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METHOD OF MOMENTS ANALYSIS OF ELECTRICALLY LARGE THIN HEXAGONAL LOOP TRANSCEIVER ANTENNAS: NEAR- AND FAR-ZONE FIELDS

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Abstract–This paper presents a method of moments (MoM) analysis, obtains the non-uniform current distribution in closed form, and computes the resulted radiated patterns in both near and far zones, of regular hexagonal loop antennas with electrically large perimeter. An oblique incident field in its general form is considered in the formulation of the non-uniform current distributions. In the Galerkin's MoM analysis, the Fourier exponential series is considered as the full-domain basis function series. As a result, the current distributions along the hexagonal loops are expressed analytically in terms of the azimuth angle for various sizes of large loops. Finally, an alternative vector analysis of the electromagnetic (EM) fields radiated from thin hexagonal loop antennas of arbitrary length a is introduced. This method which employs the dyadic Green's function (DGF) in the derivation of the EM radiated fields makes the analysis general, compact and straightforward in both near- and far-zones. The EM radiated fields are expressed in terms of the vector wave eigenfunctions. Not only the exact solution of the EM fields in the near and far zones outside a are derived by use of the spherical Bessel and Hankel functions of the first kind respectively, but also the inner regions between $\frac{\sqrt{3}a}{2}$ and a are characterized by both the spherical Bessel and Hankel functions of the first kind. Validity of the numerical results is discussed and clarified.

1. INTRODUCTION

Thin loop antennas of an arbitrary shape carrying different forms of currents and their radiation characteristics have been investigated by many researchers over the last several decades. The recognized contributions to the field are made by many researchers, e.g., [1–20], to the authors' best knowledge. Typical shapes of loop antenna are circular, triangular, and rectangular. It is well-known [5, 22, 23] that a circular loop antenna has higher directive gain, while loop antennas of triangular and rectangular structures have broad-band input impedance and eases in mechanical construction. So, this paper is motivated by studying the current distribution, near- and far-zone fields of hexagonal antennas. The radiation characteristics of the small circular loop antennas can also be readily found from antenna text books [23, 24] and literature [17–20]. However, literature on hexagonal loop antennas located in free-space is limited [22, 23, 26, 27]. Also, most commercially available software packages do not provide near-field features. Therefore, this paper aims to employ MoM to obtain current distribution of the hexagonal loop antenna and derive its field distribution using DGF.

2. GENERAL FORMULATION OF CURRENT DISTRIBUTIONS

Fig. 1 shows the geometry of a thin hexagonal loop antenna located at $z = 0$ and fed at $\phi_i = 0$ with the delta-function voltage generator $V_0\delta(\phi)$ for an active antenna system. Also, an incident wave is considered here for a passive antenna system. This makes the loop a transceiver antenna.

According to the boundary condition, the loop current carried by the transceiver antenna satisfies the integral equation [1, 5, 10, 27] as

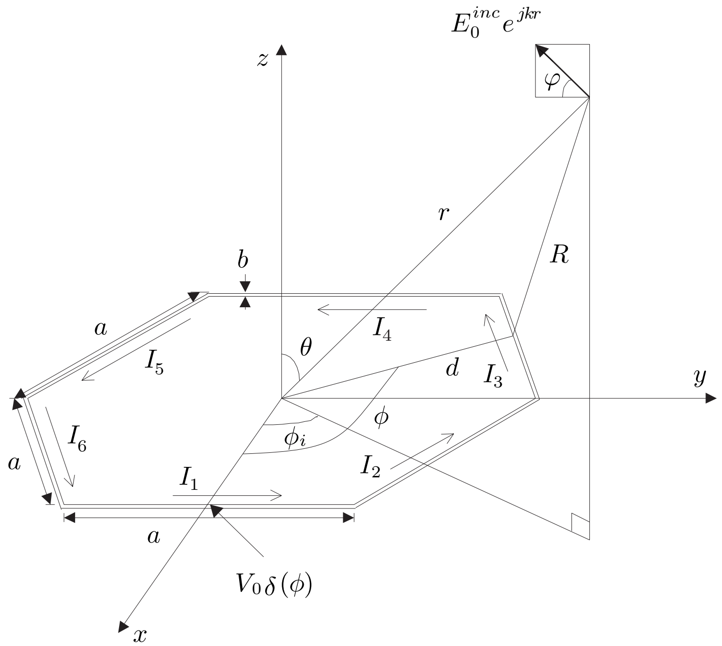

Figure 1. Geometry of a thin hexagonal loop antenna.

follows:

$$V_0\delta(\phi) + dE_\phi^{inc}(d,\phi) = \frac{j\eta_o}{4\pi}\sum_{t=1}^{6}\int_{\text{region t}} K_t(\phi-\phi')I_t(\phi')\,d\phi' \quad (1)$$

where E_ϕ^{inc} denotes the incident field, d stands for the distance between origin and side of the loop and $\eta_o = 120\pi\Omega$ is the free-space intrinsic impedance. The current $I_t(\phi)$ can be expressed by

$$I_t(\phi) = \sum_{m=0}^{\infty}(2-\delta_{m0})I_m^t e^{-jm\phi} \quad (2)$$

where t takes the form of integers from 1 to 6 for hexagonal loop, δ_{m0} denotes the Kronecker symbol and I_m^t stands for the series expansion coefficients. The integral kernel in (1) is represented by the following Fourier series expansion

$$K_t(\phi-\phi') = \sum_{m=-\infty}^{\infty} g_m^t e^{-jm(\phi-\phi')}, \quad (3)$$

$$g_m^t = \int_{\text{region } t} \left(\frac{k_0 d}{2} \left(N_{m+1}^t + N_{m-1}^t \right) - \frac{m^2}{k_0 d} N_m^t \right) d\phi', \tag{4}$$

$$d = \begin{bmatrix} \dfrac{a}{|\cos\phi'|}, & \text{for region 1 (R1) \& 4 (R4)} \\ \dfrac{a}{\left|\sin\left(\phi' + \dfrac{\pi}{6}\right)\right|}, & \text{for region 2 (R2) \& 5 (R5)} \\ \dfrac{a}{\left|\cos\left(\phi' + \dfrac{\pi}{3}\right)\right|}, & \text{for region 3 (R3) \& 6 (R6)} \end{bmatrix} \tag{5}$$

$$N_m^t = \frac{1}{4\pi^2} \int_{\text{region } t} \int_{-\pi}^{\pi} \frac{e^{jm(\phi-\phi')} e^{(-jk_0 r')}}{r'} \, d\alpha \, d\phi', \tag{6}$$

$$r' = \begin{bmatrix} \sqrt{\left(\dfrac{a}{|\cos\phi'|}\right)^2 + 4b^2 \sin^2 \dfrac{\alpha}{2}}, & \text{for R1 \& R4} \\ \sqrt{\left(\dfrac{a}{\left|\sin\left(\phi' + \dfrac{\pi}{6}\right)\right|}\right)^2 + 4b^2 \sin^2 \dfrac{\alpha}{2}}, & \text{for R2 \& R5} \\ \sqrt{\left(\dfrac{a}{\left|\cos\left(\phi' + \dfrac{\pi}{3}\right)\right|}\right)^2 + 4b^2 \sin^2 \dfrac{\alpha}{2}}, & \text{for R3 \& R6} \end{bmatrix}, \tag{7}$$

$$\begin{bmatrix} \text{R1} \\ \text{R2} \\ \text{R3} \\ \text{R4} \\ \text{R5} \\ \text{R6} \end{bmatrix} \Longrightarrow \begin{bmatrix} -\dfrac{\pi}{6} < \phi' \le \dfrac{\pi}{6} \\ \dfrac{\pi}{6} < \phi' \le \dfrac{\pi}{2} \\ \dfrac{\pi}{2} < \phi' \le \dfrac{5\pi}{6} \\ \dfrac{5\pi}{6} < \phi' \le \dfrac{7\pi}{6} \\ \dfrac{7\pi}{6} < \phi' \le \dfrac{9\pi}{6} \\ \dfrac{9\pi}{6} < \phi' \le \dfrac{11\pi}{6} \end{bmatrix}. \tag{8}$$

The incident field E_ϕ^{inc} at the angle ϕ is given by

$$E_\phi^{inc} = E_0^{inc} [\cos\psi \cos(\phi - \phi_i) + \sin\psi \sin(\phi - \phi_i) \cos\theta] e^{jk_0 d \cos(\phi-\phi_i) \sin\theta}. \tag{9}$$

It can also be expressed in terms of the Fourier series as follows:

$$E_{\phi}^{inc} = \sum_{m=-\infty}^{\infty} \sum_{t=1}^{6} f_m^t e^{-jm\phi} \tag{10}$$

where

$$f_m^t = \frac{1}{2\pi} \int_{\text{region } t} E_0^{inc} \left\{ j^{m-1} \cos\psi e^{jm\phi_0} J_m'(k_0 d \sin\theta) + j^m \sin\psi \cos\theta e^{jm\phi_0} \frac{m J_m(k_0 d \sin\theta)}{k_0 d \sin\theta} \right\} d\phi'. \tag{11}$$

To obtain maximum electric and magnetic responses, the loop orientation is made at $\psi = 0$, $\theta = \frac{\pi}{2}$, and $\phi_0 = 0$ which simplifies f_m^t and I_m^t to

$$f_m^t = \frac{1}{2\pi} \int_{\text{region } t} \frac{j^{m-1}}{2} \left\{ J_{m-1}(k_0 d) - J_{m+1}(k_0 d) \right\} d\phi' \tag{12}$$

and

$$I_m^t = -j \frac{V(0) + 2\pi f_m^t}{g_m^t \pi \eta_o}. \tag{13}$$

3. GENERAL FORMULATION OF EM RADIATED FIELDS

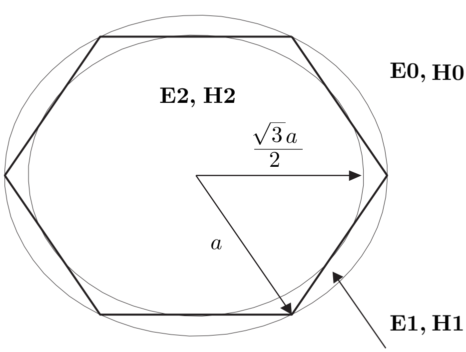

Figure 2. Intermediated zones.

The detailed formulation of the EM radiated fields of circular loop antennas has been shown in [19, (1)–(7)]. In view of this, the authors will not repeat the procedure. The inner, outer and intermediate regions defined in this paper are depicted in Fig. 2. With the current distribution given in (2), we can obtain the EM field expressions for the four zones as follows:

$$\begin{aligned}\begin{bmatrix} E_r^0 \\ E_r^1 \\ E_r^2 \end{bmatrix} &= -\frac{\eta_0 k_0^2}{4\pi}\sum_{n=1}^{\infty}\sum_{m=0}^{n} D_{mn}P_n^m(\cos\theta)\begin{matrix}\cos\\ \sin\end{matrix}(m\phi)\\ &\quad\cdot\frac{n(n+1)}{k_0 r}\left\{ j_n(k_0r)\begin{bmatrix} 0 \\ (\Phi^{N,r,1>}_{\substack{e\\o}mn} + \Phi^{N,\phi,1>}_{\substack{e\\o}mn}) \\ (\Phi^{N,r,2>}_{\substack{e\\o}mn} + \Phi^{N,\phi,2>}_{\substack{e\\o}mn}) \end{bmatrix}\right.\\ &\quad\left. + h_n^{(1)}(k_0r)\begin{bmatrix} (\Phi^{N,r,0<}_{\substack{e\\o}mn} + \Phi^{N,\phi,0<}_{\substack{e\\o}mn}) \\ (\Phi^{N,r,1<}_{\substack{e\\o}mn} + \Phi^{N,\phi,1<}_{\substack{e\\o}mn}) \\ 0 \end{bmatrix}\right\},\end{aligned} \tag{14a}$$

$$\begin{aligned}\begin{bmatrix} E_\theta^0 \\ E_\theta^1 \\ E_\theta^2 \end{bmatrix} &= -\frac{\eta_0 k_0^2}{4\pi}\sum_{n=1}^{\infty}\sum_{m=0}^{n} \mp D_{mn}\frac{m}{\sin\theta}P_n^m(\cos\theta)\\ &\quad\cdot\begin{matrix}\sin\\ \cos\end{matrix}(m\phi)\left\{ j_n(k_0r)\begin{bmatrix} 0 \\ \Phi^{M,1>}_{\substack{e\\o}mn} \\ \Phi^{M,2>}_{\substack{e\\o}mn} \end{bmatrix} + h_n^{(1)}(k_0r)\begin{bmatrix} \Phi^{M,0<}_{\substack{e\\o}mn} \\ \Phi^{M,1<}_{\substack{e\\o}mn} \\ 0 \end{bmatrix}\right\}\\ &\quad + D_{mn}\frac{dP_n^m(\cos\theta)}{d\theta}\begin{matrix}\cos\\ \sin\end{matrix}(m\phi)\\ &\quad\cdot\left\{\frac{d[rj_n(k_0r)]}{k_0rdr}\begin{bmatrix} 0 \\ (\Phi^{N,r,1>}_{\substack{e\\o}mn} + \Phi^{N,\phi,1>}_{\substack{e\\o}mn}) \\ (\Phi^{N,r,2>}_{\substack{e\\o}mn} + \Phi^{N,\phi,2>}_{\substack{e\\o}mn}) \end{bmatrix}\right.\\ &\quad\left. + \frac{d[rh_n^{(1)}(k_0r)]}{k_0rdr}\begin{bmatrix} (\Phi^{N,r,0<}_{\substack{e\\o}mn} + \Phi^{N,\phi,0<}_{\substack{e\\o}mn}) \\ (\Phi^{N,r,1<}_{\substack{e\\o}mn} + \Phi^{N,\phi,1<}_{\substack{e\\o}mn}) \\ 0 \end{bmatrix}\right\},\end{aligned} \tag{14b}$$

$$\begin{bmatrix} E_\phi^0 \\ E_\phi^1 \\ E_\phi^2 \end{bmatrix} = -\frac{\eta_0 k_0^2}{4\pi}\sum_{n=1}^{\infty}\sum_{m=0}^{n} -D_{mn}\frac{dP_n^m(\cos\theta)}{d\theta}$$

$$
\begin{aligned}
&\cdot \frac{\cos}{\sin}(m\phi)\left\{ j_n(k_0 r)\begin{bmatrix} 0 \\ \Phi^{M,1>}_{\substack{e\\o}mn} \\ \Phi^{M,2>}_{\substack{e\\o}mn} \end{bmatrix} + h_n^{(1)}(k_0 r)\begin{bmatrix} \Phi^{M,0<}_{\substack{e\\o}mn} \\ \Phi^{M,1<}_{\substack{e\\o}mn} \\ 0 \end{bmatrix}\right\} \\
&\mp D_{mn}\frac{m}{\sin\theta}P_n^m(\cos\theta)\frac{\sin}{\cos}(m\phi) \\
&\cdot \left\{ \frac{d[rj_n(k_0 r)]}{k_0 r dr}\begin{bmatrix} 0 \\ (\Phi^{N,r,1>}_{\substack{e\\o}mn} + \Phi^{N,\phi,1>}_{\substack{e\\o}mn}) \\ (\Phi^{N,r,2>}_{\substack{e\\o}mn} + \Phi^{N,\phi,2>}_{\substack{e\\o}mn}) \end{bmatrix} \right. \\
&\left. + \frac{d[rh_n^{(1)}(k_0 r)]}{k_0 r dr}\begin{bmatrix} (\Phi^{N,r,0<}_{\substack{e\\o}mn} + \Phi^{N,\phi,0<}_{\substack{e\\o}mn}) \\ (\Phi^{N,r,1<}_{\substack{e\\o}mn} + \Phi^{N,\phi,1<}_{\substack{e\\o}mn}) \\ 0 \end{bmatrix}\right\}; \qquad (14c)
\end{aligned}
$$

and

$$
\begin{aligned}
\begin{bmatrix} H_r^0 \\ H_r^1 \\ H_r^2 \end{bmatrix} &= \frac{ik_0^2}{4\pi}\sum_{n=1}^{\infty}\sum_{m=0}^{n} D_{mn}P_n^m(\cos\theta)\frac{\cos}{\sin}(m\phi) \\
&\cdot \frac{n(n+1)}{k_0 r}\left\{ j_n(k_0 r)\begin{bmatrix} 0 \\ \Phi^{M,1>}_{\substack{e\\o}mn} \\ \Phi^{M,2>}_{\substack{e\\o}mn} \end{bmatrix} + h_n^{(1)}(k_0 r)\begin{bmatrix} \Phi^{M,0<}_{\substack{e\\o}mn} \\ \Phi^{M,1<}_{\substack{e\\o}mn} \\ 0 \end{bmatrix}\right\}, \qquad (15a)
\end{aligned}
$$

$$
\begin{aligned}
\begin{bmatrix} H_\theta^0 \\ H_\theta^1 \\ H_\theta^2 \end{bmatrix} &= \frac{ik_0^2}{4\pi}\sum_{n=1}^{\infty}\sum_{m=0}^{n} D_{mn}\frac{m}{\sin\theta}P_n^m(\cos\theta)\frac{\sin}{\cos}(m\phi) \\
&\cdot \left\{ \mp j_n(k_0 r)\begin{bmatrix} 0 \\ (\Phi^{N,r,1>}_{\substack{e\\o}mn} + \Phi^{N,\phi,1>}_{\substack{e\\o}mn}) \\ (\Phi^{N,r,2>}_{\substack{e\\o}mn} + \Phi^{N,\phi,2>}_{\substack{e\\o}mn}) \end{bmatrix} \right. \\
&\left. \mp h_n^{(1)}(k_0 r)\begin{bmatrix} (\Phi^{N,r,0<}_{\substack{e\\o}mn} + \Phi^{N,\phi,0<}_{\substack{e\\o}mn}) \\ (\Phi^{N,r,1<}_{\substack{e\\o}mn} + \Phi^{N,\phi,1<}_{\substack{e\\o}mn}) \\ 0 \end{bmatrix}\right\} \\
&+ D_{mn}\frac{dP_n^m(\cos\theta)}{d\theta}\frac{\cos}{\sin}(m\phi) \\
&\cdot \left\{ \frac{d\left[rj_n(k_0 r)\right]}{k_0 r dr}\begin{bmatrix} 0 \\ \Phi^{M,1>}_{\substack{e\\o}mn} \\ \Phi^{M,2>}_{\substack{e\\o}mn} \end{bmatrix} + \frac{d[rh_n^{(1)}(k_0 r)]}{k_0 r dr}\begin{bmatrix} \Phi^{M,0<}_{\substack{e\\o}mn} \\ \Phi^{M,1<}_{\substack{e\\o}mn} \\ 0 \end{bmatrix}\right\}, \qquad (15b)
\end{aligned}
$$

$$\begin{aligned}\begin{bmatrix} H_\phi^0 \\ H_\phi^1 \\ H_\phi^2 \end{bmatrix} &= \frac{ik_0^2}{4\pi}\sum_{n=1}^{\infty}\sum_{m=0}^{n} -D_{mn}\frac{dP_n^m(\cos\theta)}{d\theta}\,\substack{\cos\\ \sin}(m\phi) \\ &\cdot\left\{ j_n(k_0r)\begin{bmatrix} 0 \\ (\Phi^{N,r,1>}_{\substack{e\\o}mn}+\Phi^{N,\phi,1>}_{\substack{e\\o}mn}) \\ (\Phi^{N,r,2>}_{\substack{e\\o}mn}+\Phi^{N,\phi,2>}_{\substack{e\\o}mn}) \end{bmatrix}\right. \\ &\left. +h_n^{(1)}(k_0r)\begin{bmatrix} (\Phi^{N,r,0<}_{\substack{e\\o}mn}+\Phi^{N,\phi,0<}_{\substack{e\\o}mn}) \\ (\Phi^{N,r,1<}_{\substack{e\\o}mn}+\Phi^{N,\phi,1<}_{\substack{e\\o}mn}) \\ 0 \end{bmatrix}\right\} \\ &\mp D_{mn}\frac{m}{\sin\theta}P_n^m(\cos\theta)\,\substack{\sin\\ \cos}(m\phi) \\ &\cdot\left\{ \frac{d[rj_n(k_0r)]}{k_0rdr}\begin{bmatrix} 0 \\ \Phi^{M,1>}_{\substack{e\\o}mn} \\ \Phi^{M,2>}_{\substack{e\\o}mn} \end{bmatrix} + \frac{d[rh_n^{(1)}(k_0r)]}{k_0rdr}\begin{bmatrix} \Phi^{M,0<}_{\substack{e\\o}mn} \\ \Phi^{M,1<}_{\substack{e\\o}mn} \\ 0 \end{bmatrix}\right\}; \quad (15c)\end{aligned}$$

where $j_n(k_0r)$ and $h_n^{(1)}(k_0r)$ are the spherical Bessel and Hankel functions of the first kind respectively, $P_n^m(\cos\theta)$ is the associated Legendre function, and the normalization coefficient D_{mn} is defined by

$$D_{mn} = \frac{(2n+1)}{n(n+1)}\frac{(n-m)!}{(n+m)!}.$$

The coefficients of the EM fields are expressed by

$$\begin{bmatrix} \Phi^{M,0<}_{\substack{e\\o}mn} \\ \Phi^{M,1<}_{\substack{e\\o}mn} \\ \Phi^{M,1>}_{\substack{e\\o}mn} \\ \Phi^{M,2>}_{\substack{e\\o}mn} \end{bmatrix} = \begin{bmatrix} \Psi^{M,0<}_{\substack{e\\o}14}+\Psi^{M,0<}_{\substack{e\\o}25}+\Psi^{M,0<}_{\substack{e\\o}36} \\ \Psi^{M,1<}_{\substack{e\\o}14}+\Psi^{M,1<}_{\substack{e\\o}25}+\Psi^{M,1<}_{\substack{e\\o}36} \\ \Psi^{M,1>}_{\substack{e\\o}14}+\Psi^{M,1>}_{\substack{e\\o}25}+\Psi^{M,1>}_{\substack{e\\o}36} \\ \Psi^{M,2>}_{\substack{e\\o}14}+\Psi^{M,2>}_{\substack{e\\o}25}+\Psi^{M,2>}_{\substack{e\\o}36} \end{bmatrix}, \qquad (16a)$$

$$\begin{bmatrix} \Phi^{N,r,0<}_{\substack{e\\o}mn} \\ \Phi^{N,r,1<}_{\substack{e\\o}mn} \\ \Phi^{N,r,1>}_{\substack{e\\o}mn} \\ \Phi^{N,r,2>}_{\substack{e\\o}mn} \end{bmatrix} = \begin{bmatrix} \Psi^{N,r<}_{\substack{e\\o}14}+\Psi^{N,r<}_{\substack{e\\o}25}+\Psi^{N,r<}_{\substack{e\\o}36} \\ \Psi^{N,r,1<}_{\substack{e\\o}14}+\Psi^{N,r,1<}_{\substack{e\\o}25}+\Psi^{N,r,1<}_{\substack{e\\o}36} \\ \Psi^{N,r,1>}_{\substack{e\\o}14}+\Psi^{N,r,1>}_{\substack{e\\o}25}+\Psi^{N,r,1>}_{\substack{e\\o}36} \\ \Psi^{N,r,2>}_{\substack{e\\o}14}+\Psi^{N,r,2>}_{\substack{e\\o}25}+\Psi^{N,r,2>}_{\substack{e\\o}36} \end{bmatrix}, \qquad (16b)$$

$$\begin{bmatrix} \Phi^{N,\phi<}_{\substack{e\\o}mn} \\ \Phi^{N,\phi,1<}_{\substack{e\\o}mn} \\ \Phi^{N,\phi,1>}_{\substack{e\\o}mn} \\ \Phi^{N,\phi,2>}_{\substack{e\\o}mn} \end{bmatrix} = \begin{bmatrix} \Psi^{N,\phi<}_{\substack{e\\o}14} + \Psi^{N,\phi<}_{\substack{e\\o}25} + \Psi^{N,\phi<}_{\substack{e\\o}36} \\ \Psi^{N,\phi,1<}_{\substack{e\\o}14} + \Psi^{N,\phi,1<}_{\substack{e\\o}25} + \Psi^{N,\phi,1<}_{\substack{e\\o}36} \\ \Psi^{N,\phi,1>}_{\substack{e\\o}14} + \Psi^{N,\phi,1>}_{\substack{e\\o}25} + \Psi^{N,\phi,1>}_{\substack{e\\o}36} \\ \Psi^{N,\phi,2>}_{\substack{e\\o}14} + \Psi^{N,\phi,2>}_{\substack{e\\o}25} + \Psi^{N,\phi,2>}_{\substack{e\\o}36} \end{bmatrix}. \tag{16c}$$

The current distributions are obtained using the MoM where the relationship between the loop and wire cross-section radii is $2\ln\frac{2\pi d}{b} \geq 10$. The analytical expressions of the current distribution are provided in the Appendix A and the convergence of the analysis is checked in details.

To check the results of our numerical computations, we compared our results with the radiation patterns in [23, Fig. 5.52] for square loop antenna [26, 28], and those by Rao [13] for circular one [20, 27]. Excellent agreements between their works and ours are shown. This confirms partially the correctness of our theoretical derivation and numerical algorithm.

4. NUMERICAL RESULTS

The current distributions are obtained using the MoM where the relationship between the loop and wire cross-section radii is $2\ln\frac{2\pi d}{b} \geq 10$. The analytical expressions of the current distribution are provided in the Appendix A and the convergence of the analysis is checked in details.

Forty terms are considered for the numerical computation of the summation of the spherical Bessel and Hankel functions (i.e., with respect to the index n). The 3-D normalized EM radiation patterns in the intermediate zones of the hexagonal loop antennas are depicted in Figs. 3, 4 and 5, respectively. The figures clearly show that the magnitude of the field intensities increases as the observation point moves closely to the source and decreases as it moves away. Besides, it is obvious that the directivity increases too, as the observation point is close to the source. All these observations, by instinct and intuition, are correct. At the co-ordinate origin (center of the square loop), the electromagnetic fields become null, which is in agreement with that of Bourt theorem. From Figs. 3 to 5, it is seen that around the loop, the electric fields are maximum and the field distribution due to the

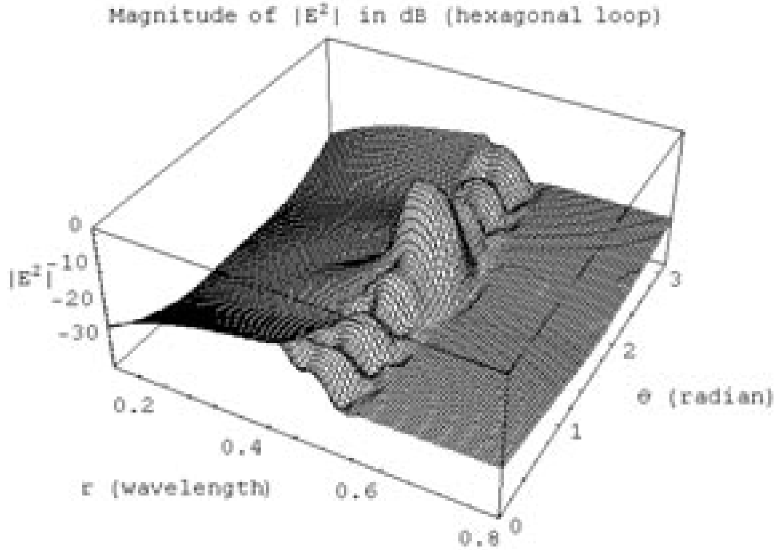

Figure 3. 3-D plot of power intensity (in dB) of the hexagonal loop antenna of length $(a = 0.5\lambda)$ as a function of r and θ in E plane.

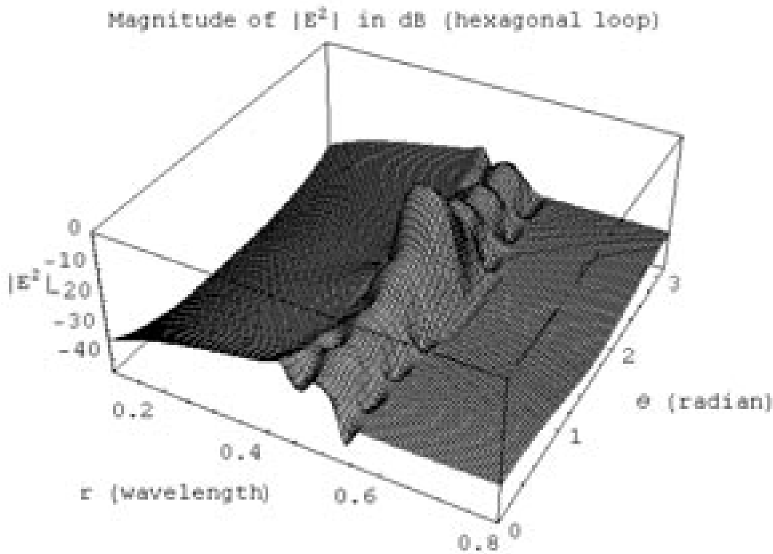

Figure 4. 3-D plot of power intensity (in dB) of the hexagonal loop antenna of length $(a = 0.5\lambda)$ as a function of r and θ in H plane.

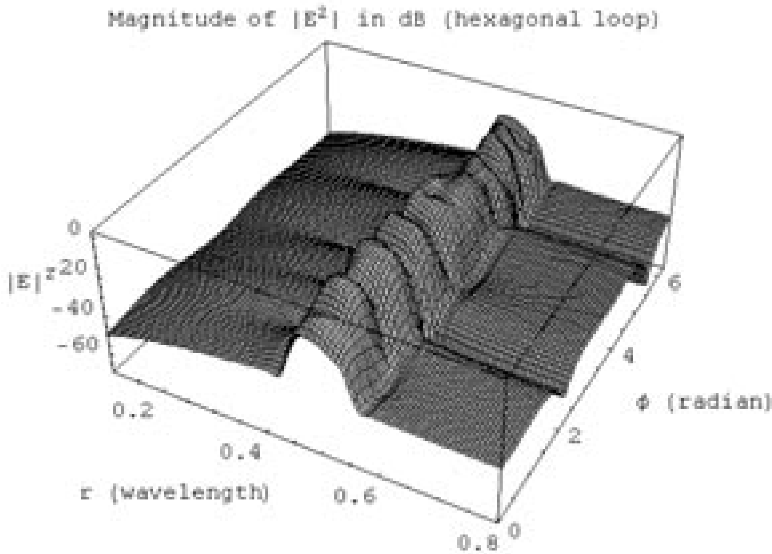

Figure 5. 3-D plot of power intensity (in dB) of the hexagonal loop antenna of length $(a = 0.5\lambda)$ as a function of r and ϕ in $X\,Y$ plane.

loop varies dramatically with the location, especially within the area of region 1. This phenomenon is primarily due to the interaction of the source current and radiated field or strong coupling within the area. Another interesting point to note is when the observation point moves away from the source, the null point occurs at $\theta = \frac{\pi}{2}$ and this is due to the fact that the r component of the E field, E_r is no longer dominant. Besides this, we can observe from Figs. 3 to 5 that continuities are shown between regions (i.e., Region 0, 1 and 2) though all were characterized quite differently. It should be pointed out that all numerical results are normalized by respectively their maxima.

5. CONCLUSIONS

This paper presents a MoM analysis of electrically large hexagonal loop antennas. From the analysis, the radiated EM fields in the inner, outer and intermediate regions, of thin hexagonal loop antennas are obtained. The trigonometric functions are chosen as the full-domain basis and weighting functions in the Galerkin's approach. Also, the dyadic

Green's function in spherical coordinates is utilized in the derivation of the analytical expressions of the EM fields in the near, far and intermediate zones. The radiation patterns due to the derived current distributions are plotted in polar coordinates. Some new results are also presented in the paper for the EM fields due to large hexagonal loop antennas.

APPENDIX A. ANALYTICAL EXPRESSIONS OF URRENT DISTRIBUTIONS

In this appendix, the current distributions along the hexagonal loop antenna wires of length $a = 0.5\lambda$ are provided in closed form. They are expressed in terms of the spherical azimuth angle ϕ for different loop dimensions. These formulas are extremely useful for scientists and engineers who would like to obtain full-wave characteristics of the loop antennas since they can be straightforwardly used these formulas without repeating the MoM analysis.

Hexagonal loop of length $a = \frac{\lambda}{2}$

All the current coefficients are normalized by a factor of 10^{-8}.

$$\begin{aligned}
I_1(\phi) = \\
&(2.129j)e^{j11.00\phi} + (6.621 + 2.253j)e^{j10\phi} + (12.00 - 26.11j)e^{j9\phi} \\
&+ (-114.0 - 71.30j)e^{j8\phi} + (-562.1 + 485.7j)e^{j7\phi} + (1535 + 3179j)e^{j6\phi} \\
&+ (858.7 + 3818j)e^{j5\phi} + (3931 - 5.251j)e^{j4\phi} + (226.1 - 2718j)e^{j3\phi} \\
&+ (949.3 + 117.6j)e^{j2\phi} + (713.9 - 497.1j)e^{j\phi} + (-2587 - 386.8j) \\
&+ (713.9 - 497.1j)e^{-j\phi} + (949.3 + 117.6j)e^{-j2\phi} + (226.1 - 2718j)e^{-j3\phi} \\
&+ (3931 - 5.251j)e^{-j4\phi} + (858.7 + 3818j)e^{-j5\phi} + (1535 + 3179j)e^{-j6\phi} \\
&+ (-562.1 + 485.7j)e^{-j7\phi} + (-114.0 - 71.30j)e^{-j8\phi} \\
&+ (12 - 26.11j)e^{-j9\phi} + (6.621 + 2.253j)e^{-j10\phi} + (2.129j)e^{-j11.00\phi}
\end{aligned} \tag{17a}$$

$$\begin{aligned}
I_2(\phi) = \\
&(1.145j)e^{j12\phi} + (2.000)e^{j11\phi} + (-13.25 - 2.650j)e^{j10\phi} \\
&+ (15.16 + 25.89j)e^{j9\phi} + (183.7 - 60.50j)e^{j8\phi} + (2143 - 544.3j)e^{j7\phi} \\
&+ (1772 + 3133j)e^{j6\phi} + (3386 - 1263j)e^{j5\phi} + (-6860 + 36.66j)e^{j4\phi} \\
&+ (2202 + 1798j)e^{j3\phi} + (-600.8 + 1510j)e^{j2\phi} + (6099 - 10000j)e^{j\phi} \\
&+ (1862 + 1993j) + (-6460 - 6715j)e^{-j\phi} + (-190.1 - 1752j)e^{-j2\phi}
\end{aligned}$$

$$
\begin{aligned}
&+ (2261 + 4124j)e^{-j3\phi} + (-6284 + 1948j)e^{-j4\phi} + (-4930 - 2684j)e^{-j5\phi} \\
&+ (89.20 - 4105j)e^{-j6\phi} + (1631 + 140.2j)e^{-j7\phi} + (77.88 + 302.9j)e^{-j8\phi} \\
&+ (-36.19 + 48.34j)e^{-j9\phi} + (-12.45 + 1.152j)e^{-j10\phi} \\
&+ (-1.870 - 1.967j)e^{-j11\phi}
\end{aligned} \tag{17b}
$$

$$
\begin{aligned}
&I_3(\phi) = \\
&(-1.139 + 1.447j)e^{j10\phi} + (-1.348 - 5.217j)e^{j9\phi} + (63.36 - 30.93j)e^{j8\phi} \\
&+ (73.95 - 105.9j)e^{-j7\phi} + (139.0 + 638.5j)e^{-j6\phi} + (419.8 - 878.6j)e^{j5\phi} \\
&+ (-239.2 + 926.7j)e^{j4\phi} + (-288.3 - 418.1j)e^{j3\phi} \\
&+ (-349.3 + 69.12j)e^{j2\phi} + (1169 + 1337j)e^{j\phi} + (-418.8 + 250.1j) \\
&+ (1167 + 1437j)e^{-j\phi} + (325.1 + 60.97j)e^{-j2\phi} + (-721.4 + 434.9j)e^{-j3\phi} \\
&+ (-256.1 + 957.4j)e^{-j4\phi} + (358.7 + 563.8j)e^{-j5\phi} \\
&+ (333.4 + 29.11j)e^{-j6\phi} + (77.44 - 104.0j)e^{-j7\phi} \\
&+ (-16.04 - 35.01j)e^{-j8\phi} + (-9.134)e^{-j9\phi} + (-1.207 + 1.491j)e^{-j10\phi}
\end{aligned} \tag{17c}
$$

$$
\begin{aligned}
&I_4(\phi) = \\
&\quad (-1.309j)e^{j10\phi} + 5.268 + 1.501j)e^{j9\phi} + (-10.21 + 23.61j)e^{j8\phi} \\
&+ (-107.5 - 93.52j)e^{j7\phi} + (627.2 + 207.1j)e^{j6\phi} + (-682.2 + 269.0j)e^{j5\phi} \\
&+ (-116.8 - 732.0j)e^{j4\phi} + (514.2 - 38.24j)e^{j3\phi} + (-6.133 - 180.9j)e^{j2\phi} \\
&+ (951.0 - 13.82j)e^{j\phi} + (4.315 + 495.4j) + (951.0 - 13.83j)e^{-j\phi} \\
&+ (-6.115 - 180.9j)e^{-j2\phi} + (514.3 - 38.13j)e^{-j3\phi} \\
&+ (-116.6 - 732.2j)e^{-j4\phi} + (-682.7 + 268.7j)e^{-j5\phi} \\
&+ (627.8 + 207.3j)e^{-j6\phi} + (-107.5 - 93.52j)e^{-j7\phi} \\
&+ (-10.22 + 23.60j)e^{-j8\phi} + (5.267 + 1.505j)e^{-j9\phi} + (-1.309j)e^{-j10\phi}
\end{aligned} \tag{17d}
$$

$$
\begin{aligned}
&I_5(\phi) = \\
&\quad (-1.232j)e^{j12\phi} + (2.219j)e^{j11\phi} + (14.41 - 2.509j)e^{j10\phi} \\
&+ (21.74 + 25.36j)e^{j9\phi} + (-259.1 + 62.38j)e^{j8\phi} + (1417 - 90.15j)e^{j7\phi} \\
&+ (-2450 - 3.292j)e^{j6\phi} + (3.674 - 17.97j)e^{j5\phi} + (713.1 - 25.96j)e^{j4\phi} \\
&+ (2709 + 1552j)e^{j3\phi} + (596.3 - 2185j)e^{j2\phi} + (8076 - 6300j)e^{j\phi} \\
&+ (1655 - 2504j) + (-8869 - 1049j)e^{-j\phi} + (-59.49 + 2524j)e^{-j2\phi} \\
&+ (-4534 + 4795j)e^{-j3\phi} + (8040 - 492.7j)e^{-j4\phi}
\end{aligned}
$$

$$
\begin{aligned}
&+(-4350-3901j)e^{-j5\phi}+(-172.4+3428j)e^{-j6\phi}\\
&+(1628-635.0j)e^{-j7\phi}+(-235.7-491.1j)e^{-j8\phi}\\
&+(-69.28+68.41j)e^{-j9\phi}+(16.16+2.799j)e^{-j10\phi}\\
&+(-1.468-2.473j)e^{-j11\phi}
\end{aligned}
\tag{17e}
$$

$$
\begin{aligned}
I_6(\phi) =&\\
&(-1.264)e^{j12\phi}+(-1.863-2.000j)e^{j11\phi}+(-8.379-6.889j)e^{j10\phi}\\
&+(-29.64+5.327j)e^{j9\phi}+(76.68+303.0j)e^{j8\phi}+(-615.0-561.9j)e^{j7\phi}\\
&+(-3712+542.8j)e^{j6\phi}+(-4931-2697j)e^{j5\phi}+(-5312-1641j)e^{j4\phi}\\
&+(-2467+1409j)e^{j3\phi}+(-189.0-1752j)e^{j2\phi}+(8380-7405j)e^{j\phi}\\
&+(-1196-2446j)+(7819-4411j)e^{-j\phi}+(-600.6+1511j)e^{-j2\phi}\\
&+(1369+4000j)e^{-j3\phi}+(-4557-2390j)e^{-j4\phi}\\
&+(3383-1268j)e^{-j5\phi}+(453.5+1.798j)e^{-j6\phi}\\
&+(-511.9-471.5j)e^{-j7\phi}+(183.7-60.81j)e^{-j8\phi}\\
&+(-11.92+43.47j)e^{-j9\phi}+(-6.082-7.599j)e^{-j10\phi}+(1.998)e^{-j11\phi}
\end{aligned}
\tag{17f}
$$

APPENDIX B. EXPLICIT EXPRESSIONS OF COEFFICIENTS OF THE SERIES

The intermediate expressions of the coefficients defined in (14) and (15) are determined as follows:

$$
\begin{aligned}
\begin{bmatrix}
\Psi^{M,0<}_{\substack{e\\o}14}\\
\Psi^{M,1<}_{\substack{e\\o}14}\\
\Psi^{M,1>}_{\substack{e\\o}14}\\
\Psi^{M,2>}_{\substack{e\\o}14}
\end{bmatrix}
=&
\begin{bmatrix}
\left(\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} I_1-\int_{\frac{5\pi}{6}}^{\frac{7\pi}{6}} I_4\right)\\
\left(\int_{-\phi_1}^{\phi_1} I_1-\int_{\pi-\phi_1}^{\pi+\phi_1} I_4\right)\\
\left(\left(\int_{\frac{11\pi}{6}}^{2\pi-\phi_1}+\int_{\phi_1}^{\frac{\pi}{6}}\right)I_1-\left(\int_{\frac{5\pi}{6}}^{\pi-\phi_1}+\int_{\pi+\phi_1}^{\frac{7\pi}{6}}\right)I_4\right)\\
\left(\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} I_1-\int_{\frac{5\pi}{6}}^{\frac{7\pi}{6}} I_4\right)
\end{bmatrix}
\begin{bmatrix}
j_n\left(\frac{k_0a}{|\cos\phi'|}\right)\\
j_n\left(\frac{k_0a}{|\cos\phi'|}\right)\\
h_n^{(1)}\left(\frac{k_0a}{|\cos\phi'|}\right)\\
h_n^{(1)}\left(\frac{k_0a}{|\cos\phi'|}\right)
\end{bmatrix}\\
&\cdot\left[-(2-\delta_{m0})\frac{dP_n^m(0)}{d\theta}\begin{matrix}\cos\\ \sin\end{matrix}(m\phi')\cos(\phi')\left(\frac{a}{|\cos\phi'|}\right)^2\right]d\phi',
\end{aligned}
\tag{18a}
$$

$$\begin{bmatrix} \Psi^{M,0<}_{\substack{e\\o}25} \\ \Psi^{M,1<}_{\substack{e\\o}25} \\ \Psi^{M,1>}_{\substack{e\\o}25} \\ \Psi^{M,2>}_{\substack{e\\o}25} \end{bmatrix} = \begin{bmatrix} (\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} I_2 - \int_{\frac{7\pi}{6}}^{\frac{3\pi}{2}} I_5) \\ (\int_{\frac{\pi}{3}-\phi_1}^{\frac{\pi}{3}+\phi_1} I_2 - \int_{\frac{4\pi}{3}-\phi_1}^{\frac{4\pi}{3}+\phi_1} I_5) \\ \left((\int_{\frac{\pi}{6}}^{\frac{\pi}{3}-\phi_1} + \int_{\frac{\pi}{3}+\phi_1}^{\frac{\pi}{2}})I_2 - (\int_{\frac{7\pi}{6}}^{\frac{4\pi}{3}-\phi_1} + \int_{\frac{4\pi}{3}+\phi_1}^{\frac{3\pi}{2}})I_5\right) \\ (\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} I_2 - \int_{\frac{7\pi}{6}}^{\frac{3\pi}{2}} I_5) \end{bmatrix}$$
$$\cdot \begin{bmatrix} j_n(\frac{k_0 a}{|\sin(\phi'+\frac{\pi}{6})|}) \\ j_n(\frac{k_0 a}{|\sin(\phi'+\frac{\pi}{6})|}) \\ h_n^{(1)}(\frac{k_0 a}{|\sin(\phi'+\frac{\pi}{6})|}) \\ h_n^{(1)}(\frac{k_0 a}{|\sin(\phi'+\frac{\pi}{6})|}) \end{bmatrix} \left[(2-\delta_{m0})\frac{dP_n^m(0)}{d\theta}\frac{\cos}{\sin}(m\phi')\right]$$
$$\cdot \left[-\left(\frac{\sqrt{3}}{2}\sin\phi' + \frac{1}{2}\cos\phi'\right)\left(\frac{a}{\left|\sin\left(\phi'+\frac{\pi}{6}\right)\right|}\right)^2\right] d\phi', \tag{18b}$$

$$\begin{bmatrix} \Psi^{M,0<}_{\substack{e\\o}36} \\ \Psi^{M,1<}_{\substack{e\\o}36} \\ \Psi^{M,1>}_{\substack{e\\o}36} \\ \Psi^{M,2>}_{\substack{e\\o}36} \end{bmatrix} = \begin{bmatrix} (\int_{\frac{2\pi}{2}-\phi_1}^{\frac{5\pi}{6}} I_3 - \int_{\frac{3\pi}{2}}^{\frac{11\pi}{6}} I_6) \\ (\int_{\frac{2\pi}{3}-\phi_1}^{\frac{2\pi}{3}+\phi_1} I_3 - \int_{\frac{5\pi}{3}-\phi_1}^{\frac{5\pi}{3}+\phi_1} I_6) \\ \left((\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}-\phi_1} + \int_{\frac{2\pi}{3}+\phi_1}^{\frac{5\pi}{6}})I_3 - (\int_{\frac{3\pi}{2}}^{\frac{5\pi}{3}-\phi_1} + \int_{\frac{5\pi}{3}+\phi_1}^{\frac{11\pi}{6}})I_6\right) \\ (\int_{\frac{2\pi}{3}-\phi_1}^{\frac{5\pi}{6}} I_3 - \int_{\frac{3\pi}{2}}^{\frac{11\pi}{6}} I_6) \end{bmatrix}$$
$$\cdot \begin{bmatrix} j_n(\frac{k_0 a}{|\cos(\phi'+\frac{\pi}{3})|}) \\ j_n(\frac{k_0 a}{|\cos(\phi'+\frac{\pi}{3})|}) \\ h_n^{(1)}(\frac{k_0 a}{|\cos(\phi'+\frac{\pi}{3})|}) \\ h_n^{(1)}(\frac{k_0 a}{|\cos(\phi'+\frac{\pi}{3})|}) \end{bmatrix} \left[(2-\delta_{m0})\frac{dP_n^m(0)}{d\theta}\frac{\cos}{\sin}(m\phi')\right]$$
$$\cdot \left[-\left(\frac{\sqrt{3}}{2}\sin\phi' - \frac{1}{2}\cos\phi'\right)\left(\frac{a}{\left|\cos\left(\phi'+\frac{\pi}{3}\right)\right|}\right)^2\right] d\phi', \tag{18c}$$

$$\begin{bmatrix} \Psi^{N,r,0<}_{\substack{e\\o}14} \\ \Psi^{N,r,1<}_{\substack{e\\o}14} \\ \Psi^{N,r,1>}_{\substack{e\\o}14} \\ \Psi^{N,r,2>}_{\substack{e\\o}14} \end{bmatrix} = \begin{bmatrix} (\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} I_1 - \int_{\frac{5\pi}{6}}^{\frac{7\pi}{6}} I_4) \\ (\int_{-\phi_1}^{\phi_1} I_1 - \int_{\pi-\phi_1}^{\pi+\phi_1} I_4) \\ \left((\int_{\frac{11\pi}{6}}^{2\pi-\phi_1} + \int_{\phi_1}^{\frac{\pi}{6}}) I_1 - (\int_{\frac{5\pi}{6}}^{\pi-\phi_1} + \int_{\pi+\phi_1}^{\frac{7\pi}{6}}) I_4\right) \\ (\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} I_1 - \int_{\frac{5\pi}{6}}^{\frac{7\pi}{6}} I_4) \end{bmatrix}$$
$$\cdot \begin{bmatrix} j_n(\frac{k_0 a}{|\cos\phi'|}) \\ j_n(\frac{k_0 a}{|\cos\phi'|}) \\ h_n^{(1)}(\frac{k_0 a}{|\cos\phi'|}) \\ h_n^{(1)}(\frac{k_0 a}{|\cos\phi'|}) \end{bmatrix} \left[(2-\delta_{m0})\frac{n(n+1)P_n^m(0)}{k_0} \begin{matrix}\cos\\ \sin\end{matrix}(m\phi')\right]$$
$$\cdot \left[\sin(\phi')\left(\frac{a}{|\cos\phi'|}\right)\right] d\phi', \tag{18d}$$

$$\begin{bmatrix} \Psi^{N,r,0<}_{\substack{e\\o}25} \\ \Psi^{N,r,1<}_{\substack{e\\o}25} \\ \Psi^{N,r,1>}_{\substack{e\\o}25} \\ \Psi^{N,r,2>}_{\substack{e\\o}25} \end{bmatrix} = \begin{bmatrix} (\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} I_2 - \int_{\frac{7\pi}{6}}^{\frac{3\pi}{2}} I_5) \\ (\int_{\frac{\pi}{3}-\phi_1}^{\frac{\pi}{3}+\phi_1} I_2 - \int_{\frac{4\pi}{3}-\phi_1}^{\frac{4\pi}{3}+\phi_1} I_5) \\ \left((\int_{\frac{\pi}{6}}^{\frac{\pi}{3}-\phi_1} + \int_{\frac{\pi}{3}+\phi_1}^{\frac{\pi}{2}}) I_2 - (\int_{\frac{7\pi}{6}}^{\frac{4\pi}{3}-\phi_1} + \int_{\frac{4\pi}{3}+\phi_1}^{\frac{3\pi}{2}}) I_5\right) \\ (\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} I_2 - \int_{\frac{7\pi}{6}}^{\frac{3\pi}{2}} I_5) \end{bmatrix}$$
$$\cdot \begin{bmatrix} j_n(\frac{k_0 a}{|\sin(\phi'+\frac{\pi}{6})|}) \\ j_n(\frac{k_0 a}{|\sin(\phi'+\frac{\pi}{6})|}) \\ h_n^{(1)}(\frac{k_0 a}{|\sin(\phi'+\frac{\pi}{6})|}) \\ h_n^{(1)}(\frac{k_0 a}{|\sin(\phi'+\frac{\pi}{6})|}) \end{bmatrix} \left[(2-\delta_{m0})\frac{n(n+1)P_n^m(0)}{k_0} \begin{matrix}\cos\\ \sin\end{matrix}(m\phi')\right]$$
$$\cdot \left[\left(\frac{1}{2}\sin\phi' - \frac{\sqrt{3}}{2}\cos\phi'\right)\left(\frac{a}{\left|\sin\left(\phi'+\frac{\pi}{6}\right)\right|}\right)\right] d\phi', \tag{18e}$$

$$\begin{bmatrix} \Psi_{\substack{e\\o}36}^{N,r,0<} \\ \Psi_{\substack{e\\o}36}^{N,r,1<} \\ \Psi_{\substack{e\\o}36}^{N,r,1>} \\ \Psi_{\substack{e\\o}36}^{N,r,2>} \end{bmatrix} = \begin{bmatrix} (\int_{\frac{2\pi}{3}-\phi_1}^{\frac{5\pi}{6}} I_3 - \int_{\frac{3\pi}{2}}^{\frac{11\pi}{6}} I_6) \\ (\int_{\frac{2\pi}{3}-\phi_1}^{\frac{2\pi}{3}+\phi_1} I_3 - \int_{\frac{5\pi}{3}-\phi_1}^{\frac{5\pi}{3}+\phi_1} I_6) \\ \left((\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}-\phi_1} + \int_{\frac{2\pi}{3}+\phi_1}^{\frac{5\pi}{6}}) I_3 - (\int_{\frac{3\pi}{2}}^{\frac{5\pi}{3}-\phi_1} + \int_{\frac{5\pi}{3}+\phi_1}^{\frac{11\pi}{6}}) I_6 \right) \\ (\int_{\frac{2\pi}{3}-\phi_1}^{\frac{5\pi}{6}} I_1 - \int_{\frac{3\pi}{2}}^{\frac{11\pi}{6}} I_6) \end{bmatrix}$$
$$\cdot \begin{bmatrix} j_n(\frac{k_0 a}{|\cos(\phi'+\frac{\pi}{3})|}) \\ j_n(\frac{k_0 a}{|\cos(\phi'+\frac{\pi}{3})|}) \\ h_n^{(1)}(\frac{k_0 a}{|\cos(\phi'+\frac{\pi}{3})|}) \\ h_n^{(1)}(\frac{k_0 a}{|\cos(\phi'+\frac{\pi}{3})|}) \end{bmatrix} \left[(2-\delta_{m0}) \frac{n(n+1)P_n^m(0)}{k_0} \begin{matrix}\cos\\ \sin\end{matrix} (m\phi') \right]$$
$$\cdot \left[\left(-\frac{\sqrt{3}}{2}\cos\phi' - \frac{1}{2}\sin\phi' \right) \left(\frac{a}{\left|\cos\left(\phi' + \frac{\pi}{3}\right)\right|} \right) \right] d\phi', \tag{18f}$$

$$\begin{bmatrix} \Psi_{\substack{e\\o}14}^{N,\phi,0<} \\ \Psi_{\substack{e\\o}14}^{N,\phi,1<} \\ \Psi_{\substack{e\\o}14}^{N,\phi,1>} \\ \Psi_{\substack{e\\o}14}^{N,\phi,2>} \end{bmatrix} = \begin{bmatrix} (\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} I_1 - \int_{\frac{5\pi}{6}}^{\frac{7\pi}{6}} I_4) \\ (\int_{-\phi_1}^{\phi_1} I_1 - \int_{\pi-\phi_1}^{\pi+\phi_1} I_4) \\ \left((\int_{\frac{11\pi}{6}}^{2\pi-\phi_1} + \int_{\phi_1}^{\frac{\pi}{6}}) I_1 - (\int_{\frac{5\pi}{6}}^{\pi-\phi_1} + \int_{\pi+\phi_1}^{\frac{7\pi}{6}}) I_4 \right) \\ (\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} I_1 - \int_{\frac{5\pi}{6}}^{\frac{7\pi}{6}} I_4) \end{bmatrix}$$
$$\cdot \begin{bmatrix} \frac{d[\frac{a}{|\cos\phi'|} j_n(\frac{k_0 a}{|\cos\phi'|})]}{k_0 d(\frac{a}{|\cos\phi'|})} \\ \frac{d[\frac{a}{|\cos\phi'|} j_n(\frac{k_0 a}{|\cos\phi'|})]}{k_0 d(\frac{a}{|\cos\phi'|})} \\ \frac{d[\frac{a}{|\cos\phi'|} h_n^{(1)}(\frac{k_0 a}{|\cos\phi'|})]}{k_0 d(\frac{a}{|\cos\phi'|})} \\ \frac{d[\frac{a}{|\cos\phi'|} h_n^{(1)}(\frac{k_0 a}{|\cos\phi'|})]}{k_0 d(\frac{a}{|\cos\phi'|})} \end{bmatrix} \left[(2-\delta_{m0}) m P_n^m(0) \begin{matrix}\sin\\ \cos\end{matrix} (m\phi') \right]$$
$$\cdot \left[\mp\cos(\phi') \left(\frac{a}{|\cos\phi'|} \right) \right] d\phi', \tag{18g}$$

$$\begin{bmatrix} \Psi^{N,\phi,0<}_{\substack{e\\o}25} \\ \Psi^{N,\phi,1<}_{\substack{e\\o}25} \\ \Psi^{N,\phi,1>}_{\substack{e\\o}25} \\ \Psi^{N,\phi,2>}_{\substack{e\\o}25} \end{bmatrix} = \begin{bmatrix} (\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} I_2 - \int_{\frac{7\pi}{6}}^{\frac{3\pi}{2}} I_5) \\ (\int_{\frac{\pi}{3}-\phi_1}^{\frac{\pi}{3}+\phi_1} I_2 - \int_{\frac{4\pi}{3}-\phi_1}^{\frac{4\pi}{3}+\phi_1} I_5) \\ \left((\int_{\frac{\pi}{6}}^{\frac{\pi}{3}-\phi_1} + \int_{\frac{\pi}{3}+\phi_1}^{\frac{\pi}{2}}) I_2 - (\int_{\frac{7\pi}{6}}^{\frac{4\pi}{3}-\phi_1} + \int_{\frac{4\pi}{3}+\phi_1}^{\frac{3\pi}{2}}) I_5 \right) \\ (\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} I_2 - \int_{\frac{7\pi}{6}}^{\frac{3\pi}{2}} I_5) \end{bmatrix}$$
$$\cdot \begin{bmatrix} \frac{d[\frac{a}{|\sin(\phi'+\frac{\pi}{6})|} j_n(\frac{k_0 a}{|\sin(\phi'+\frac{\pi}{6})|})]}{k_0 d(\frac{a}{|\sin(\phi'+\frac{\pi}{6})|})} \\ \frac{d[\frac{a}{|\sin(\phi'+\frac{\pi}{6})|} j_n(\frac{k_0 a}{|\sin(\phi'+\frac{\pi}{6})|})]}{k_0 d(\frac{a}{|\sin(\phi'+\frac{\pi}{6})|})} \\ \frac{d[\frac{a}{|\sin(\phi'+\frac{\pi}{6})|} h_n^{(1)}(\frac{k_0 a}{|\sin(\phi'+\frac{\pi}{6})|})]}{k_0 d(\frac{a}{|\sin(\phi'+\frac{\pi}{6})|})} \\ \frac{d[\frac{a}{|\sin(\phi'+\frac{\pi}{6})|} h_n^{(1)}(\frac{k_0 a}{|\sin(\phi'+\frac{\pi}{6})|})]}{k_0 d(\frac{a}{|\sin(\phi'+\frac{\pi}{6})|})} \end{bmatrix} \left[(2-\delta_{m0}) m P_n^m(0) \begin{matrix} \sin \\ \cos \end{matrix} (m\phi') \right]$$
$$\cdot \left[\mp \left(\frac{\sqrt{3}}{2} \cos\phi' + \frac{1}{2} \sin\phi' \right) \left(\frac{a}{\left|\sin\left(\phi' + \frac{\pi}{6}\right)\right|} \right) \right] d\phi'; \tag{18h}$$

$$\begin{bmatrix} \Psi^{N,\phi,0<}_{\substack{e\\o}36} \\ \Psi^{N,\phi,1<}_{\substack{e\\o}36} \\ \Psi^{N,\phi,1>}_{\substack{e\\o}36} \\ \Psi^{N,\phi,2>}_{\substack{e\\o}36} \end{bmatrix} = \begin{bmatrix} (\int_{\frac{2\pi}{3}-\phi_1}^{\frac{5\pi}{6}} I_3 - \int_{\frac{3\pi}{2}}^{\frac{11\pi}{6}} I_6) \\ (\int_{\frac{2\pi}{3}-\phi_1}^{\frac{2\pi}{3}+\phi_1} I_3 - \int_{\frac{5\pi}{3}-\phi_1}^{\frac{5\pi}{3}+\phi_1} I_6) \\ \left((\int_{\frac{\pi}{2}}^{\frac{2\pi}{3}-\phi_1} + \int_{\frac{2\pi}{3}+\phi_1}^{\frac{5\pi}{6}}) I_3 - (\int_{\frac{3\pi}{2}}^{\frac{5\pi}{3}-\phi_1} + \int_{\frac{5\pi}{3}+\phi_1}^{\frac{11\pi}{6}}) I_6 \right) \\ (\int_{\frac{2\pi}{3}-\phi_1}^{\frac{5\pi}{6}} I_3 - \int_{\frac{3\pi}{2}}^{\frac{11\pi}{6}} I_6) \end{bmatrix}$$
$$\cdot \begin{bmatrix} \frac{d[\frac{a}{|\cos(\phi'+\frac{\pi}{3})|} j_n(\frac{k_0 a}{|\cos(\phi'+\frac{\pi}{3})|})]}{k_0 d(\frac{a}{|\cos\phi'|})} \\ \frac{d[\frac{a}{|\cos(\phi'+\frac{\pi}{3})|} j_n(\frac{k_0 a}{|\cos(\phi'+\frac{\pi}{3})|})]}{k_0 d(\frac{a}{|\cos(\phi'+\frac{\pi}{3})|})} \\ \frac{d[\frac{a}{|\cos(\phi'+\frac{\pi}{3})|} h_n^{(1)}(\frac{k_0 a}{|\cos(\phi'+\frac{\pi}{3})|})]}{k_0 d(\frac{a}{|\cos(\phi'+\frac{\pi}{3})|})} \\ \frac{d[\frac{a}{|\cos(\phi'+\frac{\pi}{3})|} h_n^{(1)}(\frac{k_0 a}{|\cos(\phi'+\frac{\pi}{3})|})]}{k_0 d(\frac{a}{|\cos(\phi'+\frac{\pi}{3})|})} \end{bmatrix} \left[(2-\delta_{m0}) m P_n^m(0) \begin{matrix} \sin \\ \cos \end{matrix} (m\phi') \right]$$
$$\cdot \left[\mp \left(\frac{\sqrt{3}}{2} \sin\phi' - \frac{1}{2} \cos\phi' \right) \left(\frac{a}{\left|\cos\left(\phi' + \frac{\pi}{3}\right)\right|} \right) \right] d\phi', \tag{18i}$$

where $\phi_1 = \cos^{-1}(a/r)$. The associated Legendre function $P_n^m(0)$ and its first-order derivative $dP_n^m(0)/d\theta$ are given by

$$\frac{dP_n^m(0)}{d\theta} = -\frac{2^{m+1}\sin\left[\frac{1}{2}(n+m)\pi\right]\Gamma\left(\frac{n+m}{2}+1\right)}{\sqrt{\pi}\Gamma\left(\frac{n-m+1}{2}\right)}, \tag{19a}$$

$$P_n^m(0) = \frac{2^m\cos\left[\frac{1}{2}(n+m)\pi\right]\Gamma\left(\frac{n+m+1}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{n-m}{2}+1\right)}. \tag{19b}$$

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