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MULTILAYERED MEDIA GREEN'S FUNCTIONS FOR MPIE WITH GENERAL ELECTRIC AND MAGNETIC SOURCES BY THE HERTZ POTENTIAL APPROACH

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Abstract—A complete set of three dimensional multilayered media Green's functions is presented for general electric and magnetic sources. The Green's functions are derived in the mixed potential form, which is identical with the Michalski-Zheng C-formulation. The approach applied in this paper is based on the classical Hertz potential representation. A special emphasis is on the formulation of the dyadic Green's functions $\bar{\mathcal{G}}^{HJ}$ and $\bar{\mathcal{G}}^{EM}$. In these functions the derivatives due to the curl operator are taken in the spectral domain. This avoids the need of the numerical differentiation. Furthermore, it is found that the Hertzian potentials satisfy several useful duality and reciprocity relations. By these relations the computational efficiency of the Hertz potential approach can be significantly improved and the number of required Sommerfeld integrals can be essentially reduced. We show that all spectral domain Green's functions can be obtained from only two spectral domain Hertzian potentials, which correspond to the TE component of a vertical magnetic dipole and the TM component of a vertical electric dipole. The derived formulas are verified by numerical examples.

1. INTRODUCTION

Electromagnetic wave propagation in layered media plays a crucial role in many applications, e.g., in design and analysis of microstrip circuits and antennas [1–5], and geophysical prospecting and remote sensing [6–10]. As well known, the theory and numerical methods in layered media are more complicated than in the free space, because the Green's functions are dyadic and involve evaluation of highly oscillating and slowly converging Sommerfeld integrals [11, 12]. When applying the integral equation method in layered media one first has to find the associated multilayered media Green's functions [13–15]. The classical way of determining the Green's functions is based on the Hertz potential representation [16, 17]. However, this method has not been applied in recent years, because sufficiently efficient methods have been obtained by decomposing the fields into TE and TM components [18–20, 12], or by applying the transmission line analogy [13, 15].

In [21] Michalski and Zheng derived the multilayered media Green's functions for the electromagnetic scattering by conducting objects of arbitrary shape. In that case only the electric current is needed to solve the problem. Moreover, they consider three different mixed potential integral equation (MPIE) formulations. More recently [13], these formulations are extended for the scattering by homogeneous objects where both the electric and magnetic sources are needed.

In this paper we consider multilayered media Green's functions for the electric and magnetic MPIE formulations with general electric and magnetic sources. The derived MPIE formulation coincides with the Michalski-Zheng C-formulation [21, 13]. The Green's functions are derived by applying the classical Hertz potential approach [16, 17]. Special emphasis is on the formulation of the dyadic Green's functions $\bar{\mathcal{G}}^{HJ}$ for the magnetic field of an electric source and $\bar{\mathcal{G}}^{EM}$ for the electric field of a magnetic source. Here the derivatives due to the curl operator in $\bar{\mathcal{G}}^{HJ}$ and $\bar{\mathcal{G}}^{EM}$ are taken in the spectral domain. This avoids numerical differentiation, but leads to the calculation of the first and second order Sommerfeld integrals. The second order Sommerfeld integrals, however, can be reduced to the first and zero order Sommerfeld integrals.

The Hertz potentials satisfy several useful duality and reciprocity relations. For example the correction terms of the Michalski-Zheng C-formulation due to the electric or magnetic sources are, up to a multiplicative constant, the Hertzian potentials due to the vertical component of a horizontal magnetic or electric dipole, respectively. By these relations, we are able to show that the Hertz potentials can be expressed in terms of two functions. These functions correspond to the vertical component of a vertical magnetic dipole, which is of the TE type, and to the vertical component of a vertical electric dipole, which is of the TM type. In particular, our formulation leads to a very efficient method, since in the spectral domain the iterative formulas are required for two functions only. In addition, by applying the properties of the Hertz potentials and by appropriately formulating the Green's functions componentwise, we can essentially reduce the number of required spectral domain Green's functions and Sommerfeld integrals. Only seven spectral domain Green's functions and ten Sommerfeld integrals are required to represent 32 non-zero components of six (two scalar and four dyadic) electric and magnetic mixed potential Green's functions. The derived formulas are verified by numerical examples.

2. STATEMENT OF THE PROBLEM

Consider a medium consisting of $N+1$ homogeneous dielectric layers separated by N planar interfaces parallel to the x, y-plane of a Cartesian coordinate system and located at $z = z_j$, $j = 1, \ldots, N$. The layers are assumed to be laterally infinite. The medium of the layer j is characterized by constant electromagnetic parameters ε_j and μ_j. The layers may be lossy, i.e., ε may be complex and the first and last layer may be perfectly conducting. Let a dielectric or a perfectly conducting object D (or a collection of several objects) of arbitrary shape be embedded in the medium as illustrated in Figure 1.

In the sequel, $\mathbf{r} = (x, y, z)$ denotes a field point in the layer j and $\mathbf{r}' = (x', y', z')$ denotes a source point in the layer n. Let ε, μ and ε', μ' denote the electromagnetic parameters of the field and source layer, respectively. The time-harmonic fields due to an arbitrary current distribution ($\mathbf{J}$, $\mathbf{M}$), $\mathbf{J}$ being an electric current and $\mathbf{M}$ being

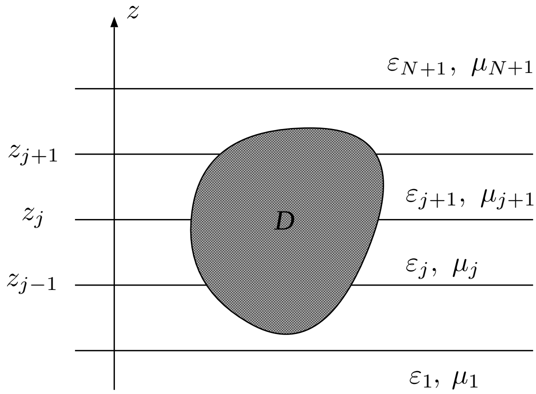

Figure 1. A body D buried in a layered media.

a magnetic current, can be expressed as

$$\mathbf{E}(\mathbf{r}) = -\frac{1}{i\omega\varepsilon}(\nabla\nabla\cdot+k^2)\left\langle\bar{\mathcal{G}}^J;\mathbf{J}\right\rangle_{S_J} + \nabla\times\left\langle\bar{\mathcal{G}}^M;\mathbf{M}\right\rangle_{S_M}, \tag{1}$$

$$\mathbf{H}(\mathbf{r}) = \frac{1}{i\omega\mu}(\nabla\nabla\cdot+k^2)\left\langle\bar{\mathcal{G}}^M;\mathbf{M}\right\rangle_{S_M} + \nabla\times\left\langle\bar{\mathcal{G}}^J;\mathbf{J}\right\rangle_{S_J}, \tag{2}$$

where $k=\omega\sqrt{\mu\varepsilon}$ is the wavenumber, S_J is the support of $\mathbf{J}$, S_M is the support of $\mathbf{M}$ and the Hertz potentials are

$$\mathbf{\Pi} := \langle\bar{\mathcal{G}};\mathbf{F}\rangle_S = \int_S \bar{\mathcal{G}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{F}(\mathbf{r}')d(\mathbf{r}').$$

In this paper the time-factor $e^{-i\omega t}$ is supposed and suppressed. Operators $\bar{\mathcal{G}}^J$ and $\bar{\mathcal{G}}^M$ are the electric and magnetic type *dyadic Green's functions* due to the electric and magnetic sources. Once the Green's functions for a layered medium are available, we can use (1) and (2) to compute the incident fields and to express the scattered fields in terms of the unknown surface currents $\mathbf{J}=\mathbf{n}\times\mathbf{H}$ and $\mathbf{M}=-\mathbf{n}\times\mathbf{E}$, where $\mathbf{n}$ is the exterior unit normal of the boundary of D. Then by combining the integral equations of the exterior and interior problems, in a similar fashion as in free space [22], a set of coupled integral equations for solving the surface currents $\mathbf{J}$ and $\mathbf{M}$ can be obtained [10].

3. MULTILAYERED MEDIA GREEN'S FUNCTIONS

The major aim of this paper is to derive the multilayered media Green's functions for the mixed potential integral equations (MPIE) with general electric and magnetic sources. We will apply the classical Hertz potential representation [17]. The Hertz potential approach has not been very popular method for determining the Green's functions in a general layered medium, because the previous Hertz potential approaches yield to complicated algorithms and more efficient algorithms were obtained when the fields are decomposed into TE and TM components [18, 12] or when the transmission line analogy [13, 15] is applied. In this paper we, however, show that the computational efficiency of the Hertz potential approach can be significantly improved, being at least of the same order as that for the other methods.

In a layered medium the Green's functions can be found in four steps as follows:

1. Choose the form of the dyadic Green's functions. Here we apply the traditional form

$$\bar{\mathcal{G}}^{J/M} = (\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}})G_{xx}^{J/M} + \hat{\mathbf{z}}\hat{\mathbf{x}}G_{zx}^{J/M} + \hat{\mathbf{z}}\hat{\mathbf{y}}G_{zy}^{J/M} + \hat{\mathbf{z}}\hat{\mathbf{z}}G_{zz}^{J/M}, \quad (3)$$

   where $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ are unit vectors in the Cartesian coordinate system.
2. Transform the Green's functions into the spectral domain by taking the Fourier trans- formation with respect to the x and y coordinates:

$$\tilde{G}(k_x, k_y) := \mathcal{F}(G(x, y))(k_x, k_y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(x, y)e^{i(x,y)\cdot(k_x,k_y)}dxdy.$$

3. Find the *spectral domain Green's functions* $\tilde{G}$.
4. Take the inverse Fourier transformation to obtain the spatial domain Green's functions G

$$\begin{aligned} G(x, y) :=\ & \mathcal{F}^{-1}(\tilde{G}(k_x, k_y))(x, y) \\ =\ & \frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tilde{G}(k_x, k_y)e^{-i(k_x,k_y)\cdot(x,y)}dk_xdk_y. \end{aligned}$$

3.1. Hertz Potentials

Consider an electric dipole $\mathbf{J} = J\hat{\mathbf{j}}\delta(\mathbf{r} - \mathbf{r}')$ and a magnetic (current) dipole $\mathbf{M} = M\hat{\mathbf{m}}\delta(\mathbf{r} - \mathbf{r}')$ embedded in layer n at point $\mathbf{r}'$. Here $\hat{\mathbf{j}}$ and $\hat{\mathbf{m}}$ are arbitrary unit vectors and for simplicity we suppose that the dipoles have unit amplitudes, i.e., $J = M = 1$. We want to calculate the electromagnetic fields $\mathbf{E}$ and $\mathbf{H}$ at an arbitrary point $\mathbf{r}$ in the layer j due to the dipoles $\mathbf{J}$ and $\mathbf{M}$. We express the fields in each layer by appropriate Hertz potentials. The potentials on the adjacent layers are coupled via the boundary conditions at the interfaces. As well known [17], two potential functions are required to present the field of a horizontal dipole and one potential function is required to present the field of a vertical dipole. Assuming that the horizontal dipoles are oriented along the x-axis, we write the electric and magnetic Hertz potentials as follows

$$\mathbf{\Pi}^{HED} = \Psi^J \hat{\mathbf{x}} + \partial_x \Phi^J \hat{\mathbf{z}}, \qquad \mathbf{\Pi}^{VED} = \Theta^J \hat{\mathbf{z}}, \tag{4}$$

$$\mathbf{\Pi}^{HMD} = \Psi^M \hat{\mathbf{x}} + \partial_x \Phi^M \hat{\mathbf{z}}, \qquad \mathbf{\Pi}^{VMD} = \Theta^M \hat{\mathbf{z}}. \tag{5}$$

Here HED, VED, HMD and VMD stand for a horizontal electric dipole, vertical electric dipole, horizontal magnetic dipole and vertical magnetic dipole, respectively. Thus, altogether six potential functions, $\Psi^{J/M}$, $\Phi^{J/M}$ and $\Theta^{J/M}$ are required to express the fields of general electric and magnetic sources. These potentials are functions of $\mathbf{r}$ and $\mathbf{r}'$ and they are solutions of the Helmholtz equations separately in each layer [17]. By requiring that the corresponding electromagnetic fields have continuous tangential components across the interfaces of the layers, we find that

$$\begin{gathered} \Phi^{J/M}, \ \partial_z \Psi^{J/M}, \ \frac{1}{a^{J/M}} \left(\Psi^{J/M} + \partial_z \Phi^{J/M} \right), \ a^{M/J} \Psi^{J/M} \ \text{and} \\ \Theta^{J/M}, \ \frac{1}{a^{J/M}} \partial_z \Theta^{J/M}, \end{gathered} \tag{6}$$

must be continuous across the interfaces $z = z_1, \ldots, z_N$, and at the perfectly conducting boundaries

$$\partial_z \Theta^J, \quad \Theta^M, \quad \Psi^J, \quad \partial_z \Phi^J, \quad \partial_z \Psi^M \quad \text{and} \quad \Phi^M \tag{7}$$

must vanish. Here we have denoted $a^J = \varepsilon$ and $a^M = \mu$.

By generalizing first the Hertz potential representation for arbitrary oriented electric and magnetic dipoles and then for current distributions, the dyadic Green's functions in (1) and (2) can be given by

$$\bar{\mathcal{G}}^{J/M} = (\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}})\Psi^{J/M} + (\hat{\mathbf{z}}\hat{\mathbf{x}}\partial_x + \hat{\mathbf{z}}\hat{\mathbf{y}}\partial_y)\Phi^{J/M} + \hat{\mathbf{z}}\hat{\mathbf{z}}\Theta^{J/M}. \tag{8}$$

Note that (8) is identical with the traditional form (3) of the dyadic Green's functions.

3.2. Mixed Potential Green's Functions

Due to the hyper singular behavior of the operator $\nabla\nabla\cdot$, usually the mixed potential form of the original equations (1) and (2) is applied, where the divergence is moved to the source current. In a layered medium the derivation of the mixed potential representation requires a special attention. Changing the order of integration and differentiation and by applying the Gauss divergence theorem, we want to find such a scalar function $G^{VJ/VM}$ which satisfies

$$\nabla \cdot \left\langle \bar{\mathcal{G}}^{J/M}; \mathbf{F} \right\rangle = -\left\langle \nabla' G^{VJ/VM}; \mathbf{F} \right\rangle = \left\langle G^{VJ/VM}, \nabla' \cdot \mathbf{F} \right\rangle,$$

where ∇' denotes gradient in the primed source coordinates and $\mathbf{F}$ is either the electric or magnetic current. In a layered medium, however, such a $G^{VJ/VM}$ does not exist generally [23]. This is due to the fact that the scalar potential Green's functions of the horizontal and vertical dipoles in a layered medium are in general different [24]. Furthermore, if the scalar potential Green's function is not continuous on the interfaces of the layers, the contour integrals on the interfaces do not disappear [21]. A usual remedy to these problems is to define so called *correction factor* $\mathbf{C}^{J/M}$ according to

$$\nabla \cdot \bar{\mathcal{G}}^{J/M} = -\nabla' G^{VJ/VM} + \mathbf{C}^{J/M}. \tag{9}$$

However, as pointed out in [21], the choice of $\mathbf{C}^{J/M}$ is not unique. A choice

$$\mathbf{C}^{J/M} = C^{J/M}\hat{\mathbf{z}},$$

with the traditional form of the dyadic Green's function (8) gives the Michalski-Zheng *C-formulation* [21].

Next we show that the Hertz potential approach with the traditional form of the dyadic Green's functions (8) naturally leads to a correction term which turns out to coincide with the Michalski-Zheng C-formulation correction term. From (8) we get

$$\nabla \cdot \bar{\mathcal{G}}^{J/M} = \begin{bmatrix} \partial_x(\Psi^{J/M} + \partial_z \Phi^{J/M}) \\ \partial_y(\Psi^{J/M} + \partial_z \Phi^{J/M}) \\ \partial_z \Theta^{J/M} \end{bmatrix}. \tag{10}$$

Because the horizontal derivatives of a Green's function G satisfy $\partial_x G = -\partial_{x'} G$ and $\partial_y G = -\partial_{y'} G$, we can change the x and y derivatives as x' and y' derivatives and then move the horizontal derivatives into the source current. Thus,

$$G^{VJ/VM} = \frac{1}{a^{J/M}}(\Psi^{J/M} + \partial_z \Phi^{J/M}) \tag{11}$$

gives the continuous scalar potential Green's function due to the horizontal dipoles. Note that the electromagnetic constants $\varepsilon(= a^J)$ and $\mu(= a^M)$ must be included in the formulas in order to have continuous scalar potential Green's functions (see (6)). In a layered medium this, however, does not generally hold for the z derivative. In order to obtain the same scalar potential Green's function for both the horizontal and vertical dipoles, we have to modify the z-component of the right hand side of (10) as

$$\partial_z \Theta^{J/M} = -\partial_{z'}(\Psi^{J/M} + \partial_z \Phi^{J/M}) + k^2 \Upsilon^{J/M},$$

where

$$\Upsilon^{J/M} = \frac{1}{k^2}\left(a^{J/M}\partial_{z'} G^{VJ/VM} + \partial_z \Theta^{J/M}\right)$$

is the *correction factor*. We can easily show that $\Upsilon^{J/M}$ coincides with the correction term of the Michalski-Zheng C-formulation. Now by integrating by parts and by applying the Gauss divergence theorem, we can write

$$\frac{1}{a^{J/M}}\nabla\cdot\left\langle \bar{\mathcal{G}}^{J/M};\mathbf{F}\right\rangle = \left\langle G^{VJ/VM}, \nabla'\cdot\mathbf{F}\right\rangle + \frac{k^2}{a^{J/M}}\left\langle \Upsilon^{J/M}\hat{\mathbf{z}};\mathbf{F}\right\rangle.$$

By incorporating the correction factor into the vector potential Green's function, we get the mixed potential representations

$$\mathbf{E}(\mathbf{r}) = -\frac{1}{i\omega}\nabla\left\langle G^{VJ}, \nabla'\cdot\mathbf{J}\right\rangle_{S_J} + i\omega\left\langle \bar{\mathcal{G}}^{AJ};\mathbf{J}\right\rangle_{S_J} + \left\langle \bar{\mathcal{G}}^{EM};\mathbf{M}\right\rangle_{S_M}, \quad (12)$$

$$\mathbf{H}(\mathbf{r}) = \frac{1}{i\omega}\nabla\left\langle G^{VM}, \nabla'\cdot\mathbf{M}\right\rangle_{S_J} - i\omega\left\langle \bar{\mathcal{G}}^{AM};\mathbf{M}\right\rangle_{S_M} + \left\langle \bar{\mathcal{G}}^{HJ};\mathbf{J}\right\rangle_{S_J}. \quad (13)$$

Here we have denoted

$$\begin{aligned} \bar{\mathcal{G}}^{AJ/AM} &= a^{M/J}\left(\bar{\mathcal{G}}^{J/M} + \nabla\Upsilon^{J/M}\hat{\mathbf{z}}\right), \\ \bar{\mathcal{G}}^{HJ/EM} &= \nabla\times\bar{\mathcal{G}}^{J/M}. \end{aligned}$$

Note that the correction factor has lower singularity than the other terms of $\bar{\mathcal{G}}^{AJ/AM}$ [21] and therefore, $\nabla\Upsilon^{J/M}$ does not increase the singularity of $\bar{\mathcal{G}}^{AJ/AM}$.

To summarize, we note that the mixed potential Green's functions (MPGF) in (12) and (13) are given by (11) and by

$$\bar{\mathcal{G}}^{AJ/AM} = a^{M/J}\Big((\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}})\Psi^{J/M} + (\hat{\mathbf{z}}\hat{\mathbf{x}}\partial_x + \hat{\mathbf{z}}\hat{\mathbf{y}}\partial_y)\Phi^{J/M}$$

$$+ (\hat{\mathbf{x}}\hat{\mathbf{z}}\partial_x + \hat{\mathbf{y}}\hat{\mathbf{z}}\partial_y)\Upsilon^{J/M} + \hat{\mathbf{z}}\hat{\mathbf{z}}(\Theta^{J/M} + \partial_z \Upsilon^{J/M})\Big), \quad (14)$$

$$\begin{aligned}\bar{\mathcal{G}}^{HJ/EM} &= (\hat{\mathbf{x}}\hat{\mathbf{x}} - \hat{\mathbf{y}}\hat{\mathbf{y}})\partial_x\partial_y\Phi^{J/M} + \hat{\mathbf{x}}\hat{\mathbf{y}}(-\partial_z\Psi^{J/M} + \partial_y^2\Phi^{J/M}) \\ &\quad +\hat{\mathbf{y}}\hat{\mathbf{x}}(\partial_z\Psi^{J/M} - \partial_x^2\Phi^{J/M}) + (-\hat{\mathbf{z}}\hat{\mathbf{x}}\partial_y + \hat{\mathbf{z}}\hat{\mathbf{y}}\partial_x)\Psi^{J/M} \\ &\quad +(\hat{\mathbf{x}}\hat{\mathbf{z}}\partial_y - \hat{\mathbf{y}}\hat{\mathbf{z}}\partial_x)\Theta^{J/M}. \end{aligned} \quad (15)$$

3.3. Spectral Domain Green's Functions

In a layered medium the Green's functions can be found in closed form only in the spectral domain. Therefore, the Green's functions are usually transformed into the spectral domain via the Fourier transformation. Next we work out the spectral domain counterpart of the MPGF (11), (14) and (15). Using the fact that nabla is given in the spectral domain as

$$\tilde{\nabla} = [-ik_x, -ik_y, \partial_z],$$

we write the spectral domain MPGF in terms of the spectral domain Hertz potentials $\tilde{\Psi}^{J/M}, \tilde{\Phi}^{J/M}, \tilde{\Theta}^{J/M}$ and the correction factors as follows

$$\begin{aligned}\tilde{G}^{VJ/VM} &= \frac{1}{a^{J/M}}\left(\tilde{\Psi}^{J/M} + \partial_z\tilde{\Phi}^{J/M}\right), \\ \tilde{G}_{xx}^{AJ/AM} &= a^{M/J}\tilde{\Psi}^{J/M} = \tilde{G}_{yy}^{AJ/AM}, \\ \tilde{G}_{rz}^{AJ/AM} &= a^{M/J}\tilde{\Upsilon}^{J/M}, \\ \tilde{G}_{zr}^{AJ/AM} &= a^{M/J}\tilde{\Phi}^{J/M}, \\ \tilde{G}_{zz}^{AJ/AM} &= a^{M/J}\left(\tilde{\Theta}^{J/M} + \partial_z\tilde{\Upsilon}^{J/M}\right). \end{aligned} \quad (16)$$

The xz, yz, zx and zy components of $\tilde{\mathcal{G}}^{AJ/AM}$ are

$$\tilde{G}_{tz}^{AJ/AM} = -ik_t\tilde{G}_{rz}^{AJ/AM},\ \tilde{G}_{zt}^{AJ/AM} = -ik_t\tilde{G}_{zr}^{AJ/AM},\ t = x, y.$$

In a similar fashion, the spectral domain components of $\bar{\mathcal{G}}^{HJ/EM}$ are given by

$$\begin{aligned}\tilde{G}_{xx}^{HJ/EM} &= -k_xk_y\tilde{\Phi}^{J/M} = -\tilde{G}_{yy}^{HJ/EM}, \\ \tilde{G}_{xy}^{HJ/EM} &= -\partial_z\tilde{\Psi}^{J/M} - k_y^2\tilde{\Phi}^{J/M}, \\ \tilde{G}_{yx}^{HJ/EM} &= \partial_z\tilde{\Psi}^{J/M} + k_x^2\tilde{\Phi}^{J/M}, \\ \tilde{G}_{\varphi z}^{HJ/EM} &= \tilde{\Theta}^{J/M}, \\ \tilde{G}_{z\varphi}^{HJ/EM} &= -\tilde{\Psi}^{J/M}, \end{aligned} \quad (17)$$

and the xz, yz, zx and zy components are

$$\begin{array}{llll} \tilde{G}_{xz}^{HJ/EM} = -ik_y \tilde{G}_{\varphi z}^{HJ/EM}, & \tilde{G}_{zx}^{HJ/EM} = -ik_y \tilde{G}_{z\varphi}^{HJ/EM}, \\ \tilde{G}_{yz}^{HJ/EM} = ik_x \tilde{G}_{\varphi z}^{HJ/EM}, & \tilde{G}_{zy}^{HJ/EM} = ik_x \tilde{G}_{z\varphi}^{HJ/EM}. \end{array}$$

For a general multilayered medium with arbitrary number of layers it is difficult to get exact expressions for the spectral domain Green's functions. This is usually possible only in the source layer. On the other layers the Green's functions are obtained via an iterative scheme [20, 12, 25]. In a similar fashion the spectral domain Hertz potentials are obtained by iterative formulas. Here we, however, do not need to apply iterative algorithms for all six spectral domain potentials $\tilde{\Psi}^{J/M}$, $\tilde{\Phi}^{J/M}$ and $\tilde{\Theta}^{J/M}$ because the formulas can be significantly simplified by applying the following identities (see Appendix A)

$$\tilde{\Psi}^{J/M} = b^{M/J} \tilde{\Theta}^{M/J}, \tag{18}$$

$$\tilde{\Phi}^{J/M} = -\frac{1}{k_\rho^2} \left(b^{M/J} \partial_z \tilde{\Theta}^{M/J} + \partial_{z'} \tilde{\Theta}^{J/M} \right), \tag{19}$$

where $b^J = \varepsilon'/\varepsilon,\ b^M = \mu'/\mu,\ k_\rho = \sqrt{k_x^2 + k_y^2}$ and $\partial_{z'}$, is the derivative with respect to the source z-coordinate. By using (19) the expression for the correction terms $\tilde{\Upsilon}^{J/M}$ can be simplified and can be given in terms of $\tilde{\Theta}^J$ and $\tilde{\Theta}^M$ as

$$\tilde{\Upsilon}^{J/M} = \frac{1}{k_\rho^2} b^{M/J} \left(\partial_{z'} \tilde{\Theta}^{M/J} + b^{J/M} \partial_z \tilde{\Theta}^{J/M} \right). \tag{20}$$

Hence, by applying (18)–(20) we find that the spectral domain Hertz potentials and the correction terms can be expressed in terms of $\tilde{\Theta}^J$ and $\tilde{\Theta}^M$. Furthermore, because $\tilde{\Theta}^J$ corresponds to a TM field component of a VED and $\tilde{\Theta}^M$ corresponds to a TE field component of a VMD [12], all six Hertz potentials and the correction terms can be expressed in terms of the TE and TM field components. In other words, the iterative algorithms are required for $\tilde{\Theta}^J$ and $\tilde{\Theta}^M$ only, and the same computational efficiency as with the TE/TM-field decomposition can be achieved.

In addition to the identities (18)–(20), the potentials satisfy several other very useful duality and reciprocity relations, see Appendix A. For example, the identities

$$\tilde{\Upsilon}^J = -b^M \tilde{\Phi}^M \qquad \text{and} \qquad \tilde{\Upsilon}^M = -b^J \tilde{\Phi}^J, \tag{21}$$

show the duality of the correction terms and the potentials $\tilde{\Phi}^{J/M}$. An other important result is the reciprocity of $\tilde{\Phi}^J$ and $\tilde{\Phi}^M$

$$\tilde{\Phi}^M(z,z') = \tilde{\Phi}^J(z',z). \tag{22}$$

By applying formulas (18), (19) and (22) in (16) and (17), we may conclude that only seven spectral domain functions are required to represent 32 non-zero components of six (two scalar and four dyadic) electric and magnetic spectral domain MPGF. The functions are

$$\tilde{\Theta}^{J/M},\ \ \partial_z\tilde{\Theta}^{J/M},\ \ \tilde{\Phi}^J \ \text{ and } \ \partial_z\tilde{\Phi}^{J/M}. \tag{23}$$

Note that

$$\partial_z\tilde{\Phi}^M(z,z') \neq \partial_z\tilde{\Phi}^J(z',z), \quad \text{but} \quad \partial_z\tilde{\Phi}^M(z,z') = \partial_{z'}\tilde{\Phi}^J(z',z)$$

and that the z and z' derivatives can be evaluated analytically, because the spectral domain potentials are exponential functions of z and z', see Appendix B.

3.4. Spatial Domain Green's Functions

Once the spectral domain potentials are found, the spatial domain potentials are obtained by the inverse Fourier transformation. For a function $\tilde{G}$ depending on k_ρ the inverse Fourier transformation reduces to a Hankel transformation according to the following formula

$$G(r) = \mathcal{F}^{-1}(\tilde{G}(k_\rho)e^{in\alpha})(r) = (-i)^n\mathcal{H}_n(\tilde{G}(k_\rho))(r)e^{in\beta}, \tag{24}$$

where $\alpha = \arctan(k_y/k_x)$, $\beta = \arctan((y-y')/(x-x'))$ and $r = \sqrt{(x-x')^2+(y-y')^2}$. The nth order Hankel transformation is defined by

$$\mathcal{H}_n(\tilde{G}(k_\rho))(r) = \frac{1}{2\pi}\int_0^\infty J_n(rk_\rho)\tilde{G}(k_\rho)k_\rho dk_\rho, \tag{25}$$

where J_n is the Bessel function of order n. To simplify the equations, we define the following notation

$$\mathcal{S}_n(\tilde{G}) = \mathcal{H}_n(\tilde{G}k_\rho^n).$$

Now in the aid of (25) we can write the spatial domain Green's functions as follows

$$G^{VJ/VM} = \frac{1}{a^{J/M}}\left(\mathcal{S}_0(\tilde{\Psi}^{J/M}) + \mathcal{S}_0(\partial_z\tilde{\Phi}^{J/M})\right),$$

$$\bar{\mathcal{G}}^{AJ/AM} = a^{M/J}\begin{bmatrix} \mathcal{S}_0(\tilde{\Psi}^{J/M}) & 0 & -\cos\beta\mathcal{S}_1(\tilde{\Upsilon}^{J/M}) \\ 0 & \mathcal{S}_0(\tilde{\Psi}^{J/M}) & -\sin\beta\mathcal{S}_1(\tilde{\Upsilon}^{J/M}) \\ -\cos\beta\mathcal{S}_1(\tilde{\Phi}^{J/M}) & -\sin\beta\mathcal{S}_1(\tilde{\Phi}^{J/M}) & \mathcal{S}_0(\tilde{\Theta}^{J/M})+\mathcal{S}_0(\partial_z\tilde{\Upsilon}^{J/M}) \end{bmatrix},$$

$$\bar{\mathcal{G}}^{HJ/EM} = \begin{bmatrix} \frac{1}{2}\sin 2\beta\mathcal{S}_2(\tilde{\Phi}^{J/M}) & G_{xy}^{HJ/EM} & -\sin\beta\mathcal{S}_1(\tilde{\Theta}^{J/M}) \\ G_{yx}^{HJ/EM} & -\frac{1}{2}\sin 2\beta\mathcal{S}_2(\tilde{\Phi}^{J/M}) & \cos\beta\mathcal{S}_1(\tilde{\Theta}^{J/M}) \\ \sin\beta\mathcal{S}_1(\tilde{\Psi}^{J/M}) & -\cos\beta\mathcal{S}_1(\tilde{\Psi}^{J/M}) & 0 \end{bmatrix},$$

where

$$\begin{aligned} G_{xy}^{HJ/EM} &= -\mathcal{S}_0(\partial_z\tilde{\Psi}^{J/M}) - \frac{1}{2}\left(\cos 2\beta\mathcal{S}_2(\tilde{\Phi}^{J/M}) + \mathcal{S}_0(k_\rho^2\tilde{\Phi}^{J/M})\right), \\ G_{yx}^{HJ/EM} &= \mathcal{S}_0(\partial_z\tilde{\Psi}^{J/M}) - \frac{1}{2}\left(\cos 2\beta\mathcal{S}_2(\tilde{\Phi}^{J/M}) - \mathcal{S}_0(k_\rho^2\tilde{\Phi}^{J/M})\right). \end{aligned}$$

This leads to 20 different Sommerfeld integrals. Since the calculation of the Sommerfeld integrals is difficult due to the singular and oscillating kernels [12], it is essential to minimize the number of Sommerfeld integrals. By applying the formula [26]

$$J_2(rk_\rho) = \frac{2}{rk_\rho}J_1(rk_\rho) - J_0(rk_\rho),$$

we can get rid of the second order Sommerfeld integrals by

$$\mathcal{S}_2(\tilde{G}) = \frac{2}{r}\mathcal{S}_1(\tilde{G}) - \mathcal{S}_0(k_\rho^2\tilde{G}).$$

The second order Hankel transformation arises from the double derivatives in (15). By the above formula and the identities of Appendix A, we express the spatial domain Green's functions compactly as follows

$$G^{VJ/VM} = \frac{1}{a^{J/M}}\left(b^{M/J}G_z^{AM/AJ} + G_1^{VJ/VM}\right), \tag{26}$$

$$\begin{aligned} \bar{\mathcal{G}}^{AJ/AM} &= a^{M/J}\Big((\hat{\mathbf{x}}\hat{\mathbf{x}} + \hat{\mathbf{y}}\hat{\mathbf{y}})b^{M/J}G_z^{AM/AJ} - \hat{\mathbf{z}}\hat{\mathbf{r}}G_{zr}^{AJ/AM} \\ &\quad + \hat{\mathbf{r}}\hat{\mathbf{z}}b^{M/J}G_{zr}^{AM/AJ} + \hat{\mathbf{z}}\hat{\mathbf{z}}\left(G_z^{AJ/AM} - b^{M/J}G_1^{VM/VJ}\right)\Big), \end{aligned} \tag{27}$$

$$\begin{aligned} \bar{\mathcal{G}}^{HJ/EM} &= (\hat{\mathbf{x}}\hat{\mathbf{x}} - \hat{\mathbf{y}}\hat{\mathbf{y}})G_{xx}^{HJ/EM} + \hat{\mathbf{x}}\hat{\mathbf{y}}G_{xy}^{HJ/EM} + \hat{\mathbf{y}}\hat{\mathbf{x}}G_{yx}^{HJ/EM} \\ &\quad + \hat{\varphi}\hat{\mathbf{z}}G_{\varphi z}^{HJ/EM} - \hat{\mathbf{z}}\hat{\varphi}b^{M/J}G_{\varphi z}^{EM/HJ}, \end{aligned} \tag{28}$$

where $\hat{\mathbf{r}} = \cos\beta\hat{\mathbf{x}} + \sin\beta\hat{\mathbf{y}}$ and $\hat{\varphi} = -\sin\beta\hat{\mathbf{x}} + \cos\beta\hat{\mathbf{y}}$. Here we have denoted (after several manipulations)

$$\begin{aligned}
G_{xx}^{HJ/EM} &= \sin 2\beta \left(\frac{1}{r}G_{zr}^{AJ/AM} - \frac{1}{2}G_1^{HJ/EM}\right),\\
G_{yx}^{HJ/EM} &= b^{M/J}G_2^{EM/HJ} - \frac{1}{r}\cos 2\beta G_{zr}^{AJ/AM} + \frac{1}{2}(1+\cos 2\beta)G_1^{HJ/EM}, \quad (29)\\
G_{xy}^{HJ/EM} &= -b^{M/J}G_2^{EM/HJ} - \frac{1}{r}\cos 2\beta G_{zr}^{AJ/AM} - \frac{1}{2}(1-\cos 2\beta)G_1^{HJ/EM},
\end{aligned}$$

and the components of the Green's functions are given by the following Sommerfeld integrals

$$\begin{aligned}
G_1^{VJ/VM} &= \mathcal{S}_0(\partial_z\tilde{\Phi}^{J/M}),\\
G_z^{AJ/AM} &= \mathcal{S}_0(\tilde{\Theta}^{J/M}),\\
G_{zr}^{AJ/AM} &= \mathcal{S}_1(\tilde{\Phi}^{J/M}),\\
G_{\varphi z}^{HJ/EM} &= \mathcal{S}_1(\tilde{\Theta}^{J/M}),\\
G_1^{HJ/EM} &= \mathcal{S}_0(k_\rho^2\tilde{\Phi}^{J/M}),\\
G_2^{HJ/EM} &= \mathcal{S}_0(\partial_z\tilde{\Theta}^{J/M}).
\end{aligned} \quad (30)$$

From the above analysis and by applying the reciprocity of $\tilde{\Phi}^J$ and $\tilde{\Phi}^M$, we may conclude, that in order to obtain the spatial domain MPGF for general electric and magnetic sources, we have to compute ten different Sommerfeld integrals listed in (30).

3.5. Green's Functions for a Microstrip

Next we shortly give the Green's functions for typical microstrip applications, where only the electric field of electric sources is required. The needed Green's functions are

$$\begin{aligned}
G^{VJ} &= \frac{1}{\varepsilon}\mathcal{S}_0\left(\frac{\mu'}{\mu}\tilde{\Theta}^M + \partial_z\tilde{\Phi}^J\right),\\
G_{xx}^{AJ} &= \mu'\mathcal{S}_0(\tilde{\Theta}^M),\\
G_{zx}^{AJ} &= -\cos\beta\mu\mathcal{S}_1(\tilde{\Phi}^J),\\
G_{zy}^{AJ} &= -\sin\beta\mu\mathcal{S}_1(\tilde{\Phi}^J),\\
G_{zz}^{AJ} &= \mu\mathcal{S}_0\left(\tilde{\Theta}^J - \frac{\mu'}{\mu}\partial_z\tilde{\Phi}^M\right),
\end{aligned}$$

and G_{xz}^{AJ} and G_{yz}^{AJ} are obtained via reciprocity

$$G_{xz}^{AJ}(z,z') = -\frac{\mu'}{\mu}G_{zx}^{AJ}(z',z), \quad G_{yz}^{AJ}(z,z') = -\frac{\mu'}{\mu}G_{zy}^{AJ}(z',z).$$

Thus, four Sommerfeld integrals are required to present the electric MPGF $\bar{\mathcal{G}}^{AJ}$ and G^{VJ}.

4. NUMERICAL VERIFICATION

The derived formulas are verified by considering numerical examples and by making comparisons with results presented in the literature. The Green's functions are evaluated by calculating the Sommerfeld integrals numerically. The Sommerfeld integration path is divided into in three parts as shown in Figure 2. The curved path from 0 to $T_0 \approx 1.2 \max(k_m)$, $m = 1, \ldots, N+1$, is included to circumvent the poles. Here k_m is the wave number of the layer m. The tail, i.e., $k_\rho > T_\infty$, is first approximated by a sum of exponential functions via the generalized pencil of function (GPOF) method [28, 29] and then the Sommerfeld identity [16] is applied to get the integral in closed form [27]. Naturally the choices of T_0 and T_∞ depend on the Green's function, on the source and field layers and on the structure.

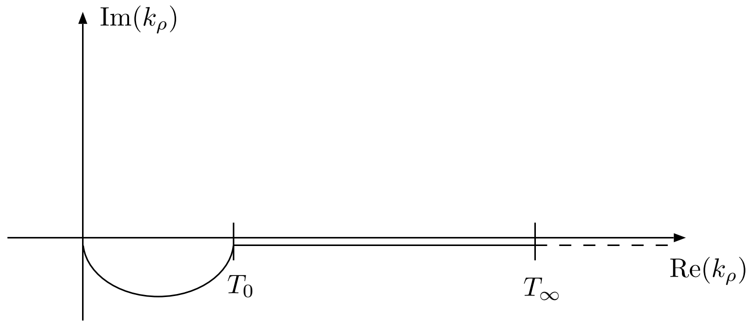

Figure 2. Integration path for the Sommerfeld integrals.

First we consider the components of the Green's functions and then, as a more practical example and to verify the Green's functions due to the curl operator, we compute the electric field of a magnetic frill.

4.1. Mixed Potential Green's Functions

First we consider a structure with five substrate layers and with a perfectly conducting ground plate [30, 5]. The geometry is displayed in Figure 3 and the frequency is 30 GHz.

Figure 4 shows the magnitude of the Green's function components G^{VJ}, $G^{AJ}_{xx} = a^M b^M G^{AM}_z$, $G^{AJ}_{zx} = -\cos\beta a^M G^{AJ}_{zr}$ with $\beta = 0$ and

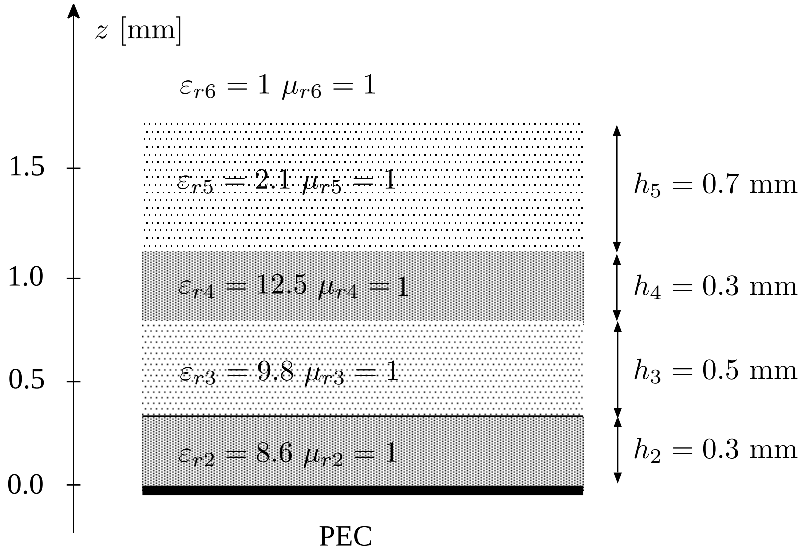

Figure 3. Geometry of a structure with five substrate layers and a perfectly conducting ground plane. The origin of the z axis is on the interface of the first substrate layer and the PEC layer. Note that the metallic ground plate is considered as the layer number 1.

$G_{zz}^{AJ} = a^M(G_z^{AJ} - b^M G_1^{VM})$ for the six-layer medium of Figure 3. The agreement with the results presented in the literature [30, 5] is very good. Figure 5 shows the corresponding Green's functions G^{VM}, $G_{xx}^{AM} = a^J b^J G_z^{AJ}$, $G_{zx}^{AM} = -\cos\beta a^J G_{zr}^{AM}$ with $\beta = 0$ and $G_{zz}^{AM} = a^J(G_z^{AM} - b^J G_1^{VJ})$ for the magnetic sources.

4.2. Electric Field of a Magnetic Frill

Coaxially fed microstrip structure is a typical case where magnetic currents are needed. The coaxial opening is usually replaced by an equivalent magnetic frill [2]. Next we calculate the electric field of a magnetic frill by the method introduced in [2] and by applying the method presented in this paper. We consider an azimuthal magnetic current

$$\mathbf{M}(r,\phi,z) = \frac{1}{\ln(r_2/r_1)}\frac{1}{r}\delta(z-z')\hat{\phi}, \quad r_1 \le r \le r_2, \quad 0 \le \phi \le 2\pi, \quad (31)$$

where r_1 and r_2 denote the inner and outer radii of the coaxial line and $\hat{\phi}$ is the angular unit vector. The radial and axial components of the

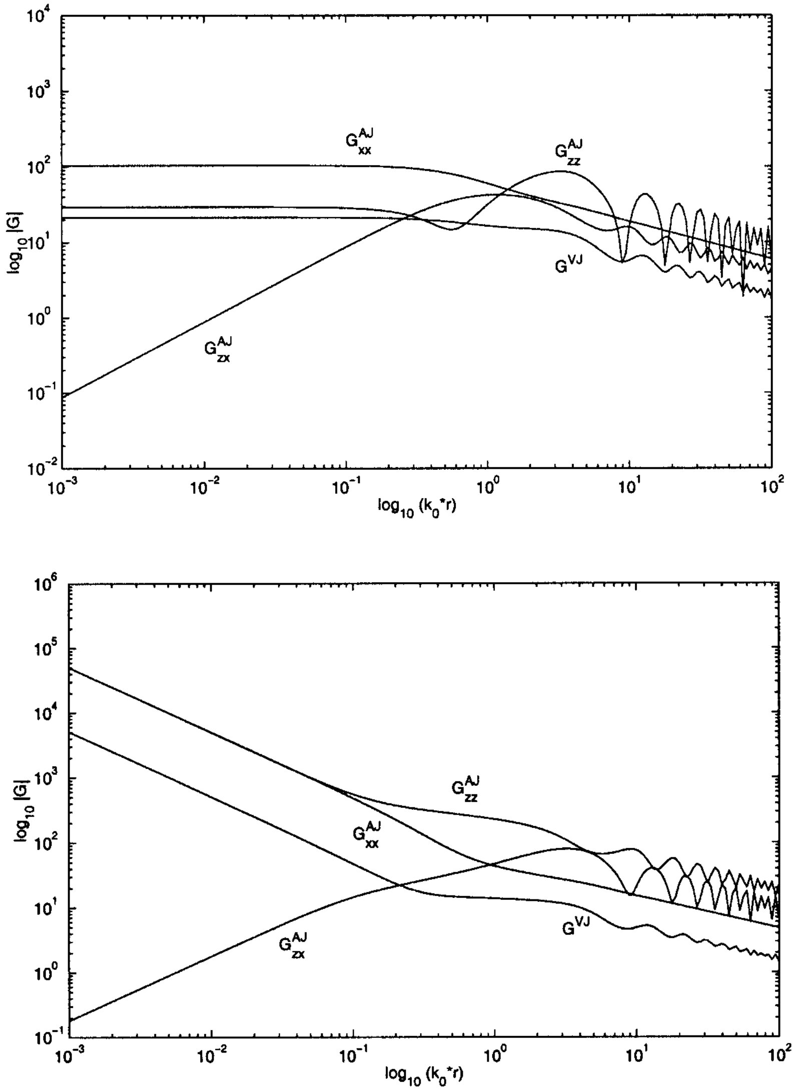

Figure 4. The magnitude of the Green's functions G^{VJ}, G_{xx}^{AJ}, G_{zx}^{AJ} and G_{zz}^{AJ} (multiplied by ε_0) in the 10 base logarithmic scale for the six layer structure of Figure 3. At the top $z' = 0.4\,\text{mm}$, $z = 1.4\,\text{mm}$ (i.e., $n = 3$, $j = 5$) and at the bottom $z' = z = 0.4\,\text{mm}$ (i.e., $n = j = 3$), respectively. Horizontal axis gives k_0 (wave number in vacuum) times r in the 10 base logarithmic scale and the frequency is 30 GHz.

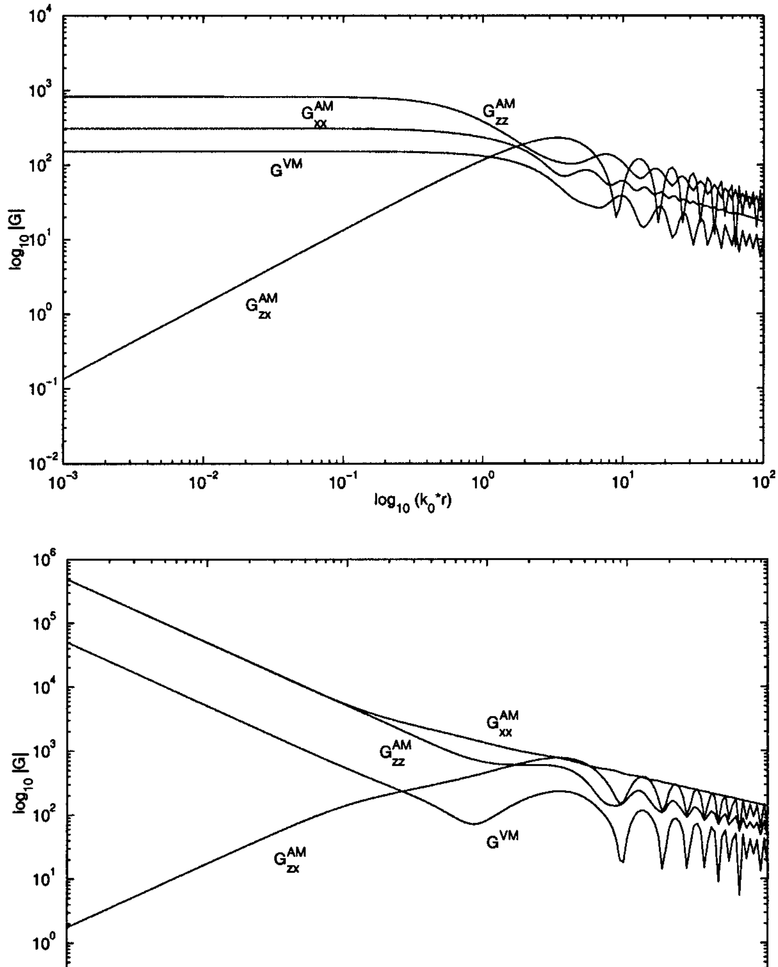

Figure 5. The magnitude of the Green's functions G^{VM}, G_{xx}^{AM}, G_{zx}^{AM} and G_{zz}^{AM} (divided by μ_0) for the six layer structure, as in Figure 4.

electric field of the corresponding magnetic frill can be given as follows

$$\begin{aligned} E_r &= -2\pi S_1(1/k_\rho^2 \partial_z \tilde{\Psi}^F), \\ E_z &= 2\pi S_0(\tilde{\Psi}^F), \end{aligned} \tag{32}$$

where $\tilde{\Psi}^F$ is of the TM type and given as $\tilde{\Theta}^J$ in Appendix B multiplied by

$$\frac{1}{\ln(r_2/r_1)}(J_0(r_1 k_\rho) - J_0(r_2 k_\rho)).$$

For comparison we compute the electric field **E** by (12), which yields that **E** due to the magnetic frill with the magnetic current (31) is

$$\mathbf{E} = \frac{1}{\ln(r_2/r_1)} \int_0^{2\pi} \int_{r_1}^{r_2} \bar{\mathcal{G}}^{EM} \cdot \hat{\phi} dr' d\phi'. \tag{33}$$

We consider a magnetic frill in a case of a four layer structure with a perfectly conducting ground plate and the frequency is 1 GHz. The frill is at the interface of PEC and the first substrate layer, i.e., $z' = 0.0$ mm, see Figure 6. The electric field is calculated by (32) and integrating numerically by (33) where $\bar{\mathcal{G}}^{EM}$ is given by (28)–(30). The results are displayed in Figure 7. The agreement is very good.

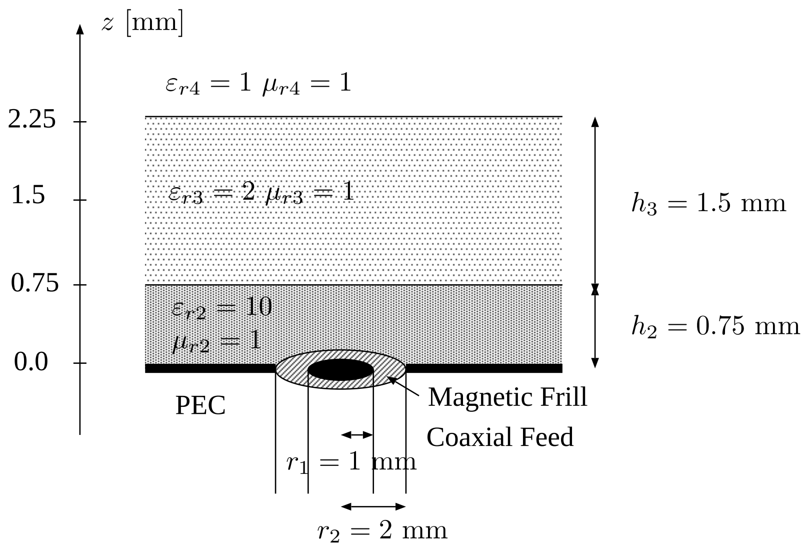

Figure 6. Magnetic frill at $z' = 0.0$ mm. Note that the metallic ground plate is considered as the layer number 1.

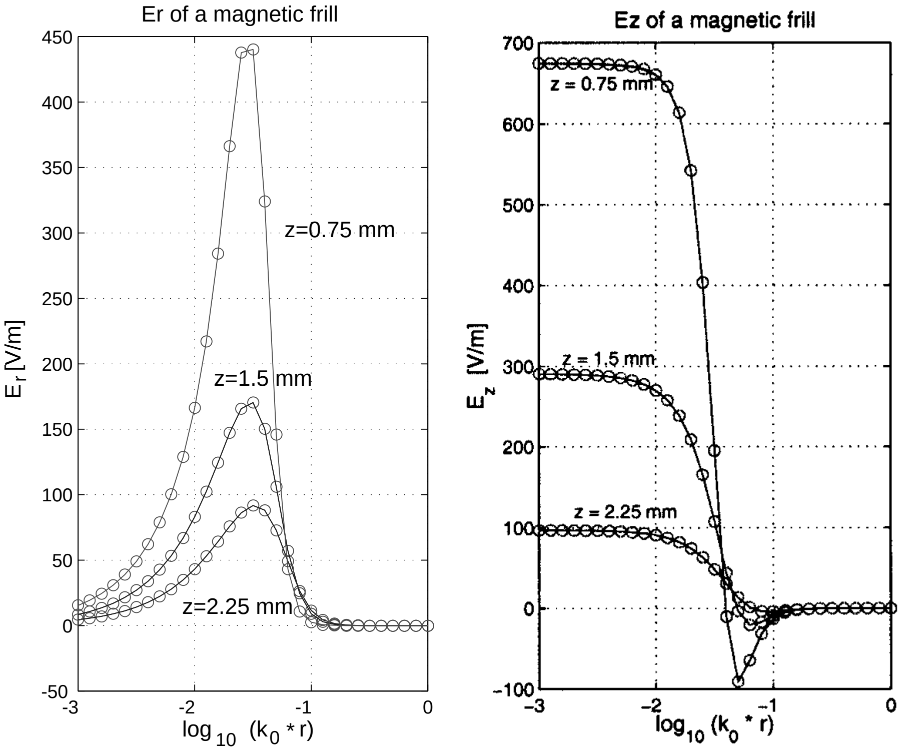

Figure 7. The radial and axial components of the electric field of a magnetic frill computed by (32) and (33) in the geometry of Figure 6. The fields computed by (33) are indicated by circles. The magnetic frill is at $z' = 0.0\,\text{mm}$ and the field points are at $z = 2.25, 1.5$ and 0.75 mm, i.e., at the interface of air and the second substrate layer, in the middle of the second substrate layer and at the interface of the substrate layers, respectively. The inner radius of the frill is 1 mm and the outer radius is 2 mm. The frequency is 1 GHz and the origin of the r axis is at the center of the coaxial line.

5. CONCLUSIONS

When both the electric and magnetic sources are present the application of the integral equation method in a layered medium becomes rather involved, because of dyadic Green's functions and coupled integral equations. In this paper we give a compact representation of the electric and magnetic type multilayered media Green's functions based on the classical Hertz potential approach.

The Green's functions are given in the mixed potential form which coincides with the Michalski-Zheng C-formulation. The derivatives due to the curl operator are taken in the spectral domain, leading to a straightforward application of the method of moments without a need for numerical differentiation. Furthermore, the Hertz potentials satisfy several duality and reciprocity relations. By these relations both the computational efficiency of the Hertz potential approach can be improved and the number of required Sommerfeld integrals and spectral domain Green's functions can be reduced. The derived formulas can be applied for the method of moments solution of electromagnetic scattering by three dimensional arbitrary shaped dielectric or perfectly conducting objects buried in layered media and for the analysis of complicated three dimensional microstrip structures.

APPENDIX A. RELATIONS FOR THE SPECTRAL DOMAIN POTENTIALS

The spectral domain Hertzian potentials satisfy the following identities

$$\begin{aligned}
\tilde{\Psi}^{J/M} &= b^{M/J}\tilde{\Theta}^{M/J}, \\
\tilde{\Upsilon}^{J/M} &= -b^{M/J}\tilde{\Phi}^{M/J}, \\
\tilde{\Phi}^{J/M} &= -\frac{1}{k_\rho^2}\left(\partial_z\tilde{\Psi}^{J/M} + \partial_{z'}\tilde{\Theta}^{J/M}\right), \\
\tilde{\Upsilon}^{J/M} &= \frac{1}{k_\rho^2}\left(b^J b^M \partial_z\tilde{\Theta}^{J/M} + \partial_{z'}\tilde{\Psi}^{J/M}\right).
\end{aligned} \tag{A1}$$

Here we have denoted $b^J = \varepsilon'/\varepsilon$ and $b^M = \mu'/\mu$. Furthermore, the potentials also satisfy the following reciprocity relations

$$\begin{aligned}
\tilde{\Psi}^{J/M}(z,z') &= b^{M/J}\tilde{\Psi}^{J/M}(z',z), & \tilde{\Psi}^{J/M}(z,z') &= \tilde{\Theta}^{M/J}(z',z), \\
\tilde{\Theta}^{J/M}(z,z') &= \frac{1}{b^{J/M}}\tilde{\Theta}^{J/M}(z',z), & \tilde{\Phi}^{J/M}(z,z') &= -b^{M/J}\tilde{\Upsilon}^{J/M}(z',z), \\
\tilde{\Phi}^{J}(z,z') &= \tilde{\Phi}^{M}(z',z), & b^M\tilde{\Upsilon}^J(z,z') &= b^J\tilde{\Upsilon}^M(z',z).
\end{aligned} \tag{A2}$$

The z derivative of $\tilde{\Phi}^{J/M}$ satisfy

$$\partial_z\tilde{\Phi}^{J/M}(z,z') = \partial_{z'}\tilde{\Phi}^{M/J}(z',z).$$

The spectral domain potentials are related to the transmission line Green's functions [13] (in an isotropic medium) as follows

$$\begin{aligned}
\tilde{\Psi}^J &= \frac{1}{j\omega\mu}V_i^h, & \tilde{\Psi}^M &= \frac{1}{j\omega\varepsilon}I_v^e,\\
\tilde{\Phi}^J &= \frac{1}{k_\rho^2}(I_i^h - I_i^e), & \tilde{\Phi}^M &= \frac{1}{k_\rho^2}(V_v^e - V_v^h),\\
\tilde{\Theta}^J &= \frac{1}{j\omega\varepsilon'}I_v^e, & \tilde{\Theta}^M &= \frac{1}{j\omega\mu'}V_i^h,\\
\tilde{\Upsilon}^J &= \frac{\mu'}{\mu k_\rho^2}(V_v^h - V_v^e), & \tilde{\Upsilon}^M &= \frac{\varepsilon'}{\varepsilon k_\rho^2}(I_i^e - I_i^h).
\end{aligned}$$

Furthermore, the scalar potential Green's functions yield the following identities

$$\begin{aligned}
\tilde{\Psi}^J + \partial_z \tilde{\Phi}^J &= -\frac{j\omega\varepsilon}{k_\rho^2}(V_i^h - V_i^e),\\
\tilde{\Psi}^M + \partial_z \tilde{\Phi}^M &= -\frac{j\omega\mu}{k_\rho^2}(I_v^e - I_v^h).
\end{aligned}$$

APPENDIX B. RECURSIVE FORMULAS FOR THE SPECTRAL DOMAIN POTENTIALS

In this appendix we give the recursive formulas for the functions $\tilde{\Theta}^J$ and $\tilde{\Theta}^M$. Once $\tilde{\Theta}^J$ and $\tilde{\Theta}^M$ are determined, the other potentials $\tilde{\Psi}^{J/M}$, $\tilde{\Phi}^{J/M}$ and $\tilde{\Upsilon}^{J/M}$ can be obtained by the formulas (A1). We express the functions $\tilde{\Theta}^{J/M}$ in the layer j due to the sources in the layer n as follows

$$\begin{aligned}
\tilde{\Theta}^{J/M} = \frac{1}{2k_{zn}}\Big(& A_{nj}^{J/M+} e^{-k_{zj}(z-z_{j-1})} e^{-k_{zn}(z_n-z')}\\
&+ A_{nj}^{J/M-} e^{-k_{zj}(z-z_{j-1})} e^{-k_{zn}(z'-z_{n-1})}\\
&+ B_{nj}^{J/M+} e^{-k_{zj}(z_j-z)} e^{-k_{zn}(z_n-z')}\\
&+ B_{nj}^{J/M-} e^{-k_{zj}(z_j-z)} e^{-k_{zn}(z'-z_{n-1})} + \delta_{nj} e^{-k_{zn}|z-z'|}\Big),
\end{aligned}$$

where $z_{n-1} \le z' \le z_n,\ z_{j-1} \le z \le z_j$,

$$k_{zj} = k_z = -i\sqrt{k_j^2 - k_\rho^2}, \qquad k_{zn} = k_{z'} = -i\sqrt{k_n^2 - k_\rho^2},$$

and $\delta_{nj} = 1$ only if $j = n$, i.e., the field and source layer coincide. Note that potentials $\tilde{\Phi}^{J/M}$ and $\tilde{\Upsilon}^{J/M}$ do not have the direct term

$e^{-k_{zn}|z-z'|}$. The coefficients $A^{J/M\pm}$ and $B^{J/M\pm}$ are given in the terms of the *generalized TE and TM reflection* and *transmission coefficients* [12]

$$\bar{R}^{TE/TM}_{m\pm} := \bar{R}^{TE/TM}_{m,m\pm 1} = \frac{R^{TE/TM}_{m,m\pm 1} + \bar{R}^{TE/TM}_{m\pm 1,m\pm 2} c_{m\pm 1}}{1 + R^{TE/TM}_{m,m\pm 1} \bar{R}^{TE/TM}_{m\pm 1,m\pm 2} c_{m\pm 1}},$$

$$\bar{T}^{TE/TM}_{m\pm} := \bar{T}^{TE/TM}_{m,m\pm 1} = \frac{1 + \bar{R}^{TE/TM}_{m,m\pm 1}}{1 + \bar{R}^{TE/TM}_{m\pm 1,m\pm 2} c_{m\pm 2}} = \frac{1 + R^{TE/TM}_{m,m\pm 1}}{1 + R^{TE/TM}_{m,m\pm 1} \bar{R}^{TE/TM}_{m\pm 1,m\pm 2} c_{m\pm 1}}$$

where $R^{TE/TM}_{m,m\pm+1}$ are the Fresnel reflection coefficients [12], $h_m = z_m - z_{m-1}$ is the height of the layer m and $c_m = e^{-2k_{zm}h_m}$. We distinguish three cases ($1 \leq j,\ n \leq N+1$):

		$j = n$	$j < n$	$j > n$
A^+_{nj}	:	$\bar{R}^+_n \bar{R}^-_n M_n d_n$	$\bar{R}^-_j \bar{R}^+_n M_n T^-_{nj} d_n d_j$	$M_n T^+_{nj}$
A^-_{nj}	:	$\bar{R}^-_n M_n$	$\bar{R}^-_j M_n T^-_{nj} d_j$	$\bar{R}^-_n M_n T^+_{nj} d_n$
B^+_{nj}	:	$\bar{R}^+_n M_n$	$\bar{R}^+_n M_n T^-_{nj} d_n$	$\bar{R}^+_j M_n T^+_{nj} d_j$
B^-_{nj}	:	A^+_{nj}	$M_n T^-_{nj}$	$\bar{R}^+_j \bar{R}^-_n M_n T^+_{nj} d_n d_j$

Here we have denoted $d_n = e^{-k_{zn}h_n}$,

$$M_n = \frac{1}{1 - \bar{R}^+_n \bar{R}^-_n c_n} \quad \text{and} \quad T^\pm_{nj} = \bar{T}^\pm_n \prod_{m=n+1/j-1}^{j-1/n+1} \bar{T}^\pm_m d_m.$$

The coefficients $A^{J\pm}$ and $B^{J\pm}$ are obtained by the above formulas with the TM reflection and transmission coefficients, i.e., $\bar{R}^\pm_m = R^{TM}_{m\pm}$ and $\bar{T}^\pm_m = \bar{T}^{TM}_{m\pm}$. Similarly, the coefficients $A^{M\pm}$ and $B^{M\pm}$ are obtained with the TE reflection and transmission coefficients i.e., $\bar{R}^\pm_m = R^{TE}_{m\pm}$ and $\bar{T}^\pm_m = \bar{T}^{TE}_{m\pm}$.

The coefficients $A^\pm$ and $B^\pm$ correspond to the upgoing and downgoing parts of the wave reflected from the interfaces as follows (see Figure B1):

$A+$: upgoing at the source point and upgoing at the field point,

$A-$: downgoing at the source point and upgoing at the field point,

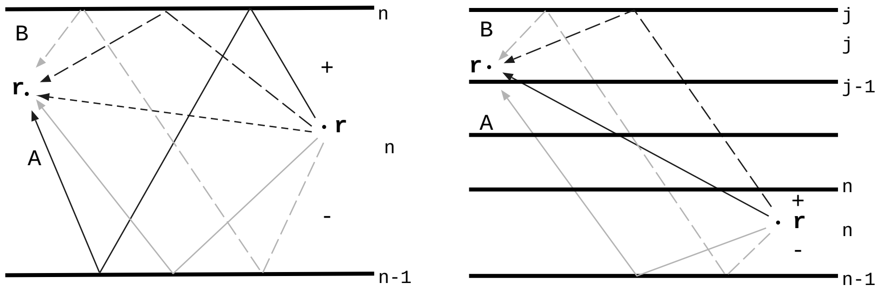

Figure B1. Geometrical interpretation of the coefficients $A^{\pm}$ (denoted by solid lines) and $B^{\pm}$ (denoted by dashed lines). A^{+} and B^{+} are denoted by dark lines and A^{-} and B^{-} are denoted by light lines. On the left the source and the field layer coincide, i.e., $j = n$ and on the right $j > n$. The dotted line on the left hand side corresponds to the direct term.

$B+$: upgoing at the source point and downgoing at the field point,

$B-$: downgoing at the source point and downgoing at the field point.

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