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THE MUELLER MATRIX OF A TWO-LAYER ECCENTRICALLY BIANISOTROPIC CYLINDER LINEAR ARRAY WITH DOUBLE HELICAL CONDUCTANCES OF THE SURFACES: CLARIFICATION OF THE MAGNETIC SYMMETRY GROUPS

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Abstract–Unique effects of the double helical conductances of the surfaces (HCS) on the Mueller matrix (Mm) of a two-layer eccentrically bianisotropic cylinder linear array are investigated in this paper. The mathematical treatment is conducted based on the boundary-value approach combined with the technique of generalized separation variables. Both the TM_z- and TE_z-polarization of the obliquely incident waves are taken into account in the analysis. To gain insight into some physical mechanisms, numerical examples are presented to show the influences of the variations of the twist angles on the behavior of Mm of a linear array of four bianisotropic cylinders. Correspondingly, various magnetic symmetry groups (such as $D_\infty(C_\infty)$, $C_{\infty v}(C_\infty)$, $D_{\infty h}(D_\infty)$, $C_{\infty h}(C_\infty)$) and some generalized symmetry and anti-symmetry relations, which govern all the elements of Mm or the scattering cross section under special chiral operations, are demonstrated. The present studies can be exploited to identify the constitutive characteristics of some bianisotropic media and to provide better understanding of the electromagnetic wave interaction with bianisotropic cylindrical objects with complex boundaries.

1. INTRODUCTION

Electromagnetic scattering of a plane wave by a singly cylinder, a system of two cylinders, and an array of many conducting and dielectric cylinders has been extensively studied by many researchers in the past several decades [1–11]. These studies have been motivated both by scientific interests in developing new techniques for solving scattering problems and by some particular engineering applications. Recently, Korshunova, Sivov, and Shatrov, etc. have studied the interaction between electromagnetic waves and cylindrical objects possessing perfectly electric or magnetic conductance along helical lines, demonstrated several novel phenomena, and proposed some practical applications at microwave frequencies [11–16].

On one hand, the boundary conditions of these cylindrical objects are directly related to the twist angle of the helical surfaces, so the radiation, the wave guiding, and the scattering characteristics can be controlled and optimized by choosing an appropriate twist angle. In some special cases of the helical conductance surfaces, it is known that strip grid structures can be used as the polarization selective surface or utilized to build hard surfaces for reducing the forward scattering of cylindrical objects [17–19]. Also, they can be exploited to make some helical antenna structures at microwave frequencies [20, 21]. On the other hand, it is interesting to note that spherically helical conducting particles can be exploited to make artificial chiral media [22, 23].

In this paper, some novel effects of double helical conductances of the surface (HCSs) on the Mueller matrix (Mm) of a linear array of eccentrically two-layered bianisotropic cylinders are found and discussed in the case of the TM_z- and TE_z-polarizations of the obliquely incident waves. Certainly, such double HCS can be made of conductor grids in terms of a multifilar helix with a twist angle in either the right-handed (RH) or the left-handed (LH) form. It is known that the electromagnetic scattering from cylindrical bianisotropic objects has been well-documented recently [24–26], and the characteristics of Mm for two bianisotropic cylinders have also been examined in one of the authors' previous studies [27, 28]. The current work actually is a further extension of the previous studies, where the Mm features related to the double HCS of inhomogeneous bianisotropic cylinders will be looked into. The motivation for this study is not only the academic importance but also the essential measurement of the bianisotropic cylindrical objects combined with complex boundaries.

2. DESCRIBTION OF THE PROBLEM

Figure 1 presents the geometry of the problem, in which N parallel, infinitely long, non-overlapping, eccentrically two-layered bianisotropic cylinders are embedded in the isotropic medium $(\varepsilon_b,\ \mu_b)$. The cross section of the qth composite cylinder is shown in Fig. 1(b), and its radii are denoted by $R_1^{(q)}$ and $R_2^{(q)}$ with respect to two local co-ordinate systems, respectively, while the eccentric distance is noted by $d^{(q)}$ $(q = 1, 2, \ldots, N)$ and the distance between the qth cylinder and the first is given by $D^{(q)}$ $(q \neq 1)$. The incident plane wave is assumed to propagate in the direction of $\overline{k}(k_b, \theta_0, \varphi_0) = k_b(\sin\theta_0\cos\varphi_o\overline{e}_x + \sin\theta_0\sin\varphi_0\overline{e}_y + \cos\theta_0\overline{e}_z)$, where $(\overline{e}_x, \overline{e}_y, \overline{e}_z)$ are the three unit vectors in the host co-ordinate system.

Here the constitutive features of the two-layer bianisotropic media are described by the linear equation as $\left(e^{j\omega t}\right)$:

$$\vec{D}^{(l,q)} = \left[\varepsilon^{(l,q)}\right]\vec{E}^{(l,q)} + \left[\xi_e^{(l,q)}\right]\vec{H}^{(l,q)} \qquad l = 1, 2 \tag{1a}$$

$$\vec{B}^{(l,q)} = \left[\mu^{(l,q)}\right]\vec{H}^{(l,q)} + \left[\xi_m^{(l,q)}\right]\vec{E}^{(l,q)} \tag{1b}$$

where $\left[\varepsilon^{(l,q)}\right]$, $\left[\mu^{(l,q)}\right]$, $\left[\xi_e^{(l,q)}\right]$, and $\left[\xi_m^{(l,q)}\right]$ are the permittivity tensor, permeability tensor, and magnetoelectric cross-coupling tensors, respectively. They can be expressed in the gyrotropic from, i.e.,

$$\left[C^{(l,q)}\right] = \begin{bmatrix} C_1^{(l,q)} & -jC_{12}^{(l,q)} & 0 \\ jC_{12}^{(l,q)} & C_1^{(l,q)} & 0 \\ 0 & 0 & C_2^{(l,q)} \end{bmatrix}, \qquad C = \varepsilon, \mu,\ \xi_e,\ \xi_m \tag{2}$$

where the case shown in reference [29] is not taken account. For certain magnetic symmetry groups there exist many kinds of possible choices for $\left[C^{(l,q)}\right]$ [30–33]. For instance, corresponding to the magnetic groups C_∞, $D_\infty(C_\infty)$, $C_{\infty v}(C_\infty)$, $C_{\infty h}(C_\infty)$, respectively, the

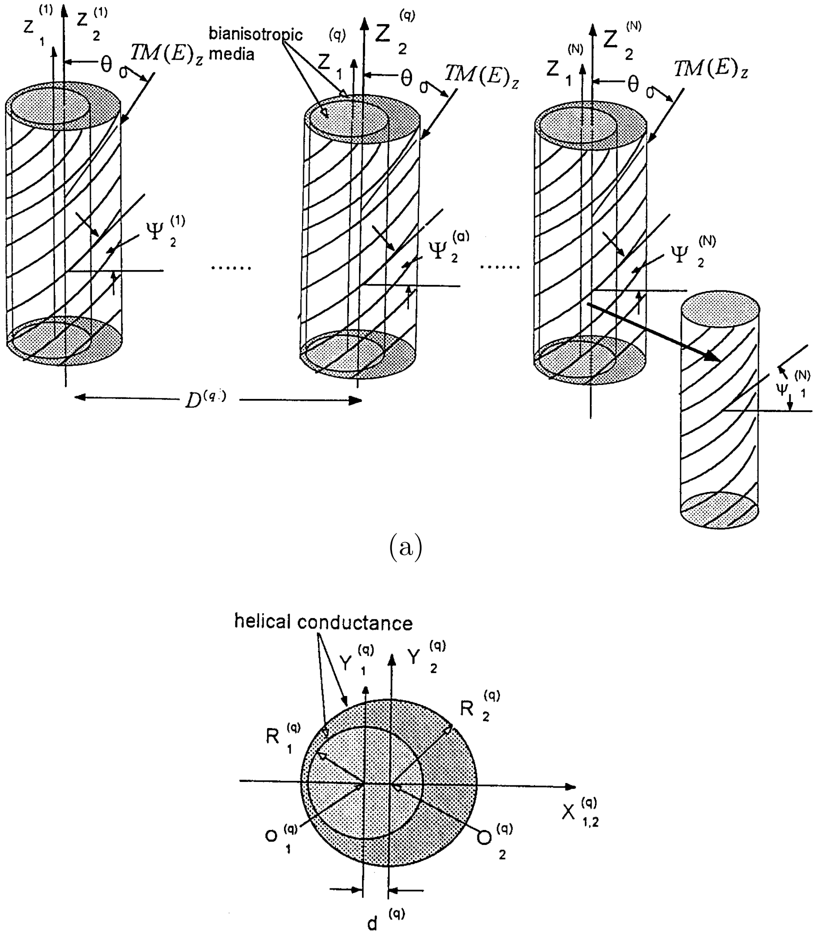


Figure 1. Geometry and co-ordinates of the linear array of N parallel eccentric bianisotropic cylinders with the helical conductances at $\rho_1^{(q)} = R_1^{(q)}$ and $\rho_2^{(q)} = R_2^{(q)}$, respectively, The separation between the cylinders $C^{(q)}$ and $C^{(N)}$ is noted by $D^{(Nq)} = |D^{(N)} - D^{(q)}|$.

magnetoelectric cross-coupling tensors $\left[\xi^{(q)}\right]$, and $\left[\eta^{(q)}\right]$ could be

$$1^\circ.\quad \left[\xi^{(q)}\right]=\begin{bmatrix} \xi_{xx}^{(q)} & \xi_{xy}^{(q)} & 0 \\ -\xi_{xy}^{(q)} & \xi_{xx}^{(q)} & 0 \\ 0 & 0 & \xi_{zz}^{(q)} \end{bmatrix},\quad \left[\eta^{(q)}\right]=\begin{bmatrix} \eta_{xx}^{(q)} & \eta_{xy}^{(q)} & 0 \\ -\eta_{xy}^{(q)} & \eta_{xx}^{(q)} & 0 \\ 0 & 0 & \eta_{zz}^{(q)} \end{bmatrix} \tag{3a}$$

$$2°.\ \left[\xi^{(q)}\right]=\begin{bmatrix}\xi_{xx}^{(q)} & \xi_{xy}^{(q)} & 0\\ -\xi_{xy}^{(q)} & \xi_{xx}^{(q)} & 0\\ 0 & 0 & \xi_{zz}^{(q)}\end{bmatrix},\ \left[\eta^{(q)}\right]=\begin{bmatrix}-\xi_{xx}^{(q)} & -\xi_{xy}^{(q)} & 0\\ \xi_{xy}^{(q)} & -\xi_{xx}^{(q)} & 0\\ 0 & 0 & -\xi_{zz}^{(q)}\end{bmatrix} \tag{3b}$$

$$3°.\ \left[\xi^{(q)}\right]=\begin{bmatrix}\xi_{xx}^{(q)} & \xi_{xy}^{(q)} & 0\\ -\xi_{xy}^{(q)} & \xi_{xx}^{(q)} & 0\\ 0 & 0 & \xi_{zz}^{(q)}\end{bmatrix},\ \left[\eta^{(q)}\right]=\begin{bmatrix}\xi_{xx}^{(q)} & \xi_{xy}^{(q)} & 0\\ -\xi_{xy}^{(q)} & \xi_{xx}^{(q)} & 0\\ 0 & 0 & \xi_{zz}^{(q)}\end{bmatrix} \tag{3c}$$

$$4°.\ \left[\xi^{(q)}\right]=\begin{bmatrix}\xi_{xx}^{(q)} & \xi_{xy}^{(q)} & 0\\ -\xi_{xy}^{(q)} & \xi_{xx}^{(q)} & 0\\ 0 & 0 & \xi_{zz}^{(q)}\end{bmatrix},\ \left[\eta^{(q)}\right]=\begin{bmatrix}\xi_{xx}^{(q)} & -\xi_{xy}^{(q)} & 0\\ \xi_{xy}^{(q)} & \xi_{xx}^{(q)} & 0\\ 0 & 0 & \xi_{zz}^{(q)}\end{bmatrix} \tag{3d}$$

The twist angles of the helical lines on the inner surface $\rho_1^{(q)} = R_1^{(q)}$ and the outer surface $\rho_2^{(q)} = R_2^{(q)}$ above are assumed to be $\Psi_1^{(q)}$ and $\Psi_2^{(q)}$, respectively. When $0 < \Psi_{1,2}^{(q)} < \frac{\pi}{2}$, the helical lines are the RH, while $\frac{\pi}{2} < \Psi_{1,2}^{(q)} < \pi$ correspond to the LH. Especially, when $\Psi_{1,2}^{(q)} = 0, \frac{\pi}{2}$ the helical surfaces are reduced to the T- and L-strips, respectively [19].

3. FIELD DISTRIBUTION

At first, a plane wave of TM with respect to $z_2^{(q)}$ axis obliquely incident on the linear array and the incident electric field components in the cylindrical co-ordinate system $O_2^{(q)}\left(\rho_2^{(q)}, \varphi_2^{(q)}, z_2^{(q)}\right)$ are expressed as

$$E_{zinc}^{(q)} = E_0 \sin\theta_0 e^{j\delta^{(q)}} \sum_{m=-\infty}^{\infty} j^m J_{m2}^{(q)} e^{-jm\left(\varphi_2^{(q)}-\varphi_0\right)} e_2^{(q)},\ q=1,2,\ldots,N \tag{4}$$

$$E_{\varphi inc}^{(q)} = E_0 \sin\theta_0 e^{j\delta^{(q)}} \sum_{m=-\infty}^{\infty} \frac{m\cos\theta_0}{k_{b0}\rho_2^{(q)}} j^m J_{m2}^{(q)} e^{-jm\left(\varphi_2^{(q)}-\varphi_0\right)} e_2^{(q)} \tag{5}$$

$$H_{\varphi inc}^{(q)} = -E_0 \sin\theta_0 \frac{j\omega\varepsilon_b}{k_{b0}} e^{j\delta^{(q)}} \sum_{m=-\infty}^{\infty} j^m J_{m2}^{(q)\prime} e^{-jm\left(\varphi_2^{(q)}-\varphi_0\right)} e_2^{(q)} \tag{6}$$

where $J_{m2}^{(q)} = J_m\left(k_{b0}\rho_2^{(q)}\right)$ is the mth-order Bessel function of the first kind, $J_{m2}^{(q)\prime} = J_m'\left(k_{b0}\rho_2^{(q)}\right)$ denotes the derivative of $J_{m2}^{(q)}$ with respect to its argument, $k_{b0}^2 = k_b^2 - \beta^2$, $\beta = k_b\cos\theta_0$, $k_b = \omega\sqrt{\mu_b\varepsilon_b}$, $\delta^{(q)} = k_{b0}\cos\varphi_0 D^{(q)}$, $e_2^{(q)} = e^{j\beta z_2^{(q)}}$.

By using the method of generalized separation variables and following the similar procedure adopted in [28], it is found that the tangential filed components in the bianisotropic core $\rho_1^{(q)} \leq R_1^{(q)}$ and in the second layer $\rho_2^{(q)} \leq R_2^{(q)}$ under the co-ordinate system $O_1^{(q)}\left(\rho_1^{(q)}, \varphi_1^{(q)}, z_1^{(q)}\right)$ can be written as

$$E_{zl}^{(q)} = \sum_{\pm} E_{zl\pm}^{(q)}, H_{zl}^{(q)} = \sum_{\pm} H_{zl\pm}^{(q)}, E_{\varphi l}^{(q)} = \sum_{\pm} E_{\varphi l\pm}^{(q)}, H_{\varphi l}^{(q)} = \sum_{\pm} H_{\varphi l\pm}^{(q)}, \tag{7a}$$

and

$$\begin{bmatrix} E_{z1\pm}^{(q)} \\ H_{z1\pm}^{(q)} \\ E_{\varphi 1\pm}^{(q)} \\ H_{\varphi 1\pm}^{(q)} \end{bmatrix} = \sum_{m=-\infty}^{\infty} \begin{bmatrix} X_{1\pm}^{(q,1)} \\ X_{3\pm}^{(q,1)} \\ X_{5\pm}^{(q,1)} \\ X_{7\pm}^{(q,1)} \end{bmatrix} D_{1m\pm}^{(q,1)} e^{-jm\varphi_1^{(q)}} \tag{7b}$$

$$\begin{bmatrix} E_{z2\pm}^{(q)} \\ H_{z2\pm}^{(q)} \\ E_{\varphi 2\pm}^{(q)} \\ H_{\varphi 2\pm}^{(q)} \end{bmatrix} = \sum_{m=-\infty}^{\infty} \begin{bmatrix} X_{1\pm}^{(q,2)} & X_{2\pm}^{(q,2)} \\ X_{3\pm}^{(q,2)} & X_{4\pm}^{(q,2)} \\ X_{5\pm}^{(q,2)} & X_{6\pm}^{(q,2)} \\ X_{7\pm}^{(q,2)} & X_{8\pm}^{(q,2)} \end{bmatrix} \begin{bmatrix} D_{1m\pm}^{(q,2)} \\ D_{2m\pm}^{(q,2)} \end{bmatrix} e^{-jm\varphi_1^{(q)}} \tag{7c}$$

where $X_{s\pm}^{(q,l)}$ $(s = 1, \ldots, 8)$ can be referred in [28]. However, it must be noted that the time harmonic factor here is $e^{j\omega t}$, and $D_{1,2m\pm}^{(q,l)}$ are the unknown mode expanding coefficients. The boundary conditions for the HCS on the surface $\rho_1^{(q)} = R_1^{(q)}$ are

$$\begin{aligned} &E_{z1}^{(q)} = E_{z2}^{(q)}, \quad E_{\varphi 1}^{(q)} = E_{\varphi 2}^{(q)}, \quad E_{\varphi 1}^{(q)} + E_{z1}^{(q)} \tan \Psi_1^{(q)} = 0, \\ &H_{\varphi 1}^{(q)} + H_{z1}^{(q)} \tan \Psi_1^{(q)} = H_{\varphi 2}^{(q)} + H_{z2}^{(q)} \tan \Psi_1^{(q)} \end{aligned} \tag{8}$$

Using the addition theorem for the cylindrical Bessel functions, (7c) can be transformed into the following form in the co-ordinate system $O_2^{(q)}\left(\rho_2^{(q)}, \varphi_2^{(q)}, z_2^{(q)}\right)$:

$$\begin{bmatrix} E_{z2}^{(q)} \\ H_{z2}^{(q)} \\ E_{\varphi 2}^{(q)} \\ H_{\varphi 2}^{(q)} \end{bmatrix} = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \begin{bmatrix} V_{nm}^{(q,1)}\left(\rho_2^{(q)}\right) & V_{nm}^{(q,2)}\left(\rho_2^{(q)}\right) \\ V_{nm}^{(q,3)}\left(\rho_2^{(q)}\right) & V_{nm}^{(q,4)}\left(\rho_2^{(q)}\right) \\ V_{nm}^{(q,5)}\left(\rho_2^{(q)}\right) & V_{nm}^{(q,6)}\left(\rho_2^{(q)}\right) \\ V_{nm}^{(q,7)}\left(\rho_2^{(q)}\right) & V_{nm}^{(q,8)}\left(\rho_2^{(q)}\right) \end{bmatrix} \begin{bmatrix} D_{1m+}^{(q,2)} \\ D_{1m-}^{(q,2)} \end{bmatrix} e^{-jm\varphi_2^{(q)}} \tag{9}$$

where $V_{nm}^{(q,s)}\left(\rho_2^{(q)}\right)$ $(s = 1, 2, \ldots, 8)$ are presented in reference [28] and suppressed $(-j \Rightarrow j)$. But, it is assumed here that $\beta = k_b \cos\theta_0 \neq 0$ and the four constitutive tensors are all in the gyrotropic form.

Furthermore, the total tangential field components outside the bian-isotropic cylinders $\left(\rho_2^{(q)} \geq R_2^{(q)}\right)$ with respect to $O_2^{(q)}\left(\rho_2^{(q)}, \varphi_2^{(q)}, z_2^{(q)}\right)$ can be written as:

$$E_{zb}^{(q)} = E_{\varphi inc}^{(q)} + \sum_{m=-\infty}^{\infty} \left\{ a_m^{(q)} H_{m2}^{(q)} + J_{m2}^{(q)} \left[\sum_{g=1}^{q-1} A_{mq}^{(g)} + \sum_{h=q+1}^{n} A_{mq}^{(h)} \right] \right\} e_{\varphi 2}^{(q)} \tag{10a}$$

$$H_{zb}^{(q)} = \sum_{m=-\infty}^{\infty} \left\{ b_m^{(q)} H_{m2}^{(q)} + J_{m2}^{(q)} \left[\sum_{g=1}^{q-1} B_{mq}^{(g)} + \sum_{h=q+1}^{n} B_{mq}^{(h)} \right] \right\} e_{\varphi 2}^{(q)} \tag{10b}$$

$$\begin{aligned} E_{\varphi b}^{(q)} = E_{\varphi inc}^{(q)} + \sum_{m=-\infty}^{\infty} \frac{\beta\cos\theta_0}{k_b^2 \sin^2\theta_0} \Bigg\{ \Bigg[a_m^{(q)} \frac{m}{\rho_2^{(q)}} H_{m2}^{(q)} + J_{m2}^{(q)} \sum_{\substack{k=1\\k\neq q}}^{N} A_{ma}^{(k)} \Bigg] \\ + \frac{j\omega\mu_b}{k_{b0}} \Bigg[b_m^{(q)} H_m^{(q)\prime} + J_{m2}^{(q)\prime} \Bigg[\sum_{g=1}^{q-1} B_{mq}^{(g)} + \sum_{h=q+1}^{N} B_{mq}^{(h)} \Bigg]\Bigg]\Bigg\} e_{\varphi_2}^{(q)} \end{aligned} \tag{10c}$$

$$\begin{aligned} H_{\varphi_b}^{(q)} \\ = H_{\varphi inc}^{(q)} + \sum_{m=-\infty}^{\infty} \left(-\frac{j\omega\varepsilon_b}{k_{b0}}\right) \Bigg\{ \Bigg[a_m^{(q)} H_{m2}^{(q)\prime} + J_{m2}^{(q)\prime} \Bigg[\sum_{g=1}^{q-1} A_{mq}^{(q)} + \sum_{g=h+1}^{N} A_{mq}^{(h)} \Bigg]\Bigg] \\ + \frac{\cos\theta_0}{k_b \sin^2\theta_0} \Bigg[\sum_{m=-\infty}^{\infty} b_m^{(q)} \frac{m}{\rho_2^{(q)}} H_{m2}^{(q)} + J_{m2}^{(q)} \Bigg[\sum_{g=1}^{q-1} B_{mq}^{(g)} + \sum_{g=h+1}^{N} B_{mq}^{(h)} \Bigg]\Bigg]\Bigg\} e_{\varphi_2}^{(q)} \end{aligned} \tag{10d}$$

and

$$A_{mq}^{(g)}\left(B_{mq}^{(g)}\right) = \sum_{l=-\infty}^{\infty} a_m^{(g)} \left(b_m^{(g)}\right) H_{l-m}^{(2)} \left(k_{b0} D^{(gq)}\right) e^{j(l-m)\varphi_0}, \quad g < q, \tag{10e}$$

$$A_{mq}^{(h)}\left(B_{mq}^{(h)}\right) = \sum_{l=-\infty}^{\infty} (-1)^{(l+m)} a_m^{(h)} \left(b_m^{(h)}\right) H_{l-m}^{(2)} \left(k_{b0} D^{(hq)}\right) e^{j(l-m)\varphi_0}, \ h < q \tag{10f}$$

in which the factor $e_2^{(q)}$ is understood and suppressed in (10a–d), and the translation addition theorem for cylindrical wave function is also employed here. $H_{m2}^{(q)} = H_m^{(2)}\left(k_{b0}R_2^{(q)}\right)$ is the mth-order Hankel function of the second kind, $H_{m2}^{(q)'} = H_m^{(2)'}\left(k_{b0}\rho_2^{(q)}\right)$ denotes the derivative of $H_{m2}^{(q)}$ with respect to its argument, $e_{\varphi_2}^{(q)} = e^{jm\varphi_2^{(q)}}$, and $a(b)_m^{(q)}$ are the unknown scattering coefficients.

Similarly, the boundary conditions are given as follows: for the HCS at $\rho_2^{(q)} = R_2^{(q)}$:

$$\begin{aligned} &E_{z2}^{(q)} = E_{zb}^{(q)}, \quad E_{\varphi_2}^{(q)} = E_{\varphi_b}^{(q)}, \quad E_{\varphi_2}^{(q)} + E_{z2}^{(q)}\tan\Psi_2^{(q)} = 0, \\ &H_{\varphi_2}^{(q)} + H_{z2}^{(q)}\tan\Psi_2^{(q)} = H_{\varphi b}^{(q)} + H_{zb}^{(q)}\tan\Psi_2^{(q)} \end{aligned} \tag{11a}$$

and for the T-strips $\left(\Psi_2^{(q)} = 0\right)$ at $\rho_2^{(q)} = R_2^{(q)}$,

$$E_{z2}^{(q)} = E_{zb}^{(q)}, \quad E_{\varphi_2}^{(q)} = E_{\varphi_b}^{(q)} = 0, \quad H_{\varphi_2}^{(q)} = H_{\varphi_b}^{(q)} \tag{11b}$$

while for L-strips $\left(\Psi_2^{(q)} = 90°\right)$, we get

$$E_{z2}^{(q)} = E_{zb}^{(q)} = 0, \quad E_{\varphi_2}^{(q)} = E_{\varphi_b}^{(q)}, \quad H_{z2}^{(q)} = H_{zb}^{(q)} \tag{11c}$$

Introducing (10) into (11a), (11b), and (11c) yields a system of equations of infinite series for determining the unknown coefficients $\left\{D_{1m\pm}^{(q)}, a_m^{(q)}, b_m^{(q)}, q = 1, 2, \ldots, N\right\}$, respectively. For instance, for the case of $\psi_2^{(q)} \neq 0°, 90°$, we have

$$\begin{aligned} &\sum_{m=-\infty}^{\infty}\left[D_{1m+}^{(q)}V_{nm}^{(q,1)}\left(R_2^{(q)}\right) + D_{1m-}^{(q)}V_{nm}^{(q,2)}\left(R_2^{(q)}\right)\right] \\ &= \left[S_1^{(q)}J_{m2}^{(q)} + a_m^{(q)}H_{m2}^{(q)} + J_{m2}^{(q)}\left[\sum_{g=1}^{q-1}A_{mq}^{(g)} + \sum_{g=h+1}^{N}A_{mq}^{(h)}\right]\right]e^{jm\varphi_0} \end{aligned} \tag{12a}$$

$$\begin{aligned} &\sum_{m=-\infty}^{\infty}\left[D_{1m+}^{(q)}V_{nm}^{(q,3)}\left(R_2^{(q)}\right) + D_{1m-}^{(q)}V_{nm}^{(q,4)}\left(R_2^{(q)}\right)\right] \\ &= \frac{1}{k_{b0}^2}\left\{\frac{m\beta}{R_2^{(q)}}\left[S_1^{(q)}J_{m2}^{(q)} + a_m^{(q)}H_{m2}^{(q)} + J_{m2}^{(q)}\left[\sum_{g=1}^{q-1}A_{mq}^{(g)} + \sum_{g=h+1}^{N}A_{mq}^{(h)}\right]\right]\right. \end{aligned}$$

$$+ j\omega\mu_b k_{b0}\left[b_m^{(q)}H_{m2}^{(q)'}+J_{m2}^{(q)'}\left[\sum_{g=1}^{q-1}B_{mq}^{(g)}+\sum_{g=h+1}^{N}B_{mq}^{(h)}\right]\right]\Bigg\}e^{jm\varphi_0} \tag{12b}$$

$$\sum_{m=-\infty}^{\infty}\left\{D_{1m+}^{(q)}\left[\tan V_{nm}^{(q,1)}\left(R_2^{(q)}\right)+V_{nm}^{(q,5)}\left(R_2^{(q)}\right)\right]\right.$$
$$\left.+D_{1m-}^{(q)}\left[\tan V_{nm}^{(q,2)}\left(R_2^{(q)}\right)+V_{nm}^{(q,6)}\left(R_2^{(q)}\right)\right]\right\}e^{jm\varphi_0}=0 \tag{12c}$$

$$\sum_{m=-\infty}^{\infty}\left\{D_{1m\pm}^{(q)}\left[\tan V_{nm}^{(q,3)}\left(R_2^{(q)}\right)+V_{nm}^{(q,7)}\left(R_2^{(q)}\right)\right]\right.$$
$$\left.+D_{1m-}^{(q)}\left[\tan V_{nm}^{(q,4)}\left(R_2^{(q)}\right)+V_{nm}^{(q,8)}\left(R_2^{(q)}\right)\right]\right\}$$
$$=\left\{\tan\psi_2^{(q)}\left[b_m^{(q)}H_{m2}^{(q)}+J_{m2}^{(q)}\left[\sum_{g=1}^{q-1}B_{mq}^{(g)}+\sum_{g=h+1}^{N}B_{mq}^{(h)}\right]\right]\right.$$
$$+\frac{1}{k_{b0}^2}\left\{-j\omega\varepsilon_b k_{b0}\left[S_1^{(q)}J_{m2}^{(q)'}+a_m^{(q)}H_2^{(q)'}+J_{m2}^{(q)'}\left[\sum_{g=1}^{q-1}A_{mq}^{(q)}+\sum_{g=h+1}^{N}A_{mq}^{(h)}\right]\right]\right\}$$
$$\left.+\frac{m\beta}{R_2^{(q)}}\left[b_m^{(q)}H_{m2}^{(q)}+J_{m2}^{(q)}\left[\sum_{g=1}^{q-1}B_{mq}^{(g)}+\sum_{g=h+1}^{N}B_{mq}^{(h)}\right]\right]\right\}e^{m\varphi_0} \tag{12d}$$

where $S_1^{(q)}=j^m\sin\theta_0 e^{j\delta(q)}$. It should be noted that the above system is solved for each m independently. To find a numerical solution of (12), we must truncate it to a finite size. Physical insight into the problem suggests that the series-truncation number N_0 $(m,n,l=-N_0,\ldots,N_0)$, depends on the electric size $\max\left\{k_bR_2^{(q)}\right\}$. Including the terms $(2N_0+1)$ leads to $4N(2N_0+1)$ equations, i.e.,

$$\left[\tilde{D}\right]_{4N(2N_0+1)\times 4N(2N_0+1)}\left[\tilde{a}\right]_{4N(2N_0+1)\times 1}=\left[\tilde{f}\right]_{4N(2N_0+1)\times 1} \tag{13}$$

where $[\tilde{a}]$ is the unknown coefficient matrix, $\left[\tilde{f}\right]$ the incident matrix, and $\left[\tilde{D}\right]$ a sparse matrix. For instance, when $N=4$, $\left[\tilde{D}\right]$ contains 144 zero sub-matrixes of $(2N_0+1)\times(2N_0+1)$, and the sparsity is increased if $\theta_0=90^\circ$. Due to the sparsity property of $\left[\tilde{D}\right]$ the dimension of the system can be easily reduced prior to numerical solution by eliminating variables which yields a reduction in dimension, and hence, in the computation time, especially for the large N. With the θ_0 approaching to 0° or 180°, the system matrix $\left[\tilde{D}\right]$ becomes quite ill-conditioned because strongly guided modes (with respect to $z_2^{(q)}$-axis) are excited. On the other hand, the truncation term number N_0 should be chosen to be large enough when $k_{b0}R_2^{(q)} \gg 1$. Under such circumstances the asymptotic technique of geometric optics must be developed. For the obliquely incident wave of a TE_z-polarization plane wave, the related field components and a system matrix equations for determining the unknown coefficients are presented in Appendix A.

Of particular interest here is the Mueller scattering matrix elements which contain enough information of co- and cross-polarized wave scattering by the objects. Furthermore, they can be exploited for determining the constitutive parameters of various complex media [34, 35], and the Mueller matrix is

$$[M] = [m_{uv}]_{4\times 4}, \quad m_{uv} = m_{uv}(T_1, T_2, T_3, T_4) \tag{14a}$$

with [24]

$$\begin{aligned} T_1 &= \sum_{m=-\infty}^{\infty}\sum_{q=1}^{N}\frac{j^m}{\sin\theta_0}a_m^{(q)}e^{jk_bD^{(q)}\cos\varphi}e^{jm(\varphi-\varphi_0)}, \\ T_2 &= T_1|_{a\to c}, \qquad T_3 = \frac{T_1|_{a\to d}k}{\omega\mu_b}, \qquad T_4 = \frac{T_1|_{a\to c}\,\omega\mu_b}{k_b} \end{aligned} \tag{14b}$$

where $(u, v) = 1,\ 2,\ 3,$ and 4, and all the 16 elements can be calculated based on the formulas shown in (10) and in Appendix A, These elements of $[M]$ contain very sensitive co- and cross-polarized scattering information for identifying the constitutive features of bianisotropic objects with different magnetic symmetry groups, and certainly, significant cross depolarizing effects can be expected in the scattered fields.

On the other hand, the normalized scattering cross section per unit length of the above linear array can be derived following the procedure used in [3]. For example, for four parallel two-layered eccentric bianisotropic cylinders with equal interval $D^{(2)}$, we find

$$C_{sca}^{TM} = \frac{4\eta\omega}{k_{b0}}\left(\varepsilon_b C_{sca}^{TMI} + \mu_b C_{sca}^{TMII}\right) \tag{15a}$$

$$C_{sca}^{TE} = \frac{4\omega}{\eta k_{b0}^2}\left(\mu_b C_{sca}^{TMI} + \varepsilon_b C_{sca}^{TMII}\right) \tag{15b}$$

and

$$\begin{aligned}
C_{sca}^{TMI} =& \sum_{m=-\infty}^{\infty}\sum_{q=1}^{4}\left|a_m^{(q)}\right|^2 + \mathrm{Re}\sum_{n=-\infty}^{\infty}\gamma\left[e_- J_{n-m}\left(k_{b0}D^{(2)}\right)\left(a_m^{(1)}a_n^{(2)^*}\right.\right.\\
&\left.+a_m^{(2)}a_n^{(3)^*} + a_m^{(3)}a_n^{(4)^*}\right) + e_+ J_{n-m}\left(k_{b0}D^{(2)}\right)\left(a_m^{(2)}a_n^{(1)^*} + a_m^{(3)}a_n^{(2)^*}\right.\\
&\left.+a_m^{(4)}a_n^{(3)^*}\right) + e_- J_{n-m}\left(2k_{b0}D^{(2)}\right)\left(a_m^{(1)}a_n^{(3)^*} + a_m^{(2)}a_n^{(4)^*}\right)\\
&+ e_+ J_{n-m}\left(2k_{b0}D^{(2)}\right)\left(a_m^{(3)}a_n^{(1)^*} + a_m^{(4)}a_n^{(2)^*}\right)\\
&\left.+J_{n-m}\left(3k_{b0}D^{(2)}\right)\left(e_- a_m^{(1)}a_n^{(4)^*} + e_+ a_m^{(4)}a_n^{(1)^*}\right)\right] \qquad (15c)
\end{aligned}$$

$$\begin{aligned}
C_{sca}^{TMI} =& \sum_{m=-\infty}^{\infty}\sum_{q=1}^{4}\left|b_m^{(q)}\right|^2 + \mathrm{Re}\sum_{n=-\infty}^{\infty}\gamma\left[e_- J_{n-m}\left(k_{b0}D^{(2)}\right)\left(b_m^{(1)}b_n^{(2)^*}\right.\right.\\
&\left.+b_m^{(2)}b_n^{(3)^*} + b_m^{(3)}b_n^{(4)^*}\right) + e_+ J_{n-m}\left(k_{b0}D^{(2)}\right)\left(b_m^{(2)}b_n^{(1)^*} + b_m^{(3)}b_n^{(2)^*}\right.\\
&\left.+b_m^{(4)}b_n^{(3)^*}\right) + e_- J_{n-m}\left(2k_{b0}D^{(2)}\right)\left(b_m^{(1)}b_n^{(3)^*} + b_m^{(2)}b_n^{(4)^*}\right)\\
&+ e_+ J_{n-m}\left(2k_{b0}D^{(2)}\right)\left(b_m^{(3)}b_n^{(1)^*} + b_m^{(4)}b_n^{(2)^*}\right)\\
&\left.+J_{n-m}\left(3k_{b0}D^{(2)}\right)\left(e_- b_m^{(1)}b_n^{(4)^*} + e_+ b_m^{(4)}b_n^{(1)^*}\right)\right] \qquad (15d)
\end{aligned}$$

and $C_{sca}^{TEI}=C_{sca}^{TMI}|_{a\to d}$, $C_{sca}^{TEII}=C_{sca}^{TMII}|_{b\to c}$, $\gamma=(-1)^n j^{(m+n)}e^{j(m-n)\varphi_0}$, $\beta=(n-m)\pi/2$, $e_\pm=e^{\pm j\beta}$. The aster above denotes complex conjugate, and Re the real part.

4. NUMERICAL RESULTS AND DISCUSSION

Obviously, there are countless numbers of interesting cases that we can investigate, however, here we pay our major attention to the effects of double HCS at $\rho_l^{(q)} = R_l^{(q)}$ on the Mm of the above eccentric bianisotropic cylinder array. For practical consideration, we let $\varepsilon_b = \varepsilon_0$ and $\mu_b = \mu_0$ in the following numerical examples.

At first, the convergence behavior of the truncation terms in the series summation has been checked. Table 1 lists the elements of m_{nv} ($uv = 11,\ 12,$ and 13) as a function of the integer N_0, which is the absolute value of the upper limit of the index in the series summation. The parameters chosen in the analysis are $f = 10\,\mathrm{GHz}$, $k_bR_1^{(q)} = 0.5k_bR_2^{(q)} = 2.0k_bd_c^{(q)} = 1.0$, $k_bD^{(1)} = 20.0$, $k_bD^{(2)} = 40.0$,

Table 1. m_{uv} as a function of the integer N_0 for four eccentric bianisotropic cylinders with double helical conductances.

N_0	$\log m_{11}$	m_{12}/m_{11}	m_{13}/m_{11}
4	7.638923	-0.703052	0.055291
5	6.407612	-0.550661	0.396088
6	7.100031	-0.717271	0.126303
7	6.680107	-0.669196	0.240703
8	6.697588	-0.673163	0.234063
9	6.696777	-0.672981	0.234374
10	6.696824	-0.672991	0.234357
11	6.696825	-0.672991	0.234357
12	6.696826	-0.672991	0.234357

$k_b D^{(3)} = 60.0$. The inner cylinder ($q = 1$, 2, 3, and 4) are characterized by the ordinary uniaxial form with the magnetic symmetry group $D_{\infty h}$, i.e.,

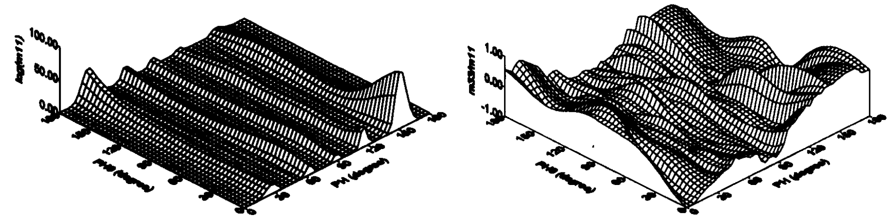
$$\left[\varepsilon^{(1,q)}\right] = \varepsilon_0 \begin{bmatrix} 2.0 & 0 & 0 \\ 0 & 2.0 & 0 \\ 0 & 0 & 2.5 \end{bmatrix}, \quad \left[\mu^{(1,q)}\right] = \mu_0 \begin{bmatrix} 1.0 & 0 & 0 \\ 0 & 1.0 & 0 \\ 0 & 0 & 1.5 \end{bmatrix},$$
$$\left[\xi_e^{(1,q)}\right] = \left[\xi_m^{(1,q)}\right] = j10^{-5}\overline{\overline{I}},$$

while the outer layer of the cylinders are assumed to be the uniaxial bianisotropic form with the magnetic symmetry group D_∞:

$$\left[\varepsilon^{(2,q)}\right] = \varepsilon_0 \begin{bmatrix} 4.0 & 0 & 0 \\ 0 & 4.0 & 0 \\ 0 & 0 & 4.5 \end{bmatrix}, \quad \left[\mu^{(2,q)}\right] = \mu_0 \begin{bmatrix} 2.0 & 0 & 0 \\ 0 & 2.0 & 0 \\ 0 & 0 & 2.5 \end{bmatrix},$$
$$\left[\xi_e^{(2,q)}\right] = -\left[\xi_m^{(2,q)}\right] = j\sqrt{\mu_0\varepsilon_0} \begin{bmatrix} 0.6 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0.8 \end{bmatrix}.$$

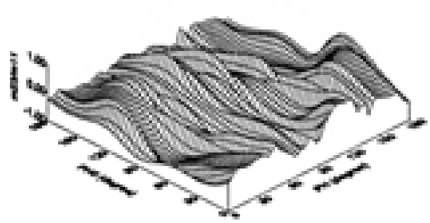
The twist angles are chosen to be $\psi_1^{(q)} = 30°$ (RH), $\psi_2^{(q)} = 150°$ (LH), and $\theta_0 = 45°$, $\varphi = \varphi_0 = 60°$.

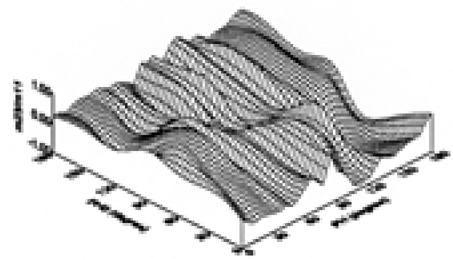
In Table 1, the outer radii of the four parallel cylinders are comparable with the incident wavelength since we choose $k_b R_2^{(q)} = 2.0$ here. It



(a)

$\theta_0 = 45°(135°),\ \psi_1^{(q)} = 45°(135°),\ \psi_2^{(q)} = 135°(45°)$





(b) (c)

$\theta_0 = 45°, \psi_1^{(q)} = 45°, \psi_2^{(q)} = 135°;$ $\quad \theta_0 = 135°, \psi_1^{(q)} = 135°, \psi_2^{(q)} = 45°;$

Figure 2. m_{vu} (uv = 11, 23, and 33) as functions of φ_0 and φ ($N_0 \geq 8$).

is obvious that not many terms yields a convergent solution. However, as θ_0 tends to $0°$ or $180°$, more terms should be kept in the series summations, while the twist angles $\psi_{1,2}^{(q)}$ has little effect on the series convergence.

Furthermore, Fig. 2 depicts the three-dimensional variation of $[m_{uv}]$ with the changing of φ_0 and φ for the above magnetic symmetry group case. The parameters are the same as in Table 1, except that the incident direction and twist angles are varied.

In Fig. 2, only three elements of [M] are presented and another elements are also examined but are suppressed here to save space. The values of m_{uv} ($uv = 23, 33$) are normalized to m_{11} at each angle. Physically, m_{11} represents the total signal intensity which is the scattered intensity resulting in from un-polarized incident radiation ($|m_{11}|^2 > 1,\ |m_{uv}/m_{11}| < 1,\ u, v \neq 1$). So the minimum and maximum values of the vertical scale of the normalized elements are -1.0 and $+1.0$, which represent -100% and $+100\%$ polarization, respectively. It is well known that there exist several simplified relationships among the elements of [M] for the isotropic objects [3]. However, all the 16 elements here are independent and are directly governed by the twist angles $\psi_l^{(q)}$, which introduce significant cross-depolarized effect in the most scattering directions, and strong cross depolarized effect can be expected. On the other hand, by comparing (c) with (b) or by another numerical test, we at first find,

$$m_{uv}\left(\theta_0,\psi_1^{(q)},\psi_2^{(q)},\left[\xi_e^{(l,m)}\right],\left[\xi_m^{(l,q)}\right]\right)$$
$$=m_{uv}\left(180^\circ-\theta_0,180^\circ-\psi_1^{(q)},180^\circ-\psi_2^{(q)},\left[\xi_m^{(l,m)}\right],\left[\xi_e^{(l,q)}\right]\right) \quad (16a)$$

with $uv=\{11,\ 12,\ 21,\ 22,\ 33,\ 34,\ 43,\ \text{and}\ 44\}$, and

$$m_{\tilde{u}\tilde{v}}\left(\theta_0,\psi_1^{(q)},\psi_2^{(q)},\left[\xi_e^{(l,m)}\right],\left[\xi_m^{(l,q)}\right]\right)$$
$$=-m_{\tilde{u}\tilde{v}}\left(180^\circ-\theta_0,180^\circ-\psi_1^{(q)},180^\circ-\psi_2^{(q)},\left[\xi_m^{(l,m)}\right],\left[\xi_e^{(l,q)}\right]\right) \quad (16b)$$

with $\tilde{u}\tilde{v}=\{13,\ 14,\ 23,\ 24,\ 31,\ 32,\ 41,\ \text{and}\ 42\}$. Physically, (16a, b) demonstrate that the co-polarized components uv are symmetric while the cross-polarized components $\tilde{u}\tilde{v}$ are anti-symmetric. It should be noted that (16a, b) have no relation to the geometrical size, location and number of the cylinders, or the loss degree of UBMs even if the magnetoelectric cross coupling elements $C_{1,2}^{(l,q)}(C=\xi_e,\xi_m)$ are in the form of $C_{1,2}^{(l,q)}=C_{e1,2}^{(l,q)}-jC_{m1,2}^{(l,q)}\left\{\mathrm{Re}(C_{e,m1,2}^{(l,q)})\geq 0\right\}$. On the other hand, we know that there exist several kinds of UBM, such as

$$1^\circ\ (D_\infty)C_1^{(l,q)}\neq 0,\ C_2^{(l,q)}\neq 0,\ C_{12}^{(l,q)}=0,\ C=\varepsilon,\mu,\xi_e,\xi_m,\ \text{and}\ \xi_m=-\xi_e \quad (17a)$$

$$2^\circ\ (D_h(D))\ \text{as}\ 1^\circ,\ \text{except that}\ \xi_m=\xi_e \quad (17b)$$

$$3^\circ\ (D)\ \text{as}\ 1^\circ,\ \text{except that}\ \xi_m\neq\xi_e \quad (17c)$$

$$4^\circ\ (C_v)C_1^{(l,q)}\neq 0,\ C_2^{(l,q)}\neq 0,\ C_{12}^{(l,q)}=0,\ C=\varepsilon,\mu;$$
$$\xi_{1,2e}^{(l,q)}=0,\ \xi_{m1,2}^{(l,q)}=0,\ \xi_{e12}^{(l,q)}\neq 0,\ \xi_{m12}^{(l,q)}\neq 0,\ \xi_{m12}^{(l,q)}=-\xi_{e12}^{(l,q)} \quad (18a)$$

$$5^\circ\ (C_v)\ \text{as}\ 4^\circ,\ \text{except that}\ \xi_{m12}^{(l,q)}=\xi_{e12}^{(l,q)} \quad (18b)$$

$$6^\circ\ (C_v)\ \text{as}\ 4^\circ,\ \text{except that}\ \xi_{m12}^{(l,q)}\neq\xi_{e12}^{(l,q)} \quad (18c)$$

All these models could be constructed by embedding helical elements into a homogeneous medium using some special technologies. Structurally, the above UBM (1°) in Fig. 2 possess three (helical) chiralities. Exchanging the locations of $\left[\xi_{e,m}^{(l,q)}\right]$ in (1a, b) and replacing $\Psi_1^{(q)}$

by $180° - \psi_1^{(q)}$ in (16a, b) mean the simultaneously reversed rotations of helices. While additional numerical tests prove that (16a,b) only hold true for the UBM (4°) with the magnetic symmetry group $C_{\infty v}$. For models 2°, 3°, 5°, and 6° (16a, b) are not satisfied. Furthermore, for 1° and 2° (16a, b) can be extended to more general case:

$$m_{uv}\left(\theta_0, \psi_1^{(R)}, \psi_1^{(L)}, \psi_2^{(R)}, \psi_2^{(L)}, \left[\xi_e^{(l,m)}\right], \left[\xi_m^{(l,q)}\right]\right) = m_{uv}\Big(180° - \theta_0,$$
$$180° - \psi_1^{(R)}, 180° - \psi_1^{(L)}, 180° - \psi_2^{(R)}, 180° - \psi_2^{(L)}, \left[\xi_m^{(l,m)}\right], \left[\xi_e^{(l,q)}\right]\Big) \tag{19a}$$

$$m_{\tilde{u}\tilde{v}}\left(\theta_0, \psi_1^{(R)}, \psi_1^{(L)}, \psi_2^{(R)}, \psi_2^{(L)}, \left[\xi_e^{(l,m)}\right], \left[\xi_m^{(l,q)}\right]\right) = -m_{\tilde{u}\tilde{v}}\Big(180° - \theta_0,$$
$$180° - \psi_1^{(R)}, 180° - \psi_1^{(L)}, 180° - \psi_2^{(R)}, 180° - \psi_2^{(L)}, \left[\xi_m^{(l,m)}\right], \left[\xi_e^{(l,q)}\right]\Big) \tag{19b}$$

where R is the number of cylinders with the RH surface conductance, and L is the number of cylinders with the LH surface conductance $(R + L = N)$. Naturally, (19a, b) hold true for the linear array of biaxial bianisotropic cylinders with one special form of the magnetic symmetry group C_{2v} as [33]:

$$\left[C^{(l,q)}\right] = \begin{bmatrix} C_1^{(l,q)} & 0 & 0 \\ 0 & C_2^{(l,q)} & 0 \\ 0 & 0 & C_3^{(l,q)} \end{bmatrix}, \quad C = \varepsilon, \mu,$$
$$\left[\xi_e^{(l,q)}\right] = -\left[\xi_m^{(l,q)}\right] = j\begin{bmatrix} 0 & -\xi_{e12}^{(l,q)} & 0 \\ \xi_{e12}^{(l,q)} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{20}$$

For the ordinary uniaxial case $\left(\left[\xi_e^{(l,q)}\right] = \left[\xi_m^{(l,q)}\right] = 0\overline{\overline{I}}\right)$, numerical tests prove that, when

1. $\theta_0 = 90°$ (normal incidence), and $\psi_1^{(q)} = 0°$ $(l = 1, 2,\ q = 1, \ldots, N)$, i.e., the helical surfaces are reduced to be the T-strips, we have,

$$[M] = \begin{bmatrix} m_{11} & m_{12} & 0 & 0 \\ m_{12} & m_{11} & 0 & 0 \\ 0 & 0 & m_{33} & m_{34} \\ 0 & 0 & -m_{34} & m_{33} \end{bmatrix} \tag{21}$$

and there exist only four independent elements in $[M]$. Since $m_{34} = -m_{43} = 0$, the scattered fields still contain circularly polarized component while $m_{13,14,23,24,31,32,41,42} = 0$ indicates that the cylinders with T-strips do not depolarize the incident waves.

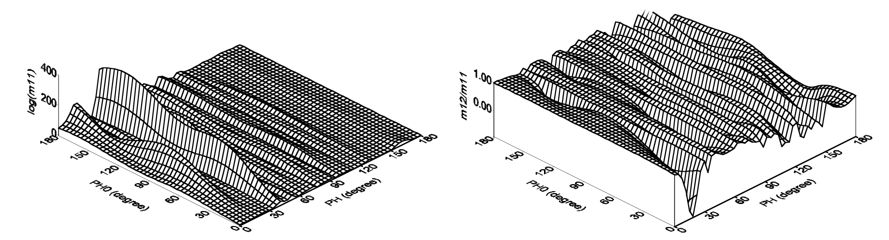
2. $\theta_0 \neq 90^\circ$, but $\psi_1^{(q)} = 0^\circ$. Under such circumstances, $[M]$ takes the general form. In some special directions, we always find,

$$\begin{aligned}
&m_{11} = m_{22}|_{\varphi_0=\varphi=0^\circ}, && m_{11} = m_{22}|_{\varphi_0=\varphi=180^\circ},\\
&m_{11} = m_{22}|_{\varphi_0=0^\circ,\varphi=180^\circ}, && m_{11} = m_{22}|_{\varphi_0=180^\circ,\varphi=0^\circ};\\
&m_{12} = m_{21}|_{\varphi_0=\varphi=0^\circ}, && m_{12} = m_{21}|_{\varphi_0=\varphi=180^\circ},\\
&m_{12} = m_{21}|_{\varphi_0=0^\circ,\varphi=180^\circ}, && m_{12} = m_{21}|_{\varphi_0=180^\circ,\varphi=0^\circ};\\
&m_{33} = m_{44}|_{\varphi_0=\varphi=0^\circ}, && m_{33} = m_{44}|_{\varphi_0=\varphi=180^\circ},\\
&m_{33} = m_{44}|_{\varphi_0=0^\circ,\varphi=180^\circ}, && m_{33} = m_{44}|_{\varphi_0=180^0,\varphi=0^\circ};\\
&m_{34} = -m_{43}|_{\varphi_0=\varphi=0^\circ}, && m_{34} = -m_{43}|_{\varphi_0=\varphi=180^\circ},\\
&m_{34} = -m_{43}|_{\varphi_0=0^\circ,\varphi=180^\circ}, && m_{34} = -m_{43}|_{\varphi_0=180^\circ,\varphi=0^\circ}\\
&m_{31} = m_{42}|_{\varphi_0=\varphi=0^\circ}, && m_{31} = m_{42}|_{\varphi_0=\varphi=180^\circ},\\
&m_{31} = m_{42}|_{\varphi_0=0^\circ,\varphi=180^\circ}, && m_{31} = m_{42}|_{\varphi_0=180^\circ,\varphi=0^\circ};\\
&m_{13} = m_{24}|_{\varphi_0=\varphi=0^\circ}, && m_{13} = m_{24}|_{\varphi_0=\varphi=180^\circ},\\
&m_{13} = m_{24}|_{\varphi_0=0^\circ,\varphi=180^\circ}, && m_{13} = m_{24}|_{\varphi_0=180^\circ,\varphi=0^\circ}
\end{aligned} \tag{22}$$

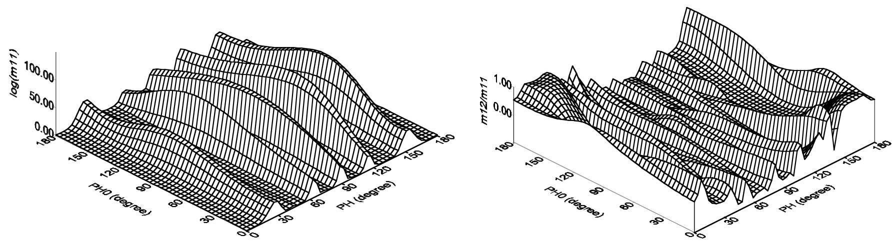
It is pointed out that (22) has no relation to the value of $\psi_1^{(q)}$ but we should have $\psi_2^{(q)} = 0^\circ$.

For the uniaxial bianisotropic cylinders (D_∞) in Fig. 2, when the outer surfaces are L-strips, i.e., $\psi_2^{(q)} = 90^\circ$ at $\rho_2^{(q)} = R_2^{(q)}$, $[M]$ keeps the same simple form as in (21) and no-depolarized effect in the scattered fields can be expected even if $\psi_1^{(q)} \neq 0^\circ$ and $\theta_0 \neq 90^\circ$. So, under such circumstances, the magnetoelectric cross coupling effects in bianisotropic cylinders are completely shielded by the L-strips and the cylinders are just "isotropic".

Figure 3 depicts $[m_{uv}]$ as a function of φ for four parallel eccentric bianisotropic cylinders corresponding to the magnetic symmetry groups $D_\infty(C_\infty)$, and both T-strips and L-strips at $\rho_2^{(q)} = R_2^{(q)}$ $(q = 1, 2, 3, 4)$ are considered, respectively. All the geometrical parameters and the operating frequency are the same as in Table 1. In addition, we have $k_bR_1^{(q)} = 0.5k_bR_2^{(q)} = 2.0k_bd_c^{(q)} = 1.0$, $k_bD^{(1)} = 20.0$,



(a) $\theta_0 = 90°,\ \psi_1^{(q)} = 45°,\ \psi_2^{(q)} = 0°$ (T-strips)



(b) $\theta_0 = \psi_1^{(q)} = 45°,\ \psi_2^{(q)} = 90°$ (L-strips)

Figure 3. m_{uv} $(uv = 11, 12)$ as functions of φ_0 and φ.

$k_b D^{(2)} = 40.0,\ k_b D^{(3)} = 60.0)$,

$$\left[\mu^{(1,q)}\right] = \mu_0 \overline{\overline{I}}, \qquad \left[\xi_e^{(1,q)}\right] = \left[\xi_m^{(1,q)}\right] = j10^{-5}\overline{\overline{I}},$$

$$\left[\varepsilon^{(1,q)}\right] = \varepsilon_0 \begin{bmatrix} 2.0 & -j0.3 & 0 \\ j0.3 & 2.0 & 0 \\ 0 & 0 & 2.5 \end{bmatrix},$$

$$\left[\varepsilon^{(2,q)}\right] = \varepsilon_0 \begin{bmatrix} 5.0 & -j0.6 & 0 \\ j0.6 & 5.0 & 0 \\ 0 & 0 & 5.5 \end{bmatrix},$$

$$\mu_1^{(2,q)} = \mu_0 \left\{1 + \omega_0^{(2,q)} \omega_m^{(2,q)} \Big/ \left[\omega_0^{(2,q)^2} - \omega^2\right]\right\},$$

$$\mu_{12}^{(2,q)} = -\mu_0 \omega \omega_m^{(2,q)} \Big/ \left[\omega_0^{(2,q)^2} - \omega^2\right],$$

$$\omega_m^{(2,q)} = 2.21 \times 10^5 M_s^{(2,q)}, \qquad M_s^{(2,q)} \mu_0 = 0.275T,$$

$$\omega_0^{(2,q)} / \omega_m^{(2,q)} = 0.3, \qquad \mu_2^{(2,q)} = \mu_0,$$

and

$$\left[\xi_e^{(2,q)}\right] = -\left[\xi_m^{(2,q)}\right] = j\sqrt{\mu_0 \varepsilon_0} \begin{bmatrix} 0.6 & -0.5 & 0 \\ 0.5 & 0.6 & 0 \\ 0 & 0 & 0.8 \end{bmatrix} (D_\infty(C_\infty)).$$

In Figs. 3a, b, the inner layer of four parallel eccentric cylinders is assumed to be the ordinary gyroelectric medium, respectively, while the outer layer is bianisotropic and the magnetic anisotropy is just in the gyrotropic from [23]. In cases (a) and (b) there only exist the co-polarized components in the scattered fields ($m_{11} = m_{22}$, $m_{12} = m_{21}$, $m_{33} = m_{44}$, $m_{34} = -m_{43}$, $m_{13,14,23,24,31,32,41,42} = 0$). Since $\theta_0 = 90°$ in case (a), fewer resonant peaks in the total scattered field intensity m_{11} are developed, but the maximum of peak is much more larger than that of the oblique incidence in case (b). Also, various numerical tests prove that (19a, b) only hold true for the eccentric bianisotropic cylinders with the magnetic symmetry group $D_\infty(C_\infty)$; and they are not satisfied for the magnetic symmetry groups $C_{\infty v}(C_\infty)$ and $C_{\infty h}(C_\infty)$. For instance, in case (a),

$$m_{uv}\left(\theta_0 = 90°, \psi_1^{(R)}, \psi_1^{(L)}, \left[\xi_e^{(l,m)}\right], \left[\xi_m^{(l,q)}\right]\right) = m_{uv}\Big(\theta_0 = 90°,$$

$$180° - \psi_1^{(R)}, 180° - \psi_1^{(L)}, \left[\xi_m^{(l,m)}\right], \left[\xi_e^{(l,q)}\right]\Big)\Big|_{\psi_2^{(q)}=0°} (D_\infty(C_\infty)) \qquad (23a)$$

$$m_{uv}\left(\theta_0 = 90°, \psi_1^{(R)}, \psi_1^{(L)}, \left[\xi_e^{(l,m)}\right], \left[\xi_m^{(l,q)}\right]\right) \neq m_{uv}\Big(\theta_0 = 90°,$$

$$180° - \psi_1^{(R)}, 180° - \psi_1^{(L)}, \left[\xi_m^{(l,m)}\right], \left[\xi_e^{(l,q)}\right]\Big)(C_{\infty v}(C_\infty), C_{\infty h}(C_\infty)) \qquad (23b)$$

and for the oblique incidence, besides (23a, b) we have

$$m_{\tilde{u}\tilde{v}}\left(\theta_0, \psi_1^{(R)}, \psi_1^{(L)}, \left[\xi_e^{(l,m)}\right], \left[\xi_m^{(l,q)}\right]\right) = -m_{\tilde{u}\tilde{v}}\Big(180^0 - \theta_0,$$

$$180° - \psi_1^{(R)}, 180° - \psi_1^{(L)}, \left[\xi_m^{(l,m)}\right], \left[\xi_e^{(l,q)}\right]\Big)\Big|_{\psi_2^{(q)}=0°} (D_\infty(C_\infty)) \qquad (24a)$$

$$m_{\tilde{u}\tilde{v}}\left(180° - \theta_0, \psi_1^{(R)}, \psi_1^{(L)}, \left[\xi_e^{(l,m)}\right], \left[\xi_m^{(l,q)}\right]\right) \neq -m_{\tilde{u}\tilde{v}}\Big(180° - \theta_0,$$

$$180° - \psi_1^{(R)}, 180° - \psi_1^{(L)}, \left[\xi_m^{(l,m)}\right], \left[\xi_e^{(l,q)}\right]\Big) (C_{\infty v}(C_\infty), C_{\infty h}(C_\infty)) \qquad (24b)$$

In case (b), only (23a, b) exist for any incident direction $\theta_0(\neq 0°, 180°)$. So under such circumstances, m_{uv} are good indicators for distinguishing the bianisotropics between $D_\infty(C_\infty)$ and $C_{\infty v}(C_\infty)$ or $C_{\infty h}(C_\infty)$. On the other hand, it can be predicated that similar conclusions can be drawn for another kinds of linear arrays made of inhomogeneous cylinders such as impedance cylinders eccentrically coated with these bianisotropic media or ordinary gyrotropic media $\left(\text{i.e., } \left[\varepsilon^{(l,q)}\right] = \overline{\overline{I}}, \left[\xi_e^{(l,q)}\right] = \left[\xi_m^{(l,q)}\right] = 0\overline{\overline{I}}\right)$.

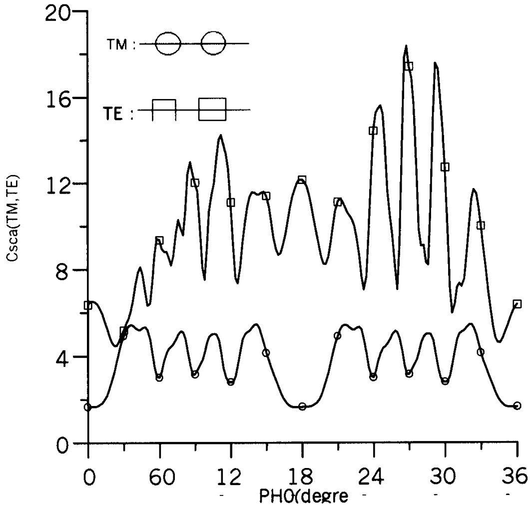
Finally, Fig. 4 depicts the extinction and scattering cross sections per unit length of four parallel eccentric bianisotropic cylinders as a function of $\varphi_0 (\varphi = 180° + \varphi_0)$ corresponding to different $\psi_l^{(q)}$, and all the constitutive parameters are the same as in Fig. 3 for the magnetic symmetry group $D_\infty(C_\infty)$.

In Fig. 4, both C_{sca}^{TM} and C_{sca}^{TE} are normalized to $4R_2^{(1)}$ each other, and the convergence behavior of the truncation terms in the series summation has also been checked here $(N_0 \geq 8)$. In case (a) we choose $\psi_2^{(q)} = 90°$ (L-strips) and in case (b) the helical conductances on $\rho_1^{(q)} = R_1^{(q)}$ and $\rho_2^{(q)} = R_2^{(q)}$ are all in the RH form $(\psi_1^{(q)} = \psi_2^{(q)} = 45°)$. In case (c) we let $\psi_2^{(q)} = 45°$ (RH), $\psi_1^{(q)} = 135°$ (LH), and just $\psi_1^{(q)} + \psi_2^{(q)} = 180°$. It is shown that in case (a) $C_{sca}^{TM} < C_{sca}^{TE}$, and conversely, in cases (b), (c) $C_{sca}^{TM} > C_{sca}^{TE}$. On the other hand, comparing case (c) with (b) or (a), it is clear that very high resonant peaks are observed and strong resonant scattering takes place under the conditions of $\theta_0 = \psi_2^{(q)} = 45°$, $\psi_1^{(q)} = 135°$. At low-frequency, we know that the resonance phenomenon for a magnetodielectric rod with an anisotropic helical conductance surface has already been examined in [4]. However, such resonant scattering can be damped by introducing the T- or L-strips at $\rho_2^{(q)} = R_2^{(q)}$. Changing the magnetic symmetry groups from $D_\infty(C_\infty)$ to $C_{\infty v}(C_\infty)$ or $C_{\infty h}(C_\infty)$, numerical tests show that,

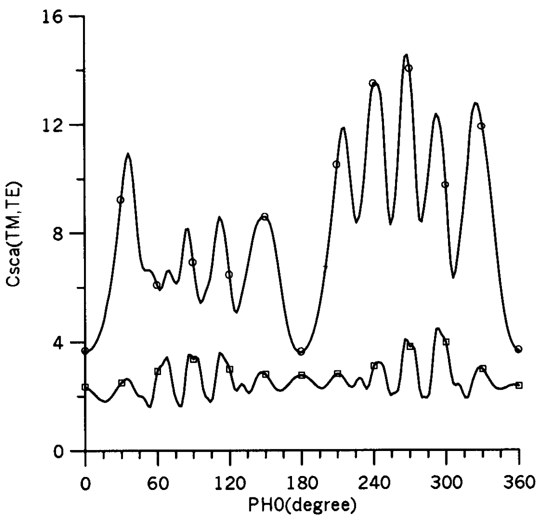
$$\begin{aligned} &C_{sca}^{TM(TE)}\left(\theta_0, \psi_1^{(R)}, \psi_1^{(L)}, \psi_2^{(R)}, \psi_2^{(L)}, \left[\xi_e^{(l,m)}\right], \left[\xi_m^{(l,q)}\right]\right) \\ &\quad = C_{sca}^{TM(TE)}\Big(180^0 - \theta_0, 180° - \psi_1^{(R)}, 180° - \psi_1^{(L)}, 180° - \psi_2^{(R)}, \\ &\qquad 180° - \psi_2^{(L)}, \left[\xi_m^{(l,m)}\right], \left[\xi_e^{(l,q)}\right]\Big)\,(D_\infty(C_\infty)) \end{aligned} \tag{25}$$

$$\begin{aligned} &C_{sca}^{TM(TE)}\left(\theta_0, \psi_1^{(R)}, \psi_1^{(L)}, \psi_2^{(R)}, \psi_2^{(L)}, \left[\xi_e^{(l,m)}\right], \left[\xi_m^{(l,q)}\right]\right) \\ &\quad \neq C_{sca}^{TM(TE)}\Big(180° - \theta_0, 180° - \psi_1^{(R)}, 180° - \psi_1^{(L)}, 180° - \psi_2^{(R)}, \\ &\qquad 180° - \psi_2^{(L)}, \left[\xi_m^{(l,m)}\right], \left[\xi_e^{(l,q)}\right]\Big)\,(C_{\infty v}(C_\infty), C_{\infty h}(C_\infty)) \end{aligned} \tag{26}$$

Here, (25) is applicable for the magnetic symmetry groups D_∞ $(1°)$ in (17a) and $C_{\infty v}$ $(4°)$ in (18a). For other groups such as $2°$ (17b), $3°$ (17c), $5°$ (18b) and $6°$ (18c), (25) is not obtainable by the above chiral operation and only (26) is true. Furthermore, same conclusions can be

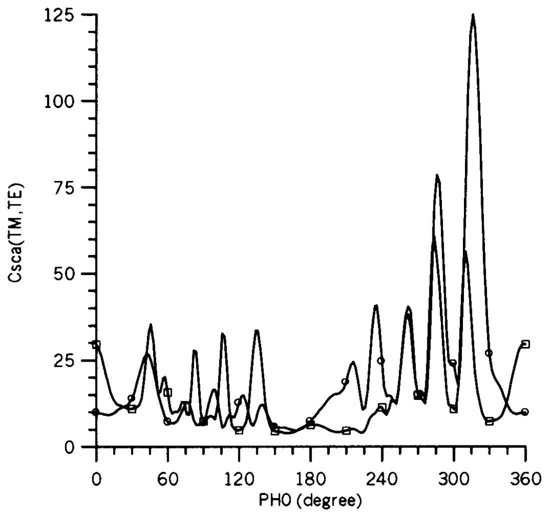


(a) $\theta_0 = \psi_1^{(q)} = 45°$, and $\psi_2^{(q)} = 90°$ (L-strips)



(b) $\theta_0 = \psi_1^{(q)} = \psi_2^{(q)} = 45°$

Figure 4. The scattering cross sections for four parallel eccentric bian-isotropic cylinders $\left(\text{circular dot : } C_{sca}^{TM}/\left(4R_2^{(1)}\right);\ \text{square dot: } C_{sca}^{TE}/\left(4R_2^{(1)}\right)\right)$.



(c) $\theta_0 = \psi_2^{(q)} = 45°,\ \psi_1^{(q)} = 135°$

Figure 4. Continued.

drawn for the excitation section for these group sets. So, (25) and (26) provide another ways for diagnosing the constitutive characteristics of bianisotropic cylindrical objects.

5. CONCLUSION

In the present contribution, our attention has been paid to the multiple scattering by a linear array of bianisotropic eccentric two-layered cylinders possessing double RH or LH helical conductances of the surfaces. The globe effects of bianisotropics on the Mueller matrix and scattering cross section are explored. Corresponding to various magnetic symmetry groups, some unique and novel relations that govern the 16 elements of Mueller matrix or scattering and excitation cross sections in the far-field scattering region are developed under certain chiral operations. These general relations are not restricted to the bianisotropic cylinder linear array and the the random array. They are also applicable to the ordinary gyroelectromagnetic case as long as the cylinders are parallel and infinitely long with eccentric or concentric circular cross sections.

APPENDIX A

For the obliquely incident wave of TE-polarization with respect to the z-axis, we have

$$H_{zinc}^{(q)} = H_0 \sin\theta_0 e^{j\delta(q)} \sum_{m=-\infty}^{\infty} j^m J_{m2}^{(q)} e^{-jm\left(\varphi_2^{(q)}-\varphi_0\right)} e_2^{(q)}, \quad q = 1, 2, \ldots, N \tag{A1}$$

so in the region $\rho_2^{(q)} \geq R_2^{(q)}$,

$$H_{zb}^{(q)} = H_{zinc}^{(q)} + \sum_{m=-\infty}^{\infty} \left\{ c_m^{(q)} H_{m2}^{(q)} + J_{m2}^{(q)} \left[\sum_{g=1}^{q-1} C_{mq}^{(g)} + \sum_{h=q+1}^{N} C_{mq}^{(h)} \right] \right\} e_{\varphi 2}^{(q)} \tag{A2}$$

$$E_{zb}^{(q)} = \sum_{m=-\infty}^{\infty} \left\{ d_m^{(q)} H_{m2}^{(q)} + J_{m2}^{(q)} \left[\sum_{g=1}^{q-1} D_{mq}^{(g)} + \sum_{h=q+1}^{N} D_{mq}^{(h)} \right] \right\} e_{\varphi 2}^{(q)} \tag{A3}$$

$$\begin{aligned} E_{\varphi b}^{(q)} = \sum_{m=-\infty}^{\infty} \frac{\beta \cos\theta_0}{k_b^2 \sin^2\theta_0} & \left[d_m^{(q)} \frac{m}{\rho_2^{(q)}} H_{m2}^{(q)} + J_{m2}^{(q)} \left[\sum_{g=1}^{q-1} D_{mq}^{(g)} + \sum_{h=q+1}^{N} D_{mq}^{(h)} \right] \right] \\ & + \frac{1}{k_{b0}^2} \left\{ j\omega\mu_0 k_{b0} \left[H_0 j^m \sin\theta_0 e^{j\delta(q)} J_m^{(q)\prime} + c_m^{(q)} H_{m2}^{(q)\prime} \right.\right. \\ & \left.\left. + J_{m2}^{(q)\prime} \left[\sum_{g=1}^{q-1} C_{mq}^{(g)} + \sum_{h=q+1}^{N} C_{mq}^{(h)} \right] \right] \right\} e_{\varphi 2}^{(q)} \end{aligned} \tag{A4}$$

$$\begin{aligned} H_{\varphi b}^{(q)} = \sum_{m=-\infty}^{\infty} \frac{1}{k_{b0}^2} & \left\{ j\omega\mu_b k_{b0} \left[d_m^{(q)} H_{m2}^{(q)\prime} + J_{m2}^{(q)\prime} \left[\sum_{g=1}^{q-1} D_{mq}^{(g)} + \sum_{g=h+1}^{N} D_{mq}^{(h)} \right] \right. \right. \\ & + \frac{m \cos\theta_0}{\rho_2^{(q)}} \left[H_0 j^m \sin\theta_0 e^{j\delta(q)} J_m^{(q)} + c_m^{(q)} H_{m2}^{(q)} \right] \\ & \left.\left. + J_{m2}^{(q)} \left[\sum_{g=1}^{q-1} C_{mq}^{(g)} + \sum_{h=q+1}^{N} C_{mq}^{(h)} \right] \right] \right\} \end{aligned} \tag{A5}$$

and

$$C_{mq}^{(g)} \left(D_{mq}^{(g)} \right) = \sum_{l=-\infty}^{\infty} c_m^{(g)} \left(d_m^{(g)} \right) H_{l-m}^{(2)} \left(k_{b0} D^{(gq)} \right) e^{j(l-m)\varphi_0}, \quad g < q, \tag{A6}$$

$$C_{mq}^{(h)}\left(D_{mq}^{(h)}\right)=\sum_{l=-\infty}^{\infty}(-1)^{l+m}c_m^{(h)}\left(d_m^{(h)}\right)H_{l-m}^{(2)}\left(k_{b0}D^{(hq)}\right)e^{j(l-m)\varphi_0},\quad h<q \tag{A7}$$

On the other hand, the field components in bianisotropic coatings can be expressed in a similar form as (8). At $\rho_2^{(q)}=R_2^{(q)}$, we have

$$\sum_{n=-\infty}^{\infty}\left[D_{1n+}^{(q)'}V_{nm}^{(q,1)}\left(R_2^{(q)}\right)+D_{1n-}^{(q)'}V_{nm}^{(q,2)}\left(R_2^{(q)}\right)\right]=\left[d_n^{(q)}J_2^{(q)}+J_2^{(q)}D_a^{(q)}\right]e^{jn\varphi_0} \tag{A8}$$

$$\begin{aligned}\sum_{n=-\infty}^{\infty}&\left[D_{1n+}^{(q)}V_{nm}^{(q,3)}\left(R_2^{(q)}\right)+D_{1n-}^{(q)}V_{nm}^{(q,4)}\left(R_2^{(q)}\right)\right]\\&=\frac{1}{k_{b0}^2}\left\{\frac{n\beta}{R_2^{(q)}}\left[S_1^{(q)}J_2^{(q)}+a_n^{(q)}H_2^{(q)}+J_2^{(q)}A_a^{(q)}\right]\right.\\&\quad\left.+\frac{n\beta}{R_2^{(q)}}\left[d_n^{(q)}H_2^{(q)}+J_2^{(q)}D_a^{(q)}\right]\right\}e^{jn\varphi_0}\end{aligned} \tag{A9}$$

$$\begin{aligned}\sum_{n=-\infty}^{\infty}&\left\{D_{1n+}^{(q)'}\left[\tan V_{nm}^{(q,1)}\left(R_2^{(q)}\right)+V_{nm}^{(q,5)}\left(R_2^{(q)}\right)\right]\right.\\&\left.+D_{1n-}^{(q)'}\left[\tan V_{nm}^{(q,2)}\left(R_2^{(q)}\right)+V_{nm}^{(q,6)}\left(R_2^{(q)}\right)\right]\right\}e^{jn\varphi_0}=0\end{aligned} \tag{A10}$$

$$\begin{aligned}\sum_{n=-\infty}^{\infty}&\left\{D_{1n+}^{(q)'}\left[\tan V_{nm}^{(q,3)}\left(R_2^{(q)}\right)+V_{nm}^{(q,7)}\left(R_2^{(q)}\right)\right]\right.\\&\left.+D_{1n-}^{(q)'}\left[\tan V_{nm}^{(q,4)}\left(R_2^{(q)}\right)+V_{nm}^{(q,8)}\left(R_2^{(q)}\right)\right]\right\}\\&=\left\{\tan\psi_2^{(q)}\left[S_2^{(q)}J_2^{(q)}+c_n^{(q)}H_2^{(q)}+J_2^{(q)}C_a^{(q)}\right]\right.\\&\quad+\frac{1}{k_{b0}^2}\left\{-j\omega\varepsilon_b k_{b0}\left[d_n^{(q)}H_2^{(q)'}+J_2^{(q)'}D_a^{(q)}\right]\right\}\\&\quad\left.+\frac{n\beta}{R_2^{(q)}}\left[S_1^{(q)}J_2^{(q)}+c_n^{(q)}H_2^{(q)}+J_2^{(q)}C_a^{(q)}\right]\right\}e^{jn\varphi_0}\end{aligned} \tag{A11}$$

To solve the unknown scattering coefficients $c(d)_m^{(q)}$ from (A8)–(A11), the summation should be also truncated to a finite size.

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