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A METHOD OF MOMENTS ANALYSIS OF A MICROSTRIP PHASED ARRAY IN THREE-LAYERED STRUCTURES

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Abstract–In this paper, the scanning characteristics of an infinite stacked microstrip phased array in a three-layered structure are analysed. In the analysis of the field distribution, the spectral-domain Galerkin Method of Moments together with the planar dyadic Green's function is applied. An attachment mode current is employed to model the singularity of currents nearby the feed point so as to facilitate the fast convergence. The currents on all patches are expanded in terms of the trigonometric basis and weighting functions in the entire domain. The normalized patterns of the infinite microstrip array are computed in this paper and the scanning features of the antenna against the scanning angle and frequency are discussed in both the E- and H-planes.

1. INTRODUCTION

Microstrip antennas and phased arrays are widely used in telecommunication and radar systems due to their many well-known advantages, such as small size, light-weight, flush-mountability, inexpensive fabrication, and easy conformability, although having narrow impedance bandwidth. Over the past 20 years, a tremendous amount of theoretical and experimental work on microstrip antennas has been done. Many new ideas and design methods have been proposed to broaden the bandwidth of microstrip antennas [1]. Among these, the most straightforward way is to increase the separation between the patch and the ground plane by using a thicker substrate, although a grounded thick dielectric slab results in surface-wave mode losses. Also, the concept of stacking microstrip patches is also employed to improve bandwidth as is well known nowadays. Theoretical treatments of various stacked patch geometries have been conducted and their results published in literature [2–5].

The motivation of this paper is to combine the two techniques, i.e., increasing the thickness of the substrate (with three dielectric layers) and stacking microstrip patches (extended to an infinite microstrip phased array), so as to further broaden the bandwidth of the microstrip antennas. In other words, a microstrip phased array in a three-layered geometry is dealt with and analysed in this paper by the Garlerkin Method of Moments. Each element of the array is comprised of two-layer stacked rectangular patches fed by a small capacitor-plate mounted on the top of the coaxial probe. This structure is utilized to increase significantly the bandwidth of the antennas [6].

This paper is organized in the following order. Section 2 presents the theoretical analysis of an infinite array of capacitor-probe-fed rectangular patches by the Garlerkin Method of Moments. In section 3, the numerical results of reflection coefficients and the normalized power patterns as a function of the scanning angle in the E- and H-planes are obtained and presented. The spectral domain dyadic Green's function components are necessarily self-contained in Appendex A for completeness.

2. ANALYSIS TECHNIQUES

Consider an infinite periodic array arranged in rectangular grids as shown in Fig. 1. For the ease of formulation, the Cartesian coordinate system is set as in Fig. 1. Fig. 2 shows the construction of the microstrip antenna array elements where the excitation is made by means of a small planar capacitor plate mounted on the top of the coaxial probe, of which the parameters are given in Fig. 2(a). This structure is general and it can be reduced to the structure used in [9] by letting $h_3 = 0$, $\varepsilon_{r3} = 1$, and $W_{x3} = W_{y3} = 0$. It is assumed

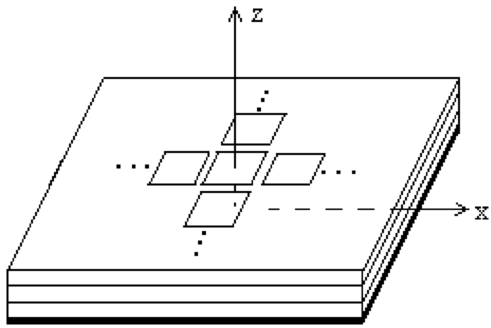
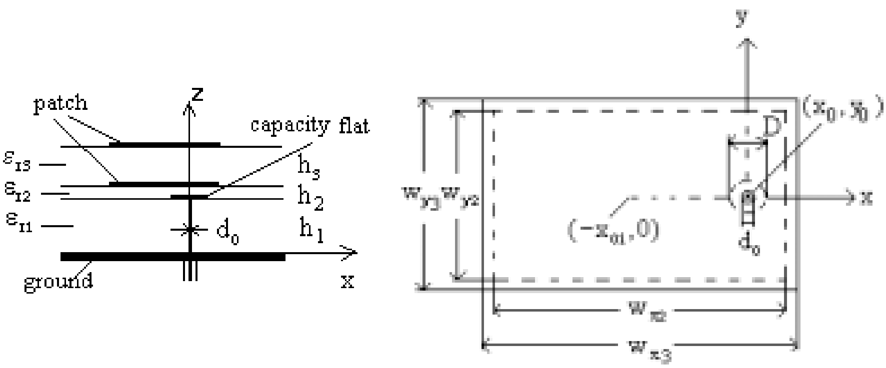


Figure 1. Infinite microstrip phased array.



(a) side view (b) plan view

Figure 2. The antenna element and coordinate system.

that the thickness of the intermediate layer, h_2, is very small as compared with the thickness of the bottom, h_1. The capacitance of the planar capacitor plate can be determined from its diameter and the characteristics of the dielectric substrate [6].

It is assumed that the patches, the feed capacitance plate, the feed probe and the grounded plate are perfect conductors, the thickness of the patches is infinitely thin, and the substrates are homogeneous in the x-y plane. Also, it is assumed that each element of the infinite array is excited by a 1 volt δ voltage source. In this analysis, the diameter of the feed probe is considered as that of the inner conductor rather than the outer conductor of the probe.

It is known that the field $\boldsymbol{E}_{pq}$ at $(x,\ y,\ z)$ in the p-direction produced by the q-directional electric source moment located at $(x_0,$

$y_0,\ z_0)$ can be written as:

$$\boldsymbol{E}^0_{pq}\left(x,y,z|x_0,y_0,z_0\right)=\frac{1}{4\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\boldsymbol{G}_{pq}\left(k_x,k_y,z|z_0\right)\times e^{jk_x(x-x_0)+jk_y(y-y_0)}dk_xdk_y, \tag{1}$$

where p and q can be x, y or z, and $\boldsymbol{G}_{pq}$ are the Green's functions produced by a unit electric source moment and their components can be found in Appendix A. For an infinite microstrip patch array, we can simplify the problem into a cell in a unit space by using the Floquet theorem [7–9].

In order to analyze the feature of the elements of the phased array, we first formulate the dyadic Green's function in this structure. The components of the Green's functions are derived by the Floquet theorem in a periodic structure using the Poisson summation equation.

By assuming that each element is phased at a scanning angle (θ, φ) in an infinite array fed by infinitesimal q-direction current, then the electric field at $(x,\ y,\ z)$ in the p-direction produced by the current element $m=0$ and $n=0$ at $(x_0,\ y_0,\ z_0)$ can be written as [7]:

$$\boldsymbol{E}_{pq}\left(x,y,z|x_0,y_0,z_0\right)=\frac{1}{ab}\sum_{m,=-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\boldsymbol{G}_{pq}\left(-k_{xm},-k_{yn},z|z_0\right)\times e^{-jk_x(x-x_0)-jk_y(y-y_0)}, \tag{2}$$

where a and b are the inter-element spacings along the x- and y-axes, respectively, and

$$k_x=k_0u+\frac{2\pi m}{a}, \tag{3a}$$

$$k_y=k_0v+\frac{2\pi n}{b}, \tag{3b}$$

$$u=\sin\theta\cos\varphi, \tag{3c}$$

$$v=\sin\theta\sin\varphi, \tag{3d}$$

$$k_0^2=\omega^2\mu_0\varepsilon_0. \tag{3e}$$

For the sake of convenience, the dyadic Green's function can be expressed in component form as follows:

$$\begin{aligned}\overline{\boldsymbol{G}}=&\ \widehat{\boldsymbol{x}}G_{xx}\widehat{\boldsymbol{x}}+\widehat{\boldsymbol{x}}G_{xy}\widehat{\boldsymbol{y}}+\widehat{\boldsymbol{x}}G_{xz}\widehat{\boldsymbol{z}}\\&+\widehat{\boldsymbol{y}}G_{yx}\widehat{\boldsymbol{x}}+\widehat{\boldsymbol{y}}G_{yy}\widehat{\boldsymbol{y}}+\widehat{\boldsymbol{y}}G_{yz}\widehat{\boldsymbol{z}}\\&+\widehat{\boldsymbol{z}}G_{zx}\widehat{\boldsymbol{x}}+\widehat{\boldsymbol{z}}G_{zy}\widehat{\boldsymbol{y}}+\widehat{\boldsymbol{z}}G_{zz}\widehat{\boldsymbol{z}}.\end{aligned} \tag{4}$$

From the boundary conditions, the total tangential electric field components on the top and bottom patches should vanish, i.e.,

$$\boldsymbol{E}_{tan}^{tot} = \boldsymbol{E}_{tan}^{e} + \boldsymbol{E}_{tan}^{s} = 0, \tag{5}$$

where $\boldsymbol{E}_{tan}^{e}$ and $\boldsymbol{E}_{tan}^{s}$ indicate the induced and scattered fields, and

$$\boldsymbol{E}^s = \iiint \overline{\boldsymbol{G}}\,(x, y, z|x_0, y_0, z_0) \cdot \boldsymbol{J}\,(x_0, y_0, z_0)\, dx_0 dy_0 dz_0, \tag{6}$$

where $\boldsymbol{J} = \boldsymbol{J}_{p1} + \boldsymbol{J}_{p2} + \boldsymbol{J}_{p3}$ with $\boldsymbol{J}_{p1}$, $\boldsymbol{J}_{p2}$ and $\boldsymbol{J}_{p3}$ being the unknown current density vectors on the patches respectively.

$$\boldsymbol{J}\,(x, y, z) = \sum_{j=1}^{N_1} I_j \boldsymbol{J}_j(x, y, z) \sum_{j=N_1+1}^{N_1+N_2} I_j \boldsymbol{J}_j(x, y, z) + \sum_{j=N_1+N_2+1}^{N_1+N_2+N_3} I_j \boldsymbol{J}_j(x, y, z), \tag{7}$$

where $\boldsymbol{J}_j$ ($j = 1, \cdots, N_1$; $N_1{+}1, \cdots, N_1{+}N_2$; and $N_1{+}N_2{+}1, \cdots, N_1{+}N_2 + N_3$) are the basis functions on the bottom patch (feed capacitor plate), the middle patch, and the top patch, respectively.

For the bottom patch, there is a single pole at the junction point since the capacitor plate is fed by the coaxial probe. Now, let us use $\boldsymbol{\phi}$ to indicate the singularity term, so that

$$\boldsymbol{J}_1 = \boldsymbol{\phi} + \boldsymbol{J}_1^R, \tag{8}$$

where $\boldsymbol{J}_1^R$ is the regular current on the lowest patch.

For the coordinates system shown in this paper, the singularity term of the current expansion is thus given by [10]:

$$\begin{aligned}
\boldsymbol{\phi}\,(x, y) = &-\frac{1}{2w_{y1}} \sum_{0}^{\infty} \epsilon_n \cos\frac{n\pi}{2} \left[\cos\frac{n\pi}{w_{y1}}\left(y + \frac{w_{y1}}{2}\right)\right. \\
&\times \widehat{\boldsymbol{e}}_x f_s\,(Z, w_{x1}, x_0, x) + \widehat{\boldsymbol{e}}_y f_c\,(Z, w_{x1}, x_0, x) \\
&\times \left.\left(\frac{n\pi}{w_{y1}}\right) \sin\frac{n\pi}{w_{y1}}\left(y + \frac{w_{y1}}{2}\right)\right],
\end{aligned} \tag{9}$$

where

$$f_s(Z, w_{x1}, x_0, x) = \frac{\sin Z\,(x + x_0)}{\sin Z w_{x1}} - \mathrm{sgn}(x - x_0) \times \frac{\sin Z\,(w_{y1} - |x - x_0|)}{\sin Z w_{x1}}, \tag{10a}$$

$$f_c\,(Z, w_{x1}, x_0, x) = \frac{\cos Z\,(x + x_0)}{Z \sin Z w_{x1}} + \frac{\cos Z\,(w_{y1} - |x - x_0|)}{Z \sin Z w_{x1}}, \tag{10b}$$

$$\mathrm{sgn}(x) = \begin{cases} 1, & x \succ 0 \\ 0, & x = 0 \\ -1, & x \prec 0 \end{cases}, \tag{10c}$$

$$\epsilon_n = \begin{cases} 1, & n = 0 \\ 0, & n \succ 0 \end{cases}, \tag{10d}$$

$$Z = \sqrt{k^2 - \left(\frac{n\pi}{w_{y1}}\right)^2}, \tag{10e}$$

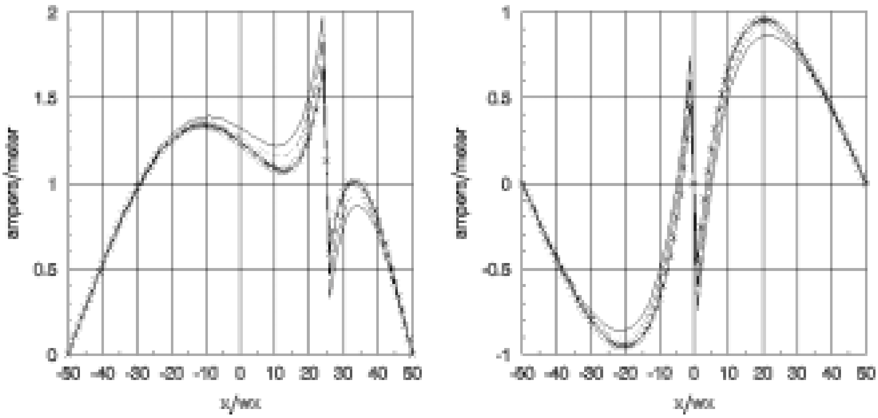
$$k = \sqrt{\varepsilon_{r1}}k_0, \tag{10f}$$

and where $\widehat{\boldsymbol{e}}_x$, $\widehat{\boldsymbol{e}}_y$ and the $\widehat{\boldsymbol{e}}_z$ are $\widehat{\boldsymbol{x}}$-, $\widehat{\boldsymbol{y}}$- and $\widehat{\boldsymbol{z}}$-directional unit vectors, respectively. Figs. 3(a) and (b) show, respectively, the x-directional currents on the patch when the feed point is located at different places (i.e., (a) at $x_p = 0.25w_{x1}$ and $y_p = 0$; and (b) at $x_p = y_p = 0$), where the solid, dotted and cross curves represent the currents when $y = 0.002w_{y1}$, $y = 0.02w_{y1}$, and $y = 0.04w_{y1}$, respectively. The curves in Fig. 3 reveal that the attachment mode current meets the requirements of boundary and edge conditions. The currents along both x- and y-directions are symmetric with respect to the feed point, but in opposite directions.

Now, let us expand the currents on patches in terms of $\boldsymbol{f}_{1n}(x, y)$, $\boldsymbol{f}_{2n}(x, y)$ and $\boldsymbol{f}_{3n}(x, y)$, i.e.,

$$\boldsymbol{J}_i = \sum_{i=1}^{\infty} I_{in}\boldsymbol{f}_{in}\,(x, y)\;(i = 2, 3), \tag{11a}$$

$$\boldsymbol{J}_1^R = \sum_{i=1}^{\infty} I_{1n}\boldsymbol{f}_{1n}\,(x, y), \tag{11b}$$



(a) The feed point is at $x_p = 0.25w_{x1}$ and $y_p = 0$.

(b) The feed point is at $x_p = y_p = 0$.

Figure 3. The attachment mode current along x-direction on the patch.

where I_{in} $(i = 1, 2, 3)$ are unknown coefficients. Based on the cavity mode theory, the currents on the patches can be expanded in terms of the basis functions in the entire domain. For the first patch (the feed capacitance), the current expansion function is:

$$\begin{aligned}\boldsymbol{f}_{1n} = &\,\widehat{\boldsymbol{e}}_x\left(\frac{m_x\pi}{w_{x1}}\right)\sin\left[\frac{m_x\pi}{w_{x1}}\left(x+\frac{w_{x1}}{2}\right)\right]\cos\left[\frac{m_y\pi}{w_{y1}}\left(y+\frac{w_{y1}}{2}\right)\right]\\ &+\widehat{\boldsymbol{e}}_y\left(\frac{m_y\pi}{w_{y1}}\right)\cos\left[\frac{m_x\pi}{w_{x1}}\left(x+\frac{w_{x1}}{2}\right)\right]\sin\left[\frac{m_y\pi}{w_{y1}}\left(y+\frac{w_{y1}}{2}\right)\right].\end{aligned}\tag{13}$$

For the second and third patches, there is a current displacement along x-direction. Thus, the expansion function of the current distribution becomes:

$$\begin{aligned}\boldsymbol{f}_{in} = &\,\widehat{\boldsymbol{e}}_x\left(\frac{m_x\pi}{w_{xi}}\right)\sin\left[\frac{m_x\pi}{w_{xi}}\left(x-x_{0i}+\frac{w_{xi}}{2}\right)\right]\cos\left[\frac{m_y\pi}{w_{yi}}\left(y+\frac{w_{yi}}{2}\right)\right]\\ &+\widehat{\boldsymbol{e}}_y\left(\frac{m_y\pi}{w_{yi}}\right)\cos\left[\frac{m_x\pi}{w_{xi}}\left(x_0-x_{0i}+\frac{w_{xi}}{2}\right)\right]\sin\left[\frac{m_y\pi}{w_{yi}}\left(y+\frac{w_{yi}}{2}\right)\right],\\ &(i=2,3;m_x=0,1,\cdots;m_y=0,1,2,\cdots.)\end{aligned}\tag{13}$$

where x_{0i} $(i = 2, 3)$ are the displacements from the coordinate origin between the second and third patches.

In the previous formulation, the Fourier transformation has been used in Equations (9), (12) and (13). Considering the difference of the integration interval, we then obtain for $i=2$ and 3:

$$\begin{aligned}
\boldsymbol{\phi}^T =& \frac{-1}{2w_{y1}}\sum_0^\infty \epsilon_n\ cos\left[\frac{n\pi}{w_{y1}}\left(y_0+\frac{w_{y1}}{2}\right)\right] \\
&\times\Bigg[\widehat{\boldsymbol{e}}_x F_c\left(n, w_{y1}, k_y\right) F_{ss}\left(Z, w_{x1}, x_0, k_x\right) \\
&+\widehat{\boldsymbol{e}}_y\left(\frac{n\pi}{w_{y1}}\right) F_s\left(n, w_{y1}, k_y\right) F_{cc}\left(Z, w_{x1}, x_0, k_x\right)\Bigg], \quad (14a)\\
\boldsymbol{F}_1^T\left(k_x, k_y\right) =& \widehat{\boldsymbol{e}}_x\left(\frac{m_x\pi}{w_{x1}}\right) F_s\left(m_x, w_{x1}, k_x\right) F_c\left(m_y, w_{y1}, k_y\right) \\
&+\widehat{\boldsymbol{e}}_y\left(\frac{m_y\pi}{w_{y1}}\right) F_c\left(m_x, w_{x1}, k_x\right) F_s\left(m_y, w_{y1}, k_y\right), \quad (14b)\\
\boldsymbol{F}_i^T\left(k_x, k_y\right) =& \widehat{\boldsymbol{e}}_x\left(\frac{m_x\pi}{w_{xi}}\right) F_{1x}\left(m_x, w_{xi}, k_x\right) F_c\left(m_y, w_{yi}, k_y\right) \\
&+\widehat{\boldsymbol{e}}_y\left(\frac{m_y\pi}{w_{yi}}\right) F_{2x}\left(m_x, w_{xi}, k_x\right) F_s\left(m_y, w_{yi}, k_y\right), \quad (14c)
\end{aligned}$$

where

$$\begin{aligned}
F_{1x}\left(m_x, w_{xi}, k_x\right) =& \frac{\dfrac{m_x\pi}{w_{xi}}}{k_x^2-\left(\dfrac{m_x\pi}{w_{xi}}\right)^2} \\
&\times\left[(-1)^{m_x} e^{-jk_x\left(\frac{w_{xi}}{2}+x_{0i}\right)}-e^{jk_x\left(\frac{w_{xi}}{2}-x_{0i}\right)}\right], \quad (15a)\\
F_{2x}\left(m_x, w_{yi}, k_x\right) =& \frac{jk_x}{k_x^2-\left(\dfrac{m_x\pi}{w_{xi}}\right)^2} \\
&\times\left[(-1)^{m_x} e^{-jk_x\left(\frac{w_{xi}}{2}+x_{0i}\right)}-e^{jk_x\left(\frac{w_{xi}}{2}-x_{0i}\right)}\right], \quad (15b)\\
F_s\left(m, w_{xi}, k_x\right) =& \frac{\dfrac{m\pi}{w_{xi}}}{k_x^2-\left(\dfrac{m\pi}{w_{xi}}\right)^2} \\
&\times\left[(-1)^{m} e^{-jk_x\left(\frac{w_{xi}}{2}\right)}-e^{jk_x\left(\frac{w_{xi}}{2}\right)}\right], \quad (15c)
\end{aligned}$$

$$F_c\left(m, w_{xi}, k_x\right) = \frac{jk_x}{k_x^2 - \left(\dfrac{m\pi}{w_{xi}}\right)^2} \times \left[(-1)^m e^{-jk_x\left(\frac{w_{xi}}{2}\right)} - e^{jk_x\left(\frac{w_{xi}}{2}\right)}\right], \tag{15d}$$

$$F_{ss}(Z, w_{xi}, x_0, k_x) = \frac{-2j}{(k_x^2 - Z^2)\sin Z w_{xi}} \left\{ Z\left[e^{jZx_0} \sin\frac{w_{xi}(k_x - Z)}{2} + e^{-jZx_0} \sin\frac{w_{xi}(k_x + Z)}{2}\right] - k_x e^{-jk_x} \sin Z w_{xi} \right\}, \tag{15e}$$

$$F_{cc}(Z, w_{xi}, x_0, k_x) = \frac{2}{(k_x^2 - Z^2)\sin Z w_{xi}} \left\{ k_x\left[e^{jZx_0} \sin\frac{w_{xi}(k_x - Z)}{2} + e^{-jZx_0} \sin\frac{w_{xi}(k_x + Z)}{2}\right] - Z e^{-jk_x} \sin Z w_{xi} \right\}, \tag{15f}$$

and w_{xi} and w_{yi} ($i = 1, 2$, and 3) are the x- and y-dimensions of the patches, respectively. For the patch between the first and second substrates, the same diameter D is used to represent its length and width, i.e., $w_{x1} = w_{y1} = D$. By assuming that the radiation element is phased at the scanning angle $(\theta,\ \varphi)$, the electric field integral equation (EFIE) can then be obtained and solved by Galerkin Method of Moments. With this technique, the equation in matrix form can be rewritten as [7]:

$$[Z]\,[I] = [V], \tag{16}$$

where $[Z]$ is the impedance matrix of the size $(N_1 + N_2 + N_3) \times (N_1 + N_2 + N_3)$, expressed as:

$$[Z] = \begin{bmatrix} Z_{11}^{FF} + Z_{11}^{FP} + Z_{11}^{PF} + Z_{11}^{PP} & Z_{12}^{FP} + Z_{12}^{PP} & \cdots & Z_{1N_1+N_2+N_3}^{FP} + Z_{1N_1+N_2+N_3}^{PP} \\ Z_{21}^{PF} + Z_{21}^{PP} & Z_{22}^{PP} & \cdots & Z_{2N_1+N_2+N_3}^{PP} \\ \vdots & \vdots & \vdots & \vdots \\ Z_{N_1+N_2+N_3 1}^{PF} & \cdots & \cdots & Z_{N_1+N_2+N_3 N_1+N_2+N_3}^{PP} \end{bmatrix}, \tag{17}$$

where $[V]$ is a voltage vector or a column matrix of $(N_1 + N_2 + N_3)$ rows, $[I]$ is a matrix consisting of $(N_1+N_2+N_3)$ unknown coefficients of the basis functions.

When the antenna is fed by a δ voltage source, the element of the voltage matrix can be written as:

$$V_i = \begin{cases} 1, & i = 1, \\ 0, & i \neq 1. \end{cases} \tag{18}$$

The elements of the impedance matrix (18) can be further expressed as:

$$Z_{11} = Z_{11}^{FF} + Z_{11}^{FP} + Z_{11}^{PF} + Z_{11}^{PP}, \tag{19a}$$

$$Z_{1j} = Z_{1j}^{FP} + Z_{1j}^{PP}, \ (j = 2, \cdots, N_1 + N_2 + N_3), \tag{19b}$$

$$Z_{i1} = Z_{i1}^{PF} + Z_{i1}^{PP}, \ (i = 2, \cdots, N_1 + N_2 + N_3), \tag{19c}$$

$$Z_{ij} = Z_{ij}^{PP}, \ (i, j = 2, \cdots, N_1 + N_2 + N_3), \tag{19d}$$

where the components of the matrix can be found in Appendix B.

The input impedance is

$$Z_{in} = \frac{1}{I_1} + jX_L, \tag{20}$$

where X_L is the inductance of the probe, given by [3]

$$X_L = 60k_0h_1 \ln\left(\frac{2}{k_0h_1\sqrt{\epsilon_{r1}}}\right). \tag{21}$$

After we have obtained the impedance, the reflection coefficient as a function of scanning angle $(\theta, \ \varphi)$ is [11]

$$\Gamma(\theta, \varphi) = \frac{Z_{in}(\theta, \varphi) - Z(0, 0)}{Z_{in}(\theta, \varphi) - Z^*(0, 0)}, \tag{22}$$

where $Z_0(0, 0)$ is the input broadside impedance. The element gain pattern $G(\theta, \varphi)$ is related to the reflection coefficient by

$$G(\theta, \varphi) = \left(1 - |\Gamma(\theta, \varphi)|^2\right) \cos\theta. \tag{23}$$

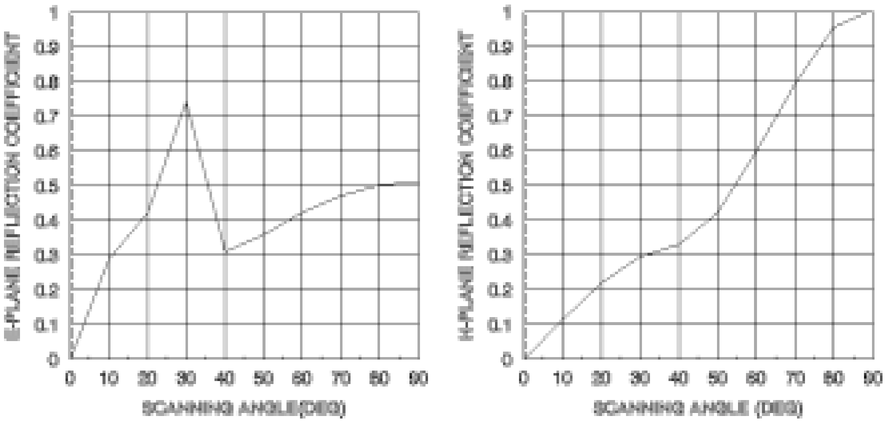
3. NUMERICAL RESULTS

We have considered three independent problems and their relevant convergences. First, although (9) presents the attachment mode current on the patch as an infinite summation, ten terms are shown to be sufficient for the summation to achieve the needed accuracy since the speed of convergence has been accelerated in the analysis. Second, the elements of the matrix are determined by (19), (24)–(27) and each term in the infinite summation represents a Floquet mode of periodic structure, thus we should truncate the infinite summation. In our calculation, 301 Floquet modes $(-150 < m, n < 150)$ are adopted. This mode number is more than that chosen in [7] because the selected attachment mode presents not only the rapid variation of the feed current distribution but also the rapid variation of the current distribution near the feed point. Thus, it has a broader bandwidth. Lastly, we must make proper expansion of the currents on the patches so as to include the edge effects of the currents, based on the results in [7–9]. We have found the current $(m_x, m_y) = (1, 0)$, $(3, 0)$, $(5, 0)$, $(7, 0)$ in the x-direction and $(m_x, m_y) = (0, 2)$ in the y-direction both meeting the prescribed resolution, in which the mode (1, 0) is the main cavity mode while the mode (0, 1) represent the component of cross section, so that six or seven modes are needed only in this computation (for the lowest patch one attached mode should be included).

In our calculation, we assumed that the impedance is matched when the scanning angle is $(\theta, \varphi) = (0, 0)$, so the reflection coefficient is 0. The spacing between the two adjacent elements is $\frac{\lambda_0}{2}$ (where λ_0 is the wave number in free space). The physical and electrical parameters of the antenna are given by

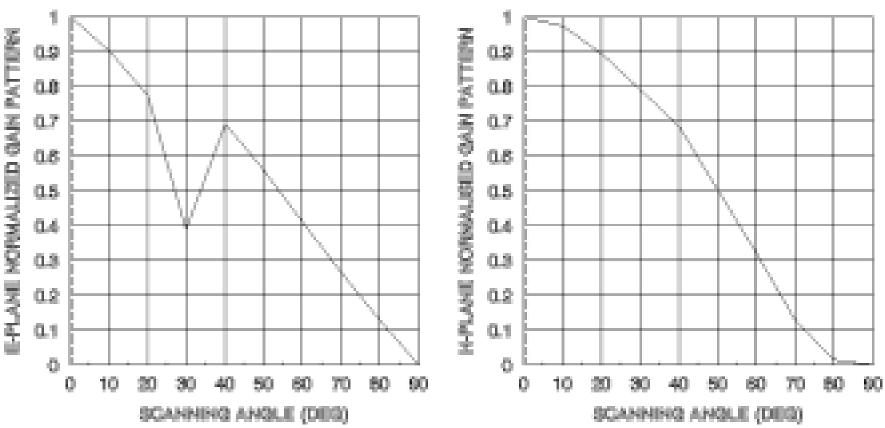
- $W_{x2} = W_{y2} = 62.5\,\mathrm{mm}$, and $W_{x3} = W_{y3} = 62.5\,\mathrm{mm}$;
- $h_1 = 8\,\mathrm{mm}$, $h_2 = 1.5\,\mathrm{cm}$, and $h_3 = 9.5\,\mathrm{mm}$;
- $D = 12$ mm, and $d_0 = 1$ mm;
- $\epsilon_{r1} = 2.2$, $\epsilon_{r2} = 2.6$, and $\epsilon_{r3} = 2.3$; and
- $x_{02} = x_{03} = 27\,\mathrm{mm}$, and $x_0 = y_0 = 0$.

Fig. 4. shows the reflection coefficients of E- and H-plane against scanning angle and Fig. 5. shows the normalized gain pattern against the scanning angle. Furthermore, Fig. 4. reveals that the scanning performance in the H-plane is much better than that of the E-plane. The scanning behavior deteriorates in the E-plane when the scanning angle $\theta = 30°$, but this phenomenon does not exist in the H-plane.



(a) (b)

Figure 4. Reflection coefficient against scanning angle.



(a) (b)

Figure 5. Normalized gain pattern against scanning angle.

The results given above are very similar to the results in [7] and [9]. The model developed in this paper can be easily simplified to that in [7] and [9].

4. CONCLUDING REMARKS

This paper has presented a Method of Moments solution to the problem of an infinite stacked microstrip phased array in a three-layered structure. Validation of the theoretical analysis has been indirectly examined by reducing the model to a one-layered and two-layered structures and comparing with some existing results. Numerical results are obtained in this paper in order to show the performance of the microstrip phased array antenna of infinite elements. It is shown from the studies that the phased array has wider scanning performance than a conventional patch array.

APPENDIX A. GREEN'S FUNCTION COMPONENTS

By using the SDT technique [12–15] and a lengthy algebraic manipulation (for which detailed procedure is available in [16]), we can obtain and list the necessary dyadic Green's function components that used in this paper.

$$\begin{aligned} &G_{xx}(k_x, k_y, h_1|h_1) \\ &\qquad = \frac{1}{j\omega\varepsilon_0}\frac{\sin k_1 h_1}{\beta^2 T_m T_e}\left[k_1 k_2 k_x^2 T_e + k_0^2 k_y^2 T_m\right], \end{aligned} \tag{24a}$$

$$\begin{aligned} &G_{xx}(k_x, k_y, h_1+h_2|h_1) \\ &\qquad = G_{xx}\left(k_x, k_y, h_1|h_1+h_2\right) \\ &\qquad = \frac{1}{j\omega\varepsilon_0}\frac{\sin k_1 h_1}{\beta^2 T_m T_e}\left[k_1 k_2 k_x^2 T_e\left(\cos k_2 h_2 - jP_m \sin k_2 h_2\right)\right. \\ &\qquad\quad \left. + \, k_0^2 k_y^2 T_m\left(\cos k_2 h_2 - jP_e \sin k_2 h_2\right)\right], \end{aligned} \tag{24b}$$

$$\begin{aligned} &G_{xx}(k_x, k_y, h_1+h_2+h_3|h_1) \\ &\qquad = G_{xx}\left(k_x, k_y, h_1|h_1+h_2+h_3\right) \\ &\qquad = \frac{1}{j\omega\varepsilon_0}\frac{\sin k_1 h_1}{\beta^2 T_m T_e}\left\{k_1 k_2 k_x^2 T_e\left[\left(\cos k_2 h_2 - jP_m \sin k_2 h_2\right)\cos k_3 h_3\right.\right. \\ &\qquad\quad \left. - \frac{\varepsilon_{r2} k_3}{\varepsilon_{r3} k_2}\left(\sin k_2 h_2 + jP_m \cos k_2 h_2\right)\sin k_3 h_3\right] \\ &\qquad\quad + k_0^2 k_y^2 T_m\left[\left(\cos k_2 h_2 - jP_e \sin k_2 h_2\right)\cos k_3 h_3\right. \\ &\qquad\quad \left.\left. - \frac{k_2}{k_3}\left(\sin k_2 h_2 + jP_e \cos k_2 h_2\right)\sin k_3 h_3\right]\right\}, \end{aligned} \tag{24c}$$

$$\begin{aligned}
&G_{yx}(k_x, k_y, h_1|h_1) \\
&\qquad = G_{xy}\left(k_x, k_y, h_1|h_1\right) \\
&\qquad = \frac{1}{j\omega\varepsilon_0}\frac{\sin k_1 h_1}{\beta^2 T_m T_e} k_x k_y \left(k_1 k_2 T_e - k_0^2 T_m\right),
\end{aligned} \tag{24d}$$

$$\begin{aligned}
&G_{yx}(k_x, k_y, h_1+h_2|h_1) \\
&\qquad = G_{xy}\left(k_x, k_y, h_1|h_1+h_2\right) \\
&\qquad = G_{yx}\left(k_x, k_y, h_1|h_1+h_2\right) \\
&\qquad = G_{xy}\left(k_x, k_y, h_1+h_2|h_1\right) \\
&\qquad = \frac{1}{j\omega\varepsilon_0}\frac{\sin k_1 h_1}{\beta^2 T_m T_e} k_x k_y \left[k_1 k_2 T_e \left(\cos k_2 h_2 - jP_m \sin k_2 h_2\right)\right. \\
&\qquad\quad \left. - \, k_0^2 T_m \left(\cos k_2 h_2 - jP_e \sin k_2 h_2\right)\right],
\end{aligned} \tag{24e}$$

$$\begin{aligned}
&G_{yx}(k_x, k_y, h_1+h_2+h_3|h_1) \\
&\qquad = G_{xy}\left(k_x, k_y, h_1|h_1+h_2+h_3\right) \\
&\qquad = G_{yx}\left(k_x, k_y, h_1|h_1+h_2+h_3\right) \\
&\qquad = G_{xy}\left(k_x, k_y, h_1+h_2+h_3|h_1\right) \\
&\qquad = \frac{1}{j\omega\varepsilon_0}\frac{\sin k_1 h_1}{\beta^2 T_m T_e} k_x k_y \left\{ k_1 k_2 T_e \left[\left(\cos k_2 h_2 - jP_m \sin k_2 h_2\right) \cos k_3 h_3 \right.\right. \\
&\qquad\quad \left. - \, \frac{\varepsilon_{r2} k_3}{\varepsilon_{r3} k_2} \left(\sin k_2 h_2 + jP_m \cos k_2 h_2\right) \sin k_3 h_3 \right] \\
&\qquad\quad - \, k_0^2 T_m \left[\left(\cos k_2 h_2 - jP_e \sin k_2 h_2\right) \cos k_3 h_3 \right. \\
&\qquad\quad \left.\left. - \, \frac{k_2}{k_3} \left(\sin k_2 h_2 + jP_e \cos k_2 h_2\right) \sin k_3 h_3 \right] \right\},
\end{aligned} \tag{24f}$$

$$\begin{aligned}
&G_{xx}(k_x, k_y, h_1+h_2|h_1+h_2) \\
&\qquad = \frac{1}{j\omega\varepsilon_0}\frac{1}{\beta^2 T_m T_e} \left[k_1 k_2 k_x^2 T_e \left(\cos k_2 h_2 - jP_m \sin k_2 h_2\right) \right. \\
&\qquad\quad \times \left(\sin k_1 h_1 \cos k_2 h_2 + \frac{\varepsilon_{r1} k_2}{\varepsilon_{r2} k_2} \cos k_1 h_1 \sin k_2 h_2 \right) \\
&\qquad\quad + \, k_0^2 k_y^2 T_m \left(\cos k_2 h_2 - jP_e \sin k_2 h_2\right) \\
&\qquad\quad \left. \times \left(\sin k_1 h_1 \cos k_2 h_2 + \frac{k_1}{k_2} \cos k_1 h_1 \sin k_2 h_2 \right) \right],
\end{aligned} \tag{24g}$$

$$
\begin{aligned}
&G_{xx}(k_x, k_y, h_1+h_2+h_3|h_1+h_2) \\
&\quad = G_{xx}\left(k_x, k_y, h_1+h2|h_1+h_2+h_3\right) \\
&\quad = \frac{1}{j\omega\varepsilon_0}\frac{1}{\beta^2 T_m T_e}\left\{k_1 k_2 k_x^2 T_e\left[\left(\cos k_2 h_2 - jP_m \sin k_2 h_2\right)\cos k_3 h_3\right.\right. \\
&\qquad \left. - \frac{\varepsilon_{r2} k_3}{\varepsilon_{r3} k_2}\left(\sin k_2 h_2 + jP_m \cos k_2 h_2\right)\sin k_3 h_3\right] \\
&\qquad \times \left(\sin k_1 h_1 \cos k_2 h_2 + \frac{\varepsilon_{r1} k_2}{\varepsilon_{r2} k_1}\cos k_1 h_1 \sin k_2 h_2\right) \\
&\qquad + k_0^2 k_y^2 T_m\left[\left(\cos k_2 h_2 - jP_e \sin k_2 h_2\right)\cos k_3 h_3\right. \\
&\qquad \left. - \frac{k_2}{k_3}\left(\sin k_2 h_2 + jP_e \cos k_2 h_2\right)\sin k_3 h_3\right] \\
&\qquad \left. \times \left(\sin k_1 h_1 \cos k_2 h_2 + \frac{k_1}{k_2}\cos k_1 h_1 \sin k_2 h_2\right)\right\},
\end{aligned} \tag{24h}
$$

$$
\begin{aligned}
&G_{xx}(k_x, k_y, h_1+h_2+h_3|h_1+h_2+h_3) \\
&\quad = \frac{1}{j\omega\varepsilon_0}\frac{1}{\beta^2 T_m T_e}\left\{\frac{k_x^2 k_3 k_4 T_e}{\varepsilon_{r2} k_3 T_{mo}^c \cos k_2 h_2 + j\varepsilon_{r3} k_2 T_{me}^c \sin k_2 h_2}\right. \\
&\qquad \times \left[\varepsilon_{r3} k_2\left(\varepsilon_{r1} k_2 \cos k_1 h_1 \sin k_2 h_2 + \varepsilon_{r2} k_1 \sin k_1 h_1 \cos k_2 h_2\right)\right. \\
&\qquad \times \cos k_3 h_3 + \varepsilon_{r2} k_3\left(\varepsilon_{r1} k_2 \cos k_1 h_1 \cos k_2 h_2\right. \\
&\qquad \left.\left. - \varepsilon_{r2} k_1 \sin k_1 h_1 \sin k_2 h_2\right)\sin k_3 h_3\right] \\
&\qquad + \frac{k_0^2 k_y^2 T_m}{k_2 T_{ee}^c \cos k_2 h_2 + jk_3 T_{eo}^c \sin k_2 h_2}\left[k_3\left(k_1 \cos k_1 h_1 \sin k_2 h_2\right.\right. \\
&\qquad \left. + k_2 \sin k_1 h_1 \cos k_2 h_2\right)\cos k_3 h_3 + k_2\left(k_1 \cos k_1 h_1 \cos k_2 h_2\right. \\
&\qquad \left.\left.\left. - k_2 \sin k_1 h_1 \sin k_2 h_2\right)\sin k_3 h_3\right]\right\},
\end{aligned} \tag{24i}
$$

$$
\begin{aligned}
&G_{yy}(k_x, k_y, h_1|h_1) \\
&\quad = \frac{1}{j\omega\varepsilon_0}\frac{\sin k_1 h_1}{\beta^2 T_m T_e}\left(k_1 k_2 k_y^2 T_e + k_0^2 k_x^2 T_m\right),
\end{aligned} \tag{24j}
$$

$$
\begin{aligned}
&G_{yy}(k_x, k_y, h_1+h_2|h_1) \\
&\quad = G_{yy}\left(k_x, k_y, h_1|h_1+h_2\right) \\
&\quad = \frac{1}{j\omega\varepsilon_0}\frac{\sin k_1 h_1}{\beta^2 T_m T_e}\left[k_1 k_2 k_y^2 T_e\left(\cos k_2 h_2 - jP_m \sin k_2 h_2\right)\right. \\
&\qquad \left. + k_0^2 k_x^2 T_m\left(\cos k_2 h_2 - jP_e \sin k_2 h_2\right)\right],
\end{aligned} \tag{24k}
$$

$$\begin{aligned}
&G_{yy}(k_x,k_y,h_1+h_2+h_3|h_1)\\
&\quad= G_{yy}\left(k_x,k_y,h_1|h_1+h_2+h_3\right)\\
&\quad= \frac{1}{j\omega\varepsilon_0}\frac{\sin k_1h_1}{\beta^2T_mT_e}\left\{k_1k_2k_y^2T_e\left[(\cos k_2h_2-jP_m\sin k_2h_2)\cos k_3h_3\right.\right.\\
&\qquad\left.-\frac{\varepsilon_{r2}k_3}{\varepsilon_{r3}k_2}\left(\sin k_2h_2+jP_m\cos k_2h_2\right)\sin k_3h_3\right]\\
&\qquad+k_0^2k_x^2T_m\left[(\cos k_2h_2-jP_e\sin k_2h_2)\cos k_3h_3\right.\\
&\qquad\left.\left.-\frac{k_2}{k_3}\left(\sin k_2h_2+jP_e\cos k_2h_2\right)\sin k_3h_3\right]\right\},
\end{aligned}\tag{24l}$$

$$\begin{aligned}
&G_{yy}(k_x,k_y,h_1+h_2|h_1+h_2)\\
&\quad= \frac{1}{j\omega\varepsilon_0}\frac{1}{\beta^2T_mT_e}\left[k_1k_2k_y^2T_e\left(\cos k_2h_2-jP_m\sin k_2h_2\right)\right.\\
&\qquad\times\left(\sin k_1h_1\cos k_2h_2+\frac{\varepsilon_{r1}k_2}{\varepsilon_{r2}k_1}\cos k_1h_1\sin k_2h_2\right)\\
&\qquad+k_0^2k_x^2T_m\left(\cos k_2h_2-jP_e\sin k_2h_2\right)\\
&\qquad\left.\times\left(\sin k_1h_1\cos k_2h_2+\frac{k_1}{k_2}\cos k_1h_1\sin k_2h_2\right)\right],
\end{aligned}\tag{24m}$$

$$\begin{aligned}
&G_{yy}(k_x,k_y,h_1+h_2+h_3|h_1+h_2)\\
&\quad= G_{yy}\left(k_x,k_y,h_1+h_2|h_1+h_2+h_3\right)\\
&\quad= \frac{1}{j\omega\varepsilon_0}\frac{1}{\beta^2T_mT_e}\left\{k_1k_2k_y^2T_e\left[(\cos k_2h_2-jP_m\sin k_2h_2)\cos k_3h_3\right.\right.\\
&\qquad\left.-\frac{\varepsilon_{r2}k_3}{\varepsilon_{r3}k_2}\left(\sin k_2h_2+jP_m\cos k_2h_2\right)\sin k_3h_3\right]\\
&\qquad\times\left(\sin k_1h_1\cos k_2h_2+\frac{\varepsilon_{r1}k_2}{\varepsilon_{r2}k_1}\cos k_1h_1\sin k_2h_2\right)\\
&\qquad+k_0^2k_x^2T_m\left[(\cos k_2h_2-jP_e\sin k_2h_2)\cos k_3h_3\right.\\
&\qquad\left.-\frac{k_2}{k_3}\left(\sin k_2h_2+jP_e\cos k_2h_2\right)\sin k_3h_3\right]\\
&\qquad\left.\times\left(\sin k_1h_1\cos k_2h_2+\frac{k_1}{k_2}\cos k_1h_1\sin k_2h_2\right)\right\},
\end{aligned}\tag{24n}$$

$$\begin{aligned}
&G_{yy}(k_x, k_y, h_1+h_2+h_3|h_1+h_2+h_3)\\
&\quad = \frac{1}{j\omega\varepsilon_0}\frac{1}{\beta^2 T_m T_e}\left\{\frac{k_y^2 k_3 k_4 T_e}{\varepsilon_{r2} k_3 T_{mo}^c \cos k_2 h_2 + j\varepsilon_{r3} k_2 T_{me}^c \sin k_2 h_2}\right.\\
&\qquad \times \left[\varepsilon_{r3} k_2 \left(\varepsilon_{r1} k_2 \cos k_1 h_1 \sin k_2 h_2 + \varepsilon_{r2} k_1 \sin k_1 h_1 \cos k_2 h_2\right)\right.\\
&\qquad \times \cos k_3 h_3 + \varepsilon_{r2} k_3 \left(\varepsilon_{r1} k_2 \cos k_1 h_1 \cos k_2 h_2 - \varepsilon_{r2} k_1 \sin k_1 h_1\right.\\
&\qquad \times \left.\left.\sin k_2 h_2\right) \sin k_3 h_3\right] + \frac{k_0^2 k_x^2 T_m}{k_2 T_{ee}^c \cos k_2 h_2 + j k_3 T_{eo}^c \sin k_2 h_2}\\
&\qquad \times \left[k_3 \left(k_1 \cos k_1 h_1 \sin k_2 h_2 + k_2 \sin k_1 h_1 \cos k_2 h_2\right) \cos k_3 h_3 + k_2\right.\\
&\qquad \times \left.\left.\left(k_1 \cos k_1 h_1 \cos k_2 h_2 - k_2 \sin k_1 h_1 \sin k_2 h_2\right) \sin k_3 h_3\right]\right\}, \qquad (24\text{o})
\end{aligned}$$

$$\begin{aligned}
&G_{xy}(k_x, k_y, h_1+h_2|h_1+h_2)\\
&\quad = G_{xy}\left(k_x, k_y, h_1+h_2|h_1+h_2\right)\\
&\quad = \frac{1}{j\omega\varepsilon_0}\frac{k_x k_y}{\beta^2 T_m T_e}\left\{k_1 k_2 T_e \left(\cos k_2 h_2 - jP_m \sin k_2 h_2\right)\right.\\
&\qquad \times \left(\sin k_1 h_1 \cos k_2 h_2 + \frac{\varepsilon_{r1} k_2}{\varepsilon_{r2} k_1} \cos k_1 h_1 \sin k_2 h_2\right)\\
&\qquad - k_0^2 T_e \left(\cos k_2 h_2 - jP_e \sin k_2 h_2\right)\\
&\qquad \times \left.\left(\sin k_1 h_1 \cos k_2 h_2 + \frac{k_1}{k_2} \cos k_1 h_1 \sin k_2 h_2\right)\right\}, \qquad (24\text{p})
\end{aligned}$$

$$\begin{aligned}
&G_{xy}(k_x, k_y, h_1+h_2+h_3|h_1+h_2)\\
&\quad = G_{xy}\left(k_x, k_y, h_1+h_2|h_1+h_2+h_3\right)\\
&\quad = G_{yx}\left(k_x, k_y, h_1+h_2+h_3|h_1+h_2\right)\\
&\quad = G_{yx}\left(k_x, k_y, h_1+h_2|h_1+h_2+h_3\right)\\
&\quad = \frac{1}{j\omega\varepsilon_0}\frac{k_x k_y}{\beta^2 T_m T_e}\left\{k_1 k_2 T_e \left[\left(\cos k_2 h_2 - jP_m \sin k_2 h_2\right) \cos k_3 h_3\right.\right.\\
&\qquad - \left.\frac{\varepsilon_{r2} k_3}{\varepsilon_{r3} k_2} \left(\sin k_2 h_2 + jP_m \cos k_2 h_2\right) \sin k_3 h_3\right]\\
&\qquad \times \left(\sin k_1 h_1 \cos k_2 h_2 + \frac{\varepsilon_{r1} k_2}{\varepsilon_{r2} k_1} \cos k_1 h_1 \sin k_2 h_2\right)\\
&\qquad - k_0^2 T_m \left[\left(\cos k_2 h_2 - jP_e \sin k_2 h_2\right) \cos k_3 h_3\right.\\
&\qquad - \left.\frac{k_2}{k_3} \left(\sin k_2 h_2 + jP_e \cos k_2 h_3\right) \sin k_3 h_3\right]\\
&\qquad \times \left.\left(\sin k_1 h_1 \cos k_2 h_2 + \frac{k_1}{k_2} \cos k_1 h_1 \sin k_3 h_3\right)\right\}, \qquad (24\text{q})
\end{aligned}$$

$$\begin{aligned}
&G_{xy}(k_x, k_y, h_1+h_2+h_3|h_1+h_2+h_3)\\
&\qquad = G_{xy}\left(k_x, k_y, h_1+h_2+h_3|h_1+h_2+h_3\right)\\
&\qquad = \frac{1}{j\omega\varepsilon_0}\frac{k_xk_y}{\beta^2T_mT_e}\left\{\frac{k_3k_4T_e}{\varepsilon_{r2}k_3T^c_{mo}\cos k_2h_2+j\varepsilon_{r3}k_2T^c_{me}\sin k_2h_2}\right.\\
&\qquad\quad \times\left[\varepsilon_{r3}k_2\left(\varepsilon_{r1}k_2\cos k_1h_1\sin k_2h_2+\varepsilon_{r2}k_1\sin k_1h_1\cos k_2h_2\right)\right.\\
&\qquad\quad \times\cos k_3h_3+\varepsilon_{r2}k_3\left(\varepsilon_{r1}k_2\cos k_1h_1\cos k_2h_2-\varepsilon_{r2}k_1\sin k_1h_1\right.\\
&\qquad\quad \times\left.\left.\sin k_2h_2\right)\sin k_3h_3\right]-\frac{k_0^2T_m}{k_2T^c_{ee}\cos k_2h_2+jk_3T^c_{eo}\sin k_2h_2}\\
&\qquad\quad \times\left[k_3\left(k_1\cos k_1h_1\sin k_2h_2+k_2\sin k_1h_1\cos k_2h_2\right)\cos k_3h_3\right.\\
&\qquad\quad \left.\left.+k_2\left(k_1\cos k_1h_1\cos k_2h_2-k_2\sin k_1h_1\sin k_2h_2\right)\sin k_3h_3\right]\right\}, \qquad (24r)
\end{aligned}$$

$$\begin{aligned}
&G_{xx}(k_x, k_y, h_1|0\preceq z_0\preceq h_1)\\
&\qquad = \frac{-1}{\omega\varepsilon_0}\frac{k_xk_2\cos k_1z_0}{T_m}, \qquad (24s)
\end{aligned}$$

$$\begin{aligned}
&G_{xx}(k_x, k_y, h_1+h_2|0\preceq z_0\preceq h_1)\\
&\qquad = \frac{-1}{\omega\varepsilon_0}\frac{k_xk_2\cos k_1z_0}{T_m}\left(\cos k_2h_2-jP_m\sin k_2h_2\right), \qquad (24t)
\end{aligned}$$

$$\begin{aligned}
&G_{xx}(k_x, k_y, h_1+h_2+h_3|0\preceq z_0\preceq h_1)\\
&\qquad = \frac{-1}{\omega\varepsilon_0}\frac{k_xk_2\cos k_1z_0}{T_m}\left[\left(\cos k_2h_2-jP_m\sin k_2h_2\right)\cos k_3h_3\right.\\
&\qquad\quad \left.-\frac{\varepsilon_{r2}k_3}{\varepsilon_{r3}k_2}\left(\sin k_2h_2+jP_m\cos k_2h_2\right)\sin k_3h_3\right], \qquad (24u)
\end{aligned}$$

$$\begin{aligned}
&G_{yz}(k_x, k_y, h_1|0\preceq z_0\preceq h_1)\\
&\qquad = \frac{-1}{\omega\varepsilon_0}\frac{k_yk_2\cos k_1z_0}{T_m}, \qquad (25a)
\end{aligned}$$

$$\begin{aligned}
&G_{yz}(k_x, k_y, h_1+h_2|0\preceq z_0\preceq h_1)\\
&\qquad = \frac{-1}{\omega\varepsilon_0}\frac{k_yk_2\cos k_1z_0}{T_m}\left(\cos k_2h_2-jP_m\sin k_2h_2\right), \qquad (25b)
\end{aligned}$$

$$\begin{aligned}
&G_{yz}(k_x, k_y, h_1+h_2+h_3|0\preceq z_0\preceq h_1)\\
&\qquad = \frac{-1}{\omega\varepsilon_0}\frac{k_yk_2\cos k_1z_0}{T_m}\left[\left(\cos k_2h_2-jP_m\sin k_2h_2\right)\cos k_3h_3\right.\\
&\qquad\quad \left.-\frac{\varepsilon_{r2}k_3}{\varepsilon_{r3}k_2}\left(\sin k_2h_2+jP_m\cos k_2h_2\right)\sin k_3h_3\right], \qquad (25c)
\end{aligned}$$

$$G_{zx}(k_x, k_y, 0 \preceq z \preceq h_1 | h_1) = \frac{1}{\omega\varepsilon_0} \frac{k_x k_2 \cos k_1 z}{T_m}, \tag{25d}$$

$$G_{zx}(k_x, k_y, 0 \preceq z \preceq h_1 | h_1 + h_2) = \frac{1}{\omega\varepsilon_0} \frac{k_x k_2 \cos k_1 z}{T_m} \left(\cos k_2 h_2 - j P_m \sin k_2 h_2 \right), \tag{25e}$$

$$\begin{aligned} G_{zx}(k_x, k_y, 0 \preceq z \preceq h_1 | h_1 + h_2 + h_3) &= \frac{1}{\omega\varepsilon_0} \frac{k_x k_2 \cos k_1 z}{T_m} \bigg[\left(\cos k_2 h_2 - j P_m \sin k_2 h_2 \right) \cos k_3 h_3 \\ &\quad - \frac{\varepsilon_{r2} k_3}{\varepsilon_{r3} k_2} \left(\sin k_2 h_2 + j P_m \cos k_2 h_2 \right) \sin k_3 h_3 \bigg], \end{aligned} \tag{25f}$$

$$G_{zy}(k_x, k_y, 0 \preceq z \preceq h_1 | h_1) = \frac{1}{\omega\varepsilon_0} \frac{k_y k_2 \cos k_1 z}{T_m}, \tag{25g}$$

$$G_{zy}(k_x, k_y, 0 \preceq z \preceq h_1 | h_1 + h_2) = \frac{1}{\omega\varepsilon_0} \frac{k_y k_2 \cos k_1 z}{T_m} \left(\cos k_2 h_2 - j P_m \sin k_2 h_2 \right), \tag{25h}$$

$$\begin{aligned} G_{zy}(k_x, k_y, 0 \preceq z \preceq h_1 | h_1 + h_2 + h_3) &= \frac{1}{\omega\varepsilon_0} \frac{k_y k_2 \cos k_1 z}{T_m} \bigg[\left(\cos k_2 h_2 - j P_m \sin k_2 h_2 \right) \cos k_3 h_3 \\ &\quad - \frac{\varepsilon_{r2} k_3}{\varepsilon_{r3} k_2} \left(\sin k_2 h_2 + j P_m \cos k_2 h_2 \right) \sin k_3 h_3 \bigg], \end{aligned} \tag{25i}$$

$$G_{zz}(k_x, k_y, 0 \preceq z \preceq h_1 | 0 \preceq z_0 \preceq h_1) = \frac{-1}{j\omega\varepsilon_0\varepsilon_{r1}} \times \begin{cases} \bigg[\delta(z - z_0) + \dfrac{\beta^2}{k_1 T_m} (\varepsilon_{r1} k_2 \sin k_1 (z_0 - h_1) \\ \qquad + j P_m \varepsilon_{r2} k_1 \cos k_1 (z - z_0)) \cos k_1 z \bigg] \\ \qquad (0 \preceq z \preceq z_0), \\ \bigg[\dfrac{\beta^2}{k_1 T_m} (\varepsilon_{r1} k_2 \sin k_1 (z - h_1) \\ \qquad + j P_m \varepsilon_{r2} k_1 \cos k_1 (z - h_1)) \cos k_1 z_0 \bigg] \\ \qquad (z_0 \preceq z \preceq h_1). \end{cases} \tag{25j}$$

where

$$T_m = \varepsilon_{r1} k_2 \cos k_1 h_1 + j P_m \varepsilon_{r2} k_1 \sin k_1 h_1, \tag{26a}$$

$$T_e = k_1 \cos k_1 h_1 + j P_e k_2 \sin k_1 h_1, \tag{26b}$$

$$P_m = \frac{\varepsilon_{r3} k_2 T_{me}^c \cos k_2 h_2 + j\varepsilon_{r2} k_3 T_{mo}^c \sin k_2 h_2}{\varepsilon_{r2} k_3 T_{mo}^c \cos k_2 h_2 + j\varepsilon_{r3} k_2 T_{me}^c \sin k_2 h_2}, \tag{26c}$$

$$P_e = \frac{k_3 T_{eo}^c \cos k_2 h_2 + j k_2 T_{ee}^c \sin k_2 h_2}{k_2 T_{ee}^c \cos k_2 h_2 + j k_3 T_{eo}^c \sin k_2 h_2}, \tag{26d}$$

$$T_{mo}^c = \varepsilon_{r3} k_4 \cos k_3 h_3 + j k_3 \sin k_3 h_3, \tag{26e}$$

$$T_{me}^c = k_3 \cos k_3 h_3 + j \varepsilon_{r3} k_4 \sin k_3 h_3, \tag{26f}$$

$$T_{eo}^c = k_4 \cos k_3 h_3 + j k_3 \sin k_3 h_3, \tag{26g}$$

$$T_{ee}^c = k_3 \cos k_3 h_3 + j k_4 \sin k_3 h_3, \tag{26h}$$

$$k_1 = \sqrt{\varepsilon_{r1} k_0^2 - \beta^2}, \Im m(k_1) \preceq 0, \tag{26i}$$

$$k_2 = \sqrt{\varepsilon_{r2} k_0^2 - \beta^2}, \Im m(k_2) \preceq 0, \tag{26j}$$

$$k_3 = \sqrt{\varepsilon_{r3} k_0^2 - \beta^2}, \Im m(k_3) \preceq 0, \tag{26k}$$

$$k_4 = \sqrt{k_0^2 - \beta^2}, \Im m(k_4) \preceq 0, \tag{26l}$$

$$\beta^2 = k_x^2 + k_y^2, \tag{26m}$$

$$k_0^2 = \omega^2 \mu_0 \varepsilon_0. \tag{26n}$$

APPENDIX B. COMPONENTS OF MATRIX

$$Z_{11}^{PF} = \frac{-1}{ab} \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} Q\left(k_{xm}, k_{yn}\right) J_0\left(d_0 \sqrt{k_{xm}^2 + k_{yn}^2}\right), \tag{27a}$$

$$\begin{aligned} Z_{1j}^{FP} = \frac{-1}{ab} \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} & \boldsymbol{R}\left(k_{xm}, k_{yn}, h_1\right) \cdot \boldsymbol{F_j^*}\left(k_{xm}, k_{yn}\right) \\ & \times J_0\left(d_0 \sqrt{k_{xm}^2 + k_{yn}^2}\right) e^{-j\left(k_{xm} x_0 + k_{yn} y_0\right)} \\ & (j = 1, \cdots, N_1), \end{aligned} \tag{27b}$$

$$\begin{aligned} Z_{1j}^{FP} = \frac{-1}{ab} \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} & \boldsymbol{R}\left(k_{xm}, k_{yn}, h_1 + h_2\right) \cdot \boldsymbol{F_j^*}\left(k_{xm}, k_{yn}\right) \\ & \times J_0\left(d_0 \sqrt{k_{xm}^2 + k_{yn}^2}\right) e^{-j\left(k_{xm} x_0 + k_{yn} y_0\right)} \\ & (j = N_1 + 1, \cdots, N_1 + N_2), \end{aligned} \tag{27c}$$

$$Z_{1j}^{FP} = \frac{-1}{ab} \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \boldsymbol{R}\left(k_{xm}, k_{yn}, h_1 + h_2 + h_3\right)$$
$$\cdot \boldsymbol{F_j^*}\left(k_{xm}, k_{yn}\right) J_0\left(d_0\sqrt{k_{xm}^2 + k_{yn}^2}\right) e^{-j(k_{xm}x_0 + k_{yn}y_0)}$$
$$(j = N_1 + N_2 + 1, \cdots, N_1 + N_2 + N_3), \tag{27d}$$

$$Z_{i1}^{PF} = \frac{-1}{ab} \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \boldsymbol{R}\left(k_{xm}, k_{yn}, h_1\right) \cdot \boldsymbol{F_i^*}\left(k_{xm}, k_{yn}\right) e^{-j(k_{xm}x_0 + k_{yn}y_0)}$$
$$(i = 1, \cdots, N_1), \tag{27e}$$

$$Z_{i1}^{PF} = \frac{-1}{ab} \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \boldsymbol{R}\left(k_{xm}, k_{yn}, h_1 + h_2\right)$$
$$\cdot \boldsymbol{F_i^*}\left(k_{xm}, k_{yn}\right) e^{-j(k_{xm}x_0 + k_{yn}y_0)}$$
$$(i = N_1 + 1, \cdots, N_1 + N_2), \tag{27f}$$

$$Z_{i1}^{PF} = \frac{-1}{ab} \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \boldsymbol{R}\left(k_{xm}, k_{yn}, h_1 + h_2 + h_3\right)$$
$$\cdot \boldsymbol{F_i^*}\left(k_{xm}, k_{yn}\right) e^{-j(k_{xm}x_0 + k_{yn}y_0)}$$
$$(i = N_1 + N_2 + 1, \cdots, N_1 + N_2 + N_3), \tag{27g}$$

$$Z_{ij}^{PP} = \frac{-1}{ab} \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \boldsymbol{F}_i\left(k_{xm}, k_{yn}\right)$$
$$\cdot \overline{\boldsymbol{G}}\left(k_{xm}, k_{yn}, h_1 | h_1\right) \cdot \boldsymbol{F}_j^*\left(k_{xm}, k_{yn}\right) e^{-j(k_{xm}x_0 + k_{yn}y_0)}$$
$$(i, j = 1, \cdots, N_1), \tag{27h}$$

$$Z_{ij}^{PP} = \frac{-1}{ab} \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \boldsymbol{F}_i\left(k_{xm}, k_{yn}\right) \cdot \overline{\boldsymbol{G}}\left(k_{xm}, k_{yn}, h_1 | h_1 + h_2\right)$$
$$\cdot \boldsymbol{F}_j^*\left(k_{xm}, k_{yn}\right) e^{-j(k_{xm}x_0 + k_{yn}y_0)}$$
$$(i = 1, \cdots, N_1, j = N_1 + 1, \cdots, N_1 + N_2), \tag{27i}$$

$$Z_{ij}^{PP} = \frac{-1}{ab} \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \boldsymbol{F}_i\left(k_{xm}, k_{yn}\right) \cdot \overline{\boldsymbol{G}}\left(k_{xm}, k_{yn}, h_1 | h_1 + h_2 + h_3\right)$$
$$\cdot \boldsymbol{F}_j^*\left(k_{xm}, k_{yn}\right) e^{-j(k_{xm}x_0 + k_{yn}y_0)}$$
$$(i = 1, \cdots, N_1, \ j = N_1 + N_2 + 1, \cdots, N_1 + N_2 + N_3), \tag{27j}$$

$$Z_{ij}^{PP} = \frac{-1}{ab} \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \boldsymbol{F}_i\left(k_{xm}, k_{yn}\right) \cdot \overline{\boldsymbol{G}}\left(k_{xm}, k_{yn}, h_1 + h_2 | h_1\right) \cdot \boldsymbol{F}_j^*\left(k_{xm}, k_{yn}\right) e^{-j\left(k_{xm}x_0 + k_{yn}y_0\right)} \quad \left(i = N_1 + 1, \cdots, N_1 + N_2, j = 1, \cdots, N_1\right), \tag{27k}$$

$$Z_{ij}^{PP} = \frac{-1}{ab} \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \boldsymbol{F}_i\left(k_{xm}, k_{yn}\right) \cdot \overline{\boldsymbol{G}}\left(k_{xm}, k_{yn}, h_1 + h_2 | h_1 + h_2\right) \cdot \boldsymbol{F}_j^*\left(k_{xm}, k_{yn}\right) e^{-j\left(k_{xm}x_0 + k_{yn}y_0\right)} \quad \left(i = N_1 + 1, \cdots, N_1 + N_2,\ j = N_1 + 1, \cdots, N_1 + N_2\right), \tag{27l}$$

$$Z_{ij}^{PP} = \frac{-1}{ab} \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \boldsymbol{F}_i\left(k_{xm}, k_{yn}\right) \cdot \overline{\boldsymbol{G}}\left(k_{xm}, k_{yn}, h_1 + h_2 | h_1 + h_2 + h_3\right) \cdot \boldsymbol{F}_j^*\left(k_{xm}, k_{yn}\right) e^{-j\left(k_{xm}x_0 + k_{yn}y_0\right)} \quad \left(i = N_1 + 1, \cdots, N_1 + N_2,\ j = N_1 + N_2 + 1, \cdots, N_1 + N_2 + N_3\right), \tag{27m}$$

$$Z_{ij}^{PP} = \frac{-1}{ab} \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \boldsymbol{F}_i\left(k_{xm}, k_{yn}\right) \cdot \overline{\boldsymbol{G}}\left(k_{xm}, k_{yn}, h_1 + h_2 + h_3 | h_1\right) \cdot \boldsymbol{F}_j^*\left(k_{xm}, k_{yn}\right) e^{-j\left(k_{xm}x_0 + k_{yn}y_0\right)} \quad \left(i = N_1 + N_2 + 1, \cdots, N_1 + N_2 + N_3,\ j = 1, \cdots, N_1\right), \tag{27n}$$

$$Z_{ij}^{PP} = \frac{-1}{ab} \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \boldsymbol{F}_i\left(k_{xm}, k_{yn}\right) \cdot \overline{\boldsymbol{G}}\left(k_{xm}, k_{yn}, h_1 + h_2 + h_3 | h_1 + h_2\right) \cdot \boldsymbol{F}_j^*\left(k_{xm}, k_{yn}\right) e^{-j\left(k_{xm}x_0 + k_{yn}y_0\right)} \quad \left(i = N_1 + N_2 + 1, \cdots, N_1 + N_2 + N_3, j = N_1 + 1, \cdots, N_1 + N_2\right), \tag{27o}$$

$$Z_{ij}^{PP} = \frac{-1}{ab} \sum_{m=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} \boldsymbol{F}_i\left(k_{xm}, k_{yn}\right) \cdot \overline{\boldsymbol{G}}\left(k_{xm}, k_{yn}, h_1 + h_2 + h_3 | h_1 + h_2 + h_3\right) \cdot \boldsymbol{F}_j^*\left(k_{xm}, k_{yn}\right) e^{-j\left(k_{xm}x_0 + k_{yn}y_0\right)} \quad \left(i = N_1 + N_2 + 1, \cdots, N_1 + N_2 + N_3,\ j = N_1 + N_2 + 1, \cdots, N_1 + N_2 + N_3\right), \tag{27p}$$

$$\begin{aligned}Q(k_x, k_y) &= \int_0^{h_1}\int_0^{h_1} G_{zz}(k_x, k_y, z|z_0)\, dz_0 dz \\ &= j\frac{Z_0}{k_0}\frac{k_0^2 \sin k_1 h_1 T_m - \beta^2 k_2 \sin k_1 d_1}{k_1^3 T_m},\end{aligned} \tag{27q}$$

$$\boldsymbol{R}(k_x, k_y, z) = \widehat{\boldsymbol{x}} R_x(k_x, k_y, z) + \widehat{\boldsymbol{y}} R_y(k_x, k_y, z), \tag{27r}$$

$$\begin{aligned}R_x(k_x, k_y, h_1) &= \int_0^{h_1} G_{xx}(k_x, k_y, h_1|z_0)\, dz_0 \\ &= -\frac{Z_0}{k_0}\frac{k_x k_2 \sin k_1 h_1}{k_1 T_m},\end{aligned} \tag{27s}$$

$$\begin{aligned}&R_x(k_x, k_y, h_1+h_2) \\ &\quad= \int_0^{h_1} G_{xx}(k_x, k_y, h_1+h_2|z_0)\, dz_0 \\ &\quad= -\frac{Z_0}{k_0}\frac{k_x k_2 \sin k_1 h_1}{k_1 T_m}(\cos k_2 h_2 - jP_m \sin k_2 h_2),\end{aligned} \tag{27t}$$

$$\begin{aligned}&R_x(k_x, k_y, h_1+h_2+h_3) \\ &\quad= \int_0^{h_1} G_{xx}(k_x, k_y, h_1+h_2+h_3|z_0)\, dz_0 \\ &\quad= -\int_0^{h_1} G_{zx}(k_x, k_y, z|h_1+h_2+h_3)\, dz \\ &\qquad\times\Big[(\cos k_2 h_2 - jP_m \sin k_2 h_2)\cos k_3 h_3 \\ &\qquad- \frac{\epsilon_{r2} k_3}{\epsilon_{r3} k_2}(\sin k_2 h_2 + jP_m)\sin k_3 h_3\Big],\end{aligned} \tag{27u}$$

$$\begin{aligned}R_y(k_x, k_y, h_1) &= \int_0^{h_1} G_{yz}(k_x, k_y, h_1|z_0)\, dz_0 \\ &= -\frac{Z_0}{k_0}\frac{k_y k_2 \sin k_1 h_1}{k_1 T_m},\end{aligned} \tag{27v}$$

$$\begin{aligned}&R_y(k_x, k_y, h_1+h_2) \\ &\quad= \int_0^{h_1} G_{yz}(k_x, k_y, h_1+h_2|z_0)\, dz_0 \\ &\quad= -\frac{Z_0}{k_0}\frac{k_y k_2 \sin k_1 h_1}{k_1 T_m}(\cos k_2 h_2 - jP_m \sin k_2 h_2),\end{aligned} \tag{27w}$$

$$\begin{aligned}&R_y(k_x, k_y, h_1+h_2+h_3) \\ &\quad= \int_0^{h_1} G_{yz}(k_x, k_y, h_1+h_2+h_3|z_0)\, dz_0\end{aligned}$$

$$= -\frac{Z_0}{k_0}\frac{k_y k_2 \sin k_1 h_1}{k_1 T_m}\left[(\cos k_2 h_2 - jP_m \sin k_2 h_2)\cos k_3 h_3 \right.$$
$$\left. - \frac{\epsilon_{r2} k_3}{\epsilon_{r3} k_2}(\sin k_2 h_2 + jP_m)\sin k_3 h_3\right]. \qquad (27x)$$

$\boldsymbol{F}_j\,(k_x, k_y)$ is the fourier transformation of the spatial current distribution, i.e.,

$$\boldsymbol{F}_j\,(k_x, k_y) = \int\int_{patch} \boldsymbol{J}\,(x, y)\, e^{-jk_x x - jk_y y} dxdy. \qquad (28)$$

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REFERENCES

1. Pozar, D. M. and D. H. Schaubert, *Microstrip Antennas,* IEEE Press, New York, 1995.
2. Barlatey, L., J. R. Mosig, and T. Sphicopoulos, "Analysis of stacked microstrip patches with a mixed potential integral equation," *IEEE Trans. Antennas Propagat.,* Vol. AP-38, 608–615, May 1990.
3. Tulintseff, A. N., S. M. Ali, and J. A. Kong, "Input impedance of a probe-fed stacked circular microstrip antenna," *IEEE Trans. Antennas Propagat.,* Vol. AP-39, No. 3, 381–390, 1991.
4. Lubin, Y. and A. Hessel, "Wide-band, wide-angle microstrip stacked-patch-element phased array," *IEEE Trans. Antennas Propagat.,* Vol. AP-39, 1062–1070, August 1991.
5. Damiano, J. P. and A. Papiernik, "Survey of analytical numerical models for probe-fed microstrip antennas," *IEE Proc. Microwave Antennas Propagat.,* Vol. 141, No. 1, 15–22, Feb. 1994.
6. Liu, Z.-F., P.-S. Kooi, L.-W. Li, M.-S. Leong, and T.-S. Yeo, "A Method for designing broadband microstrip antennas in multilayered planar structures," *IEEE Trans. Antennas Propagat.,* Vol. 47, No. 9, September 1999.
7. Aberle, J. T. and D. M. Pozar, "Analysis of infinite arrays of one- and two-probe-fed circular patches," *IEEE Trans. Antennas Propagat.,* Vol. AP-38, No. 4, 381–390, 1990.

8. Aberle, J. T. and D. M. Pozar, "Analysis of infinite array of probe-fed rectangular microstrip patches using a rigorous feed model," *IEE Proc. Microwave Antennas Propagat.,* Vol. 136, No. 2, 100–119, April 1989.
9. Aberle, J. T., D. M. Pozar, and J. Manges, "Phased arrays of probe-fed stacked microstrip patches," *IEEE Trans. Antennas Propagat.,* Vol. AP-42, No. 7, 920–927, July 1994.
10. Aberle, J. T. and D. M. Pozar, "Accurate and versatile solutions for probe-fed microstrip patch antennas and arrays," *Electromagnetics,* No. 11, 1–19, 1991.
11. Vandenbosch, G. A. E. and A. R. Van De Capelle, "Reduction of coaxial feed between two parallel conductors into finite number of voltage source," *Electro. Lett.,* Vol. 27, No. 25, 2387–2389, December 1991.
12. Itoh, T., "Spectral domain immittance approach for dispersion characteristics of generalized printed transmission lines," *IEEE Trans. Microwave Theory Tech.,* Vol. MTT-28, 733–736, 1980.
13. Itoh, T. (ed.), *Planar Transmission Line Structures,* IEEE Press, New York, 1987.
14. Itoh, T., *Numerical Techniques for Microwave and Millimeter-Wave Passive Structures,* John Wiley & Sons Inc., New York, 1989.
15. Mirshekar-Syahkal, D., *Spectral Domain Method for Microwave Integrated Circuits,* John Wiley & Sons Inc., New York, 1990.
16. Liu, Z.-F., "Design of wideband microstrip antennas and analysis of multilayered phased arrays, Ph.D. dissertation, Beijing Institute of Technology, 1996, available in Beijing Library, China.