2b + PW2f). \tag{106}$$

Normalized powers of each power are expressed as follows:

$$PTW1 = \frac{PW1t}{Pi}, \tag{107a}$$

$$PBW1 = \frac{PW1r}{Pi}, \tag{107b}$$

$$PRAD = \frac{Prad}{Pi}, \tag{107c}$$

$$PBW2 = \frac{PW2b}{Pi}, \tag{107d}$$

$$PFW2 = \frac{PW2f}{Pi}. \tag{107e}$$

5. RESULTS AND DISCUSSION

In this paper, the scattering problems of the directional couplers are analyzed for the fundamental E_{11}^x mode incidence. This directional coupler is composed of three parallel rectangular waveguide cores which are placed at equal space in the dielectric medium. Periodic groove structures of finite extent are formed on two opposite surfaces of rectangular core of the central waveguide among them. The above-mentioned problems are analyzed in the region of the normalized frequency given by Eqs. (90a)–(90d). In that region, only the fundamental E_{11}^x mode of each waveguide can be coupled with the incident mode. The Bragg condition of Eq. (101) is applied in this analysis.

In Figs. 3–7, refractive indices distributions of couplers of this paper are $n_1 = 1.20$, $n_2 = 1.18$.

In Figs. 3–6, $PBW2$ and $PFW2$ denote as the normalized power, namely, coupling efficiency of E_{11}^x modes of the waveguide-2^r, which are coupled with the incident mode of the waveguide-1 through the

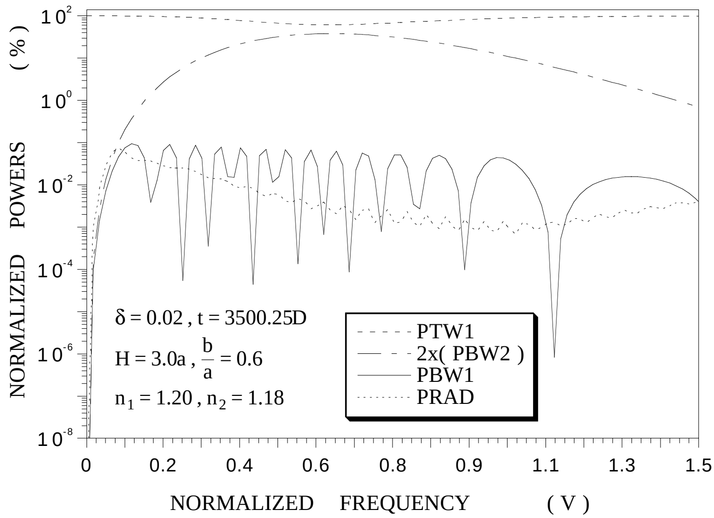

Figure 3. Normalized powers of directional coupler for rectangular waveguides shown by Fig. 1 as a function of normalized frequency V under Bragg condition of Eq. (101).

periodic structure and the propagate to negative (backward) and positive (forward) z direction, respectively. The same amount of powers are coupled in the waveguide-2^l as ones of waveguide-2^r. $PBW1$ is the normalized powers of E_{11}^x modes in the waveguide-1 which is reflected by the periodic structure and normalized by the incident power. $PRAD$ is the sum of the normalized powers of radiation fields in regions II_2^r and II_2^l. Above-mentioned quantities are derived from the first order approximate fields. $PTW1$ is the normalized power of the fundamental E_{11}^x mode which is transmitted through periodic structure in the waveguide-1 and this quantity is derived from Eqs. (106) and (107a).

Figure 3 shows $PBW1$, $PTW1$, $PRAD$ and two times of $PBW2$ as a function of the normalized frequency V when the Bragg condition given by Eq. (101) is satisfied. Therefore it is shown that the fundamental E_{11}^x modes propagating backward in the waveguide-2^r and waveguide-2^l are coupled strongly with the incident mode to the central waveguide-1. In Fig. 3, the half length of the periodic structure in the z direction, t is equal to 3500.25D. It is shown that two times $PBW2$ has maximum value nearby $V = 0.6$ and its value is about 38%. Therefore it is considered that the structure presented

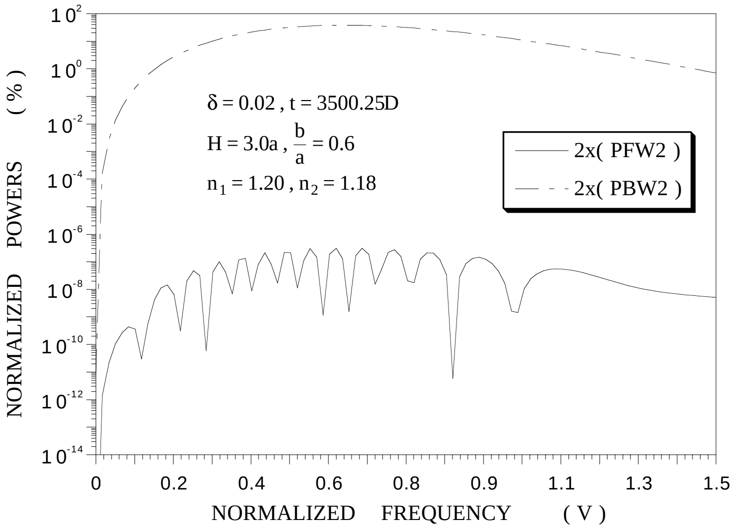

Figure 4. Comparison between normalized powers carried backward and forward by the mode in waveguide-2 as a function of normalized frequency V under the Bragg condition of Eq. (101).

in this paper has a function of the directional coupler for rectangular waveguides.

Figure 4 shows a comparison between normalized powers $PBW2$ and $PFW2$ as a function of the normalized frequency with regard to same directional coupler as one of Fig. 3 when the Bragg condition of Eq. (101) is satisfied. The incident E_{11}^x mode of the waveguide-1 is coupled with E_{11}^x modes of the waveguide-2^r and waveguide-2^l through the periodic groove structure. Then coupled modes propagate backward and forward in those waveguides. This figure shows that the power carried by the mode propagating backward in waveguide-2^r and waveguide-2^l , $PBW2$ is very large in comparison with the power carried by the mode propagating forward in those waveguides, $PFW2$ when the condition of Eq. (101) is satisfied. The result of this figure has good agreement with physical consideration qualitatively.

Figure 5 shows $PTW1$ and two times of $PBW2$ as a function of space between rectangular cores of waveguides H when the Bragg condition given by Eq. (101) is satisfied. This figure shows that $PBW2$ decreases exponentially with increase of H . Results of this analysis have good agreement with physical consideration qualitatively.

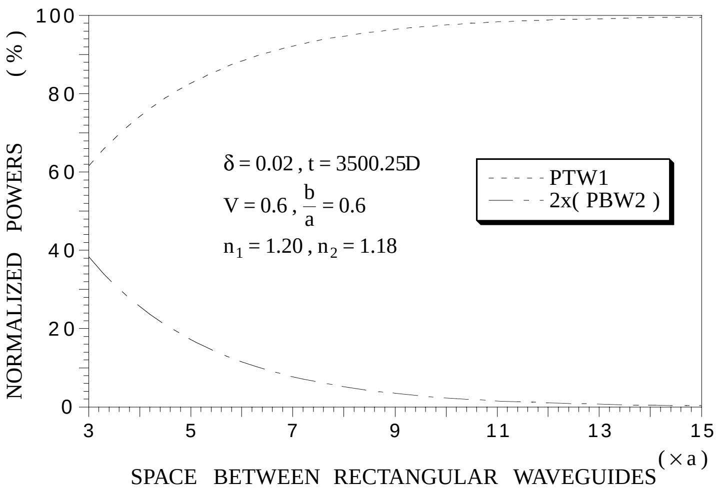

Figure 5. Normalized powers of directional coupler for rectangular waveguides as a function of space between rectangular waveguide cores H under Bragg condition of Eq. (101).

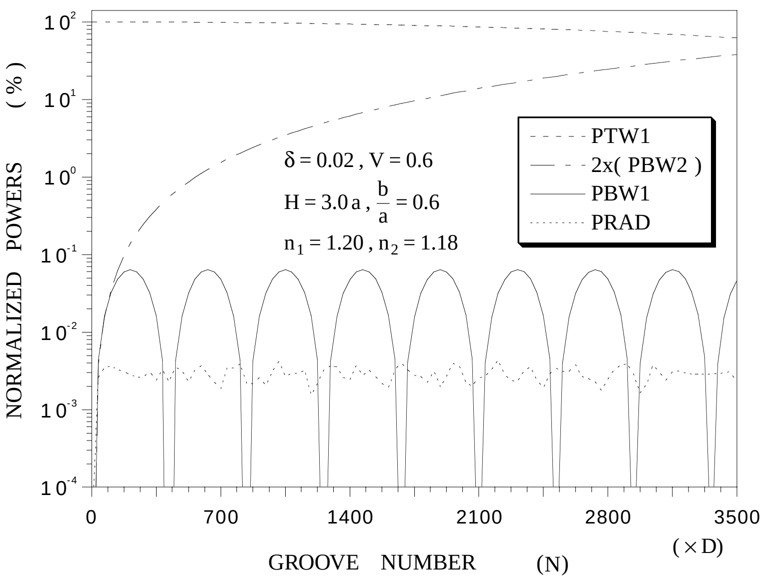

Figure 6. Normalized powers of directional coupler for rectangular waveguides as a function of groove number N under the Bragg condition of Eq. (101).

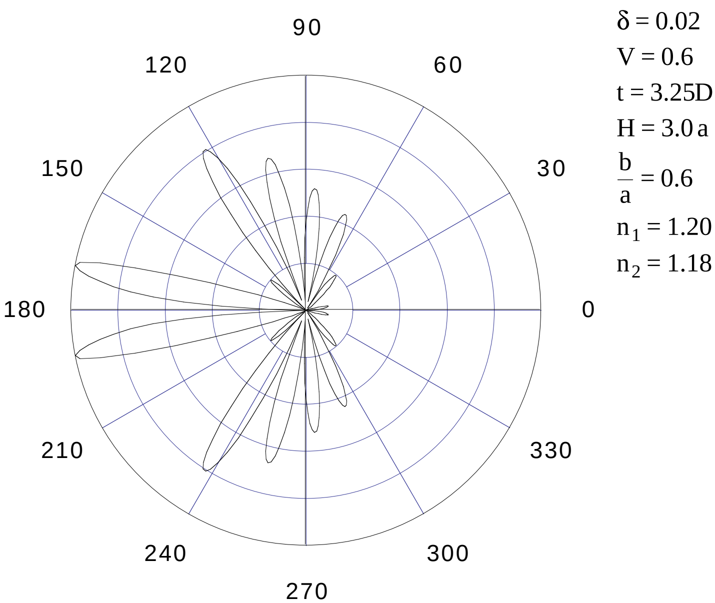

Figure 7. Far-field pattern drawn on the y-z plane in the regions II_2^r and II_2^l of the directional coupler under the condition of Eq. (101).

Figure 6 shows normalized powers as a function of the number of groove N when the Bragg condition given by Eq. (101) is satisfied. This figure shows that $PBW2$ increases monotonously with increase of groove number in the first order approximation but it is shown that fair amount of the incident power can be coupled into the other waveguides in any ratio by selecting groove number appropriately.

Figure 7 shows far-field patterns on any y-z plane in regions II_2^r and II_2^l. when the Bragg condition given by Eq. (101) is satisfied. The far-field given by Eqs. (102)–(105) depends on x coordinate. However the far-field pattern is independent on x coordinate in the regions II_2^r and II_2^l because the far-field pattern is drawn on each y-z plane in those region by normalizing by the maximum amplitude of the scattered field of each y-z plane. Therefore the profile of the far-field pattern is independent on the x coordinate in those regions. The profile of the far-field pattern of the region II_2^l is symmetry to one of II_2^r with regard to x-z plane. This figure shows that the propagation field is scattered backward with regard to direction of the incident mode strongly in this case.

6. CONCLUSIONS

A method based on the mode matching method in the sense of least squares has been applied to analyze a coupling and scattering problems of the directional coupler for rectangular waveguides. This directional coupler is composed of three parallel rectangular cores which are placed at equal space in the dielectric medium. The central rectangular core has periodic groove structures on its two surfaces which face each other and other two waveguide cores are perfect. From results based on the first order approximate solution of the integral equation, it is considered that the structure presented in this paper has function of directional coupler for rectangular waveguides when the Bragg condition is applied appropriately. Moreover it is considered that results of analyses have good agreement with the physical consideration qualitatively. Therefore this method is expected to be useful for the analysis of the rectangular waveguides which have a periodic groove structure of finite extent and have functions such as directional couplers and so on.

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