

DYNAMICS OF GAUSSIAN AND SUPER-GAUSSIAN SOLITONS IN BIREFRINGENT OPTICAL FIBERS

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Abstract—The variational principle is employed to obtain the parameters dynamics of Gaussian and super-Gaussian chirped solitons which propagates through birefringent optical fibers that is governed by the dispersion-managed vector nonlinear Schrödinger's equation. The waveform deviates from that of a classical soliton. The periodically changing strong chirp of such a soliton reduces the effective nonlinearity that is necessary for balancing the local dispersion. This study is extended to obtain the adiabatic evolution of the parameters of such a soliton in presence of perturbation terms.

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1. INTRODUCTION

The dynamics of solitons propagating in optical fibers has been a major area of research given its potential applicability in all optical

communication systems. The relevant equation is the nonlinear Schrodinger's equation with damping and periodic amplification [1]:

$$iq_z + \frac{D(z)}{2}q_{tt} + |q|^2q = -i\Gamma q + i \left[e^{\Gamma z_a} - 1 \right] \sum_{n=1}^N \delta(z - nz_a)q \quad (1)$$

Here, Γ is the normalized loss coefficient, z_a is the normalized characteristic amplifier spacing and z and t represent the normalized propagation distance and the normalized time, respectively, expressed in the usual nondimensional units.

Also, $D(z)$ is used to model strong dispersion management. We decompose the fiber dispersion $D(z)$ into two components namely a path-averaged constant value δ_a and a term representing the large rapid variation due to large local values of the dispersion [2]. Thus, we write:

$$D(z) = \delta_a + \frac{1}{z_a} \Delta(\zeta) \quad (2)$$

where $\zeta = \frac{z}{z_a}$. The function $\Delta(\zeta)$ is taken to have average zero (namely $\langle \Delta \rangle = 0$), so that the path-averaged dispersion $\langle D \rangle = \delta_a$. The proportionality factor in front of $\Delta(\zeta)$ is chosen so that both δ_a and $\Delta(\zeta)$ are quantities of order one. In practical situations, dispersion management is often performed by concatenating together two or more sections of given length with different values of fiber dispersion. In the special case of a two-step map it is convenient to write the dispersion map as a periodic extension of [2]

$$\Delta(\zeta) = \begin{cases} \Delta_1 & : 0 \leq |\zeta| < \frac{\theta}{2} \\ \Delta_2 & : \frac{\theta}{2} \leq |\zeta| < \frac{1}{2} \end{cases} \quad (3)$$

where Δ_1 and Δ_2 are given by

$$\Delta_1 = \frac{2s}{\theta} \quad \text{and} \quad \Delta_2 = \frac{2s}{1-\theta}$$

with the map strength s defined as

$$s = \frac{\theta\Delta_1 - (1-\theta)\Delta_2}{4} \quad (4)$$

Conversely, we have

$$s = \frac{\Delta_1\Delta_2}{4(\Delta_2 - \Delta_1)} \quad \text{and} \quad \theta = \frac{\Delta_2}{\Delta_2 - \Delta_1}$$

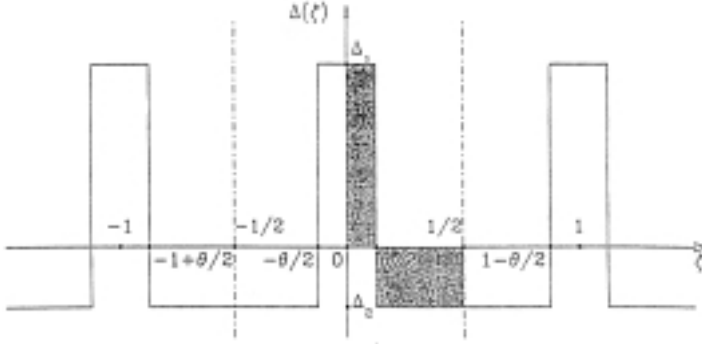


Figure 1. Schematic diagram of a two-step map.

A typical dispersion map is shown in Figure 1.

We take into account the loss and amplification cycles by looking for a solution of (1) of the form $q(z, t) = A(z)u(z, t)$ for real A . Taking A to satisfy

$$A_z + \Gamma A - \left[e^{\Gamma z_a} - 1 \right] \sum_{n=1}^N \delta(z - nz_a) A = 0 \quad (5)$$

we can show that (1) transforms to

$$iu_z + \frac{D(z)}{2} u_{tt} + g(z) |u|^2 u = 0 \quad (6)$$

where we have

$$g(z) = A^2(z) = a_0^2 e^{-2\Gamma(z - nz_a)} \quad (7)$$

for $z \in [nz_a, (n + 1)z_a)$ and $n > 0$ and also

$$a_0 = \left[\frac{2\Gamma z_a}{1 - e^{-2\Gamma z_a}} \right]^{\frac{1}{2}} \quad (8)$$

so that $\langle g(z) \rangle = 1$ over each amplification period [1]. Equation (6) governs the propagation of a dispersion managed soliton through a polarization preserved optical fiber with damping and periodic amplification.

A single mode fiber supports two degenerate modes that are polarized in two orthogonal directions. Under ideal conditions of perfect cylindrical geometry and isotropic material, a mode excited with its polarization in one direction would not couple with the mode in the orthogonal direction. However, small deviations from the

cylindrical geometry or small fluctuations in material anisotropy result in a mixing of the two polarization states and the mode degeneracy is broken. Thus the mode propagation constant becomes slightly different for the modes polarized in orthogonal directions. This properly is referred to as modal birefringence [11]. Birefringence can also be introduced artificially in optical fibers.

The propagation of solitons in birefringent nonlinear fibers has attracted much attention in recent years. It has potential applications in optical communications and optical logic devices. The equations that describe the pulse propagation through these fibers was originally derived by Menyuk [11]. They can be solved approximately in certain special cases only. The localized pulse evolution in a birefringent fiber has been studied analytically, numerically and experimentally [17] on the basis of a simplified chirp-free model without oscillating terms under the assumptions that the two polarizations exhibit different group velocities. In this paper we shall study the equations that describe the pulse propagation in birefringent fibers of the following dimensionless form:

$$i(u_z + \delta u_t) + \beta u + \frac{D(z)}{2} u_{tt} + g(z) (|u|^2 + \alpha |v|^2) u + \gamma v^2 u^* = 0 \quad (9)$$

$$i(v_z - \delta v_t) + \beta v + \frac{D(z)}{2} v_{tt} + g(z) (|v|^2 + \alpha |u|^2) v + \gamma u^2 v^* = 0 \quad (10)$$

Equations (9) and (10) are known as the Dispersion Managed Vector Nonlinear Schrodinger's Equation (DM-VNLSE). Here, u and v are slowly varying envelopes of the two linearly polarized components of the field along the x and y axis. Also, δ is the group velocity mismatch between the two polarization components and is called the birefringence parameter, β corresponds to the difference between the propagation constants, α is the cross-phase modulation coefficient and γ is the coefficient of the coherent energy coupling (four-wave mixing) term. These equations are, in general, not integrable. However, they can be solved analytically for certain specific cases [11, 17]. The first two integrals of motion of (9) and (10) are the energy and the momentum of the pulse that are respectively given by [23]:

$$W = \int_{-\infty}^{\infty} (|u|^2 + |v|^2) dt \quad (11)$$

$$M = \frac{i}{2} D(z) \int_{-\infty}^{\infty} (u^* u_t - u u_t^* + v^* v_t - v v_t^*) dt \quad (12)$$

The Hamiltonian which is given by

$$\begin{aligned}
 H = & \frac{1}{2} \int_{-\infty}^{\infty} \left[\frac{D(z)}{2} (|u_t|^2 + |v_t|^2) - \beta (|u|^2 - |v|^2) \right. \\
 & - \frac{g(z)}{2} (|u|^4 + |v|^4) - i \frac{\delta}{2} (u^* u_t - u u_t^* + v^* v_t - v v_t^*) \\
 & \left. - \alpha |u|^2 |v|^2 - \frac{1}{2} (1 - \alpha) (u^2 v^{*2} + v^2 u^{*2}) \right] dt \quad (13)
 \end{aligned}$$

is however not a constant of motion, in general. The existence of a Hamiltonian implies that we can also write (9) and (10) in the Hamiltonian form as:

$$i u_z = \frac{\delta H}{\delta u^*} \quad \text{and} \quad i v_z = \frac{\delta H}{\delta v^*}$$

This defines a Hamiltonian dynamical system on an infinite-dimensional phase space of two complex functions u and v that decrease to zero at infinity and can be analysed using the theory of Hamiltonian system.

2. LAGRANGIAN FORMULATION

Since, there is no inverse scattering solution to (9) and (10) we shall study these equations approximately by means of variational method based on the observation that it supports well-defined chirped soliton solution whose shape is that of a Gaussian [12] or a super-Gaussian (SG) [6]. For a finite dimensional problem of a single particle, the temporal development of its position is given by the Hamilton's principle of least action [11]. It states that the action given by the time integral of the Lagrangian is an extremum, namely

$$\delta \int_{t_1}^{t_2} L(x, \dot{x}) dt = 0 \quad (14)$$

where x is the position of the particle and $\dot{x} = \frac{dx}{dt}$. The variational problem (14) then leads to the familiar Euler-Lagrange's (EL) equation [11]:

$$\frac{\partial L}{\partial x} - \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) = 0 \quad (15)$$

Here, in (9) and (10), the Lagrangian density is given by:

$$\begin{aligned}
& \mathcal{L}[u, u^*, u_z, u_z^*, u_t, u_t^*; z] \\
&= \frac{i}{2}(u^*u_z - uu_z^*) + \frac{i}{2}(v^*v_z - vv_z^*) + \frac{i\delta}{2}(v^*u_t - uv_t^*) + \frac{i\delta}{2}(u^*v_t - vu_t^*) \\
&\quad - \frac{D(z)}{2}(|u_t|^2 + |v_t|^2) + \frac{g(z)}{2}(|u|^4 + |v|^4) + \alpha g(z)|u|^2|v|^2 \\
&\quad + \beta(u^*v - uv^*) + \frac{\gamma}{2}(u^2v^{*2} + v^2u^{*2}) \tag{16}
\end{aligned}$$

In this analysis we shall neglect the terms with δ as we have $\delta \leq 10^{-3}$ [17]. Also, neglecting β and the four wave mixing terms given by the γ term, we arrive at

$$iu_z + \frac{D(z)}{2}u_{tt} + g(z)(|u|^2 + \alpha|v|^2)u = 0 \tag{17}$$

$$iv_z + \frac{D(z)}{2}v_{tt} + g(z)(|v|^2 + \alpha|u|^2)v = 0 \tag{18}$$

whose Lagrangian density is

$$\begin{aligned}
& \mathcal{L}[u, u^*, u_z, u_z^*, u_t, u_t^*; z] \\
&= \frac{i}{2}(u^*u_z - uu_z^*) + \frac{i}{2}(v^*v_z - vv_z^*) - \frac{D(z)}{2}(|u_t|^2 + |v_t|^2) \\
&\quad + \frac{g(z)}{2}(|u|^4 + |v|^4) + \alpha g(z)|u|^2|v|^2 \tag{19}
\end{aligned}$$

Now, we assume that the solutions of (17) and (18) is given by a chirped pulses of the form [12]:

$$\begin{aligned}
u(z, t) &= A_1(z)f[B_1(z)\{t - t_1(z)\}] \\
&\quad \exp\left[iC_1(z)\{t - t_1(z)\}^2 - i\kappa_1(z)\{t - t_1(z)\} + i\theta_1(z)\right] \tag{20}
\end{aligned}$$

and

$$\begin{aligned}
v(z, t) &= A_2(z)f[B_2(z)\{t - t_2(z)\}] \\
&\quad \exp\left[iC_2(z)\{t - t_2(z)\}^2 - i\kappa_2(z)\{t - t_2(z)\} + i\theta_2(z)\right] \tag{21}
\end{aligned}$$

where f represents the shape of the pulse. Also, here the parameters $A_j(z)$, $B_j(z)$, $C_j(z)$, $\kappa_j(z)$, $t_j(z)$ and $\theta_j(z)$ (for $j = 1, 2$) respectively represent the soliton amplitude, the inverse width of the soliton, chirp, frequency, the center of the soliton and the center of the phase of the solitons respectively. Using the variational principle we shall

derive a set of evolution equations for the soliton parameters. We note that, this approach is only approximate and does not account for characteristics such as energy loss due to continuum radiation, damping of the amplitude oscillations and changing of the pulse shape [4, 7]. Now, integrating L , given by (19), with respect to t and using the ansatz (20) and (21) we arrive at the following Lagrangian:

$$\begin{aligned}
 L &= \int_{-\infty}^{\infty} \mathcal{L} dt \\
 &= -DA_1^2 \left(\frac{B_1}{2} I_3 + 2 \frac{C_1^2}{B_1^3} I_2 + \frac{\kappa_1^2}{2B_1} I_1 \right) \\
 &\quad + \frac{gA_1^4}{2B_1} I_4 - \frac{A_1^2}{B_1^3} I_2 \frac{dC_1}{dz} + \frac{A_1^2}{B_1} I_1 \left(t_1 \frac{d\kappa_1}{dz} - \frac{d\theta_1}{dz} \right) \\
 &\quad - DA_2^2 \left(\frac{B_2}{2} I_3 + 2 \frac{C_2^2}{B_2^3} I_2 + \frac{\kappa_2^2}{2B_2} I_1 \right) \\
 &\quad + \frac{gA_2^4}{2B_2} I_4 - \frac{A_2^2}{B_2^3} I_2 \frac{dC_2}{dz} + \frac{A_2^2}{B_2} I_1 \left(t_2 \frac{d\kappa_2}{dz} - \frac{d\theta_2}{dz} \right) \quad (22)
 \end{aligned}$$

where we have

$$\begin{aligned}
 I_1 &= \int_{-\infty}^{\infty} f^2(\tau) d\tau \\
 I_2 &= \int_{-\infty}^{\infty} \tau^2 f^2(\tau) d\tau \\
 I_3 &= \int_{-\infty}^{\infty} \left(\frac{df}{d\tau} \right) d\tau \\
 I_4 &= \int_{-\infty}^{\infty} f^4(\tau) d\tau
 \end{aligned}$$

and

$$I_5 = \int_{-\infty}^{\infty} f^2[B_1(z)(t - t_1(z))] f^2[B_2(z)(t - t_2(z))] dt$$

By the principle of least action, we have the EL equation as

$$\frac{\partial L}{\partial p} - \frac{d}{dz} \left(\frac{\partial L}{\partial p_z} \right) = 0 \quad (23)$$

where p is one of the twelve soliton parameters. Substituting A_j , B_j , C_j , κ_j , t_j and θ_j ($j = 1, 2$) for p in (23) we arrive at the

following set of equations:

$$\frac{dA_1}{dz} = -DA_1C_1 \quad (24)$$

$$\frac{dB_1}{dz} = -2DB_1C_1 \quad (25)$$

$$\frac{dC_1}{dz} = D \left(\frac{B_1^4}{2} \frac{I_3}{I_2} - 2C_1^2 \right) - \frac{gA_1^2B_1^2}{4} \frac{I_4}{I_2} - \frac{\alpha g}{2} A_2^2 B_1^3 \frac{I_5}{I_2} \quad (26)$$

$$\frac{d\kappa_1}{dz} = 0 \quad (27)$$

$$\frac{dt_1}{dz} = -D\kappa_1 \quad (28)$$

$$\frac{d\theta_1}{dz} = D \left(\frac{\kappa_1^2}{2} - \frac{I_3}{I_1} B_1^2 \right) + \frac{5gA_1^2}{4} \frac{I_4}{I_1} + \frac{3}{2} \alpha g A_2^2 B_1 \frac{I_5}{I_1} \quad (29)$$

$$\frac{dA_2}{dz} = -DA_2C_2 \quad (30)$$

$$\frac{dB_2}{dz} = -2DB_2C_2 \quad (31)$$

$$\frac{dC_2}{dz} = D \left(\frac{B_2^4}{2} \frac{I_3}{I_2} - 2C_2^2 \right) - \frac{gA_2^2B_2^2}{4} \frac{I_4}{I_2} - \frac{\alpha g}{2} A_1^2 B_2^3 \frac{I_5}{I_2} \quad (32)$$

$$\frac{d\kappa_2}{dz} = 0 \quad (33)$$

$$\frac{dt_2}{dz} = -D\kappa_2 \quad (34)$$

$$\frac{d\theta_2}{dz} = D \left(\frac{\kappa_2^2}{2} - \frac{I_3}{I_1} B_2^2 \right) + \frac{5gA_2^2}{4} \frac{I_4}{I_1} + \frac{3}{2} \alpha g A_1^2 B_2 \frac{I_5}{I_1} \quad (35)$$

Now, from (24) and (25) we conclude that $A_1 = K_1\sqrt{B_1}$ for some constant K_1 and similarly from (30) and (31) we have $A_2 = K_2\sqrt{B_2}$ for some constant K_2 . So, the number of parameters reduces by two. Thus, (24) through (35), respectively, modify to

$$\frac{dB_1}{dz} = -2DB_1C_1 \quad (36)$$

$$\frac{dC_1}{dz} = D \left(\frac{B_1^4}{2} \frac{I_3}{I_2} - 2C_1^2 \right) - \frac{K_1^2 g B_1^3}{4} \frac{I_4}{I_2} - \frac{\alpha g}{2} K_2^2 B_1^3 B_2 \frac{I_5}{I_2} \quad (37)$$

$$\frac{d\kappa_1}{dz} = 0 \quad (38)$$

$$\frac{dt_1}{dz} = -D\kappa_1 \quad (39)$$

$$\frac{d\theta_1}{dz} = D \left(\frac{\kappa_1^2}{2} - \frac{I_3}{I_1} B_1^2 \right) + \frac{5gK_1^2 B_1}{4} \frac{I_4}{I_1} + \frac{3}{2} \alpha g K_2^2 B_1 B_2 \frac{I_5}{I_1} \quad (40)$$

$$\frac{dB_2}{dz} = -2DB_2 C_2 \quad (41)$$

$$\frac{dC_2}{dz} = D \left(\frac{B_2^4 I_3}{2 I_2} - 2C_2^2 \right) - \frac{K_2^2 g B_2^3}{4} \frac{I_4}{I_2} - \frac{\alpha g}{2} K_1^2 B_2^3 B_1 \frac{I_5}{I_2} \quad (42)$$

$$\frac{d\kappa_2}{dz} = 0 \quad (43)$$

$$\frac{dt_2}{dz} = -D\kappa_2 \quad (44)$$

$$\frac{d\theta_2}{dz} = D \left(\frac{\kappa_2^2}{2} - \frac{I_3}{I_1} B_2^2 \right) + \frac{5gK_2^2 B_2}{4} \frac{I_4}{I_1} + \frac{3}{2} \alpha g K_1^2 B_1 B_2 \frac{I_5}{I_1} \quad (45)$$

2.1. Gaussian Solitons

For a Gaussian pulse we choose the form $f(\tau) = e^{-\tau^2}$ so that we have the integrals respectively

$$\begin{aligned} I_1 &= \sqrt{\frac{\pi}{2}} \\ I_2 &= \frac{1}{4} \sqrt{\frac{\pi}{2}} \\ I_3 &= \sqrt{\frac{\pi}{2}} \\ I_4 &= \frac{\sqrt{\pi}}{2} \end{aligned}$$

and

$$I_5 = \sqrt{\frac{\pi}{B_1^2 + B_2^2}} e^{-\frac{B_1^2 B_2^2}{B_1^2 + B_2^2} (t_1 - t_2)^2}$$

Thus, we have our evolution equations (36)–(45) respectively reduce to

$$\frac{dB_1}{dz} = -2DB_1 C_1 \quad (46)$$

$$\frac{dC_1}{dz} = -2\alpha g K_2^2 B_1^3 B_2 \sqrt{\frac{2}{B_1^2 + B_2^2}} e^{-\frac{B_1^2 B_2^2}{B_1^2 + B_2^2} (t_1 - t_2)^2}$$

$$-\frac{\sqrt{2}}{2}gK_1^2B_1^3 + 2D(B_1^4 - C_1^2) \quad (47)$$

$$\frac{d\kappa_1}{dz} = 0 \quad (48)$$

$$\frac{dt_1}{dz} = -D\kappa_1 \quad (49)$$

$$\begin{aligned} \frac{d\theta_1}{dz} &= \frac{5}{4\sqrt{2}}gK_1^2B_1 + \frac{3}{2}\alpha gK_2^2B_1B_2\sqrt{\frac{2}{B_1^2+B_2^2}}e^{-\frac{B_1^2B_2^2}{B_1^2+B_2^2}(t_1-t_2)^2} \\ &\quad + \frac{D}{2}(\kappa_1^2 - 2B_1^2) \end{aligned} \quad (50)$$

$$\frac{dB_2}{dz} = -2DB_2C_2 \quad (51)$$

$$\begin{aligned} \frac{dC_2}{dz} &= -2\alpha gK_1^2B_2^3B_1\sqrt{\frac{2}{B_1^2+B_2^2}}e^{-\frac{B_1^2B_2^2}{B_1^2+B_2^2}(t_1-t_2)^2} \\ &\quad - \frac{\sqrt{2}}{2}gK_2^2B_2^3 + 2D(B_2^4 - C_2^2) \end{aligned} \quad (52)$$

$$\frac{d\kappa_2}{dz} = 0 \quad (53)$$

$$\frac{dt_2}{dz} = -D\kappa_2 \quad (54)$$

$$\begin{aligned} \frac{d\theta_2}{dz} &= \frac{5}{4\sqrt{2}}gK_2^2B_2 + \frac{3}{2}\alpha gK_1^2B_1B_2\sqrt{\frac{2}{B_1^2+B_2^2}}e^{-\frac{B_1^2B_2^2}{B_1^2+B_2^2}(t_1-t_2)^2} \\ &\quad + \frac{D}{2}(\kappa_2^2 - 2B_2^2) \end{aligned} \quad (55)$$

2.2. Super-Gaussian Solitons

For a SG soliton we generalize the Gaussian pulses to $f(\tau) = e^{-\tau^{2m}}$ for $m \geq 1$ where the parameter m controls the degree of edge sharpness. With $m = 1$, we recover the case of a chirped Gaussian soliton while for larger values of m the soliton gradually becomes square shaped with sharper leading and trailing edges [3]. In Fig. 2 below, one can see the shapes of the pulses as the parameter m varies.

With this SG pulse we have

$$\begin{aligned} I_1 &= \frac{1}{m2^{\frac{1}{2m}}}\Gamma\left(\frac{1}{2m}\right) \\ I_2 &= \frac{1}{m2^{\frac{3}{2m}}}\Gamma\left(\frac{3}{2m}\right) \end{aligned}$$

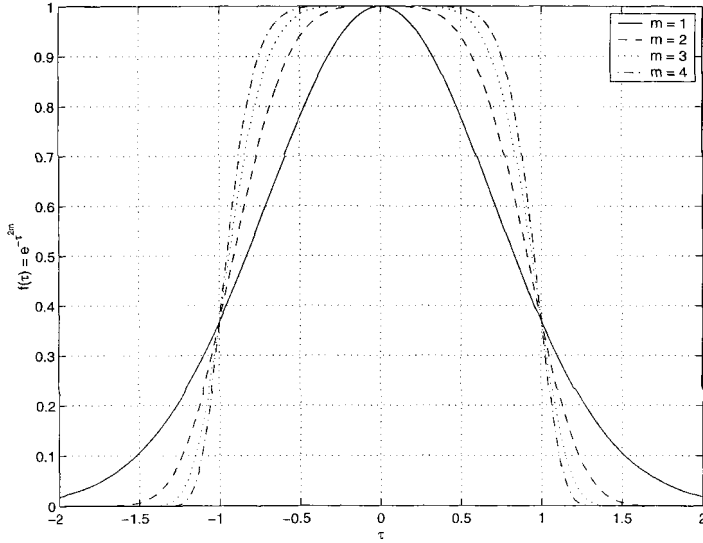


Figure 2. SG Pulse with the variation of the parameter m .

$$I_3 = m2^{\frac{1}{2m}}\Gamma\left(\frac{4m-1}{2m}\right)$$

$$I_4 = \frac{1}{m2^{\frac{1}{m}}}\Gamma\left(\frac{1}{2m}\right)$$

and

$$I_5 = \int_{-\infty}^{\infty} e^{-2\{B_1^{2m}(t-t_1)^{2m}+B_2^{2m}(t-t_2)^{2m}\}} dt$$

where $\Gamma(x)$ is the Gamma function. Here, we note that I_5 cannot be evaluated in a closed form for any m . It can be evaluated for a particular value of m that controls the degree of edge sharpness. Thus, we have our evolution equations (36)–(45) respectively reduce to

$$\frac{dB_1}{dz} = -2DB_1C_1 \tag{56}$$

$$\frac{dC_1}{dz} = D \left\{ \frac{m^2}{2^{\frac{m-2}{m}}} \frac{\Gamma\left(\frac{4m-1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)} B^4 - 2C^2 \right\} - gK_1^2 B_1^3 \frac{1}{2^{\frac{4m+1}{2m}}} \frac{\Gamma\left(\frac{1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)} - \frac{m}{2^{\frac{2m-3}{m}}} \alpha g K_2^2 B_1^3 B_2 \frac{I_5}{\Gamma\left(\frac{3}{2m}\right)} \tag{57}$$

$$\frac{d\kappa_1}{dz} = 0 \quad (58)$$

$$\frac{dt_1}{dz} = -D\kappa_1 \quad (59)$$

$$\begin{aligned} \frac{d\theta_1}{dz} = & D \left\{ \frac{\kappa^2}{2} - m^2 2^{\frac{1}{m}} \frac{\Gamma\left(\frac{4m-1}{2m}\right)}{\Gamma\left(\frac{1}{2m}\right)} B_1^2 \right\} + \frac{5}{2^{\frac{4m+1}{2m}}} g K_1 B_1 \\ & + \frac{3m}{2^{\frac{2m-1}{2m}}} \alpha g K_2^2 B_1 B_2 \frac{I_5}{\Gamma\left(\frac{1}{2m}\right)} \end{aligned} \quad (60)$$

$$\frac{dB_2}{dz} = -2DB_2 C_2 \quad (61)$$

$$\begin{aligned} \frac{dC_2}{dz} = & D \left\{ \frac{m^2}{2^{\frac{m-2}{m}}} \frac{\Gamma\left(\frac{4m-1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)} B^4 - 2C^2 \right\} - g K_2^2 B_2^3 \frac{1}{2^{\frac{4m+1}{2m}}} \frac{\Gamma\left(\frac{1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)} \\ & - \frac{m}{2^{\frac{2m-3}{m}}} \alpha g K_1^2 B_2^3 B_1 \frac{I_5}{\Gamma\left(\frac{3}{2m}\right)} \end{aligned} \quad (62)$$

$$\frac{d\kappa_2}{dz} = 0 \quad (63)$$

$$\frac{dt_2}{dz} = -D\kappa_2 \quad (64)$$

$$\begin{aligned} \frac{d\theta_2}{dz} = & D \left\{ \frac{\kappa^2}{2} - m^2 2^{\frac{1}{m}} \frac{\Gamma\left(\frac{4m-1}{2m}\right)}{\Gamma\left(\frac{1}{2m}\right)} B_2^2 \right\} + \frac{5}{2^{\frac{4m+1}{2m}}} g K_2 B_2 \\ & + \frac{3m}{2^{\frac{2m-1}{2m}}} \alpha g K_1^2 B_1 B_2 \frac{I_5}{\Gamma\left(\frac{1}{2m}\right)} \end{aligned} \quad (65)$$

We note that (56)–(65) respectively reduce to (46)–(55) for the case $m = 1$, namely the case of Gaussian solitons.

3. PERTURBATION TERMS

We, now, consider the perturbed DM-VNLSE that is given by

$$iu_z + \frac{D(z)}{2} u_{tt} + g(z) (|u|^2 + \alpha|v|^2) u = i\epsilon R_1[u, u^*; v, v^*] \quad (66)$$

$$iv_z + \frac{D(z)}{2}v_{tt} + g(z) \left(|v|^2 + \alpha|u|^2 \right) v = i\epsilon R_2[u, u^*; v, v^*] \quad (67)$$

Here, R_1 and R_2 represent the perturbation terms and the perturbation parameter ϵ , called the relative width of the spectrum, arises due to quasi-monochromaticity [3, 11]. Also, we have $0 < \epsilon \ll 1$. The perturbation terms could, very well, represent the third order dispersion, the Raman scattering, nonlinear damping and saturable amplifiers just to name a few. In presence of the perturbation terms we have the EL equation modify to [6]:

$$\frac{\partial L}{\partial p} - \frac{d}{dz} \left(\frac{\partial L}{\partial p_z} \right) = i\epsilon \int_{-\infty}^{\infty} \left(R_1 \frac{\partial u^*}{\partial p} - R_1^* \frac{\partial u}{\partial p} \right) dt \quad (68)$$

and

$$\frac{\partial L}{\partial p} - \frac{d}{dz} \left(\frac{\partial L}{\partial p_z} \right) = i\epsilon \int_{-\infty}^{\infty} \left(R_2 \frac{\partial v^*}{\partial p} - R_2^* \frac{\partial v}{\partial p} \right) dt \quad (69)$$

where p represents the soliton parameters. Once again, substituting A_j , B_j , C_j , κ_j , t_j and θ_j where $j = 1, 2$ for p in (68) and (69) we arrive at the following adiabatic evolution equations:

$$\frac{dA_1}{dz} = -DA_1C_1 - \frac{\epsilon}{2I_1I_2} \int_{-\infty}^{\infty} \Re[R_2e^{-i\phi_2}] (I_1\tau_1^2 - 3I_2) f(\tau_1) d\tau_1 \quad (70)$$

$$\frac{dB_1}{dz} = -2DB_1C_1 - \frac{\epsilon B_1}{A_1I_1I_2} \int_{-\infty}^{\infty} \Re[R_2e^{-i\phi_1}] (I_1\tau_1^2 - I_2) f(\tau_1) d\tau_1 \quad (71)$$

$$\begin{aligned} \frac{dC_1}{dz} = & D \left(\frac{B_1^4}{2} \frac{I_3}{I_2} - 2C_1^2 \right) - \frac{gA_1^2B_1^2}{4} \frac{I_4}{I_2} - \frac{\alpha g}{2} A_2^2 B_1^3 \frac{I_5}{I_2} \\ & - \frac{\epsilon B_1^2}{2A_1I_2} \int_{-\infty}^{\infty} \Im[R_2e^{-i\phi_2}] \left(f(\tau_1) + 2\tau_1 \frac{df}{d\tau_1} \right) d\tau_1 \end{aligned} \quad (72)$$

$$\frac{d\kappa_1}{dz} = \frac{2\epsilon}{A_1B_1I_1} \int_{-\infty}^{\infty} \left\{ B_1^2 \Im[Re^{-i\phi_1}] \frac{df}{d\tau_1} - 2C_1 \Re[R_2e^{-i\phi_1}] \tau_1 f(\tau_1) \right\} d\tau_1 \quad (73)$$

$$\frac{dt_1}{dz} = -D\kappa_1 + \frac{2\epsilon}{A_1B_1I_1} \int_{-\infty}^{\infty} \Re[R_2e^{-i\phi_1}] \tau_2 f(\tau_1) d\tau_1 \quad (74)$$

$$\begin{aligned} \frac{d\theta_1}{dz} = & D \left(\frac{\kappa_1^2}{2} - \frac{I_3}{I_1} B_1^2 \right) + \frac{5gA_1^2}{4} \frac{I_4}{I_1} + \frac{3}{2} \alpha g A_2^2 B_1 \frac{I_5}{I_1} \\ & + \frac{\epsilon}{2A_1B_1I_1} \int_{-\infty}^{\infty} \left\{ B_1 \Im[R_2e^{-i\phi_2}] \left(3f(\tau_1) + 2\tau_2 \frac{df}{d\tau_2} \right) \right. \\ & \left. + 4\kappa_1 \Re[R_2e^{-i\phi_1}] \tau_1 f(\tau_1) \right\} d\tau_1 \end{aligned} \quad (75)$$

$$\frac{dA_2}{dz} = -DA_2C_2 - \frac{\epsilon}{2I_1I_2} \int_{-\infty}^{\infty} \Re[R_2e^{-i\phi_2}] (I_1\tau_2^2 - 3I_2) f(\tau_2) d\tau_2 \quad (76)$$

$$\frac{dB_2}{dz} = -2DB_2C_2 - \frac{\epsilon B_2}{A_1 I_1 I_2} \int_{-\infty}^{\infty} \Re[R_2 e^{-i\phi_2}] (I_1 \tau_2^2 - I_2) f(\tau_2) d\tau_2 \quad (77)$$

$$\begin{aligned} \frac{dC_2}{dz} = & D \left(\frac{B_2^4 I_3}{2 I_2} - 2C_2^2 \right) - \frac{g A_2^2 B_2^2 I_4}{4 I_2} - \frac{\alpha g}{2} A_1^2 B_2^3 \frac{I_5}{I_2} \\ & - \frac{\epsilon B_2^2}{2 A_2 I_2} \int_{-\infty}^{\infty} \Im[R_2 e^{-i\phi_2}] \left(f(\tau_2) + 2\tau_2 \frac{df}{d\tau_2} \right) d\tau_2 \end{aligned} \quad (78)$$

$$\frac{d\kappa_2}{dz} = \frac{2\epsilon}{A_2 B_2 I_1} \int_{-\infty}^{\infty} \left\{ B_2^2 \Im[Re^{-i\phi_1}] \frac{df}{d\tau_2} - 2C_2 \Re[R_2 e^{-i\phi_2}] \tau_2 f(\tau_2) \right\} d\tau_2 \quad (79)$$

$$\frac{dt_2}{dz} = -D\kappa_2 + \frac{2\epsilon}{A_2 B_2 I_1} \int_{-\infty}^{\infty} \Re[R_2 e^{-i\phi_2}] \tau_2 f(\tau_2) d\tau_2 \quad (80)$$

$$\begin{aligned} \frac{d\theta_2}{dz} = & D \left(\frac{\kappa_2^2}{2} - \frac{I_3}{I_1} B_2^2 \right) + \frac{5g A_2^2 I_4}{4 I_1} + \frac{3}{2} \alpha g A_1^2 B_2 \frac{I_5}{I_1} \\ & + \frac{\epsilon}{2 A_2 B_2 I_1} \int_{-\infty}^{\infty} \left\{ B_2 \Im[R_2 e^{-i\phi_2}] \left(3f(\tau_2) + 2\tau_2 \frac{df}{d\tau_2} \right) \right. \\ & \left. + 4\kappa_2 \Re[R_2 e^{-i\phi_2}] \tau_2 f(\tau_2) \right\} d\tau_2 \end{aligned} \quad (81)$$

where we have used the notations

$$\tau_j = B_j(z)(t - t_j(z))$$

and

$$\phi_j = C_j(z)\{t - t_j(z)\}^2 - \kappa_j(z)\{t - t_j(z)\} + \theta_j(z)$$

for $j = 1, 2$. Also, \Re and \Im represent the real and imaginary parts respectively. We note that (24)–(35) are special cases of (70)–(81) respectively for $\epsilon = 0$. We shall now simplify the dynamics for the Gaussian and SG solitons.

3.1. Gaussian Solitons

Here, we again use $f(\tau_j) = e^{-\tau_j^2}$ where $j = 1, 2$. Also using the integrals I_j for $1 \leq j \leq 5$ from Section 2.1 we get the parameter dynamics of the solitons as

$$\frac{dA_1}{dz} = -DA_1C_1 - \frac{\epsilon}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Re[R_1 e^{-i\phi_1}] (4\tau_1^2 - 3) e^{-\tau_1^2} d\tau_1 \quad (82)$$

$$\frac{dB_1}{dz} = -2DB_1C_1 - \epsilon \sqrt{\frac{2}{\pi}} \frac{B_1}{A_1} \int_{-\infty}^{\infty} \Re[R_1 e^{-i\phi_1}] (4\tau_1^2 - 1) e^{-\tau_1^2} d\tau_1 \quad (83)$$

$$\frac{dC_1}{dz} = 2D \left(B_1^4 - C_1^2 \right) - \frac{1}{\sqrt{2}} g A_1^2 B_1^2$$

$$\begin{aligned}
& -2\alpha g A_2^2 B_1^3 \sqrt{\frac{2}{B_1^2 + B_2^2}} e^{-\frac{B_1^2 B_2^2}{B_1^2 + B_2^2} (t_1 - t_2)^2} \\
& -2\epsilon \sqrt{\frac{2}{\pi}} \frac{B_1^2}{A_1} \int_{-\infty}^{\infty} \Im[R_1 e^{-i\phi_1}] \left(1 - 4\tau_1^2\right) e^{-\tau_1^2} d\tau_1 \quad (84)
\end{aligned}$$

$$\frac{d\kappa_1}{dz} = -\frac{2\epsilon}{A_1 B_1} \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} \left\{ B_1^2 \Im[R_1 e^{-i\phi_1}] 2\tau_1 + 2C_1 \Re[R_1 e^{-i\phi_1}] \tau_1 \right\} e^{-\tau_1^2} d\tau_1 \quad (85)$$

$$\frac{dt_1}{dz} = -D\kappa_1 + \frac{2\epsilon}{A_1 B_1} \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} \Re[R_1 e^{-i\phi_1}] \tau_1 e^{-\tau_1^2} d\tau_1 \quad (86)$$

$$\begin{aligned}
\frac{d\theta_1}{dz} &= D \left(\frac{\kappa_1^2}{2} - B_1^2 \right) + \frac{5}{4\sqrt{2}} g A_1^2 \\
&+ \frac{3}{2} \alpha g A_2^2 B_1 \sqrt{\frac{2}{B_1^2 + B_2^2}} e^{-\frac{B_1^2 B_2^2}{B_1^2 + B_2^2} (t_1 - t_2)^2} \\
&+ \frac{\epsilon}{\sqrt{2\pi}} \frac{1}{A_1 B_1} \int_{-\infty}^{\infty} \left\{ B_1 \Im[R_1 e^{-i\phi_1}] \left(3 - 4\tau_1^2\right) \right. \\
&\left. + 4\kappa_1 \Re[R_1 e^{-i\phi_1}] \tau_1 \right\} e^{-\tau_1^2} d\tau_1 \quad (87)
\end{aligned}$$

$$\frac{dA_2}{dz} = -DA_2 C_2 - \frac{\epsilon}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Re[R_2 e^{-i\phi_2}] \left(4\tau_2^2 - 3\right) e^{-\tau_2^2} d\tau_2 \quad (88)$$

$$\frac{dB_2}{dz} = -2DB_2 C_2 - \epsilon \sqrt{\frac{2}{\pi}} \frac{B_2}{A_2} \int_{-\infty}^{\infty} \Re[R_2 e^{-i\phi_2}] \left(4\tau_2^2 - 1\right) e^{-\tau_2^2} d\tau_2 \quad (89)$$

$$\begin{aligned}
\frac{dC_2}{dz} &= 2D \left(B_2^4 - C_2^2 \right) - \frac{1}{\sqrt{2}} g A_2^2 B_2^2 \\
&- 2\alpha g A_1^2 B_2^3 \sqrt{\frac{2}{B_1^2 + B_2^2}} e^{-\frac{B_1^2 B_2^2}{B_1^2 + B_2^2} (t_1 - t_2)^2} \\
&- 2\epsilon \sqrt{\frac{2}{\pi}} \frac{B_2^2}{A_2} \int_{-\infty}^{\infty} \Im[R_2 e^{-i\phi_2}] \left(1 - 4\tau_2^2\right) e^{-\tau_2^2} d\tau_2 \quad (90)
\end{aligned}$$

$$\frac{d\kappa_2}{dz} = -\frac{2\epsilon}{A_2 B_2} \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} \left\{ B_2^2 \Im[R_2 e^{-i\phi_2}] 2\tau_2 + 2C_2 \Re[R_2 e^{-i\phi_2}] \tau_2 \right\} e^{-\tau_2^2} d\tau_2 \quad (91)$$

$$\frac{dt_2}{dz} = -D\kappa_2 + \frac{2\epsilon}{A_2 B_2} \sqrt{\frac{2}{\pi}} \int_{-\infty}^{\infty} \Re[R_2 e^{-i\phi_2}] \tau_2 e^{-\tau_2^2} d\tau_2 \quad (92)$$

$$\begin{aligned}
\frac{d\theta_2}{dz} &= D \left(\frac{\kappa_2^2}{2} - B_2^2 \right) + \frac{5}{4\sqrt{2}} g A_2^2 \\
&+ \frac{3}{2} \alpha g A_1^2 B_2 \sqrt{\frac{2}{B_1^2 + B_2^2}} e^{-\frac{B_1^2 B_2^2}{B_1^2 + B_2^2} (t_1 - t_2)^2} \\
&+ \frac{\epsilon}{\sqrt{2\pi}} \frac{1}{A_2 B_2} \int_{-\infty}^{\infty} \left\{ B_2 \Im [R_2 e^{-i\phi_2}] (3 - 4\tau_2^2) \right. \\
&\left. + 4\kappa_2 \Re [R_2 e^{-i\phi_2}] \tau_2 \right\} e^{-\tau_2^2} d\tau_2
\end{aligned} \tag{93}$$

3.2. Super-Gaussian Solitons

For the SG solitons we use the integrals I_l for $l = 1, 2, 3, 4, 5$ from Section 2.2 and the form of the SG soliton for $f(\tau_j)$ to finally give:

$$\begin{aligned}
\frac{dA_1}{dz} &= -DA_1 C_1 - \epsilon \frac{m^2 2^{\frac{m+2}{m}}}{\Gamma\left(\frac{1}{2m}\right) \Gamma\left(\frac{3}{2m}\right)} \int_{-\infty}^{\infty} \Re [R_1 e^{-i\phi_1}] \\
&\cdot \left\{ \frac{\tau_1^2}{m 2^{\frac{1}{2m}}} \Gamma\left(\frac{1}{2m}\right) - \frac{3}{m 2^{\frac{3}{2m}}} \Gamma\left(\frac{3}{2m}\right) \right\} e^{-\tau_1^{2m}} d\tau_1
\end{aligned} \tag{94}$$

$$\begin{aligned}
\frac{dB_1}{dz} &= -2DB_1 C_1 - \epsilon \frac{B_1}{A_1} \frac{m^2 2^{\frac{2}{m}}}{\Gamma\left(\frac{1}{2m}\right) \Gamma\left(\frac{3}{2m}\right)} \int_{-\infty}^{\infty} \Re [R_1 e^{-i\phi_1}] \\
&\cdot \left\{ \frac{\tau_1^2}{m 2^{\frac{1}{2m}}} \Gamma\left(\frac{1}{2m}\right) - \frac{3}{m 2^{\frac{3}{2m}}} \Gamma\left(\frac{3}{2m}\right) \right\} e^{-\tau_1^{2m}} d\tau_1
\end{aligned} \tag{95}$$

$$\begin{aligned}
\frac{dC_1}{dz} &= D \left\{ B_1^4 \frac{m^2}{2^{\frac{m-2}{m}}} \frac{\Gamma\left(\frac{4m-1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)} - 2C_1^2 \right\} - g A_1^2 B_1^2 \frac{1}{2^{\frac{4m+1}{2m}}} \frac{\Gamma\left(\frac{1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)} \\
&- \frac{m}{2^{\frac{2m-3}{2m}}} A_2^3 B_1^2 \frac{I_5}{\Gamma\left(\frac{3}{2m}\right)} - \epsilon \frac{B_1^2}{A_1} \frac{m 2^{\frac{2m+3}{2m}}}{\Gamma\left(\frac{3}{2m}\right)} \int_{-\infty}^{\infty} \Im [R_1 e^{-i\phi_1}] \\
&\cdot (1 - 4m\tau_1^{2m}) e^{-\tau_1^{2m}} d\tau_1
\end{aligned} \tag{96}$$

$$\frac{d\kappa_1}{dz} = -\epsilon \frac{1}{A_1 B_1} \frac{m 2^{\frac{2m-1}{2m}}}{\Gamma\left(\frac{1}{2m}\right)} \int_{-\infty}^{\infty} \left\{ 2m\tau_1^{2m-1} B_1^2 \Im [R_1 e^{-i\phi_1}] \right.$$

$$+2\tau C_1 \Re[R_1 e^{-i\phi_1}] \} e^{-\tau_1^{2m}} d\tau_1 \quad (97)$$

$$\frac{dt_1}{dz} = -D\kappa_1 + \epsilon \frac{1}{A_1 B_1} \frac{m 2^{\frac{2m+1}{2m}}}{\Gamma\left(\frac{1}{2m}\right)} \int_{-\infty}^{\infty} \Re[R_1 e^{-i\phi_1}] \tau_1 e^{-\tau_1^{2m}} d\tau_1 \quad (98)$$

$$\begin{aligned} \frac{d\theta_1}{dz} = & D \left\{ \frac{\kappa_1^2}{2} - m^2 2^{\frac{1}{m}} \frac{\Gamma\left(\frac{4m-1}{2m}\right)}{\Gamma\left(\frac{1}{2m}\right)} B_1^2 \right\} + 5g A_1^2 \frac{1}{2^{\frac{4m+1}{2m}}} \\ & - \frac{3m}{2^{\frac{2m-1}{2m}}} \alpha g A_2^2 B_1 \frac{I_5}{\Gamma\left(\frac{1}{2m}\right)} \\ & + \epsilon \frac{1}{A_1 B_1} \frac{m 2^{\frac{2m+1}{2m}}}{\Gamma\left(\frac{1}{2m}\right)} \int_{-\infty}^{\infty} \left\{ B_1 \Im[R_1 e^{-i\phi_1}] (3 - 4m\tau_1^{2m}) \right. \\ & \left. + 4\kappa_1 \Re[R_1 e^{-i\phi_1}] \tau_1 \right\} e^{-\tau_1^{2m}} d\tau_1 \quad (99) \end{aligned}$$

$$\begin{aligned} \frac{dA_2}{dz} = & -DA_2 C_2 - \epsilon \frac{m^2 2^{\frac{m+2}{m}}}{\Gamma\left(\frac{1}{2m}\right) \Gamma\left(\frac{3}{2m}\right)} \int_{-\infty}^{\infty} \Re[R_2 e^{-i\phi_2}] \\ & \cdot \left\{ \frac{\tau_2^2}{m 2^{\frac{1}{m}}} \Gamma\left(\frac{1}{2m}\right) - \frac{3}{m 2^{\frac{3}{2m}}} \Gamma\left(\frac{3}{2m}\right) \right\} e^{-\tau_2^{2m}} d\tau_2 \quad (100) \end{aligned}$$

$$\begin{aligned} \frac{dB_2}{dz} = & -2DB_2 C_2 - \epsilon \frac{B_2}{A_2} \frac{m^2 2^{\frac{2}{m}}}{\Gamma\left(\frac{1}{2m}\right) \Gamma\left(\frac{3}{2m}\right)} \int_{-\infty}^{\infty} \Re[R_2 e^{-i\phi_2}] \\ & \cdot \left\{ \frac{\tau_2^2}{m 2^{\frac{1}{m}}} \Gamma\left(\frac{1}{2m}\right) - \frac{3}{m 2^{\frac{3}{2m}}} \Gamma\left(\frac{3}{2m}\right) \right\} e^{-\tau_2^{2m}} d\tau_2 \quad (101) \end{aligned}$$

$$\begin{aligned} \frac{dC_2}{dz} = & D \left\{ B_2^4 \frac{m^2}{2^{\frac{m-2}{m}}} \frac{\Gamma\left(\frac{4m-1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)} - 2C_2^2 \right\} - g A_2^2 B_2^2 \frac{1}{2^{\frac{4m+1}{2m}}} \frac{\Gamma\left(\frac{1}{2m}\right)}{\Gamma\left(\frac{3}{2m}\right)} \\ & - \frac{m}{2^{\frac{2m-3}{2m}}} A_1^3 B_2^2 \frac{I_5}{\Gamma\left(\frac{3}{2m}\right)} - \epsilon \frac{B_2^2}{A_2} \frac{m 2^{\frac{2m+3}{2m}}}{\Gamma\left(\frac{3}{2m}\right)} \int_{-\infty}^{\infty} \Im[R_2 e^{-i\phi_2}] \\ & \cdot (1 - 4m\tau_2^{2m}) e^{-\tau_2^{2m}} d\tau_2 \quad (102) \end{aligned}$$

$$\begin{aligned} \frac{d\kappa_2}{dz} = & -\epsilon \frac{1}{A_2 B_2} \frac{m 2^{\frac{2m-1}{2m}}}{\Gamma\left(\frac{1}{2m}\right)} \int_{-\infty}^{\infty} \left\{ 2m \tau_2^{2m-1} B_2^2 \Im[R_2 e^{-i\phi_2}] \right. \\ & \left. + 2\tau_2 C_2 \Re[R_2 e^{-i\phi_2}] \right\} e^{-\tau_2^{2m}} d\tau_2 \end{aligned} \quad (103)$$

$$\frac{dt_2}{dz} = -D\kappa_2 + \epsilon \frac{1}{A_2 B_2} \frac{m 2^{\frac{2m+1}{2m}}}{\Gamma\left(\frac{1}{2m}\right)} \int_{-\infty}^{\infty} \Re[R_2 e^{-i\phi_2}] \tau_2 e^{-\tau_2^{2m}} d\tau_2 \quad (104)$$

$$\begin{aligned} \frac{d\theta_2}{dz} = & D \left\{ \frac{\kappa_2^2}{2} - m^2 2^{\frac{1}{m}} \frac{\Gamma\left(\frac{4m-1}{2m}\right)}{\Gamma\left(\frac{1}{2m}\right)} B_2^2 \right\} + 5g A_2^2 \frac{1}{2^{\frac{4m+1}{2m}}} \\ & - \frac{3m}{2^{\frac{2m-1}{2m}}} \alpha g A_1^2 B_2 \frac{I_5}{\Gamma\left(\frac{1}{2m}\right)} \\ & + \epsilon \frac{1}{A_2 B_2} \frac{m 2^{\frac{2m+1}{2m}}}{\Gamma\left(\frac{1}{2m}\right)} \int_{-\infty}^{\infty} \left\{ B_2 \Im[R_2 e^{-i\phi}] \right. \\ & \left. (3 - 4m\tau_2^{2m}) + 4\kappa_2 \Re[R_2 e^{-i\phi_2}] \tau_2 \right\} e^{-\tau_2^{2m}} d\tau_2 \end{aligned} \quad (105)$$

Here, we once again, note that (99)–(105) respectively reduce to (80)–(93) for the special case $m = 1$. Thus the parameter dynamics of chirped Gaussian and SG solitons can be very useful to study the vector solitons in a birefringent media. In particular, the equations derived in presence of the perturbation terms can be useful in various physical applications like studying the coherent energy coupling in solitons. Also it can be used to study the background radiation as well as the timing and amplitude jitter in a wavelength-division multiplexed soliton system due to perturbation of particular type.

4. CONCLUSIONS

In this paper we have derived the parameter dynamics of a chirped Gaussian and super-Gaussian soliton in a birefringent fiber. The fundamental dynamics of a dispersion managed soliton is governed by the pulse width and frequency chirp. Also, we had studied the adiabatic evolution of the soliton parameters in presence of perturbation terms of the vector DMNLSE.

These equations gives us useful estimates of the effect of perturbation terms on the soliton transmission lines. In the applied

soliton community it can be used to study the nonlinear interactions with other solitons. It can also be used to study the stochastic perturbation of optical solitons, the dispersive radiation terms just to name a few. Although, we have not included the study of radiations in birefringent optical fibers, the application of the variational principle to study radiation is awaited.

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