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VALIDATION OF A MODIFIED FDTD METHOD ON NON-UNIFORM TRANSMISSION LINES

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Abstract–In this paper, we introduce a modified FDTD model in order to investigate the behavior of the induced voltage of a non-uniform transmission line (TL) excited by lumped voltage sources or external electromagnetic wave. The parameters to be taken into account for this specific coupling phenomenon are explicitly analyzed and the affection on the behavior of the induced voltage is discussed in detail. To confirm the validity of this model, results obtained by this model, for two typical transmission line configurations, are compared to results obtained by other models, already published in the literature. Finally, several numerical calculations of the line responses are provided for non-uniform TL's that can be present in practical configurations. These results indicate that a configuration slightly different from the uniform one can cause large discrepancies on the termination voltages of the TL.

1. INTRODUCTION

The analysis of the response of a transmission line (TL) excited by lumped sources along its conductors has been extensively investigated in the past [1, 2]. Also, numerical as well as analytical models have been introduced in the literature for the analysis of the interaction between an electromagnetic field and a multiconductor transmission line, both in frequency [3–6] and time [7–10] domains. Some of the main models used for the solution of the problem are the Singularity Expansion Method (SEM), the Method of Moments (MOM), the Time Domain Frequency Domain model (TDFD), and the Finite Difference Time Domain (FDTD) technique.

The FDTD method is a general way of directly obtaining the time domain response of a transmission line by discretizing the transmission line equations in position along the line and in time, and solving the equations in a leapfrog fashion. This technique has certain advantages and drawbacks when compared to other methods. The main advantage of FDTD technique is that it is quite easy to code. Furthermore, a generalized code can include transmission line losses, as well as non-linear termination load characteristics. On the other hand, this procedure cannot be used for analysis of lines with frequency-dependent parameters. Finally, care should be exercised to choose the proper number of spatial and temporal cells, in order to obtain accurate results.

The assumption made by all authors cited in references [1–10] is that the transmission line is uniform. This condition can be relaxed if either the transmission line conductors are not parallel to each other or they are made of different materials. Similar occasions can be faced in configurations consisting of a line above an inhomogeneous ground plane. In [11], an investigation of the behavior of field-excited, non-uniform transmission lines is presented. The method used for the determination of termination voltages in the time domain is the well-known Fast Fourier Transformation (FFT) method.

In this paper, the development of a modified FDTD code is introduced for the evaluation of termination voltages of a non-uniform transmission line for both field-excitation and lumped source excitation. The numerical results obtained by the code are compared to those presented in the literature to validate the correctness and accuracy of the model. Furthermore, this model is used for the computation of the voltage induced on non-uniform TL configurations frequently met in practical cases. Some useful remarks obtained by this study are discussed below.

2. FDTD RECURSION RELATIONS FOR UNIFORM TL

For simplicity sake, the following development is based on the below mentioned assumptions:

1. The propagation mode of the line is TEM.
2. The transmission line conductors have perfect conductivity.
3. The surrounding medium is lossless.

The first condition is equivalent to the assumption that the cross-sectional dimensions of the line are much smaller than the shortest wavelength of the wave. In most practical cases, the transmission line is embedded in an inhomogeneous medium, and thus the waves that propagate along the line cannot be considered as being TEM. Moreover, even if the surrounding medium is homogeneous, the transmission line, due to per-unit-length losses, cannot support TEM propagation. Conditions two and three are set in order to be able to define the transmission line propagation as TEM (or quasi-TEM) and thus describe the transmission line behavior in terms of circuit-theory parameters. Under these conditions, the transmission line can be described by a system of two partial differential equations in the time domain (transmission line equations). A final assumption that has to be made is that the per- unit-length inductance and capacitance of the transmission line is considered as frequency- independent.

2.1 Uniform Transmission Line Excited by Lumped Sources at Its Terminations

In order to illustrate the method, consider a two-conductor, lossless transmission line of length L and distance of separation between the conductors equal to d, as depicted in Figure 1. The TEM mode model of a uniform, lossless transmission line can be expressed by the following homogeneous differential equations [2] in time domain:

$$\frac{\partial}{\partial z}V(z,t) + l\frac{\partial}{\partial t}I(z,t) = 0 \tag{1a}$$

$$\frac{\partial}{\partial z}I(z,t) + c\frac{\partial}{\partial t}V(z,t) = 0 \tag{1b}$$

where l and c is the per-unit-length inductance and capacitance of

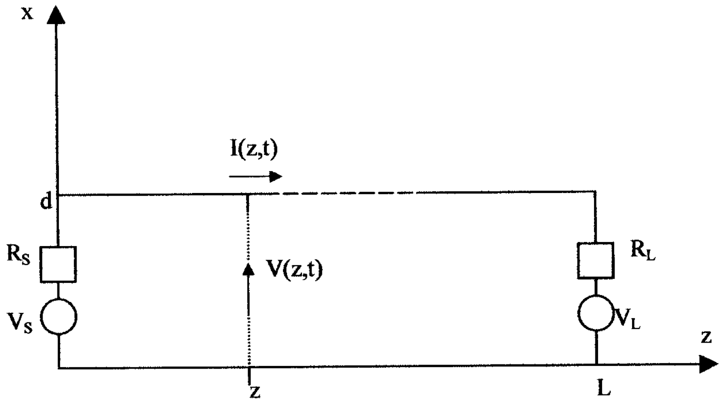

Figure 1. A uniform transmission line excited by lumped voltage sources.

the TL, respectively and $V(z,t)$, $I(z,t)$ is the line voltage and current along the TL, respectively.

The FDTD technique seeks to approximate the derivatives in (1a), (1b) with regard to the discrete solution points defined by spatial and temporal cells. According to this notation, the finite difference representation of the spatial and temporal derivative of a function $f(z,t)$ is written as:

$$\frac{\partial}{\partial z} f(z,t) = \frac{f_{k+1}^{n+1} - f_k^{n+1}}{\Delta z} \tag{2a}$$

$$\frac{\partial}{\partial t} f(z,t) = \frac{f_k^{n+1} - f_k^{n}}{\Delta t} \tag{2b}$$

To incorporate the above equations into the FDTD code, we divide the line into $KTot$ sections of length Δz $\left(\Delta z = \frac{L}{KTot}\right)$, and the total solution time into $NTot$ segments of Δt $\left(\Delta t = \frac{Total\ Solution\ Time}{NTot}\right)$. In order to insure second-order accuracy of the discretization we interlace the $KTot+1$ voltage points and $ktot$ current points as shown in Figure 2.

Consequently, each voltage and adjacent current point is separated in space and time by $\Delta z/2$ and $\Delta t/2$, respectively. Substituting (2) into (1), the finite difference approximation of (1) yields:

$$\frac{V_{k+1}^{n+1} - V_k^{n+1}}{\Delta z} + l \cdot \frac{I_k^{n+3/2} - I_k^{n+1/2}}{\Delta t} = 0 \tag{3a}$$

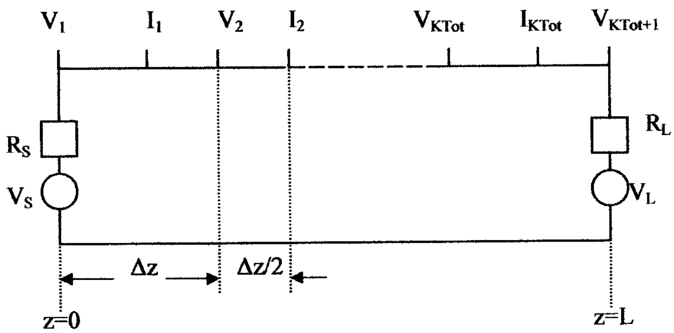

Figure 2. The spatial discretization of voltages and currents along the transmission line.

$$\frac{I_k^{n+1/2} - I_{k-1}^{n+1/2}}{\Delta z} + c \cdot \frac{V_k^{n+1} - V_k^n}{\Delta t} = 0 \tag{3b}$$

where V_k^n, I_k^n are defined as:

$$V_k^n = V((k-1) \cdot \Delta z, n \cdot \Delta t) \tag{4a}$$
$$I_k^n = I((k-1/2) \cdot \Delta z, n \cdot \Delta t) \tag{4b}$$

Equations (3a), (3b) are solved by extracting the terms V_k^{n+1}, $I_k^{n+3/2}$:

$$V_k^{n+1} = V_k^n - \frac{\Delta t}{c \cdot \Delta z} \cdot \left(I_k^{n+1/2} - I_{k-1}^{n+1/2} \right) \tag{5a}$$
$$I_k^{n+3/2} = I_k^{n+1/2} - \frac{\Delta t}{l \cdot \Delta z} \cdot \left(V_{k+1}^{n+1} - V_k^{n+1} \right) \tag{5b}$$

The above equations can be solved in a "bootstrapping" fashion [2]. At each time step n, the voltages along the line are computed in terms of the previous voltage and current values (starting with an initially relaxed line at $t = 0$). Afterwards, the currents along the line are evaluated for the next temporal cell. The final step for the determination of the FDTD code is the incorporation of terminal constraints of the TL.

Under the assumption of TEM propagation along the TL mentioned above, the lumped resistive loads at the two ends of the TL can be characterized as:

$$V(0,t) = V_S - R_S \cdot I(0,t) \tag{6a}$$
$$V(L,t) = V_L + R_L \cdot I(L,t) \tag{6b}$$

where V_S, V_L are the lumped voltage sources at the near- and far-end of the TL. Incorporating the latter equations in (5a), the final FDTD code is obtained. For each time step n, the voltage at the near- and far-end of the TL is evaluated by [12]:

$$V_1^{n+1} = \frac{1}{\left(R_S \cdot \frac{c}{2} \cdot \frac{\Delta z}{\Delta t} + \frac{1}{2}\right)} \cdot \left\{ \left(R_S \cdot \frac{c}{2} \cdot \frac{\Delta z}{\Delta t} - \frac{1}{2}\right) \cdot V_1^n - R_S \cdot I_1^{n+1/2} + \frac{V_S^{n+1} + V_S^n}{2} \right\} \quad (7a)$$

$$V_{KTot+1}^{n+1} = \frac{1}{\left(R_L \cdot \frac{c}{2} \cdot \frac{\Delta z}{\Delta t} + \frac{1}{2}\right)} \cdot \left\{ \left(R_L \cdot \frac{c}{2} \cdot \frac{\Delta z}{\Delta t} - \frac{1}{2}\right) \cdot V_{KTot+1}^n + R_L \cdot I_{KTot}^{n+1/2} + \frac{V_L^{n+1} + V_L^n}{2} \right\} \quad (7b)$$

Also, the voltage and current at each intermediate spatial segment is evaluated by equations (5a), (5b). This procedure goes on for each time cell until final solution time is reached.

2.2 Uniform Transmission Line Excited by Electromagnetic Plane Wave

Once again, consider the transmission line depicted in Figure I with voltage sources V_S, V_L removed. The transmission line is illuminated by a coplanar, vertically polarized plane wave having angle of incidence θ, as shown in Figure 3. Adopting the total field approach, the coupled, first-order, partial, non-homogeneous differential equations for the line voltage and line current are expressed as [7, 9]:

$$\frac{\partial}{\partial z} V(z,t) + l \frac{\partial}{\partial t} I(z,t) = V_F(z,t) = \frac{\partial}{\partial t} B_N(z,t) = -\frac{\partial}{\partial z} E_T(z,t) + E_L(z,t) \quad (8a)$$

$$\frac{\partial}{\partial z} I(z,t) + c \frac{\partial}{\partial t} V(z,t) = I_F(z,t) = -c \frac{\partial}{\partial t} E_T(z,t) \quad (8b)$$

where

$$E_T(z,t) = \int_0^d E_X^{inc} \cdot dx \quad (9a)$$

$$E_L(z,t) = E_Z^{inc}(x=d,z,t) - E_Z^{inc}(x=0,z,t) \tag{9b}$$

and E_X^{inc}, E_Z^{inc} are the x- and z-components of the incident electric field vector.

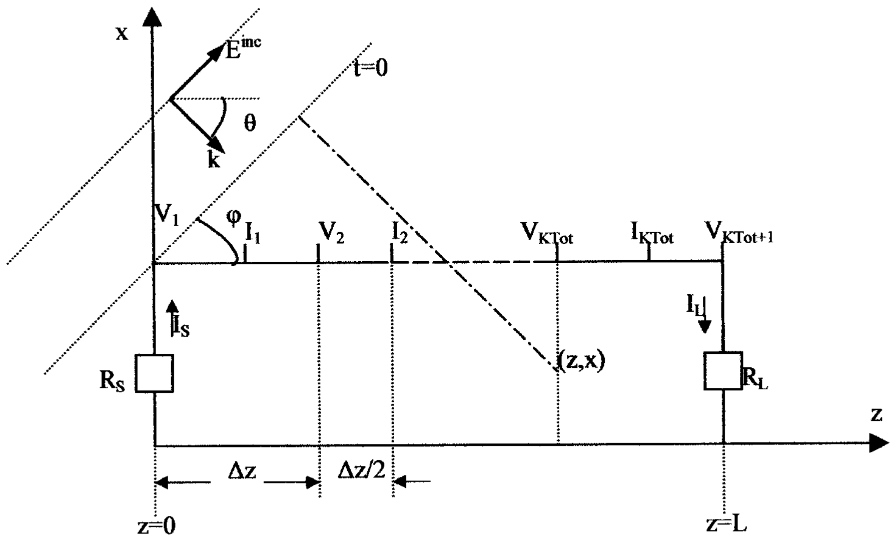

Figure 3. A uniform transmission line excited by an external electromagnetic wave.

As it is obvious from Figure 3, each spatial segment of the TL is excited by the external electromagnetic wave in different time instants. In other words, the right-hand side variables of equation (8a) and (8b) must be expressed in terms of unit step function, in order to predict the time-delay of the excitation field impinging the transmission line. Assuming the initial time instant $t = 0$ being fixed when the plane wave arrives at the point of coordinates $(z,x) = (0,d)$, an arbitrary segment $(z,x) = ((k-1)\Delta z, x)$ will be excited with a time-delay of $t_x + t_z$. It can be easily proven, from fundamental trigonometric analysis, that t_x, t_z can be written in the following form:

$$t_x = \frac{[(d-x)\cdot\cos\phi + z\cdot\sin\phi]\cdot\cos^2\phi}{u_c} \tag{10a}$$

$$t_z = \frac{[(d-x)\cdot\cos\phi + z\cdot\sin\phi]\cdot\sin^2\phi}{u_c} \tag{10b}$$

where $z = (k-1)\Delta z$, $x \in [0, d]$, and u_c is the speed of light.

Thus, the components of incident electric field vector, with respect to the angle of incidence, can be expressed as:

$$E_x^{inc}(x,z,t) = \vec{E}^{inc} \cdot \vec{x} = E^{inc}(t - t_x - t_z) \cdot \cos\theta \cdot u(t - t_x - t_z) \quad \text{(11a)}$$

$$E_z^{inc}(x,z,t) = \vec{E}^{inc} \cdot \vec{z} = E^{inc}(t - t_x - t_z) \cdot \sin\theta \cdot u(t - t_x - t_z) \quad \text{(11b)}$$

As mentioned in the previous section, the final step is the implementation of terminal conditions into FDTD algorithm. The current at the near-end of the transmission line is denoted as I_S and at the far-end as I_L (Figure 3). The terminal conditions are:

$$I_S^n = -\frac{V_1^n}{R_S} \quad \text{(12a)}$$

$$I_L^n = -\frac{V_{KTot+1}^n}{R_L} \quad \text{(12b)}$$

It is of great importance to mention that, despite intermediate ones, the termination currents are evaluated at the same temporal cell of voltages.

Substituting equations (2), (4), (9)–(12) in (8), yields the final form of FDTD code. For each temporal cell:

$$V_1^{n+1} = \frac{1}{1 + \frac{\Delta t}{c\Delta z R_S}} \cdot \left\{ \left(1 - \frac{\Delta t}{c\Delta z R_S}\right) V_1^n - \frac{2 \cdot \Delta t}{c\Delta z} I_1^{n+1/2} - \left(E_{T,1}^{n+1} - E_{T,1}^n\right) \right\} \quad \text{(13a)}$$

$$V_k^{n+1} = V_k^n - \frac{\Delta t}{c \cdot \Delta z} \cdot \left(I_k^{n+1/2} - I_{k-1}^{n+1/2}\right) - \left(E_{T,k}^{n+1} - E_{T,k}^n\right) \quad \text{(13b)}$$

$$V_{KTot}^{n+1} = \frac{1}{1 + \frac{\Delta t}{c\Delta z R_L}} \cdot \left\{ \left(1 - \frac{\Delta t}{c\Delta z R_L}\right) V_{KTot+1}^n + \frac{2 \cdot \Delta t}{c\Delta z} I_{KTot}^{n+1/2} - \left(E_{T,KTot+1}^{n+1} - E_{T,KTot+1}^n\right) \right\} \quad \text{(13c)}$$

$$I_k^{n+3/2} = I_k^{n+1/2} - \frac{\Delta t}{l \cdot \Delta z} \cdot \left(V_{k+1}^{n+1} - V_k^{n+1}\right) - \frac{\Delta t}{l \cdot \Delta z} \cdot \left(E_{T,k+1}^{n+1} - E_{T,k}^{n+1}\right) + \frac{\Delta t}{2 \cdot l} \cdot \left(E_{L,k}^{n+3/2} - E_{L,k}^{n+1/2}\right) \quad \text{(13d)}$$

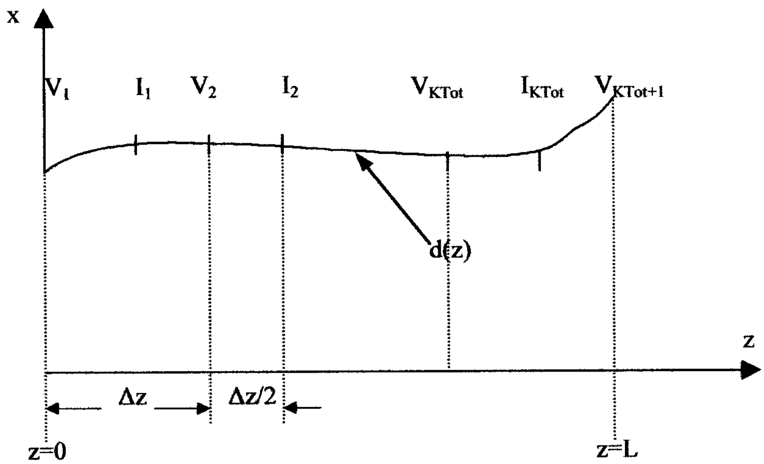

Figure 4. Arbitrary configuration of a non-uniform transmission line.

The evaluation procedure is similar to the one described in the previous section.

3. MODIFICATIONS ON FDTD CODE FOR NON-UNIFORM TL

Due to the general formulation of the FDTD code developed in the previous section, the alterations in terminal voltages caused by a non-uniform transmission line can be anticipated. Consider a transmission line where the height of its upper conductor is a function of z $(d = d(z))$. We now divide the lower (uniform) conductor of the transmission line in KTot spatial segments (Figure 4). The segments at the upper conductor are assumed as having equal length (Δz) and being parallel to z-axis. These assumptions can be set if no steep variation of the inclination of the TL exists. The differences inserted in the non-uniform configuration, compared to the uniform one, are:

a. the arrival time of incident plane wave at each spatial segment strongly depends on $d(z)$,
b. the per-unit-length inductance and capacitance of the transmission line are no longer constant and can be expressed as a function of

$d = d(z)$ [2]:

$$c = \frac{2\pi\varepsilon_0}{\ln\left(\dfrac{d^2}{a_1 a_2}\right)} \tag{14a}$$

$$l = \frac{\mu_0}{2\pi}\ln\left(\frac{d^2}{a_1 a_2}\right) \tag{14b}$$

where a_1, a_2 is the radius of the upper and lower conductor,

c. The propagation velocity, $u_{TL} = \frac{1}{\sqrt{l \cdot c}}$, along the transmission line, is constant. On the contrary, characteristic impedance, $Z_0 = \sqrt{\frac{l}{c}}$, is a function of d.

With the aforementioned notations in mind, we can readily derive the modified FDTD code for both lumped source and plane wave excitation, by applying (14) into (5), (7) and (13), respectively.

4. PRACTICAL EXAMPLES AND NUMERICAL RESULTS

Before proceeding with the verification of the modified FDTD model presented in this paper, it is of great importance to notice some serious remarks related to the computational procedure of the model and the conditions to be fulfilled in order to insure stability and accuracy of the model:

1. The basic condition for the set of recursion relations to be stable is the Courant condition [7]:

$$\Delta t \leq \frac{\Delta z}{u} \tag{15a}$$

which, equivalently, can be written as:

$$NTot \geq KTot \cdot \frac{u \cdot Final\ Solution\ Time}{TL\ Length} \tag{15b}$$

where u is the phase velocity of propagation of the wave.

2. In order to achieve computation accuracy, the spatial discretization is chosen such that each spatial segment Δz is electrically small compared to the largest significant spectral component of the excitation waveform [12].

3. The Total Solution Time is chosen such that permits the complete "temporal deployment" of the coupling phenomenon.
4. For the particular case of field excitation, the transverse incident field sources, $E_T(z,t)$, are to be evaluated at the voltage positions, whereas the longitudinal field sources, $E_L(z,t)$, are to be evaluated at the current positions [13]:

$$E^n_{T,k} = E_T((k-1)\Delta z, n\Delta t) \tag{16a}$$
$$E^n_{L,k} = E_L((k-1/2)\Delta z, n\Delta t) \tag{16b}$$

On the basis of previous notation, terminal currents, I_S, I_L, are collocated $\Delta z/2$ distance from adjacent voltages and are evaluated for the same temporal cell (equations (12a), (12b)).

To verify the computed induced voltages at both ends of a non-uniform transmission line obtained by FDTD method presented in this paper, we compare two typical configurations, already published in the literature. Moreover, the evaluated induced termination voltages for the case of nonuniform TL are compared to those obtained for uniform TL with similar characteristics.

Example 1

We consider a lossless, two-conductor transmission line excited by a lumped voltage source at its near-end $V_S(t) = 30\,\text{V}$, $V_L(t) = 0\,\text{V}$, with termination loads $R_S = 0\,\Omega$, $R_L = 100\,\Omega$, as described in [12]. The total length of the line is equal to $L = 400\,\text{m}$. For the case of a uniform transmission line, the per-unit-length inductance and capacitance are set equal to $l = 2.5 \cdot 10^{-7}\,\text{H/m}$, $c = 10^{-10}\,\text{F/m}$ and $d = 6.5 \cdot 10^{-2}\,\text{m}$, $a_1 = a_2 = a = 5\,\text{cm}$. For the case of non-uniform TL, the distance of separation between the upper and lower conductor is given by $d(z) = (6 + 0.03z) \cdot 10^{-2}$ and the corresponding per-unit-length capacitance and inductance can be evaluated from equations (14a), (14b). The transmission line is discretized in 50 spatial segments of equal length, and the total solution time (20-μsec) is discretized into 1000 temporal cells in order to fulfil Courant condition (15b). The obtained results for the induced voltage at the far-end of the transmission line are depicted in Figure 5. The corresponding results for uniform TL are identical to those presented in [12]. As it is readily shown in Figure 5, the rise time for the case of non-uniform transmission line is noticeably smaller than the one presented for uniform TL. Moreover, it is seen that in the non-uniform case, the convergence to expected constant value of 30 V is more rapid. These two conclusions can be justified by the notice that as z increases the per-unit-length inductance and capacitance of the non-uniform TL, being a function of d, are increasing and decreasing, respectively.

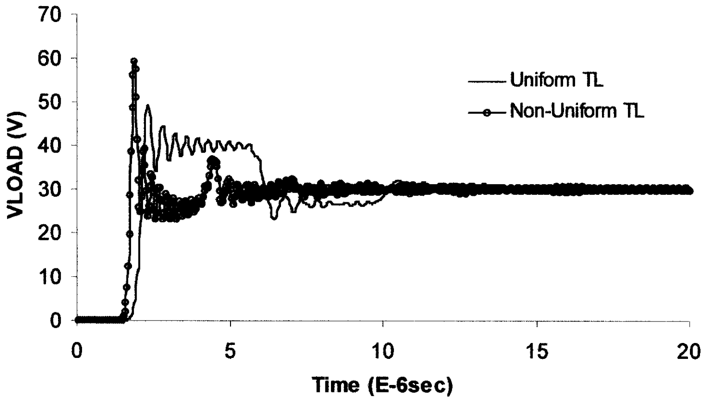

Figure 5. The induced voltage at the far-end of the uniform and non-uniform transmission line as described in Example 1.

Example 2

As a second example, we now consider the transient response behavior of the induced voltages on uniform/non-uniform TL, excited by an external electromagnetic wave, as shown in Figures 3, 4. For the case of a uniform transmission line, the characteristics of the excitation electric field and of the TL are as follows: $V_S = V_L = 0\,\mathrm{V}$, $R_S = R_L = 159\,\Omega$, $L = 30\,\mathrm{m}$, $a_1 = a_2 = a = 1.5\,\mathrm{mm}$, $d = 30\,\mathrm{cm}$ and the impinging wave is taken to be of the following form: $E^{inc}(t) = 1.05 \cdot (\exp[-4 \cdot 10^6 t] - \exp[-4.76 \cdot 10^8 t])\,\mathrm{V/m}$, with angle of incidence $\theta = 45^\circ$. Similarly, the characteristics of non-uniform TL are the same as in the case of uniform TL with the only difference that the TL height linearly increases from 10 cm to 50cm, as described in the following equation: $d(z) = (10 + 1.33z) \cdot 10^{-2}\,\mathrm{m}$. In Figure 6, the Fourier transformation of $E^{inc}(t)$ is depicted. The largest significant spectral component can be considered as being $f = 10^9\,\mathrm{Hz}$. Hence, in order to fulfill the second remark, $\Delta z (\leq \lambda/10)$ must be equal to 0.3 m. Hence, $KTot$ must be equal to 100 and from equation (15b), $Ntot = 500$, as Final Solution Time is 0.5 μsec. The results for the far-end induced voltage are shown in Figure 7 (the case of non-uniform TL gives results identical to those presented in [11], evaluated according to Time

Domain to Frequency Domain method). Although the characteristics of the transmission line as well as the characteristics of external plane wave are quite similar (or identical) for both cases, the variations between the two curves observed in Figure 7 are large. Three are the main reasons for these differences. The first one is discussed in the previous example and is related to the strong dependency of per-unit-length inductance and capacitance of the TL. The second one is that both perpendicular and longitudinal component of the external electromagnetic wave is affected due to non- uniformity if the line. The last reason is that, as z increases, the wave travels larger distances in order to impinge to the lower conductor.

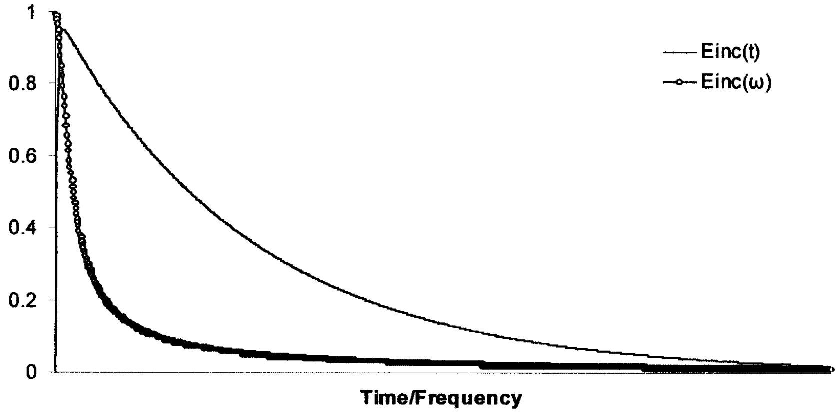

Figure 6. Time-domain and Frequency-domain representation of external electromagnetic wave (Example 2).

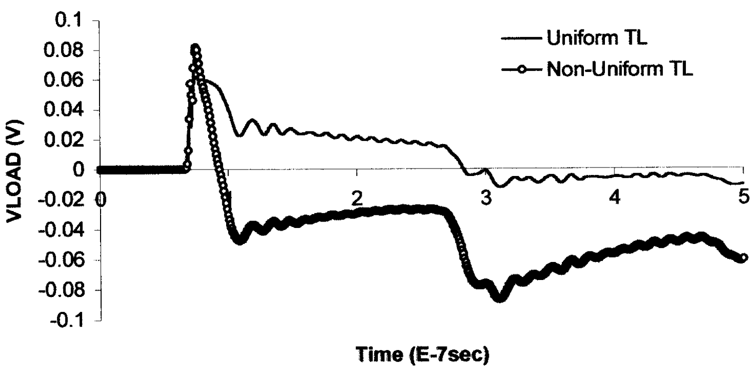

Figure 7. The induced voltage at the far-end of the uniform and non-uniform transmission line as described in Example 2.

A final remark concerning both examples is that the assumption of constant velocity of propagation along the transmission line seems to be in force; discontinuities of the waveforms are observed at the same time instant, especially for the second example.

In order to make clear the strong dependency of line response due to non-uniformity of the TL, two comparison studies are presented below (both employed for field-excited TL's):

Comparison Study 1

The far-end voltage response of a linearly ascending $(d(z) = (10 + 1.33z) \cdot 10^{-2}\,\text{m})$ transmission line, as defined in the previous section, and a linearly descending $(d(z) = (50 - 1.33z) \cdot 10^{-2}\,\text{m})$ transmission line, depicted in Figure 8, are compared to the

uniform TL $(d = 30\,\text{cm})$, also defined in the previous section. Although the inclination of both non-uniform lines is small compared to their length (40 cm in a total length of 30 m), the voltage responses are far from similar (Figure 9). As it can be readily observed, not only the rise times of the transient response of the TL's are strongly dependent on the alteration of per- unit-length inductance and capacitance, but the peak value of the induced voltage, as well.

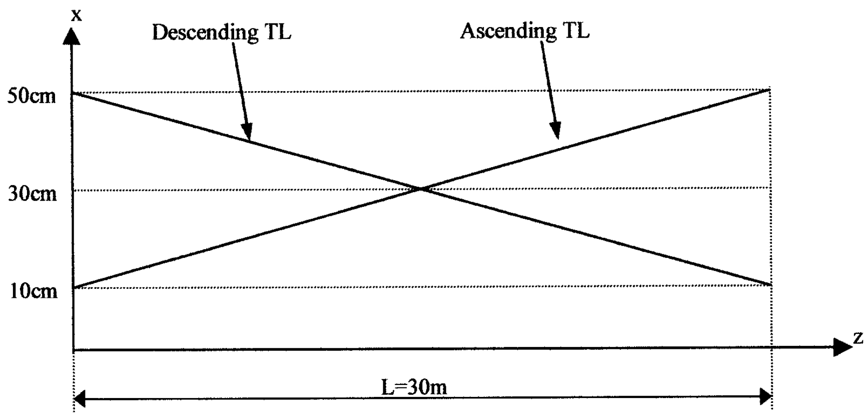

Figure 8. The configuration of non-uniform TL examined in comparison study 1.

Comparison Study 2

As a final comparison, the transmission lines having the form shown in Figure 10 are compared to the uniform transmission line defined previously. The transmission line with the form of $d(z) = (10 + 40\cdot$

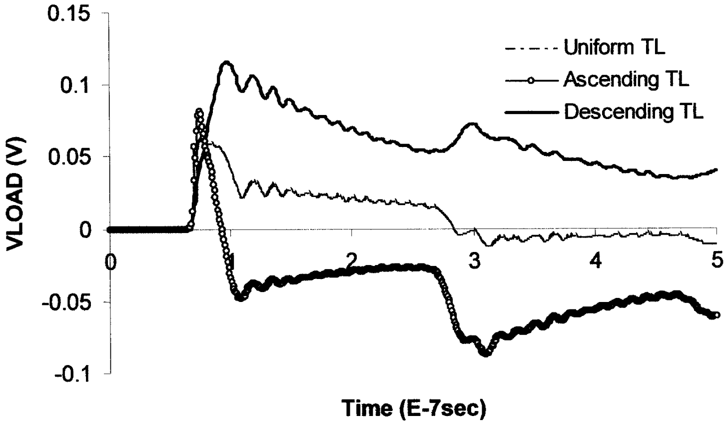

Figure 9. The comparison of far-end voltage of the transmission line configurations presented in comparison study 1.

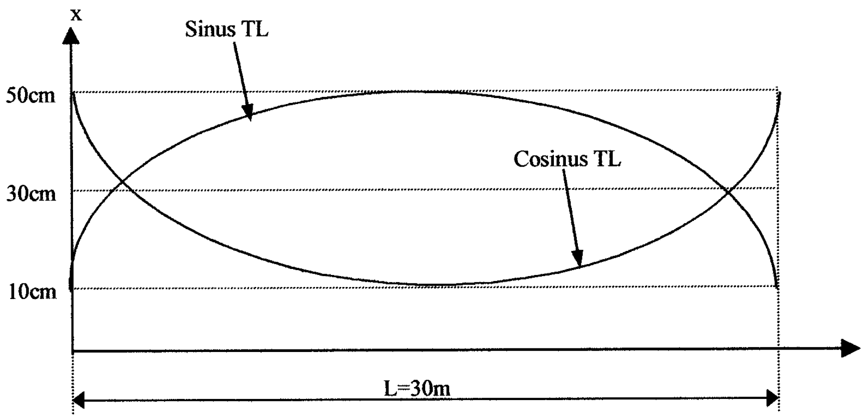

Figure 10. The configuration of non-uniform TL examined in comparison study 2.

$\sin\left[\frac{\pi}{30}\cdot z\right]\big)\cdot 10^{-2}$ m is hereinafter denoted as "Sinus TL" and the transmission line with the form of $d(z) = \big(50 + 40\cdot\cos\left[\frac{\pi}{30}\cdot z + \frac{\pi}{2}\right]\big)\cdot 10^{-2}$ m is denoted as "Cosinus TL", as well. In Figure 11, the line responses are plotted, showing great differences between the configurations. This shape of transmission line can be faced in commercial power delivery systems, where an aboveground line is rarely straight and uniform due to periodic supports (towers) and the line can meander around to conform to the earth topography. With minor modifications on the TL

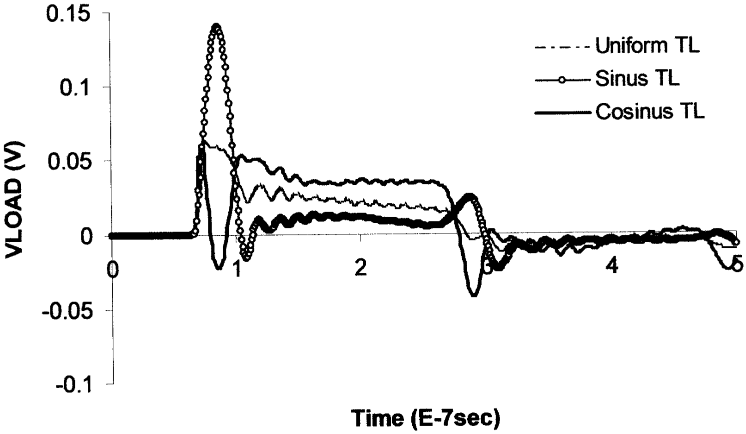

Figure 11. The comparison of far-end voltage of the transmission line configurations presented in comparison study 2.

configuration presented in this paper and on the FDTD algorithm, this model can estimate the voltage induced on an overhead line by a high altitude nuclear electromagnetic pulse (HEMP) or lightning-induced overvoltages in power lines. The interested reader can refer to [15–18] for further information on this subject. It is mentioned that the waveform of external electromagnetic pulse used in this paper is similar to the "Bell Laboratory Waveform" as described in [19] and being used by several authors [20, 21] in order to calculate the transient response of a TL over a ground plane. The important conclusion arising from the observation of Figure 11 is that an additional parameter must be taken into account when dealing with the coupling phenomenon present when a TL is illuminated by an external E/M wave; that is the non-uniformity of the TL configuration. It is of great importance to mention that if this model is used to predict the HEMP-induced voltage on overhead lines over the earth, greater differences must be expected amongst the configurations, as the amplitude of the HEMP is taken equal to about 50 kV/m, according to [19] .

A final remark arising from the observation of Figures 9 and 11 is that according to the employed coupling model, both the vertical and horizontal components of the electric field are of great importance. However, from the definition of the x- and z-component of electric field vector, one can readily note that while the contribution of the vertical field to the total voltage is of positive polarity, the horizontal field coupling may result in a contribution of positive, bipolar, or negative polarity [15]. This is due to the dependency on the configuration of the transmission line, the angle of incidence of external E/M wave and other important characteristics, such as the termination load values and the distance of separation between the transmission line conductors.

5. CONCLUSION

The FDTD technique has become popular due to its versatility and the ability to handle complex structures. In this paper, a modified FDTD algorithm presented in order to investigate the effect on the voltages induced on several non-uniform transmission-line configurations, due to lumped source or external wave excitation. The concept presented in this paper, valid for non-uniform transmission lines with frequency-independent parameters, is implemented into the general formulation of FDTD algorithm. The validation of this model confirmed by comparing the results obtained by our code with results obtained by other time domain models, already published in the literature. It is shown that the choice of FDTD model to calculate the transient response of a TL excited by an external E/M wave is useful as it permits one to incorporate, in an easy to code manner, several modifications related to the configuration of the TL, avoiding to solve analytically the set of integro-differential equations describing the coupling phenomenon. Some useful remarks on the characterization of induced voltages are discussed. The basic one is that the exact positioning of the TL is of great importance as slight inclinations can result in great alterations on the voltage induced in the TL. We must notice that this model can be easily used in multiconductor transmission line configurations. Also, although not presented in this paper, this model can be modified in order to predict transmission line losses as well as non-linear behavior of termination loads [13, 14].

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