ELECTROMAGNETIC WAVES PIER 32

Progress In Electromagnetics Research

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Progress In Electromagnetics Research

Chief Editor: J. A. Kong

Geometric Methods for Computational Electromagnetics

Editor: F. L. Teixeira

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PREFACE

This volume of the PIER series presents a collection of original as well as review papers dealing with geometric methods for computational electromagnetics.

The term geometric is used here in a broad sense to include discretization methods for Maxwell's equations which 1) do not rely solely on the vector calculus language, or 2) recognize and explore the fundamental distinction between the metric and topological discretization problems, or 3) have a strong coordinate independent flavor. As a result, we encounter here a variety of themes and perspectives. This is augmented by the fact that the authors' background is diverse, including applied mathematicians, engineers, and physicists, perhaps a consequence of the perception of computational electromagnetics as a symbiotic combination of these disciplines. Moreover, some papers are distinctively programmatic while others are more concrete in their objectives. The careful reader will nevertheless perceive many similarities and a convergence of some fundamental concepts and themes. This cannot always be appreciated from isolated journal articles, but it is our objective that in a monograph such as this, the relationships between the different perspectives can be better appreciated. It is also our hope that the collection of papers presented here will foster interactions among workers pursuing different approaches and open new research vistas.

The contributions were organized into four sections. This classification is somewhat arbitrary and the different sections are far from independent. Some papers included in one section could also fit well into a different one.

The first section contains five papers dealing with fundamental aspects of geometric methods, and includes some review papers.

In Chapter 1, Tonti presents a tutorial on his finitary formulation of electromagnetic theory from first principles. Because this formulation leads to a finite set of algebraic equations directly (i.e., without the need to resort to the usual discretization of the differential equations), it is relevant to computational electromagnetics. Also presented in detail are the so-called Tonti diagrams, particularized for electromagnetic fields.

In Chapter 2, Bossavit discusses, from a geometrical standpoint, the fundamental links between finite difference, finite element, and finite volume discretizations, stressing the special role of finite elements in the convergence and error analysis. The general advantages stemming from the use of exterior differential forms for discretization analysis are addressed. The rationale behind the use of Whitney forms as the basic interpolants, and the interpretation of the Galerkin method as a realization of discrete Hodge operators (i.e., discrete material laws incorporation all metric information) are also considered. The end result is a powerful unified picture of finite methods.

Clemens and Weiland review in Chapter 3 the main attractive features of the finite integration technique (FIT), and discuss its close connections with other discretization schemes for Maxwell's equations. They also use basic algebraic properties of the method to prove charge and energy conservation in the discrete setting.

In Chapter 4, Hyman and Shashkov review the application of mimetic finite difference methods in nonorthogonal, nonsmooth grids for Maxwell's equations. This effective discretization approach is based on the construction of discrete analogues of vector and tensor operators (and its adjoints) which automatically satisfy discrete analogs of the theorems of vector (and tensor) analysis. Both hyperbolic and parabolic diffusion regimes are considered. A convergence study for the method in smooth and non-smooth grids is included.

In Chapter 5, Mattiussi unveils basic geometric and topological concepts behind the time evolution equations usually encountered in computational electromagnetics. He advocates the use of a truly space-time approach to associate physical quantities with domains and in setting up the corresponding discrete equations. Because the balance laws do not depend on the size or shape of the (space-time) domain, these resulting space-time equations are topological in nature. This philosophy is broad enough to encompass many discretization techniques, such as FDTD, FIT or DSI, and, by recognizing the role of the constitutive equations, even higher order methods or implicit methods.

The second section of this volume includes five papers dealing with (co)homological and/or algebraic techniques for the spatial discretization problem.

Gross and Kotiuga consider in Chapter 6 the problem of exploring the topological structure of finite element algorithms via the identification of tetrahedral meshes with simplicial complexes. This leads to the construction of efficient data structures for the resulting numerical algorithms. By using a discrete form of the Poincaré duality theorem, this also allows the identification of the coboundary operators as the connection matrix for the dual complex. In addition, they discuss the role of Whitney forms as a bridge between the vector field picture and the cohomological picture and illustrate some three-dimensional applications of the theory.

In Chapter 7, Teixeira combines a geometric scheme, first developed for Chern-Simons theory, with Whitney forms to discretize

Maxwell's equations on a simplicial grid. The scheme employs a barycentric decomposition of the simplicial primal grid and the non-simplicial dual grid, leading to a natural construction of Hodge operators directly from the Whitney forms on the resulting barycentric subdivision grid.

In Chapter 8, Tarhasaari and Kettunen express Maxwell's equations as relations using concepts from naive set theory and develop an elegant algorithm based on linear algebra to tackle topological problems underlying a electromagnetic boundary value problem. One of the objectives here is to illustrate a methodology to develop data-driven approaches (instead of the usual method-driven approaches) to computational electromagnetics.

In Chapter 9, Gross and Kotiuga discuss an algorithm to make cuts for scalar magnetic potentials in three-dimensional multi-connected finite-element calculations. The algorithm is based on the algebraic structures of (co)homology theory. They also examine the computational complexity of the resulting algorithm and emphasize the fundamental distinction between the two- and three-dimensional problems.

In Chapter 10, Hiptmair discusses, from an algebraic standpoint, general properties and constraints for consistent discretizations of Hodge operators, and includes an abstract error analysis based on energy norms. The same author describes in Chapter 11 a unified and systematic approach to construct higher order finite element basis, based on interpolants of discrete differential forms (Whitney forms). This constitutes a novel and interesting foundation for p-refinement methodologies as well as for hierarchical a posteriori error estimators.

The third section contains four papers devoted to further analysis of geometric techniques discussed in the first section, with emphasis on applications.

In Chapter 12, Schuhmann and Weiland provide a detailed study of energy conservation laws under both the semi-discrete (continuous time) and the fully discrete setting of the finite integration technique of Chapter 3, as well as a study of the orthogonality of discrete eigenmodes. They also show how these results are all rooted in a few key properties of the technique.

Marrone describes and implements in Chapter 13 a geometric discretization method for Maxwell's equations, dubbed cell method, based on Tonti's formulation of Chapter 1. He includes a comparison against FDTD numerical results for cavity problems.

The contribution of van Rienen in Chapter 14 deals with an extension and frequency-domain implementation of the finite integration technique of Chapter 3 to arbitrary triangular grids.

Several numerical simulation of resonators and waveguide structures are provided to illustrate the technique.

In Chapter 15, Buksas develops an implementation of the perfectly matched layer (PML) absorbing boundary condition in conjunction with the mimetic finite difference schemes described in Chapter 4. The PML implementation is based on the anisotropic medium formulation.

Finally, the last section consists of the paper by Puska in Chapter 16, which has his own theme. The author uses Clifford's geometric algebra to tackle constitutive relations in a covariant manner. Although not written for numerical purposes in mind, this interesting paper serves to illustrate the power and adequacy of geometric techniques in a strict analytical setting as well.

As a guest editor, I wish to thank Prof. J. A. Kong for his support and encouragement. I would also like to thank P. R. Kotiuga for his suggestions, and C. O. Ao and W. Zhen for the editorial support. Finally, I would like to express my gratitude and appreciation to the authors and reviewers for their contribution to this project.

F. L. Teixeira
Columbus, Ohio

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